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PAPER

A Synthesis of Variable Wave Digital Filters

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SUMMARY It is often desired to change the cutoff frequencies of digital filters in some applications like digital electronic instruments. This paper proposes a design of variable lowpass digital filters with wider ranges of cutoff frequencies than conventional designs. Wave digital filters are used for the prototypes of variable filters. The proposed design is based on the frequency scaling in the s-domain, while the conventional ones are based on the z-domain lowpass-to-lowpass transformations. The first-order approximation by the Taylor series expansion is used to make multiplier coefficients in a wave digital filters obtained from a frequency-scaled LC filter become linear functions of the scaling parameter, which is similar to the conventional design. Furthermore this paper discusses the reduction of the approximation error. The curvature is introduced as the figure of the quality of the first-order approximation. The use of the second-order approximation to large-curvature multiplier coefficients instead of the first-order one is proposed.

key words: *variable digital filter, wave digital filters, digital filter, LC filter, signal processing*

1. Introduction

It is often desired to change instantaneously the cutoff frequencies of lowpass and highpass digital filters and to change instantaneously the center frequencies and/or the bandwidths of bandpass and bandstop ones, in some applications like digital electronic instruments. One approach to the implementation of such real-time changeable digital filters is to realize digital filters whose frequency characteristics can be controlled by a single parameter implemented as identical multiplier coefficients. The re-design of digital filters with new characteristics is not suitable for the real-time change of their characteristics because it takes a long time. It is well-known that the cutoff frequencies of lowpass and highpass filters and the center frequencies of bandpass and bandstop filters can be changed by single parameters derived from the spectral transformations for digital filters introduced by Constantinides [1].

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The circuitry realizations of his transformations includes the parameters as identical multiplier coefficients. The replacement of delay units with the realized allpass circuits is available for realizing bandpass and bandstop filters with variable center frequencies. On the contrary such replacement is not available for realizing lowpass and highpass filters with variable cutoff frequencies and bandpass and bandstop filters with variable bandwidths because it forms delay-free loops. If the cutoff frequencies of lowpass filters can be changed by single parameters, three other types of filters can also be controlled by the same parameters through the frequency transformations from the lowpass filters. Therefore, to realize variable characteristic digital filters it is the key to study lowpass filters with variable cutoff frequencies.

Mitra, Neuvo and Roivainen have proposed a design of variable IIR lowpass filters [2]. Their design uses the parallel structures of two allpass circuits. The lowpass-to-lowpass transformations are applied to the two allpass transfer functions to generate single parameters to control the cutoff frequencies. Before the transformed allpass transfer functions are realized, all their coefficients are approximated to the first-order polynomials of the generated parameters by means of the Taylor series expansion in order to eliminate delay-free loops resulting from the transformations. Because of this approximation, there always exist approximation errors. When the errors are large, the frequency responses are degraded. Therefore, the ranges of variable cutoff frequencies of this method are limited in certain bands. Especially the degradation of the amplitude responses of even-order filters is much more than odd-order ones, and the ranges of cutoff frequencies are narrow in the case of even order. Murakoshi, Watanabe and Nishihara have proposed a synthesis for expanding the ranges of cutoff frequencies of even-order filters [3]. It uses the cascade structures, whose second-order sections are the direct forms and the lattice forms. The adopted method for the control of cutoff frequencies is similar to the one by Mitra et al.

The deviation of multiplier coefficients from ideal values owing to the errors of the first-order approximation causes the degradation of the amplitude responses of realized variable filters. On the other hand, in the implementation of digital filters in finite-wordlength environments amplitude responses are also degraded by rounding multiplier coefficients. Such degradation is small in low-sensitivity digital filters. It is common between the approximation errors and the finite-wordlength effects that the deviation of multiplier coefficients leads to the degradation of their amplitude responses. Therefore excellent variable digital filters can be obtained if the first-order approximation method is applied to the structures with the low-sensitivity property.

Wave digital filters (WDFs) proposed by Fettweis [4] are wellknown as low-sensitivity digital filters. WDFs simulate the wave quantities of doubly-terminated reactance filters, and can inherit the low-sensitivity property of their reference filters. Multiplier coefficients in WDFs are expressed as the functions of reactances in their reference filters.

