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PAPER

A Log-Normal Distribution Model for Electron Multiplying Detector Signals in Charged Particle Beam Equipments

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SUMMARY We propose a stochastic model for signals generated through the electron multiplying effect of detectors in charged particle beam equipments. This model is based on a stochastic variable characterized by a log-normal type distribution. The model is simple and can be used to represent a wide dynamic range of signals from pulse-like signals when the primary beam current is small to continuous signals when the primary beam current is large. For the model base reference a normalization of actual signal detectors is presented. This base reference yields the unique stochastic parameter used in our model. The proposed model better approximates the actual signals in the power spectrum distribution as compared to the filtered Poisson method presented elsewhere.
key words: *stochastic process, charged particle beam equipment, log-normal distribution, filtered Poisson model*

1. Introduction

Charged particle beam equipments have been widely used in surface observation and analysis of specimens in nano-meter fields. Some charged particle beam equipments, such as scanning electron microscopes (SEM) and scanning ion microscopes use charged particle detectors to capture images. Figure 1 shows the structure of an SEM. The equipment scans charged particles primarily beamed at the surface of specimens, and the detector collects secondary electrons or back scattered electrons generated by the specimens to display specimen images. If a mathematical model of the detected signal is prepared, imaging system of the equipment can easily be evaluated.

A special feature of such detector signals is their wide dynamic range. For example, suppose a primary beam current is set to 1×10^{-11} A, the efficiency of secondary electron emission of a specimen is 1.0, and an image frame has 1000×1000 pixels scanned at a frame rate of 6.07 frame/s. The period of one pixel is 163 ns. The electric charge per one pixel becomes 1.6×10^{-18} C. As the electric charge of a single electron is 1.6×10^{-19} C, the number of electrons in one image pixel becomes only ten. In this case, the output signal of the detector features stochastic pulse signals. On the other hand, when the primary beam current is large enough, many pulses pile up and the output signal of the detector features continuous signal. A stochastic model must be used to simulate the signals.

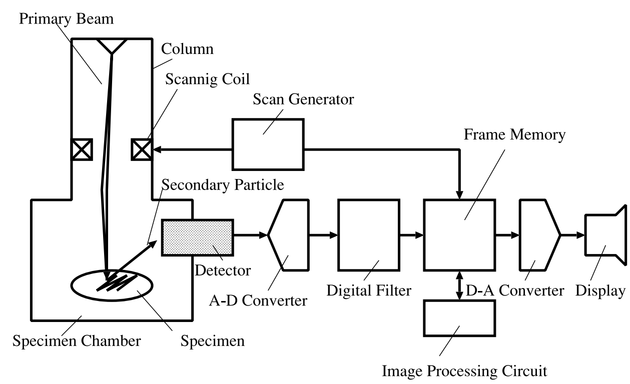

Fig. 1 Structure of scanning electron microscope.

The Poisson and the Gaussian type stochastic models have been used to analyze photo multiplier tube (PMT) detectors [1]. A filtered Poisson process which combines the general Poisson model and the impulse response of detectors [2], [3], the Polya distribution [4] and the constraint regression methods [5] have also been presented. However, those models are not so simple because multiple parameters must be determined to model an actual signal in practice. The filtered Poisson process [2], [3] naturally has a non-flat spectrum distribution caused by its filter. A flat spectrum can be obtained by decimating a sequence generated by a filtered Poisson process. Therefore, the filtered Poisson process would need larger number of Poisson impulses to obtain a flat frequency distribution.

In this paper, we propose a special one-stochastic-variable model based on the log-normal distribution [6], [7]. This is a simple model which can represent a wide dynamic range of signals from pulse-like signals to continuous signals.

The rest of this paper is organized as follows. In Sect. 2, we introduce the proposed model. In Sect. 3, a normalization of actual detector signals is derived and then used to define the unique parameter for the proposed model. Section 4 shows a comparison of signals

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generated by the proposed model and actual detector signals. Section 5 presents a comparison of the proposed model and the filtered Poisson model. Finally, we conclude in Sect. 6.

