論文 / 著書情報
Article / Book Information

Title	Wavelet Bases Obtained from the Raised-Cosine Filter
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Citation	IEICE Trans. Fundamentals, Vol. E80-A, No. 1, pp. 126-132
Pub. date	1997, 1
URL	http://search.ieice.org/
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PAPER

Wavelet Bases Obtained from the Raised-Cosine Filter

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SUMMARY In this paper, new wavelet bases are presented. We address problems associated with the proposed matched filter in multirate systems, using an optimum receiver that maximises the SNR at the sampling instant. To satisfy the Nyquist (ISI-free transmission) and matched filter (maximum SNR at the sampling instant) criteria, the overall system filtering strategy requires to split the narrowest filter equally between transmitter and receiver. In data transmission systems a raised-cosine filter is therefore often used to bandlimit signals from which wavelet bases are derived. Sampling in multiresolution subspaces is also discussed.

key words: *wavelet bases, scaling function, wave digital filters, raised-cosine filter*

1. Introduction

1.1 Wavelet Signal Processing

In conventional digital signal processing, an approximated version of the ideal low-pass filter $h(t)$ is used to bandlimit signals. In multirate systems, performing a wavelet transform by means of a filter bank, the scaling function $\varphi(t)$ (approximated version) is used to filter time continuous signals before they get sampled [1]. Using $h(t)$ in a multirate system for bandlimiting signals, the discrete signal then shows a certain error compared to a signal beeing in a multiresolution subspace. If one uses a digital pre-filter, this error can be minimised [2]. In particular, the expansion coefficients c_n can be written as [1],[16]

$$c_n = \int_{t=-\infty}^{\infty} x(t)\varphi(t-n)dt, \tag{1}$$

$n \in \mathbf{Z}$, and the related block diagram is depicted in Fig. 1. For $\varphi(t)$ having compact support in frequency domain, such as the Shannon case, $c_n = x(n)$; $c_n \neq x(n)$ otherwise, see [1],[16] for more details.

1.2 Optimum Receiver

The narrowest filter in a receiver is responsible for minimizing the system error probability. This filter is called a matched-filter if the error probability is minimum. A minimum error probability requires a maximum SNR. Optimum receivers which maximise the SNR at the sampling instants are refered to as matched filter receivers. The same error performance for bandlimited systems is achieved as that of wide band systems with a matched filter receiver if the overall system filtering strategy satisfies the Nyquist (ISI-free transmission) and matched filter (maximum SNR at the sampling instants) criteria, i.e., having a raised-cosine roll-off shaped response $H_S(f)$ equally split between transmitter $H_T(f)$ and receiver $H_R(f)$. That is $H_S(f) = H_T(f)H_R(f)$ and as we use a multirate system

$$H_S(f) = \Phi(f)\Phi(f). \tag{2}$$

Note, that an ideal channel having a transfer function $H_c(f) = 1$ is assumed in the above consideration.

2. Raised-Cosine Wavelets

Our motivation to generate raised-cosine bases stems from the two facts:
a) the scaling function $\varphi(t)$ is used to filter signals when they are processed in a system using the wavelet transform, and
b) the square-root of the raised-cosine filter is used in an optimum receiver to bandlimit signals.

After a discussion of sampling in multiresolution subspaces (approximation problem), necessary scaling function relations, wavelet relations as well as the multiresolution analysis are reviewed, which will be used douring the derivation of the raised-cosine bases.

2.1 Sampling in Multiresolution Subspaces

The WT is defined for square integrable signals ($L_2(\mathbf{R})$). And, moreover, wavelets are unconditional bases for spaces L_p, $1 < p < \infty$ [3],[16]. Furthermore, wavelets can be used to process some signals in $L_1([0,1])$ [3],[16]. If these signals are processed with realizable electrical filters (analog and digital filters), then one can only realize scaling functions that have infinite support in time as well as in frequency domain

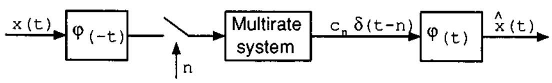

Fig. 1 Wavelet signal processing arrangement.