From the above-mentioned consideration, this paper proposes a design of variable lowpass WDFs with wider ranges of variation in cutoff frequencies than conventional designs. The proposed design is effective for both odd- and even-order cases. The first-order approximation by means of the Taylor series expansion is also used to make the first-order polynomials of control parameters for cutoff frequencies. In the proposed design such parameters are generated by the s-domain lowpass-to-lowpass transformations, in other words, the s-domain frequency scalings which are applied to the reactances existing in the equations for multiplier coefficients of WDFs, whereas in the conventional ones they are generated by the z-domain lowpass-to-lowpass transformations. The quality of the first-order approximation depends on the linearity of multiplier coefficients which are regarded as the functions of the generated parameters. The linearity of each multiplier coefficient is not always identical. This paper introduces the curvature as the measure of the linearity, and proposes to apply the second-order approximation to large-curvature coefficients instead of the first-order one.

2. On the Synthesis of WDFs

WDFs referring to doubly-terminated LC ladder filters, which are called ladder-WDFs, are consist of delay units and three kinds of adaptors such as 3-port series and parallel adaptors and 2-port ones [4]. Adaptors are circuit blocks which contain multipliers and adders. They simulate the wave quantities of the LC ladders at the point where two or three ports are connected. Assuming that in the 3-port series adaptor shown in Fig. 1 R_1, R_2 and R_3 denote the port resistances of the port 1, 2 and 3, their relation is given by

$$R_2=R_1+R_3. \tag{1}$$

The coefficient of its internal multiplier is given by

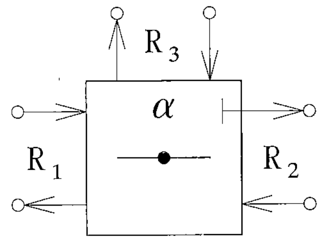


Fig. 1 Series adaptor.

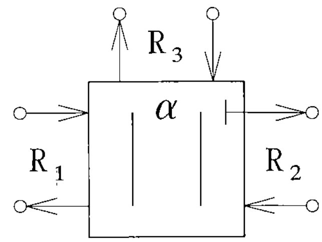


Fig. 2 Parallel adaptor.

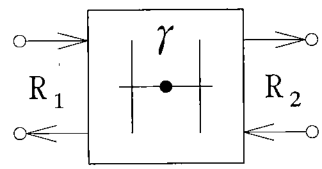


Fig. 3 2-port adaptor.

$$\alpha=\frac{R_1}{R_2}. \tag{2}$$

The relation of the port resistances R_1, R_2 and R_3 in the 3-port parallel adaptor shown in Fig. 2 is given by

$$\frac{1}{R_2}=\frac{1}{R_1}+\frac{1}{R_3}. \tag{3}$$

Its multiplier coefficient is given by

$$\alpha=\frac{R_2}{R_1}. \tag{4}$$

The two port resistances R_1 and R_2 in the 2-port adaptor shown in Fig. 3 are mutually independent. Its multiplier coefficient is given by

$$\Gamma=\frac{R_2-R_1}{R_2+R_1}. \tag{5}$$

Discrete-time circuit units of WDFs which correspond to reactance elements are called generalized delay units (GDUs) [5]. A first-order GDU corresponding to an inductor L is $-z^{-1}$, whose port resistance H is given by

$$H=L. \tag{6}$$

A first-order GDU corresponding to a capacitor C is z^{-1}, whose port resistance H is given by

$$H=\frac{1}{C}. \tag{7}$$

A second-order GDU which corresponds to LC resonant circuits can be realized by using a 3-port adaptors and two first-order GDUs [6].

A digital elementary section corresponding to analog one can be realized by connecting a GDU to the port 3 in a 3-port adaptor [7]. For such a connection the port resistance of the port 3 must be equalized to that of the GDU, that is,

$$R_3=H \tag{8}$$

must hold. Elementary sections required for the realization of ladder-WDFs are series and parallel sections, which use series and parallel adaptors respectively. Ladder-WDFs are synthesized by cascading these elementary sections. Although all the port resistances of mutually-connected ports are required to be equalized to each other for the cascade connection, only one pair of port resistances cannot be equalized to each other. In order to match the port resistances at such a port connection a 2-port adaptor is inserted.

