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PAPER Special Section on Latest Advances in Fundamental Theories of Signal Processing

Discrete Wirtinger-Type Inequalities for Gauging the Power of Sinusoids Buried in Noise

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SUMMARY Two discrete-time Wirtinger-type inequalities relating the power of a finite-length signal to that of its circularly-convolved version are developed. The usual boundary conditions that accompany the existing Wirtinger-type inequalities are relaxed in the proposed inequalities and the equalizing sinusoidal signal is free to have an arbitrary phase angle. A measure of this sinusoidal signal's power, when corrupted with additive noise, is proposed. The application of the proposed measure, calculated as a ratio, in the evaluation of the power of a sinusoid of arbitrary phase with the angular frequency $\frac{\pi}{N}$, where N is the signal length, is thoroughly studied and analyzed under additive noise of arbitrary statistical characteristic. The ratio can be used to gauge the power of sinusoids of frequency $\frac{\pi}{N}$ with a small amount of computation by referring to a ratio-versus-SNR curve and using it to make an estimation of the noise-corrupted sinusoid's SNR. The case of additive white noise is also analyzed. A sample permutation scheme followed by sign modulation is proposed for enlarging the class of target sinusoids to those with frequencies $\frac{M\pi}{N}$, where M and N are mutually prime positive integers. Tandem application of the proposed scheme and ratio offers a simple method to gauge the power of sinusoids buried in noise. The generalization of the inequalities to convolution kernels of higher orders as well as the simplification of the proposed inequalities have also been studied.

key words: discrete Wirtinger inequalities, Fan-Taussky-Todd inequalities, sinusoids, spectrum analysis, maximally flat filters, circular convolution, additive white noise, signal-to-noise ratio

1. Introduction

In 1955, Fan, Taussky, and Todd [1] published an influential paper extending the continuoues-time Wirtinger inequalities, which relate the integral of the square of functions and their first derivative, to the discrete time and derived five discrete versions that hold for sequences of finite length under certain boundary conditions. To prove the inequalities, they used the properties of the eigenvalues of matrices with respect to the maximization of the Rayleigh quotient and showed that, under specific boundary conditions on the signal, the power contained in the first- or second-order differences of a sequence is related to that of the original sequence through a sharp inequality. Interestingly, a discrete form of Wirtinger's inequality had been discovered earlier by I.J. Schoenberg in 1950 [2] but in connection with what was called the finite Fourier transform then and the discrete Fourier transform today. Schoenberg used the inequality to solve a geometric problem concerning the location of the vertices of a convergent series of polygons.

Since the publication of [1], the discrete Witinger-type inequalities caught the fancy of a number of researchers whose activities have been directed at generalizing the inequalities in various directions [3] or providing alternative insights into their nature by developing alternative proofs [4]. A curious fact about the Fan-Taussky-Todd inequalities and other related inequalities, in both their continuous- and discrete-time forms, is that they are sharp. In other words, certain functions or sequences satisfying the boundary conditions can always be found so that the two sides of the inequalities become exactly equal. In the discrete-time case, these equalizing signals are generally sinusoids, of fixed phase and frequency but with a variable amplitude, whose frequency is a fixed rational multiple of π depending on the length of the sequence only [1]–[4].

In digital signal processing, problems requiring estimation or detection of sinusoids are abundant and a large number of spectrum analysis techniques have been developed to estimate the frequency of a single-tone sinusoid or resolve multiple sinusoids in the forms of delta-function-like responses [5], [6]. See [7] for a concise yet excellent treatment of the techniques. For a new exact and direct approach see [8]. Since sinusoids play a key role in the discrete Wirtinger inequalities, from the standpoint of an engineer, we are interested in exploring their application in various signal processing problems involving the analysis of discrete-time sinusoids. The specific question we raise here is if it is possible to use these inequalities as a low-cost alternative, in terms of computational complexity, to the most common device in signal analysis, i.e., the discrete Fourier transform (DFT), in order to gauge the power of a discrete-time sinusoid buried in additive noise. We are specifically interested in devising a simple scheme to estimate the signal-to-noise ratio (SNR) of such sinusoids using a finite number of samples when their frequency is given as a rational multiple of π. To be able to provide a solution to our problem, we should first overcome some inherent obstacles in the application of the existing inequalities. These include, the limitation in the manner we form the differences of the signal, which involves simple weights only, the restriction in dealing with arbitrary discrete-time signals due to the stringent boundary conditions, and, last but not least, the constraint concerning

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the phase and frequency of the equalizing sinusoids. It is not the purpose of this paper to tout the Wirtinger inequalities as a replacement to the powerful sinusoidal analysis techniques such as Pisarenko's [7] method. Rather, we are interested in empowering the practitioner with an ultimately simple analysis tool that has the ability to extract useful information about the SNR of a sinusoid of a known fixed frequency from an observed finite-length signal.