Manuscript received April 23, 1996.
Manuscript revised August 20, 1996.

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(causality). Hence, in the realizable ideal case digital IIR filters with a finite number of filter coefficients are of interest and exactly linear phase filters are then impossible. Wavelets based on IIR-filters are e.g. the Butterworth wavelets [15]. However, one can approximate linear phase with digital IIR filters. The method of wave digital filters (WDF) is especially suited for this purpose since the well known benefits (like low-sensitivity, low roundoff noise, all aspects of stability under finite word length conditions) can be obtained in one as in multiple dimensions [14],[17]. And there is no need for a spectral factorization method when designing the digital filters, neither in one nor in multiple dimensions [12],[17].

The validity of the wavelet transform requires, that a discrete signal is obtained from the related continuous signal $x(t)$ as in (1). Nevertheless, in reality, this ideal sampling process is approximated independent whether $\varphi(t)$ has compact support or not. The question of how good the discrete-to-continuous signal representation is approximated depends on the realization of the scaling function $\varphi(t)$ in (1).

In conventional signal processing, there are many different continuous signals $x(t)$ having the same discrete sample values, unless the signal $x(t)$ is bandlimited before it undergoes the sampling process (Shannons sampling theorem).

For the wavelet transform, there exists the wavelet sampling theorem (see e.g. [9]–[11],[16]) where $\varphi(t)$ does not need to have compact support and the c_n's are obtained from $x(t)$ with a normalized maximal sampling spacing of 1 [1]. Alternatively, Shannons sampling theorem can be applied to cases, where $\varphi(t)$ has compact support either in frequency domain or in time domain, see [13]. In particular, the minimum sampling rate $f_s = 1/T_s$ for the sinc case follows as $\omega_g = \pi/2 \Rightarrow f_g = 1/4 \Rightarrow T_s \leqq \frac{1}{2f_g} = 2$ and for the Meyer case $\omega_g = 2\pi/3 \Rightarrow f_g = 1/3 \Rightarrow T_s = 3/2$, where the bandlimited signal has no spectral components above ω_g. Comparing both sampling theorems on the sinc-wavelet, Shannon's sampling theorem allows much less samples to uniquely represent the associated continuous time signal, compared to the wavelet sampling theorem. This can be particularly seen in the case of PDE's, where a bandlimited signal $x(t)$ can be uniquely recovered from samples of $x(t)$ and its derivative at half the sampling rate [11],[13].

Increasing the order of e.g. Daubechies FIR filters, splines, Butterworth filters, etc., results in higher regularity which is desired in many applcations when using the WT. By doing so, a wanted side effect occours, i.e., the asociated scaling function then has approximately compact support in frequency domain. And hence, Shannon's sampling theorem can be applied. As an example, in the Butterworth case a filter order of 9 is used in [12],[17], where in the one dimensional case 4 coefficients only are necessary. Note, B-splines can be viewed as Butterworth wavelets [15]. Thus, according to conventional digital signal processing, and because of economical reasons, one may clasify scaling functions into two groups; some examples are listed in Table 1. This classification is ment to stress the fact that the Shannon sampling theorem provides a more economic discrete representation of an associated continuous signal than the wavelet sampling theorem. Of course all the scaling functions in Table 1 are valid ones when using the wavelet sampling theorem.

Table 1 Scaling functions.