3. Derivation of Design Formulas

3.1 Realization of Variable 3-Port Adaptors

When all inductances and capacitances in an LC filter are scaled by a such as

$$L\rightarrow aL$$

and

$$C\rightarrow aC,$$

its cutoff frequency is scaled by $1/a$, and this is called frequency scaling [8]. If a WDF is synthesized from a frequency scaled LC filter and holds its scaling parameter a as a multiplier coefficient, then the cutoff frequency of such a WDF can be controlled by a. By scaling all reactances in reference filters by a, all the port resistances of their corresponding GDUs become the function of a. Since in each elementary section the port resistance R_3 of the port 3 in the 3-port adaptor to which each GDU is connected is equalized to the port resistance of the GDU, R_3 becomes the function of the scaling parameter a such as $R_3=aL$. According to Eq. (1) for series adaptors and Eq. (3) for parallel adaptor, it is shown that R_2 of the port 2 also becomes the function of a. Moreover R_1 of the port 1 is equal to the port resistance of the port 3 in the neighboring elementary section. Therefore all the port resistances in each 3-port adaptor are the functions of a, and let us represent them as $R_1(a)$, $R_2(a)$ and $R_3(a)$ respectively. It is found from Eqs. (2) and (4) that the multiplier coefficient in each 3-port adaptor is also the function of a, which is represented by $\alpha(a)$. The cutoff frequency after the frequency scaling, which is denoted by ω_c', is given by

$$\omega_c'=\frac{2}{T}\tan^{-1}\left(\frac{1}{a}\tan\frac{\omega_c T}{2}\right), \tag{9}$$

where ω_c is the original cutoff frequency and T is the sampling period.

If another WDF with another cutoff frequency is required, it is obtained by substituting a value for a and calculating the new values of multiplier coefficients. However such re-design is not suitable for the real-time change of the cutoff frequency because it takes a long time. For the realization of real-time variable digital filter it is desired that the parameter a is implemented as a multiplier coefficient which controls the cutoff frequency. Since $\alpha(a)$ is a rational function of a, $\alpha(a)$ cannot be realized in the form of a linear signal flow graph without delay-free loops. On condition that $\alpha(a)$ is a polynomial of a, delay-free loops can be eliminated. Especially the smallest number of multipliers with the coefficient a is required if $\alpha(a)$ is a linear function of a. The first-order approximation by means of the Taylor series expansion of $\alpha(a)$ around $a=1$ gives

$$\alpha(a)\cong\alpha(1)+(a-1)\alpha'(1), \tag{10}$$

where $\alpha'(1)$ is the derivative of $\alpha(a)$ evaluated at $a=1$, and $\alpha(1)$ equals the multiplier coefficient before scaling. From Eqs. (1) and (2), $\alpha(1)$ in a series adaptor is given by

$$\alpha(1)=\frac{R_1(1)}{R_2(1)}, \tag{11}$$

where

$$R_2(1)=R_1(1)+R_3(1). \tag{12}$$

On the other hand, if it is in a parallel adaptor, from Eqs. (3) and (4) it is given by

$$\alpha(1)=\frac{R_2(1)}{R_1(1)}, \tag{13}$$

where

$$\frac{1}{R_2(1)}=\frac{1}{R_1(1)}+\frac{1}{R_3(1)}. \tag{14}$$

The circuitry realization of the right-hand side of Eq. (10) is shown in Fig. 4, where a multiplier with a coefficient $a-1$ is used instead of one with a coefficient

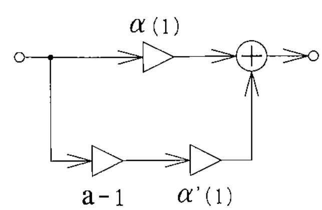


Fig. 4 Multiplier branch after the first-order approximation.

a in order to reduce the number of adders. A variable 3-port adaptor is realized by replacing a multiplier before scaling with the circuit in Fig. 4, which is practically controlled by the parameter $a-1$ instead of a. The coefficient $\alpha'(1)$ is derived as follows.

(i) Series adaptor

From Eq. (2), $\alpha'(a)$ is expressed by

$$\alpha'(a)=\frac{R_1'(a)R_2(a)-R_1(a)R_2'(a)}{R_2^2(a)}. \tag{15}$$

Accordingly $\alpha'(1)$ is given by

$$\alpha'(1)=\frac{R_1'(1)R_2(1)-R_1(1)R_2'(1)}{R_2^2(1)}. \tag{16}$$

In the proposed design, $R_1(1)$ and $R_1'(1)$, $R_3(1)$ and $R_3'(1)$ are given, and $R_2(1)$ is evaluated by Eq. (12). Since the relation between $R_1'(a)$ and $R_2'(a)$ is derived from Eq. (1) as

$$R_2'(a)=R_1'(a)+R_3'(a), \tag{17}$$

$R_2'(1)$ is given by

$$R_2'(1)=R_1'(1)+R_3'(1). \tag{18}$$

Thus $\alpha'(1)$ can be evaluated. These $R_2(1)$ and $R_2'(1)$ becomes $R_1(1)$ and $R_1'(1)$ in the next section respectively.