Our goal in this paper is to first eliminate all of the accompanying constraints that are attached to the existing inequalities by modifying them into a form that is free of boundary conditions and valid for any finite-length signal. At the same time, we generalize the inequalities by incorporating weights of arbitrary values to replace the simple difference operations used in the existing inequalities. After proposing two such inequalities that admit arbitrary weights and are free of boundary conditions, we devise a permutation and modulation scheme that enlarges the class of equalizing sinusoids. In effect, the proposed scheme transforms the fundamental equalizing signal of the proposed inequalities into one whose angular frequency can be M times higher. Next, we used the modified and generalized inequalities to propose a measure, calculated as a power ratio in the time domain, for the evaluation of the strength of a sinusoid belonging to the set of equalizing signals pertaining to the proposed inequalities. The performance of the proposed ratio is analyzed when the sinusoid is buried in additive noise. Various numerical examples show that, by plotting a performance curve that tracks the ratio versus the theoretical SNR, one can use the calculated value of the ratio to find an estimate to the target sinusoid's SNR for a wide range of values.

The paper is organized as follows. The derivation process of the proposed inequalities is presented in Sect. 2 where two discrete-time inequalities are established using Parseval's identity and their fundamental equalizing sinusoids are determined. A power ratio for gauging the power of the sinusoids is defined and analyzed in Sect. 3. Specifically, it is shown that for both the inequalities, the ratio, when calculated at various theoretical SNR levels, is bounded by simple curves from above and below. Concrete examples are presented to depict the role of these bounds in the performance of the inequalities when used for gauging the SNR of the sinusoids under additive noise. A scheme, based on permutation and sign modulation of the observed signal, is proposed in Sect. 4 for enlarging the class of equalizing sinusoids to those whose frequency is an integer multiple of that of the fundamental sinusoid. Section 5 is dedicated to the derivation of a simplified form of the proposed inequalities where the weights are canceled from both sides. Generalization of the proposed inequalities to those with three or more nonzero weights is considered in Sect. 6 where it is shown that FIR maximally flat filters having monotone magnitude responses provide a rich source of weights for forming higher-order inequalities. Conclusions are drawn in Sect. 7.

2. Proposed Inequalities and Their Derivation

Two generalized forms of the discrete inequalities of Wirtinger, also known as Fan-Taussky-Todd inequalities, are proposed in this section. From a signal processing point of view, the existing forms of these inequalities involve the time-domain power in the output of a highpass filter excited by a finite-length signal and that of the original signal. To motivate the reader, we present the following inequality, established in [1], as an example of a common existing form. If $x_1, x_2, \ldots, x_n$ are n real numbers and $x_1 = 0$, then

$$\sum_{i=1}^{n-1}(x_i - x_{i+1})^2 \geq 4\sin^2\frac{\pi}{2(2n-1)}\sum_{i=2}^{n}x_i^2, \tag{1}$$

and the equality happens if and only if

$$x_i = A\sin\frac{(i-1)\pi}{2n-1},\ i = 1, 2, \ldots, n. \tag{2}$$

The notation used above follows that of [1] but we switch to our own notation henceforth. Note that in the above inequality, the sequence x_i must be so that $x_1 = 0$, and even in that case the phase and frequency of the equalizing sinusoid are fixed by the length of the sequence. Also, the left side is formed by simple differences and there is no freedom in the choice of weights. Various generalizations of the above inequality exist [3] but the above-mentioned limitations are always present in one from or another.