"Valid" scaling functions (for Shannon sampling)	"Approximating" scaling functions (for Shannon sampling)
---------related to a digital filter having---------	
infinite number of filter coefficients	finite number of filter coefficients
sinc scaling function (IIR-filter)	Daubechies scaling function (FIR-filter)
Meyer scaling function (IIR-filter)	Haar scaling function (FIR-filter)
raised-cosine scaling function (IIR-filter)	Butterworth scaling function (IIR-filter)
	Chebyshev scaling function (IIR-filter)

In practice, one designs an "approximating" scaling function such that the aliasing error (defined below and well known from conventional signal processing [8],[13]), when compared to a "valid" scaling function, is minimized (discretization problem). Alternatively, one can use

$$h_{a0}(n) = \sqrt{2}\int_{t=-\infty}^{\infty} \varphi^*(2t-n)\varphi(t)dt \quad (3)$$

$$= \frac{1}{2\sqrt{2}\pi}\int_{\omega=-\infty}^{\infty} e^{-j\omega n}\Phi^*\left(\frac{\omega}{2}\right)\Phi(\omega)d\omega \quad (4)$$

in order to approximate a "valid" scaling function directly. (Equation (4) is obtained from (3) by using the shift property, scaling property, modulation theorem and Parsevals formula.) And since a valid scaling function has compact support in frequency domain, the integral in (4) becomes zero outside $\omega = \pm\frac{4\pi}{3}$. However, $h_{a0}(n)$ will in general not form a perfect reconstruction filter bank, since in practice, one uses only a finite set out of $\{h_{a0}(n)\}$, containing an infinite number of non-zero filter coefficients.

An aliasing error estimation is described in [10] and defined as $\| x(t) - \tilde{x}(t) \|$ for $x(t) \in L_2(R)$, where $x(t)$ must not be in a multiresolution subspace, and $\tilde{x}(t)$ is the recovered continuous signal by the interpolation formula $\tilde{x}(t) = \sum_n x(n)S(t-n)$, $S(t)$ the interpolant. In [9], for the error $\epsilon = x(t) - \tilde{x}(t)$, $x(t)$ is assumed to be in a multiresolution subspace.

So far one considered the ideal low-pass filter (sinc-scaling function) to be the interpolant, whereas one may consider e.g. in communication systems the raised-cosine filter for this purpose. Some reasons for this are discussed next.

The theoretical minimum bandwidth needed to detect R symbols per second without intersymbol interference (ISI) is R/2 hertz. This occurs in the case when using the sinc-scaling function (exactly rectangular low-pass filter, non-causal). In this case, 2 symbols per second per hertz are possible to transmit without ISI.

A matched filter is a linear filter, designed to provide the maximum signal-to-noise power ratio (SNR) at its output. In systems using a matched filter, an excess bandwidth beyond the theoretical minimum bandwidth is used and related to a roll-off factor, described below. The matched filter is a non-causal filter, which, when realized with the raised-cosine filter, combines the matched filter criteria and the Nyquist criteria (ISI). The raised-cosine filter is also a theoretical filter, but it can be better approximated. The problem of ISI degradation is not as serious as it is when using the ideal low-pass filter [8].

2.2 Multiresolution Analysis

In [1],[2]–[7] one can find the below given concept of multiresolution analysis in more detail.

The concept of multiresolution analysis consists of a sequence of nested signal subspaces V_i (Note, that the notation of the nesting order varies in the literature)

$$V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \dots \tag{5}$$

such that

$$\bigcup_{i \in \mathbf{Z}} V_i = L_2(\mathbf{R}), \quad \bigcap_{i \in \mathbf{Z}} V_i = \{0\} \tag{6}$$

$$x(t) \in V_i \Longleftrightarrow x(2^i t) \in V_0 \tag{7}$$

$$x(t) \in V_0 \Longrightarrow x(t-n) \in V_0, \quad \text{for all } n \in \mathbf{Z} \tag{8}$$