2. The Log-Normal Distribution Model

Some detectors for particle beam equipments such as scintillator-PMT systems are characterized by their electron multiplying effect. Figure 2 shows a scintillator-PMT detector. Electrons generated in the photo-cathode in a PMT are accelerated and struck to the dynodes which emit secondary electrons. The secondary electrons are struck to the next dynode stage. Since the dynodes are made of materials which have high efficiency for secondary electron emission, the number of electrons are multiplied at each stage. A PMT has about 10 stages of dynodes in general. The maximum multiplying gain is about 10^5.

Mathematical models can be classified into physical models and black-box models [8]. In the physical models, whole parameters of the structure of an actual system must be known. On the other hand, black-box models do not use a physical structure of the actual system but use a variable which fit a behavior of the actual system. The black-box model reduces the modeling effort. We use an intermediate type of these models. The actual system shown in Fig. 2 contains parameters of the energy and the current of the primary beam, and the voltage of dynodes. In the case of this actual system, we assume that the energy of electrons which hit the scintillator is almost uniform. This is because the secondary electrons which have the energy of about 20 eV are accelerated at the fixed voltage to hit the scintillator. We also assume that the voltage of the dynodes and the anode can be set arbitrarily depending on the current of the primary beam.

The whole process at a detector can be considered as a product of independent stochastic variables as shown in Fig. 3. The radio-micro-wave propagation is an example of a similar phenomenon. The magnitude distribution of scattered wave signal is expressed by the log-normal distribution. In this case, the radio-micro-wave is reflected many times on the ground.

The product of independent stochastic variables is analyzed as follows. The product of variables can be converted to the exponential of the sum of logarithm of the variables. A sufficient number of the sum of independent stochastic variables which have the same distribution and a finite variance approximates the normal distribution by the central limit theorem. The stochastic variables characterized by the log-normal-distribution are the exponential of the stochastic variables characterized by the normal-distribution. Therefore, the log-normal distribution can express the product of sufficient number of independent stochastic variables. This is the case where the electron multiplier has enough number of stages.

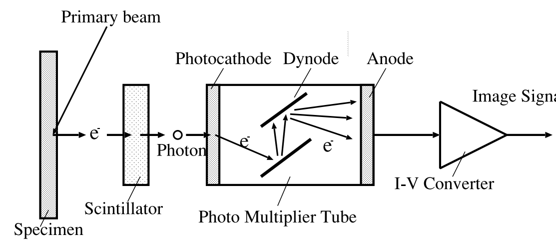


Fig. 2 Scintillator-PMT system detector.

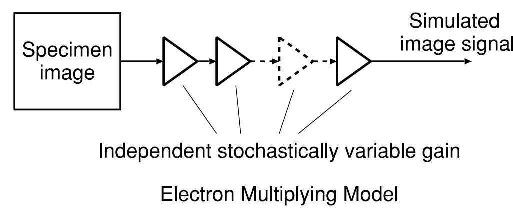


Fig. 3 Electron multiplying model of the detector.

The log-normal distribution $Q(y)$ is derived by changing variables as $y = e^x$ from normal distribution $D(x)$ [6], [7]. The normal distribution $D(x)$ is expressed as

$$D(x) = \frac{1}{\sqrt{2\pi}\hat{\sigma}} e^{-\frac{(x-\hat{\mu})^2}{2\hat{\sigma}^2}}, \tag{1}$$

where $\hat{\mu}$ denotes the mean value, and $\hat{\sigma}^2$ denotes the variance of the normal distribution. The corresponding log-normal distribution $Q(y)$ is therefore expressed as

$$Q(y) = \frac{1}{\sqrt{2\pi}\sigma} \frac{1}{y} e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}}, \quad 0 < y < \infty. \tag{2}$$