2.1 Inequality I and Its Derivation

We start by stating preliminary definitions and then proceed to establish the first inequality in detail. Let $x[n]$ be a real-valued signal of length N and $\hat{x}[n]$ be the extended version of it defined by

$$\hat{x}[n] = \begin{cases} x[n], & n = 0, 1, \ldots, N-1, \\ -x[n-N], & n = N, N+1, \ldots, 2N-1. \end{cases} \tag{3}$$

The kth DFT coefficient of $\hat{x}[n]$, given by

$$\hat{X}[k] = \sum_{n=0}^{2N-1}\hat{x}[n]W_{2N}^{nk},\ k = 0, 1, \ldots, 2N-1, \tag{4}$$

where $W_N = e^{-j\frac{2\pi}{N}}$, vanishes if k is even, i.e.,

$$\hat{X}[0] = \hat{X}[2] = \cdots = \hat{X}[2N-2] = 0. \tag{5}$$

In fact, it can be shown that the moduli of the DFT coefficients $X[k]$, of the original signal, and $\hat{X}[k]$, of the extended version, are generally related by

$$|\hat{X}[k]|^2 = 4\sin^2\left(\frac{k\pi}{2}\right)\left|X\left(\frac{k}{2}\right)\right|^2. \tag{6}$$

In this paper, brackets are used to indicate that the argument of the signal is restricted to integers while pairs of parentheses are used in the cases where a non-integer argument is also possible. Note that the evaluation of non-integer-valued arguments in $|X(\frac{k}{2})|^2$, which is not of concern in this paper, can be carried out, if necessary, using the basic definition of the DFT.

On the other hand, by Parseval's identity, the two real signals are related to each other according to

$$\begin{aligned}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2 =&2\sum_{k=1}^{N-1}|\hat{X}[k]|^2+|\hat{X}[N]|^2\\ =&4N\sum_{n=0}^{N-1}x[n]^2.\end{aligned} \tag{7}$$

Note that if N is even, then the middle coefficient $\hat{X}[N]$ vanishes.

The result of the cyclic convolution of $\hat{x}[n]$ with the $2N$-point sequence

$$h[n]=a\delta[n]+b\delta[n-2N+1],\quad n=0,1,\ldots,2N-1, \tag{8}$$

where a and b are real constants and $\delta[n]$ is the unit impulse signal, is the $2N$-point signal

$$y[n]=a\hat{x}[n]+b\hat{x}[(n+1) \bmod 2N],\quad n=0,1,\ldots,2N-1. \tag{9}$$

Since the kth DFT coefficient of $y[n]$ is the product of the kth DFT coefficients of $\hat{x}[n]$ and $h[n]$, we can write

$$|Y[k]|^2=|H[k]|^2|\hat{X}[k]|^2,\quad k=0,1,\ldots,2N-1, \tag{10}$$

where $Y[k]$ is the kth coefficient in the $2N$-point DFT of $y[n]$ and

$$|H[k]|^2=a^2+b^2+2ab\cos\frac{k\pi}{N},\quad k=0,1,\ldots,2N-1. \tag{11}$$

Application of Parseval's identity to (9) and (10) gives

$$\sum_{n=0}^{2N-1}(a\hat{x}[n]+b\hat{x}[(n+1) \bmod 2N])^2 = \frac{1}{2N}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2|H[k]|^2. \tag{12}$$

The left side of (12) can be expanded as

$$2\sum_{n=0}^{N-2}(ax[n]+bx[n+1])^2+2(ax[N-1]-bx[0])^2 \tag{13}$$

whereas, employing $\hat{X}[0]=0$ and in view of the symmetric properties of $\hat{X}[k]$ and $H[k]$, the terms on the right side of (12) can be regrouped as

$$\frac{1}{2N}\left(2\sum_{k=1}^{N-1}|\hat{X}[k]|^2|H[k]|^2+|\hat{X}[N]|^2|H[N]|^2\right). \tag{14}$$

If $ab>0$, we can show that

$$|H[1]|>|H[2]|>\cdots>|H[N]| \tag{15}$$

and hence

$$\max_{k=1,\ldots,N}|H[k]|^2=|H[1]|^2=a^2+b^2+2ab\cos\frac{\pi}{N}. \tag{16}$$

The above observation and the symmetry of the coefficients $|H[k]|(=|H[2N-k]|)$ lead to the inequality

$$\frac{1}{2N}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2|H[k]|^2 \le \frac{1}{2N}|H[1]|^2\sum_{n=0}^{2N-1}|\hat{X}[k]|^2,\ ab>0 \tag{17}$$