There exists a scaling function $\varphi(t) \in V_0$ which together with its translates

$$\varphi_n = \varphi(t-n), \quad n \in \mathbf{Z} \tag{9}$$

constitutes an orthonormal basis of the space V_0, written in the Fourier domain

$$\sum_{k=-\infty}^{\infty} \mid \Phi(\omega + 2k\pi) \mid^2 = 1. \tag{10}$$

A continuous signal $x(t) \in L_2(\mathbf{R})$, which is to undergo a multiresolution analysis, results in an orthogonal projection P_i onto subspace V_i, particularly $\lim_{i \to -\infty} P_i x(t) = x(t)$, $forall x(t) \in L_2(\mathbf{R})$. In [1] it is shown how to obtain from the continuous signal $x(t)$ the discrete signal $x(n)$ as well as the expansion coefficients. Furthermore, in [3] it is shown that:

Theorem 1:
If $\varphi(t) \in L_2(\mathbf{R})$ satisfies

$$0 < \alpha \leq \sum_{k \in \mathbf{Z}} \mid \Phi(\omega + 2\pi k) \mid^2 \leq \beta < \infty \tag{11}$$

and if the subspace V_i is spanned by vectors $\{\varphi_{i,n},\ n \in \mathbf{Z}\}$, i.e., $V_i = span\{\varphi_{i,n},\ n \in \mathbf{Z}\}$ then $\bigcap_{i \in \mathbf{Z}} V_i = \{0\}$.

Theorem 2:
If $\varphi(t) \in L_2(\mathbf{R})$ satisfies (9), V_i is defined as above, and if $\Phi(\omega)$ is bounded for all ω and continuous near $\omega = 0$, with $\Phi(0) \neq 0$, then $\bigcup_{i \in \mathbf{Z}} V_i = L_2(\mathbf{R})$.

2.3 Scaling Function and Wavelet Relations

As described above a multiresolution is composed of a scaling function $\varphi(t) \in V_0$ satisfying (3)-(8). It follows that the subspace V_0 can be constructed from the vectors in (7) and from there all the other subspaces V_i can be generated.

Designate $\varphi_{i,n}(t) = 2^{-\frac{i}{2}}\varphi(2^{-i}t - n)$, $i, n \in \mathbf{Z}$ and $\psi_{i,n}(t) = 2^{-\frac{i}{2}}\psi(2^{-i}t - n)$, $i, n \in \mathbf{Z}$ then there exists for the orthonormal basis $\{\varphi_{i,n}\}$ for V_i an orthogonal complement $\{\psi_{i,n}\}$ for W_i in V_{i-1}. Hence $V_{i-1} = V_i \oplus W_i$. Thus, if a low-pass filter $H_0(e^{j\omega})$ satisfies $\mid H_0(e^{j0}) \mid = \sqrt{2}$, $\sum_n \mid h_0(n) \mid^2 = 1$ and $\mid H_0(e^{j\pi}) \mid = 0$, then the scaling function can be expressed as

$$\Phi(\omega) = \frac{1}{\sqrt{2}} H_0(e^{\frac{j\omega}{2}}) \Phi\left(\frac{\omega}{2}\right) \tag{12}$$

and the mother wavelet as $\Psi(\omega) = \frac{1}{\sqrt{2}} H_1(e^{\frac{j\omega}{2}}) \Phi(\frac{\omega}{2})$. It is shown, e.g. in [1],[2]–[7], that the two filters $H_0(e^{j\omega})$ and $H_1(e^{j\omega})$ are related as $H_1(e^{j\omega}) = -e^{j\omega} H_0^*(e^{j(\omega+\pi)})$ where '$*$' denotes conjugate complex. Hence, in the orthonormal case, a wavelet can be constructed by $\Psi(\omega) = -e^{\frac{j\omega}{2}} H_0^*(e^{j\frac{\omega}{2}+\pi}) \Phi(\frac{\omega}{2})$. It follows that the scaling function has low-pass characteristic. Furthermore, because it is known that for orthonormal wavelet bases the scaling function satisfies (12) and $\int_{t=-\infty}^{\infty} \varphi(t) dt = 1$, it is possible to construct wavelet bases starting from a scaling function that has compact support either in time domain or in frequency domain.