The mean value μ of the log-normal distribution is expressed as

$$\mu = e^{\hat{\mu}+\frac{\hat{\sigma}^2}{2}}, \tag{3}$$

The variance σ^2 of the log-normal distribution is deduced

$$\sigma^2 = e^{2\hat{\mu}}(e^{2\hat{\sigma}^2} - e^{\hat{\sigma}^2}). \tag{4}$$

From Eqs. (3) and (4), the mean value $\hat{\mu}$ of the normal distribution and the variance $\hat{\sigma}^2$ of the normal distribution are deduced as

$$\hat{\mu} = \frac{1}{2}\ln\left(\frac{\mu^4}{\sigma^2+\mu^2}\right), \tag{5}$$

$$\hat{\sigma}^2 = \ln\left(\frac{\sigma^2}{\mu^2}+1\right). \tag{6}$$

$Q(y)$ is defined in the positive region of y. In the case of simulation of the proposed model, we calculate exponentials of random numbers characterizing the normal distribution. As examples of probability density function of the log-normal distribution, Fig. 4(a) shows the case of $\mu = 0.41$, $\sigma^2 = 0.41$ and Fig. 4(b) shows the case of $\mu = 8.2$, $\sigma^2 = 8.2$. In the figures, the solid lines indicate the log-normal distribution, and the dotted lines indicate the normal distribution corresponding to the log-normal distribution. The random numbers characterizing the log-normal distribution can be obtained by using the calculation of exponentials of the normal distribution random numbers.

The calculation steps of the signal generation for the proposed model is as follows:

step 1: Determine the mean value μ and the variance σ^2 for the desired signal.

step 2: Calculate the mean value $\hat{\mu}$ and the variance $\hat{\sigma}^2$ using Eqs. (5) and (6).

step 3: Try random numbers characterizing the normal distribution $\hat{\mu}$, $\hat{\sigma}^2$.
Let the trial results be $U(n)$, where n denotes the order of trials.

step 4: Calculate, $V(n) = e^{U(n)}$.
$V(n)$ is the output signal of the proposed model.

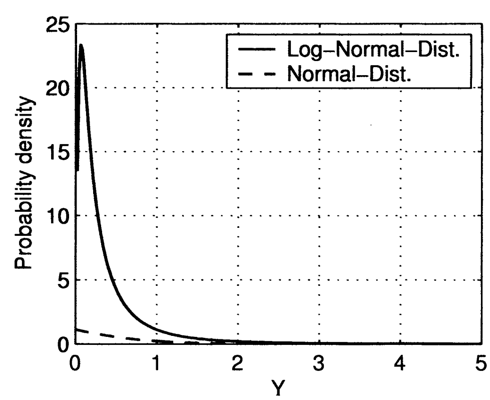
We will show the determination method for the mean value μ and the variance σ^2 of the model in the next section.

3. Normalization of Actual Signals

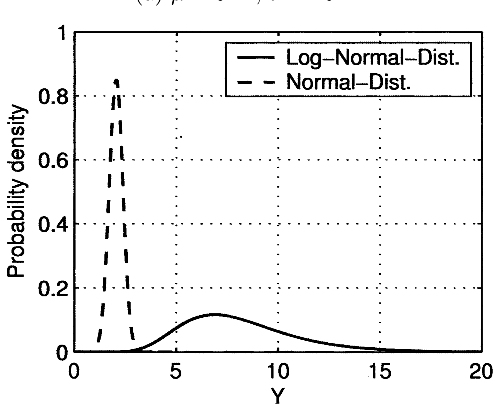
Since the primary beam current of the equipments is varied for a range of 5 decades in general, the number of electrons input to the detector changes for a range of about 5 decades. However, the gain of the following circuits usually has a limited dynamic range and can not be adjusted for that range. The electron multiplying gain is adjusted to obtain the optimum signal magnitude. Therefore, a variable to express the signal magnitude is needed independently of the electron multiplying gain.