By using the relations (12) and (7) to replace the left and right sides of (17) with equivalent time-domain expressions, we obtain

$$\sum_{n=0}^{N-2}(ax[n]+bx[n+1])^2+(ax[N-1]-bx[0])^2 \le |H[1]|^2\sum_{n=0}^{N-1}x[n]^2,\ ab>0. \tag{18}$$

The above expression becomes a strict inequality if $\hat{x}[n]$ is a real signal that has at least one pair of nonzero DFT coefficients $|\hat{X}[k]|=|\hat{X}[2N-k]|$ for $k\in\{2,3,\ldots,N\}$. The only possibility for an equality occurs when the entire spectral content of the signal $\hat{X}[k]$ for $k\in\{1,2,\ldots,N\}$ is concentrated at $\hat{X}[1]=\mu e^{j\gamma}$ which has the same nonnegative modulus μ as $\hat{X}[2N-1]$. In this case, using the inverse DFT, it turns out that the non-zero DFT coefficients of $\hat{X}[k]$ generate the equalizing signal

$$\begin{aligned}\bar{x}[n] =&\frac{\mu}{2N}(e^{j\gamma}W_{2N}^{-n}+e^{-j\gamma}W_{2N}^{-n(2N-1)})\\ =&\frac{\mu}{N}\cos\left(\frac{n\pi}{N}+\gamma\right).\end{aligned} \tag{19}$$

Since

$$\bar{x}[n+N]=-\bar{x}[n], \tag{20}$$

the equalizing signal (19) is perfectly consistent with the extension of $\bar{x}[n]$ to $\hat{\bar{x}}[n]$ as defined in (3).

The above argument establishes that for all real-valued signals of length N, and for any two real weights a and b, $ab>0$, we always have

$$\sum_{n=0}^{N-2}(ax[n]+bx[n+1])^2+(ax[N-1]-bx[0])^2$$

$$\leq \left(a^2 + b^2 + 2ab\cos\frac{\pi}{N}\right)\sum_{n=0}^{N-1}|x[n]|^2, \quad (21)$$

where equality holds if and only if

$$x[n] = \bar{x}[n] = C\cos\left(\frac{n\pi}{N} + \gamma\right) \quad (22)$$

for some real amplitude C and phase γ. Note that (21) is expressed in terms of the original signal $x[n]$ and thus there is no need to actually carry out the extension operation of (3) or compute any DFT coefficients.

2.2 Inequality II

Another generalized form of Wirtinger's inequality can be derived by considering the case where the weights a and b have opposite signs. If $ab < 0$, then it can be shown that

$$|H[1]| < |H[2]| < \cdots < |H[N]| \quad (23)$$

and hence

$$\min_{k=1,\ldots,N}|H[k]|^2 = |H[1]|^2 = a^2 + b^2 + 2ab\cos\frac{\pi}{N}. \quad (24)$$

The derivation of the second inequality is carried out by following the same steps as those taken for the first inequality but with the direction of the inequality sign reversed. Consequently, all other conditions unchanged, for any two real weights a and b, $ab < 0$, we always have

$$\sum_{n=0}^{N-2}(ax[n] + bx[n+1])^2 + (ax[N-1] - bx[0])^2$$

$$\geq \left(a^2 + b^2 + 2ab\cos\frac{\pi}{N}\right)\sum_{n=0}^{N-1}|x[n]|^2, \quad (25)$$

with the equalizing signal given by

$$\bar{x}[n] = C\cos\left(\frac{n\pi}{N} + \gamma\right). \quad (26)$$

It is interesting to note that for both inequalities the equalizing signal is a sinusoid of frequency $\frac{\pi}{N}$. The difference in the direction of the two inequalities arises from the relative sign of the weights a and b.