2.4 Raised-Cosine Bases

In the following, orthonormal wavelet bases are derived from the raised cosine filter (compact support in frequency domain) where the roll-off factor α ranges between

$$0 \leqq \alpha \leqq \frac{1}{3}. \tag{13}$$

As a special case, the sinc-scaling function follows for $\alpha = 0$ and for $\alpha = 1/3$ a Meyer scaling function.

Theorem 3:
Let W be the absolute bandwidth, $W_0 = \frac{1}{2T}$, $\alpha = \frac{(W - W_0)}{W_0}$ the roll-off factor, T the sampling period, then

$$\varphi_r(t) * \varphi_r(t) = 2W_0\ sinc(2W_0 t)\frac{\cos[2\pi(W - W_0)t]}{1 - 4(W - W_0)t^2} \tag{14}$$

where $sinc(x) = \frac{\sin(\pi x)}{\pi x}$ or in frequency domain

$$\Phi_r(\omega)\Phi_r(\omega) = \begin{cases} 1 & \text{for } \mid \frac{\omega}{2\pi} \mid < 2W_0 - W \\ \cos^2\left(\frac{\pi}{4}\frac{\mid \frac{\omega}{2\pi}\mid + W - 2W_0}{W - W_0}\right) & \text{for } 2W_0 - W < \mid \frac{\omega}{2\pi} \mid < W \\ 0 & \text{for } \mid \frac{\omega}{2\pi} \mid > W \end{cases} \tag{15}$$

$\Phi_r(\omega)$ is called the raised cosine scaling function and different α result in different wavelets.

Proof:
Sampling a bandlimited analog signal $X_a(\Omega)$ in frequency domain results in [7]

$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a\left(\Omega + \frac{2\pi k}{T}\right)\mid_{\Omega=\frac{\omega}{T}} \tag{16}$$

where the Fourier transform of $x(n)$ and $x_a(t)$ are $X(e^{j\omega})$ and $X_a(\Omega)$, respectively. In this case one can therefore write

$$H_0(e^{j\omega}) = \sqrt{2}\sum_{k\in\mathbf{Z}} \Phi_r(2\omega + 4k\pi) \tag{17}$$

for the digital low-pass filter. The wavelet follows as

$$\Psi_r(\omega) = -\frac{1}{\sqrt{2}}e^{-j\frac{\omega}{2}}\sum_{k\in\mathbf{Z}} \Phi_r(\omega + 2\pi(2k+1))\Phi_r\left(\frac{\omega}{2}\right) \tag{18}$$

and because as only for $k = 0$ and $k = -1$ the support of $\Phi_r(\frac{\omega}{2})$ and $\Phi_r(\omega + 2\pi(2k+1))$ overlaps (resulting from compact support of the scaling function in frequency domain)

$$\Psi_r(\omega) = -\frac{1}{\sqrt{2}}e^{-j\frac{\omega}{2}}[\Phi_r(\omega+2\pi)+\Phi_r(\omega - 2\pi)]\Phi_r\left(\frac{\omega}{2}\right). \tag{19}$$

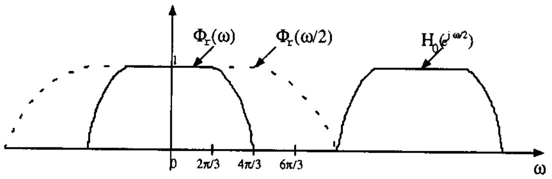

Fig. 2 $H_0(e^{j\omega})$, $\Phi_r(\omega)$ and $\Phi_r(\omega/2)$ for $\alpha = 1/3$.