In this paper, we assume that actual signals feature the same statistics as the Poisson impulse process. The mean and the variance of the Poisson process are equal to the product of the expectation and observation periods. The Poisson impulse process can be considered as a differentiation of the Poisson process, where the mean value μ and the variance σ^2 are equal to the expectation. Hence, we propose to normalize the magnitude of actually observed signals by multiplying a coefficient so that $\mu = \sigma^2$. The coefficient can be uniquely obtained because then μ is proportional to the coefficient and σ^2 is proportional to square of the coefficient. Consequently, μ expresses the mean of the normalized signal. However, if a noise of the following circuits can not be neglected for the variance of the signal, this normalized mean will be erroneous.

Figure 5 shows the normalized mean value μ of an actual detector signal. The horizontal axis denotes the primary beam current I_p of the equipment. The square points indicate measured and normalized means, and the dotted line indicates an $I_p \propto \mu$ line. Assuming that the efficiency of secondary electron generation is constant to the current of the primary beam, it is clear that $I_p \propto \mu$. The specimen for this example is a flat graphite surface with gold evaporation. So the topo-



(a) $\mu = 0.41$, $\sigma^2 = 0.41$



(b) $\mu = 8.2$, $\sigma^2 = 8.2$

Fig. 4 Stochastic probability density of log-normal distribution.

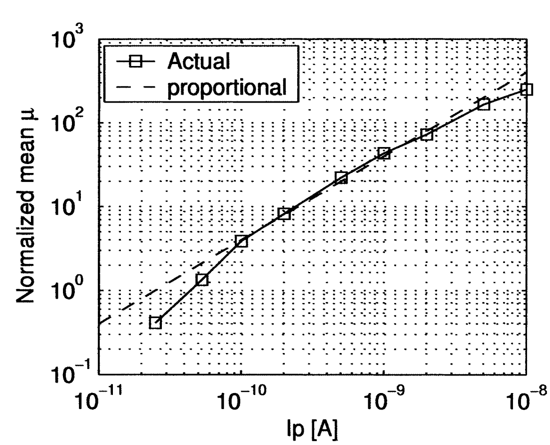


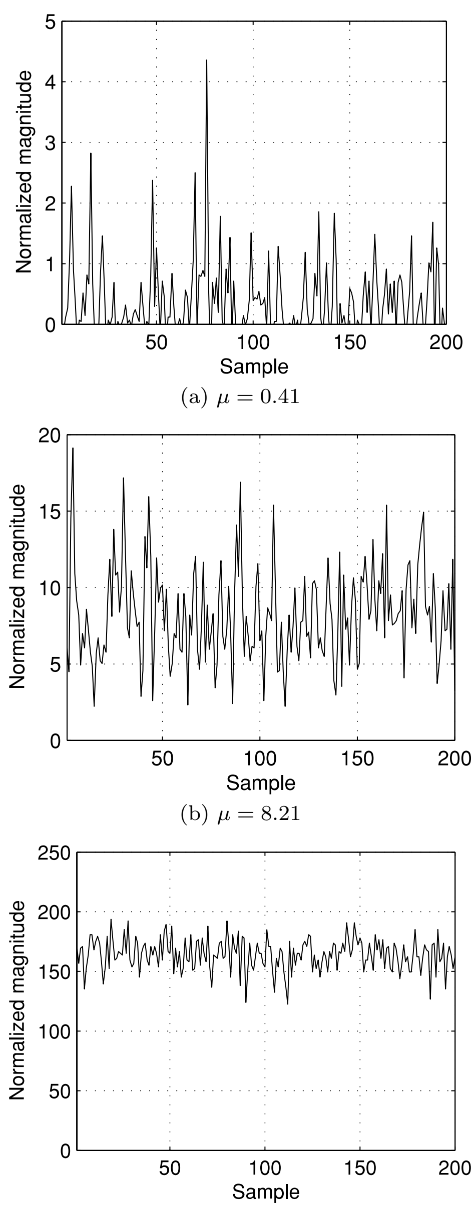
Fig. 5 Normalized mean value μ of the actual signals.

logical component does not affect the signal.