(ii) Parallel adaptor

Similarly $\alpha'(1)$ in a parallel adaptor is given by

$$\alpha'(1)=\frac{R_1(1)R_2'(1)-R_1'(1)R_2(1)}{R_1^2(1)}, \tag{19}$$

where

$$R_2'(1)=R_2^2(1)\left(\frac{R_1'(1)}{R_1^2(1)}+\frac{R_3'(1)}{R_3^2(1)}\right). \tag{20}$$

3.2 Realization of Variable GDUs

First-order GDUs which correspond to L and C scaled by a is also $-z^{-1}$ and z^{-1} respectively. Since their port resistances are denoted by

$$H(a)=aL \tag{21}$$

and

$$H(a)=\frac{1}{aC}, \tag{22}$$

the derivatives of the port resistances are given by

$$H'(a)=L \tag{23}$$

and

$$H'(a)=-\frac{1}{a^2C} \tag{24}$$

respectively. As second-order GDUs can be realized by using 3-port adaptors, second-order variable GDUs are constituted by variable 3-port adaptors and variable first-order GDUs.

3.3 Realization of Variable 2-Port Adaptors

When at least one port resistance in a 2-port adaptor becomes a function of the parameter a, its multiplier coefficient Γ also becomes a function of a, which is denoted by $\Gamma(a)$. The first-order approximation by means of the Taylor series expansion of $\Gamma(a)$ around $a=1$ gives

$$\Gamma(a)\cong\Gamma(1)+(a-1)\Gamma'(1), \tag{25}$$

where

$$\Gamma'(1)=\frac{2(R_1(1)R_2'(1)-R_1'(1)R_2(1))}{(R_1(1)+R_2(1))^2}. \tag{26}$$

The circuitry realization of Eq. (25) has the same structure as that shown in Fig. 4 where $\alpha(1)$ and $\alpha'(1)$ correspond to $\Gamma(1)$ and $\Gamma'(1)$ respectively.

3.4 Compensation for Gain Level

When the gain at a matching frequency in the passband is represented by G, it is given by

$$G=\sqrt{\frac{R_L}{R_S}}, \tag{27}$$

where R_S and R_L are the source resistance and the load resistance of the reference filter respectively. Since R_S and R_L are independent of a, G is also independent of a. Actually G is written as

$$G(a)\sqrt{\frac{R_1(a)}{R_S}\frac{R_2(a)}{R_1(a)}\cdots\frac{R_n(a)}{R_{n-1}(a)}\frac{R_L}{R_n(a)}}, \tag{28}$$

where $R_1(a), R_2(a), \cdots, R_n(a)$ are the port resistances between two neighboring ports.

Considering Eqs. (2), (4) and (5), Eq. (28) is expressed as follows;

(i) Π-ladder reference filter

When n is an odd number,

$$G(a)=\sqrt{\alpha_1(a)\frac{1}{\alpha_2(a)}\cdots\alpha_n(a)\frac{1+\Gamma(a)}{1-\Gamma(a)}} \tag{29}$$

and when n is an even number,

$$G(a)=\sqrt{\alpha_1(a)\frac{1}{\alpha_2(a)}\cdots\frac{1}{\alpha_n(a)}\frac{1+\Gamma(a)}{1-\Gamma(a)}}. \tag{30}$$

(ii) T-ladder reference filter

When n is an odd number,

$$G(a)=\sqrt{\frac{1}{\alpha_1(a)}\alpha_2(a)\cdots\frac{1}{\alpha_n(a)}\frac{1+\Gamma(a)}{1-\Gamma(a)}} \tag{31}$$

and when n is an even number,

$$G(a)=\sqrt{\frac{1}{\alpha_1(a)}\alpha_2(a)\cdots\alpha_n(a)\frac{1+\Gamma(a)}{1-\Gamma(a)}}. \tag{32}$$

In both cases it is assumed that a 2-port adaptor is used for port matching at the output port.

If the first-order approximation is not applied to $a_i(a)$ $(i=1, 2, \cdots, n)$ and $\Gamma(a)$, the value of $G(a)$ calculated by Eqs. (29)-(31) or (32) is equal to that calculated by Eq. (27). Since such equality does not hold after the approximation, the gain level varies and is denoted by $G_s(a)$. For the compensation of this variation, a multiplier with the coefficient

$$M_G(a)=\frac{G}{G_s(a)} \tag{33}$$

is connected at the input or the output terminal. Furthermore $1/G_s(a)$ is replaced with its first-order approximation by means of the Taylor series expansion of $1/G_s(a)$ around $a=1$ to approximate $M_G(a)$ to the first-order function of a. Then considering $G(1)$

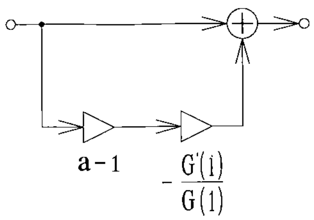


Fig. 5 Multiplier branch for gain level compensation.