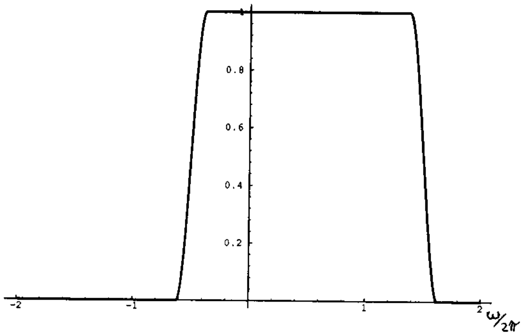

Fig. 3 $[\Phi_r(\omega)]^2 + [\Phi_r(\omega - 2\pi)]^2$ where $\Phi_r(\omega)$ has compact support.

To prove that $\Phi_r(\omega)$ satisfies (5)-(10), it is enough to check (10) in the interval $\omega \in [0, 2\pi]$, since $\Phi_r(\omega)$ is symmetric and has compact support. It follows that

$$\mid \Phi_r(\omega) \mid^2 + \mid \Phi_r(\omega - 2\pi) \mid^2 = 1 \tag{20}$$

for $\omega \in [0, 2W_0 - W]$ and for $\omega \in [2W_0 - W, W]$. And thus, orthonormality is given, see Fig. 3. Next, define V_0 to be the closed subspace spanned by this orthonormal set. V_i is similarly defined which satisfies (5) if $H_0(e^{j\omega})$ is 2π-periodic and if $H_0(e^{j\omega})$ is square integrable on $[0, 2\pi]$ such that (12) is valid. Then a 2π-periodic filter exists if $0 \leqq \alpha \leqq \frac{1}{3}$ (Compare Fig. 2, (12) and (14)). By definition, $\Phi_r(\omega)$ is zero outside W and the integral over $\Phi_r^2(\omega)$ is bounded by a finite number. It follows that a 2π-periodic, square integrable low-pass filter $H_0(e^{j\omega})$ on $[0, 2\pi]$ exists. As (5) is satisfied, then the rest of the multiresolution analysis follows from theorem 1 and theorem 2 as well as from the definitions (7) and (8) themselves. □

As an example, the one-dimensional scaling function $\Phi_r(\omega)$ and the wavelet $\Psi_r(\omega)$ for the values $T = 1$ and $\alpha = 0.25$ are shown in the Figs. 4 and 5 (time domain) and the Figs. 6 and 7 (frequency domain). And in the two-dimensional case, there exists one scaling function and three wavelets for rectangular digital/digital sampling. Namely,

$$\Phi_{rec}(\omega_1, \omega_2) = \Phi(\omega_1)\Phi(\omega_2) \tag{21}$$
$$\Psi_{1,rec}(\omega_1, \omega_2) = \Phi(\omega_1)\Psi(\omega_2) \tag{22}$$
$$\Psi_{2,rec}(\omega_1, \omega_2) = \Psi(\omega_1)\Phi(\omega_2) \tag{23}$$
$$\Psi_{3,rec}(\omega_1, \omega_2) = \Psi(\omega_1)\Psi(\omega_2), \tag{24}$$

whereas for digital/digital quincunx sampling

$$\Phi_q(\omega_1, \omega_2) = \Phi(\omega_1)\Phi(\omega_2) \tag{25}$$

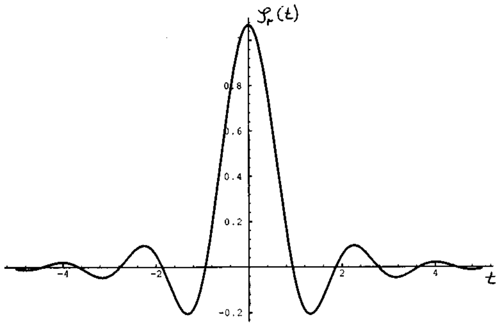

Fig. 4 Raised-cosine scaling function $\varphi_r(t)$ for $T=1, \alpha=0.25$.