The structure of the detector which we used is the same as shown in Fig. 2. The output signal bandwidth is 0 Hz to 3.0 MHz. The PMT of the detector is model R647. The detector signal is converted to an 8 bit digital signal by 163 ns sampling period. This sampling period is almost equal to the Nyquist rate [9]. The primary beam energy is 15 keV. The range of the primary beam current is from 10^{-11}A to 10^{-8} A. The electron multiplying gain is set so that the detector signal output is suited in an 8 bit dynamic range. Figures 6(a), (b) and (c) show the signals in the time domain for the cases of $\mu = 0.41$, $\mu = 8.21$ and $\mu = 165$, respectively.

In Fig. 5, when the primary beam current I_p is smaller than 10^{-10} A, the normalized mean μ is smaller than the theoretical line. This is because the actual signal has a negative value due to the undershoot of the impulse response of the detector. Since the Poisson impulse process does not assume a negative value, errors occur. When $I_p > 2 \times 10^{-9}$ A, the normalized mean μ is smaller than the theoretical line because of decrease in efficiency of secondary electron generation by contamination at the surface of the specimen. Heat by a large current density cause contamination of specimens. Other than the above, the means of normalized signal are almost proportional to the primary beam current.

Therefore, it is reasonable to settle so that the condition is $\mu = \sigma^2$. The normalized mean μ represents the general signal quantity. The proposed model uses this parameter.



(a) $\mu = 0.41$

(b) $\mu = 8.21$

(c) $\mu = 165$

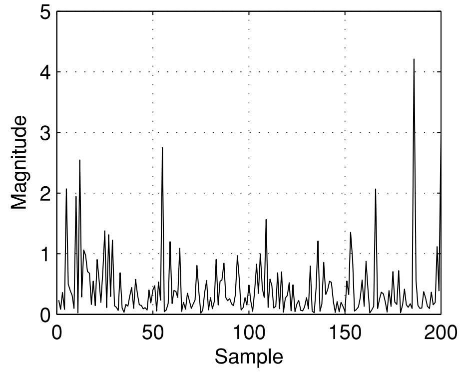
Fig. 6 Waveforms of normalized actual signal.

4. Comparison of the Log-Normal Distribution Model and the Actual Signal

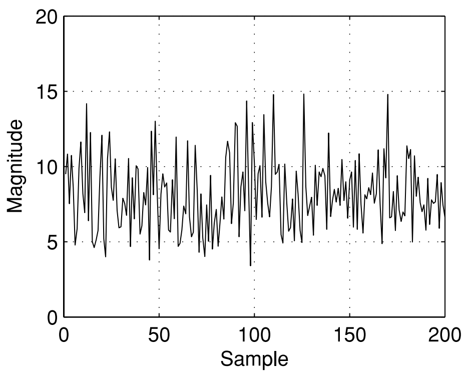
We now compare our model with actual signals in magnitude distribution. Here the condition of measurement for actual signals is the same as in Sect. 3. Figures 7(a),(b) and (c) show our model signal in the cases of $\mu = 0.41$, $\mu = 8.21$ and $\mu = 165$ corresponding to the actual signals which are shown in Figs. 6(a), (b) and (c). The calculation process of the proposed model is presented in section 2. As shown in Figs. 7(a), (b) and (c), the model can generate signals quite similar to the actual detector signals for a wide range of amplitudes.

Figures 8(a), (b) and (c) show the magnitude distribution of the signals in the cases $\mu = 0.41$, $\mu = 43.4$ and $\mu = 250$, respectively. The number of total samples is 12800 in Fig. 8(a), and 1280 for Figs. 8(b) and (c). The width of bins of histograms is 0.1 in Fig. 8(a), 3.0 in (b) and 6.4 in (c). The solid lines marked on circles indicate the proposed model signals and the dotted lines marked on points indicate the actual signals. As shown in Figs. 8(a), (b) and (c), the magnitude distribution of the proposed model agrees with that of actual signals. The mean value and the variance of the proposed model exactly match with those of the actual signal values because these numbers are provided as the model parameters.