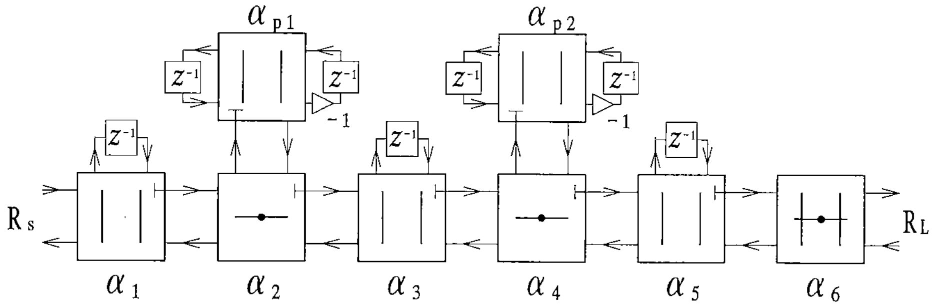
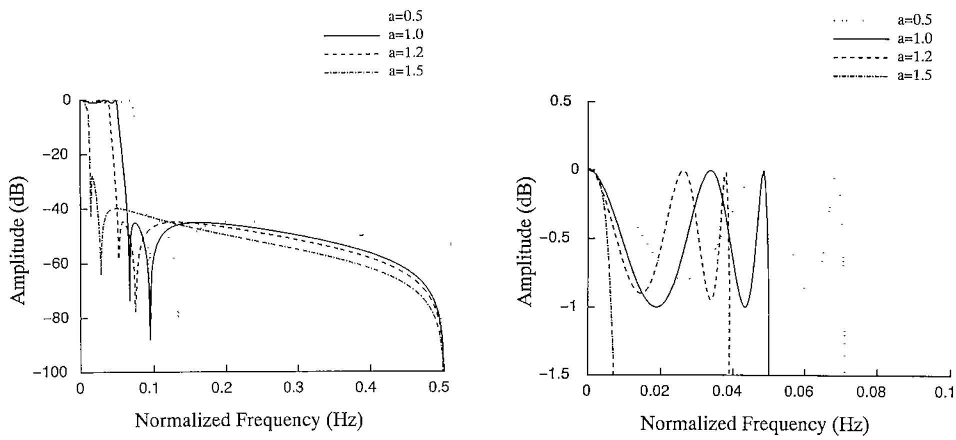


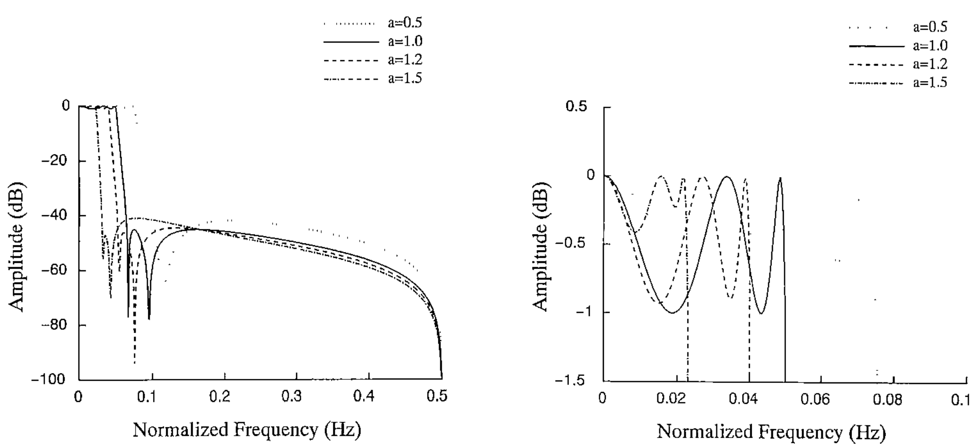
Fig. 6 Realized fifth-order WDF.



(a) Stopband

(b) Passband

Fig. 7 Amplitude response of the proposed fifth-order variable WDF.



(a) Stopband

(b) Passband

Fig. 8 Amplitude response by Mitra et al.

$$= G,$$

$$M_G \cong G\left(\frac{1}{G(1)} - (a-1)\frac{G'(1)}{G(1)^2}\right)$$

$$= 1 - (a-1)\frac{G'(1)}{G} \tag{34}$$

is obtained. The circuitry realization of Eq.(34) is shown in Fig. 5.