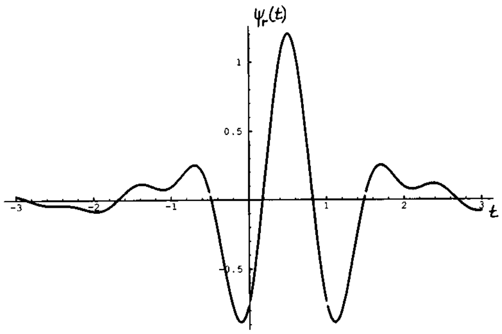

Fig. 5 Raised-cosine wavelet $\psi_r(t)$ for $T=1, \alpha=0.25$.

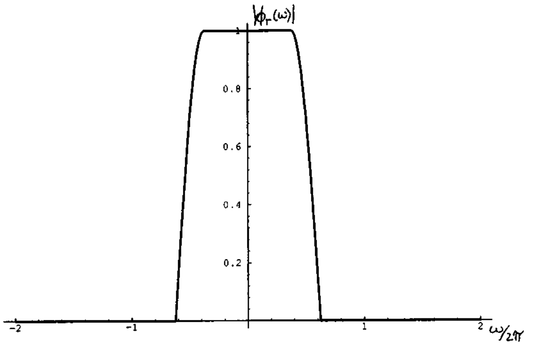

Fig. 6 Raised-cosine scaling function $\mid \Phi_r(\omega) \mid$ for $T=1$, $\alpha=0.25$.

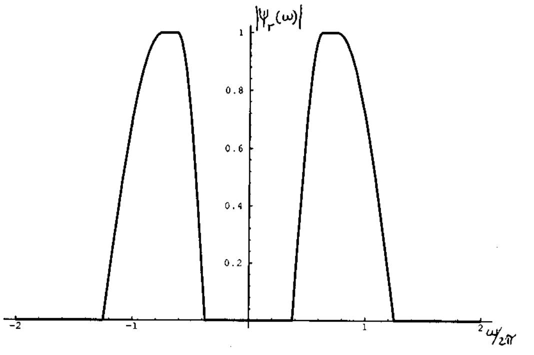

Fig. 7 Raised-cosine wavelet $\mid \Psi_r(\omega) \mid$ for $T=1, \alpha=0.25$.

$$\Psi_q(\omega_1,\omega_2) = \Phi_q(\mathbf{M}^{-1}(\omega_1,\omega_2))M_1(j\mathbf{M}^{-1}(\omega_1,\omega_2)). \tag{26}$$

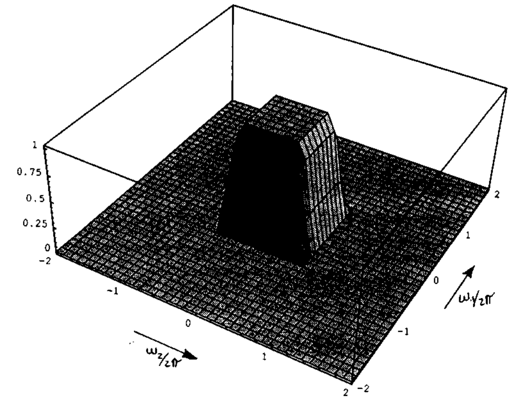

Fig. 8 Two-dimensional raised-cosine scaling function for digital/digital rectangular and digital/digital quincunx sampling, $\mid \Phi_{rec}(\omega_1,\omega_2) \mid = \mid \Phi_q(\omega_1,\omega_2) \mid$, for $\alpha=0.25$.

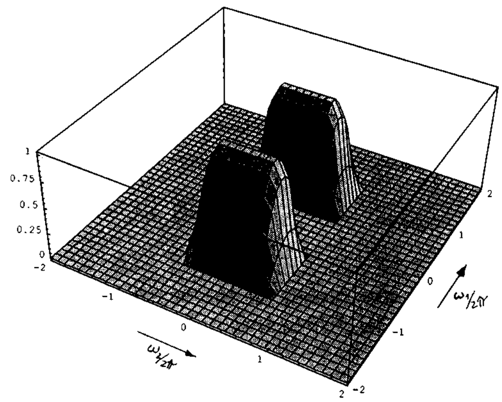

Fig. 9 Two-dimensional raised-cosine wavelet for digital/digital rectangular sampling, $\mid \Psi_{1,rec}(\omega_1,\omega_2) \mid$, for $\alpha=0.25$.