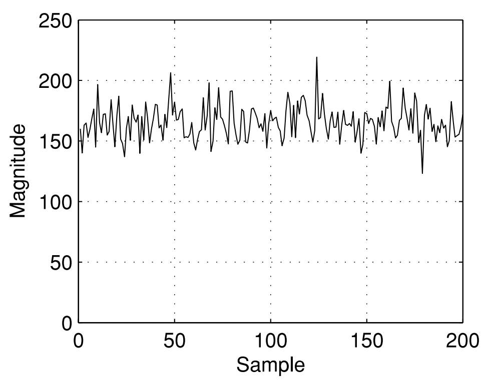
In the region around zero in Fig. 8(a), the magnitude distribution of the model is different from the magnitude distribution of the actual signal. This is because the actual signals have a negative component due to the undershoot in the impulse response of the actual detector. The order of the response is usually more than 2. The proposed model does not assume negative component of signal. Hence the difference exists.



(a) $\mu = 0.41$

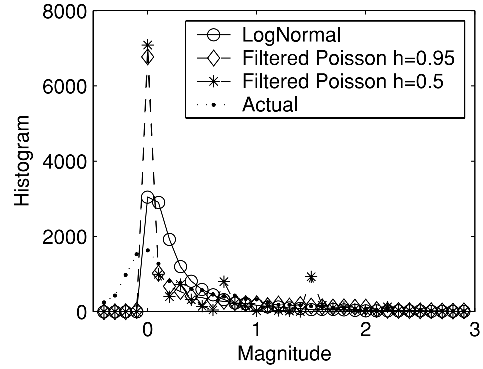


(b) $\mu = 8.21$

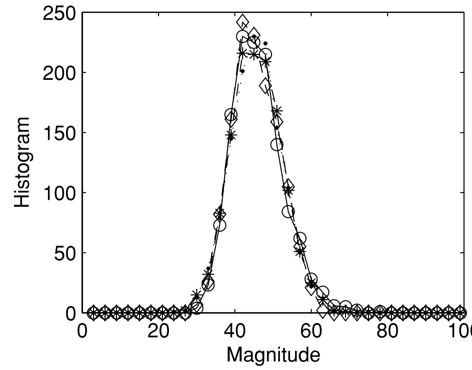


(c) $\mu = 165$

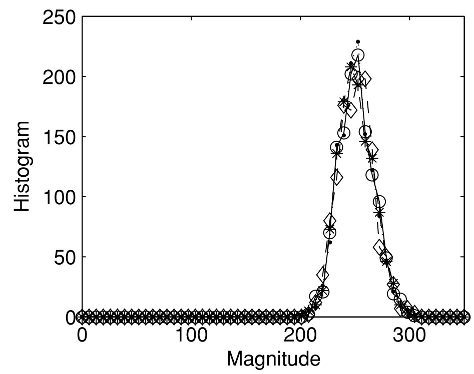
Fig. 7 Waveforms of the proposed log-normal distribution model signal.



(a) $\mu = 0.41$



(b) $\mu = 43.4$



(c) $\mu = 250$

Fig. 8 Magnitude distribution of the log-normal distribution model, filtered Poisson model ($h = 0.95$, $h = 0.5$) and the actual signal.

5. Comparison of the Log-Normal Distribution Model and the Filtered Poisson Model

Next, the proposed model and the filtered Poisson model [2], [3] are compared for their frequency distribution and magnitude distribution.