4. Illustrative Examples

In this section two illustrative examples show the effectiveness of the proposed design by comparison with Refs. [2] and [3]. First, a fifth-order elliptic transfer function is realized to compare amplitude characteristics with Ref. [2] which realizes the same transfer function. Its specification is as follows; the sampling frequency is 1 Hz, the cutoff frequency at $a=1.0$ is 0.05 Hz, the passband ripple is 1 dB and the stopband minimum attenuation is 45 dB. The realized WDF is shown in Fig. 6. Figure 7 shows the amplitude responses by the proposed design with $a=0.5$, 1.0, 1.2 and 1.5, and Fig. 8 shows those by Ref. [2] with the same values of a. It is found from these figures that the amplitude responses by the proposed design are less

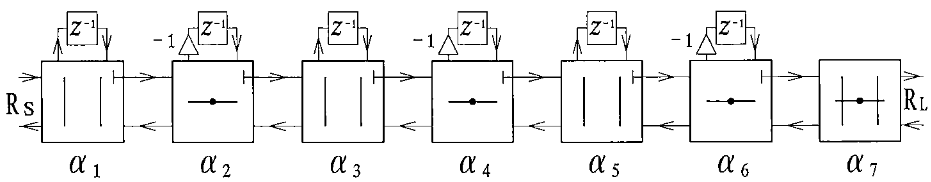
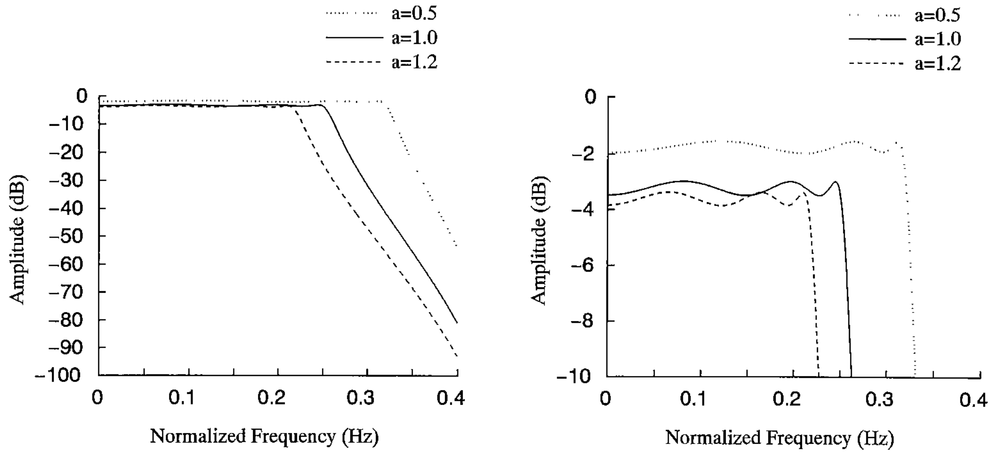
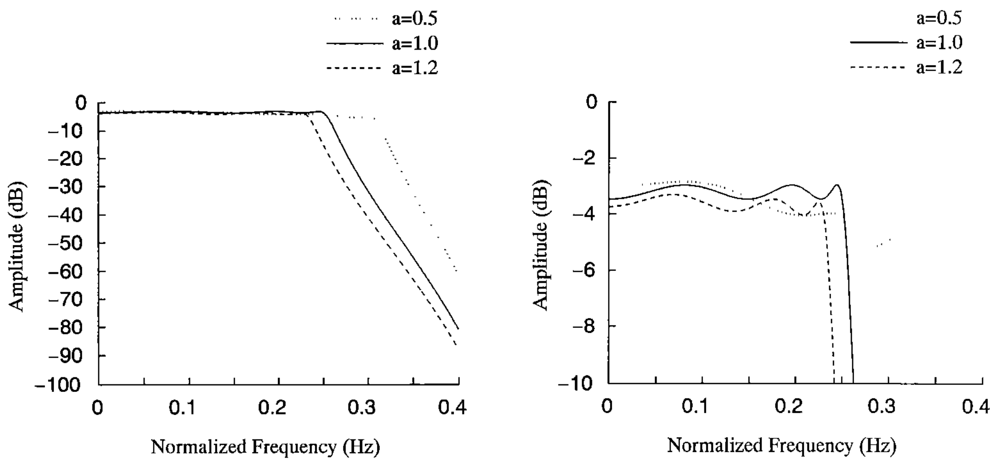


Fig. 9 Realized sixth-order WDF.



(a) Stopband (b) Passband

Fig. 10 Amplitude response of the proposed sixth-order variable WDF.