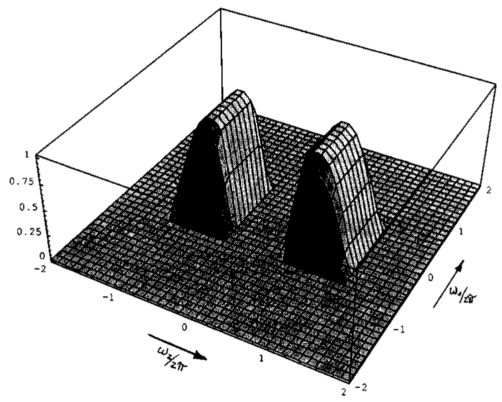

Fig. 10 Two-dimensional raised-cosine wavelet for digital/digital rectangular sampling, $\mid \Psi_{2,rec}(\omega_1,\omega_2) \mid$, for $\alpha=0.25$.

$M_1(j(\omega_1,\omega_2))$ is a two-dimensional digital high-pass filter.

Figures 8, 9, 10, 11 and 12 show two-dimensional plots of the scaling function and wavelets for digital/digital rectangular and quincunx sampling, respectively. Compare [12] for more details on the two-

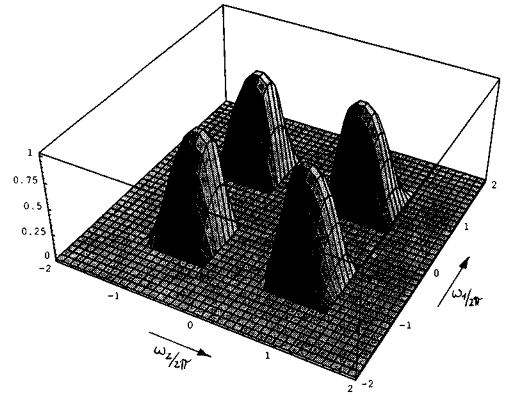

Fig. 11 Two-dimensional raised-cosine wavelet for digital/digital rectangular sampling, $|\Psi_{3,rec}(\omega_1,\omega_2)|$, for $\alpha = 0.25$.

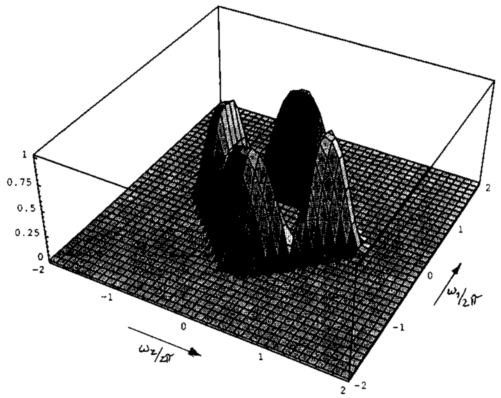

Fig. 12 Two-dimensional raised-cosine wavelet for digital/digital quincunx sampling, $|\Psi_q(\omega_1,\omega_2)|$, for $\alpha = 0.25$.

dimensional quincunx case.

However, $H_0(e^{j\omega})$ is not a rational function of $e^{j\omega}$. Hence, implementing this filter is not straight forward. In fact, it turns out that $h_0(n)$ is a non-causal filter having infinite support, which can only be approximated, e.g. with (4) or with an approximating scaling function as e.g. shown in [12],[17] and well known from the conventional signal processing.

3. Conclusions

In this paper we address problems concerning the design of wavelets as well as sampling in multiresolution subspaces. New wavelet bases have been designed, the raised-cosine bases. They are derived from the raised-cosine filter. Furthermore, the problem of sampling in multiresolution subspaces (approximation problem) has been discussed.

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