The filtered Poisson model consists of the Poisson impulse generator and a low-pass filter $H(z)$. The expectation is put into the Poisson impulse generator. The expectation is equal to the mean value and the variance of the Poisson impulse signal. The impulse signal is applied to the low-pass filter. In this paper, we choose a first order IIR digital filter for its simplicity. The system function of the filter is expressed as

$$H(z) = \frac{k(1-h)}{1-hz^{-1}}. \tag{7}$$

If the DC gain of the filter is unity, the variance of the output of the filter becomes smaller than the input. The gain k of this filter is chosen so that the variance is equal to the mean value of the filter output. The expectation is compensated to obtain the desired variance and the mean value in the filter output. In the case $\mu = 0.41$ and $h = 0.95$, the compensated expectation becomes 0.011. The comparison is carried out in the conditions of $\mu = 0.41$, 1.4, 3.9, 8.2, 22, 43, 73, 165, 250, 321, 559 and 693. In Figs. 8(a), (b) and (c), the dashed lines marked diamonds and dashed lines marked stars indicate the magnitude distribution of the filtered Poisson model in the cases of $h = 0.95$ and $h = 0.5$. The coefficient value $h = 0.95$ is found so that the evaluation function χ^2 introduced later becomes almost minimum. When $h > 0.95$, the evaluation function χ^2 increases due to an increase of fluctuation of Poisson impulse generation.

Figure 9 shows the power spectrum of signals generated by the proposed model, the filtered Poisson model and the actual signal. Figure 10 shows the autocorrelation R. In Figs. 9 and 10, the proposed model features a flat frequency distribution, but the filtered Poisson model naturally features a low-pass distribution. In the proposed model, flat frequency distribution signals up to the Nyquist rate can be obtained directly. On the other hand if a flat frequency distribution is required in the filtered Poisson model, larger number of Poisson impulses must be generated, and decimated at the filter output. Therefore, our proposed model is superior in terms of obtaining the flat frequency distribution.

In the magnitude distribution, we use the evaluation function χ^2 described as

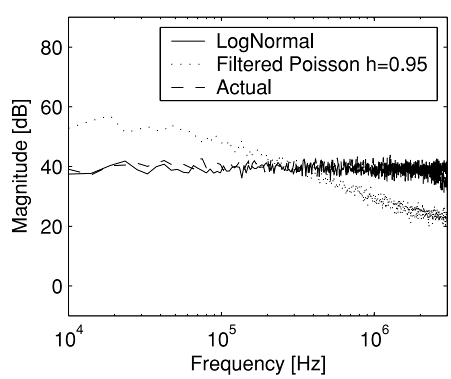


Fig. 9 Power spectrum of the log-normal distribution model, filtered Poisson model ($h = 0.95$) and the actual signal.

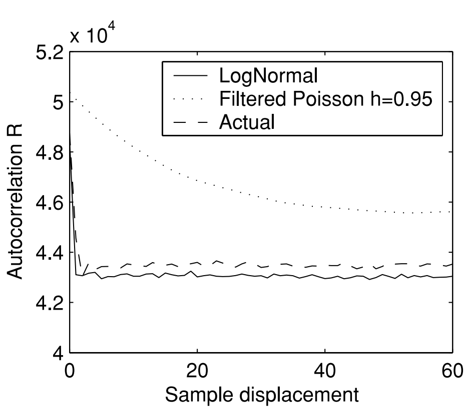


Fig. 10 Autocorrelation of the log-normal distribution model, filtered Poisson model ($h = 0.95$) and the actual signal.

$$\chi^2 = \sum_{i=-(m-1)/2}^{(m-1)/2} \frac{(O_i - E_i)^2}{E_i}, \tag{8}$$

where odd integer m denotes the number of ranges in the magnitude distribution. For the proposed model signal, O_i denotes the number of samples which have magnitude in the range between Bl_i and Bu_i. Similarly, for the actual signal, E_i denotes the number of samples which have magnitude in the same range. The lower boundary of the range Bl_i and the upper boundary of the range Bu_i are described as

$$Bl_i = \begin{cases} (wi - w/2)\sigma + \mu, & i \neq -(m-1)/2 \\ -\infty, & i = -(m-1)/2, \end{cases} \tag{9}$$

$$Bu_i = \begin{cases} (wi + w/2)\sigma + \mu, & i \neq (m-1)/2 \\ \infty, & i = (m-1)/2, \end{cases} \tag{10}$$

where w is the normalized width of the range.