(a) Stopband (b) Passband

Fig. 11 Amplitude response of cascade structure.

disturbed than Ref. [2] except for $a=1.5$ in both the passband and stopband. Since the ideal cutoff frequency in the case of $a=0.5$ is 0.0976 Hz, the ones realized by the proposed design and Ref. [2] do not reach the ideal value because of the first-order approximation. Although the cutoff frequency of the proposed method is a little lower than Ref. [2], the robustness of the amplitude response enables the proposed design to compensate the cutoff frequency by tuning the value of a. Therefore, the proposed design is better than Ref. [2] in the case that it is not required to reduce the cutoff frequency very much.

Secondly, a sixth-order Chebychev transfer function is realized to compare amplitude characteristics with Ref. [3] which realizes the same transfer function. Its specification is as follows; the sampling frequency is 1 Hz, the cutoff frequency at $a=1$ is 0.25 Hz and the passband ripple is 0.5 dB. The realized WDF is shown in Fig. 9. Figure 10 shows the amplitude responses by the proposed design with $a=0.5$, 1.0 and 1.2. Figure 11 shows those by Ref. [3] with the same values of a. It is found from these figures that the proposed design reveals more excellent equiripple characteristics than Ref. [3] in all the cases, whereas the degradation in the stopband in both Fig. 10 and Fig. 11 is small. In all the cases the gainlevel compensation is applied. However in the case of $a=0.5$ in Fig. 10 the gain level varies much, which means that the compensation does not work well. This situation is proved by Fig. 12, where the amplitude responses with and without the compensation are drawn.

These two examples show that the amplitude characteristics of the proposed design inherit the low-sensitivity property of WDFs, because the shapes of the amplitude responses by the proposed design are less disturbed than the others.

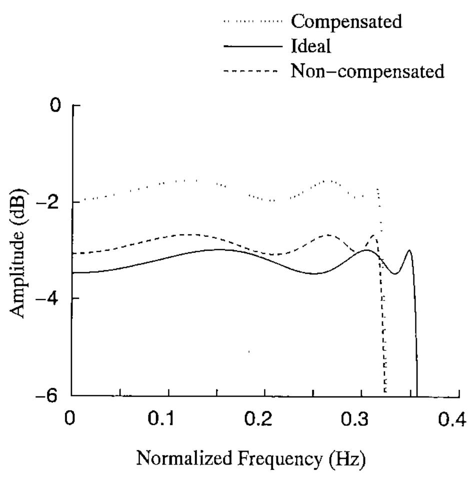


Fig. 12 Effectiveness of the gain-level compensation in the case of $a=0.5$ in the proposed design.

5. Examination of Second-Order Approximation

The discussion in the former sections is on the basis of the first-order approximation by means of the Taylor series expansion so that multiplier coefficients can become first-order functions of the parameter a. The quality of the first-order approximation depends on the linearity of multiplier coefficients. The linearity of each multiplier coefficient is not always identical. This paper introduces curvature as the measure of the linearity. Assuming that $\alpha(a)$ is a certain multiplier coefficient of the function of a, the curvature at $a=1$ is given by

$$\chi=\left.\frac{\alpha''(a)}{(1+\alpha'^2(a))^{3/2}}\right|_{a=1}. \tag{35}$$

When χ is a large number, the approximation error is large. To such multiplier coefficients as have large curvature it is proposed to apply a second-order approximation instead of the first-order one. The second-order approximation by means of the Taylor series expansion is given by

$$\alpha(a)\cong\alpha(1)+(a-1)\alpha'(1)+\frac{1}{2}(a-1)^2\alpha''(1). \tag{36}$$

The evaluation of $\alpha''(1)$ is similar to that of $\alpha'(1)$. The circuitry realization of Eq. (36) is shown in Fig. 13. The number of multipliers in Fig. 13 is more than that in Fig. 4 by 2. From the point of view of the number of multipliers, the second-order approximation is desired to be applied to a smaller number of coefficients. The numerical examples of the curvature of the multiplier coefficients in the previous fifth-order elliptic filter are as follows;

$\chi_{\alpha_{p1}}=-0.2328524171$ $\quad\chi_{\alpha_3}=0.03729291084$

$\chi_{\alpha_{p2}}=-0.4049070100$ $\quad\chi_{\alpha_4}=-0.01687862972$

$\chi_{\alpha_1}=0.1516679498$ $\quad\chi_{\alpha_5}=0.05935307659$

$\chi_{\alpha_2}=0.01691652188$ $\quad\chi_{\alpha_6}=-0.19662134875$

The amplitude response by the second-order approximation, which is applied to α_{p_1} and α_{p_2}, is shown by

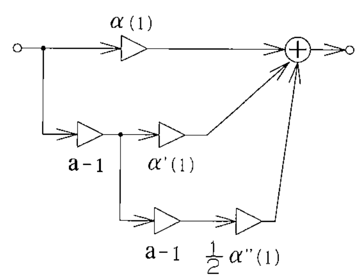


Fig. 13 Multiplier branch after the second-order approximation.