In Table 1 the evaluation function χ^2 is calculated in the case of $\mu = \sigma^2 = 165$, $m = 9$ and $w = 0.5$ for the proposed model and the actual signals. The number of total samples is 1280. In this case, $\chi^2 = 27.6$ is obtained. The chi-square goodness-of-fit test tells that the 0.02% point of χ^2 with 7 degrees of freedom is 28.2. Thus, it is not reasonable to reject the null hypothesis that the magnitude distribution of the model fit the actual signal at the 0.02% level of significance. In other words, we cannot conclude that the proposed model generates a magnitude distribution different from the actual one at the 0.02% level of significance.

Figure 11 shows the values χ^2 of magnitude distributions of the proposed model and the filtered Poisson model ($h = 0.95$, $h = 0.5$). Small values of χ^2 mean good fitness. In Fig. 11, for $\mu \geq 43$, the proposed model shows almost the same fitness with the filtered Poisson model. For $\mu \leq 22$, the proposed model does not fit to the actual distribution in comparison with the filtered Poisson model. This is because, the skewness of the log-normal distribution is larger than that of actual signal and the filtered Poisson model in this region. For $\mu \leq 3.9$, both models does not feature good fitness to actual signal, because both model have no negative overshoot of signal. However, the actual signal has a negative component as shown in Fig. 8(a). Thus, the

Table 1 Calculation of χ^2 for the proposed model in the case of $\mu = 165$.

i	Bl_i	Bu_i	O_i	E_i	$(O_i - E_i)^2/E_i$
−4	$-\infty$	142.5	39	51	3.7
−3	142.5	148.9	85	71	2.3
−2	148.9	155.4	149	181	6.9
−1	155.4	161.8	250	204	8.5
0	161.8	168.2	274	272	0.0
1	168.2	174.6	216	229	0.8
2	174.6	181.0	133	151	2.4
3	181.0	187.5	78	63	2.9
4	187.5	∞	56	58	0.1
total			1280	1280	$\chi^2 = 27.6$

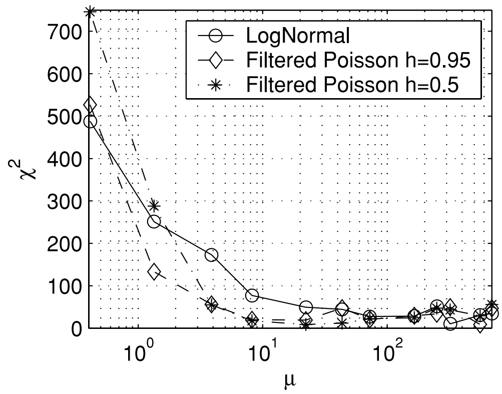


Fig. 11 Results of fitness of magnitude distribution for the models to the actual signal (small is good).

Table 2 Comparison of the log-normal distribution model and the filtered Poisson model.

	# of parameters	spectr.	mag. distri.
log-normal	1	good	good
filt. Poisson	≥ 3	bad	better

proposed model works equally to the filtered Poisson model for $\mu \geq 43$, and is slightly worse than the filtered Poisson model for $\mu < 43$ in terms of magnitude distribution.

Table 2 shows an overall comparison of the proposed model and the filtered Poisson model. The filtered Poisson model has a better feature in magnitude distribution. However it needs to determine multiple parameters and has a bad power spectrum. On the other hand the proposed model has a simple single parameter and has features of a good power spectrum and a magnitude distribution. Therefore, the proposed model is confirmed to be effective for simulating the signals of detectors in charged particle beam equipments.

6. Conclusions

We have presented a special stochastic model simulation for electron multiplying signal detectors in charged particle beam equipments. This model is based on a one stochastic parameter type log-normal distribution.

We have shown that the model is simple and can represent a wide dynamic range of signals from pulse-like signals when the primary beam current is small to continuous signals when the primary beam current is large.

A normalization method for the magnitude of the actual signal had been used to compute the proposed model characteristic parameter.

A good match of performance especially at power spectrum distribution for the actual signal shows that the proposed model is superior to other existing simulation models.

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