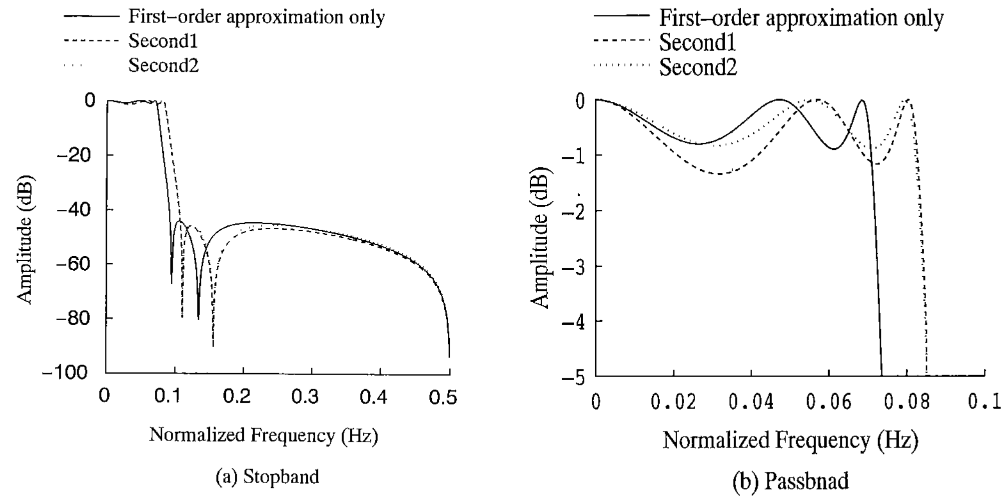


Fig. 14 Amplitude response after the second-order approximation.

the broken line and labeled as Second 1 in Fig. 14, where the response by only the first-order approximation is also plotted by the solid line. The ideal cutoff frequency of this response is 0.0976 Hz, which is obtained from $a=0.5$. Figure 14 shows that the amplitude response is much improved by the second-order approximation, because the cutoff frequency of Second 1 is 0.0831 Hz and nearer to 0.0976 Hz than that by the first-order approximation. Since α_{p_1} and α_{p_2} are related to determining the transmission zeros of the realized WDF, such an application of the higher-order approximation is interpreted in the s-domain as tuning the frequencies of the attenuation poles of an LC-ladder filter more precisely. Thus tuning the frequencies of the attenuation poles of the WDF gives this improvement. In addition to α_{p_1} and α_{p_2}, when the second-order approximation is applied to α_1 and α_6, the amplitude response is shown by the dotted line and labeled as Second 2 in Fig. 14. The passband characteristics of Second 2 is better than Second 1. Therefore the effectiveness of the second-order approximation is shown.

6. Conclusion

This paper has proposed a design of variable lowpass digital filters with wider ranges of cutoff frequencies than conventional designs. WDFs are used for the prototypes of variable filters. The proposed design is based on the frequency scaling in the s-domain, while the conventional ones are based on the z-domain lowpass-to-lowpass transformations. Multiplier coefficients in the WDF obtained from a frequency-scaled LC filter are approximated to linear functions of the scaling parameter by the Taylor series expansion, which is similar to the conventional design. Furthermore this paper has discussed the reduction of the approximation error. The curvature is introduced as the figure of the quality of the first-order approximation. The use of the second-order approximation to large-curvature multiplier coefficients instead of the first-order approximation is proposed.

Further reduction of the approximation error remains to be studied, including the derivation of more effective gain-level compensations.

Acknowledgement

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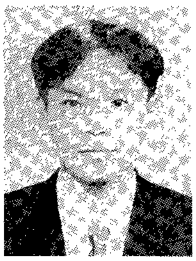

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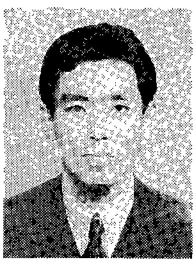

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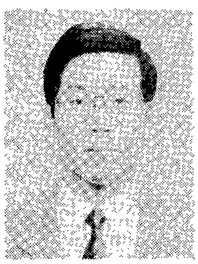

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