3. A Measure of Power for Sinusoids and Its Analysis under Additive Noise

A question of interest relating to the proposed inequalities is the behaviour of the ratio

$$\mathcal{R} \stackrel{\text{def}}{=} \frac{\sum_{n=0}^{N-2}(ax[n] + bx[n+1])^2 + (ax[N-1] - bx[0])^2}{\left(a^2 + b^2 + 2ab\cos\frac{\pi}{N}\right)\sum_{n=0}^{N-1}|x[n]|^2}, \quad (27)$$

where $ab > 0$, which is a measure of the degree of balance between the left and right side of the inequality (21). Knowing that under the conditions $ab > 0$ and $x[n] = \bar{x}[n] = C\cos(\frac{n\pi}{N} + \gamma)$, the nonnegative ratio $\mathcal{R}$ attains its maximum value of unity, i.e., $0 \leq \mathcal{R} \leq 1$, the question arises as to if it is possible to bound $\mathcal{R}$ from above and below for an arbitrary signal $x[n] \neq \bar{x}[n]$. In this section, we will see that the answer to the above question is in positive. This is especially crucial in connection with the practical applicability of $\mathcal{R}$ to the evaluation of the power of the sinusoid $\bar{x}[n]$ when it is buried in additive noise $e[n]$ in an observed signal expressed as

$$x[n] = \bar{x}[n] + e[n]. \quad (28)$$

In fact, we will see that $\mathcal{R}$ can be used as a measure to gauge the relative power of the sinusoid $\bar{x}[n]$ in the observed finite-length signal $x[n]$. In the following, we will analyze the ratio for both of the inequalities by defining a corresponding ratio $\mathcal{R}'$ for inequality II. The case of additive white noise will also be given special treatment.

3.1 Analysis of Inequality I under Noise of Unknown Characteristic

The bounds on $\mathcal{R}$ are obtained by exploiting the monotone decreasing nature of $|H[k]|$ for $0 \leq k \leq N$. Without loss of generality, we can assume that the power of the signal $\hat{x}[n]$ has been normalized to a constant value, i.e.,

$$\sum_{n=0}^{2N-1}|\hat{x}[n]|^2 = 2\sum_{n=0}^{N-1}|x[n]|^2 = \frac{1}{2N}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2 = \frac{\mu^2}{N} \quad (29)$$

for some positive and real μ. The real signal $\bar{x}[n]$ constitutes an extreme case where the entire energy of $\hat{x}[n]$ is allocated to $\hat{X}[1]$ and its symmetric pair $\hat{X}[2N-1]$. Under the same constraint on the total power, one can disperse it to the other non-vanishing bins of $\hat{X}[k]$. Suppose that of the total signal power $\frac{\mu^2}{N}$, $r\frac{\mu^2}{N}$ $(0 < r < 1)$ has been allocated evenly to $\hat{X[1]}$ and $\hat{X}[2N-1]$ to satisfy the symmetric constraints, while $(1-r)\frac{\mu^2}{N}$ has been distributed among the other non-vanishing DFT bins. For a fixed r, this is expressed by the following equivalence

$$\frac{1}{2N}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2|H[k]|^2 = \frac{1}{2N}\Bigg(2r\mu^2|H[1]|^2$$

$$+2\sum_{k=3}^{N-1}|\hat{X}[k]|^2|H[k]|^2 + |H[N]|^2|\hat{X}[N]|^2\Bigg). \quad (30)$$

In light of (15), the above relation yields the inequality

$$\frac{1}{2N}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2|H[k]|^2 \leq \frac{1}{2N}\Bigg(2r\mu^2|H[1]|^2$$

$$+ |H[3]|^2\left(2\sum_{k=3}^{N-1}|\hat{X}[k]|^2 + |\hat{X}[N]|^2\right)\Bigg) \quad (31)$$

which, in turn, leads to

$$\frac{1}{2N}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2|H[k]|^2 \leq \frac{1}{2N}\left(2r\mu^2|H[1]|^2 + 2|H[3]|^2(1-r)\mu^2\right). \quad (32)$$

Henceforth, whenever we wish to stress the dependence of $\mathcal{R}$ on the weights and energy distribution parameter r, we use the notation $\mathcal{R}(a,b,r)$. Thus, combining (12), (13), (16), (27), (29), we can write (32) as

$$\mathcal{R}(a,b,r) \leq r + (1-r)\frac{|H[3]|^2}{|H[1]|^2}, \quad ab > 0. \quad (33)$$

In the same vein, it can be established that

$$\frac{1}{2N}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2|H[k]|^2 \geq \begin{cases} \frac{1}{2N}\left(2r\mu^2|H[1]|^2 + 2|H[N]|^2(1-r)\mu^2\right), \\ N\text{odd}, \\ \\ \frac{1}{2N}\left(2r\mu^2|H[1]|^2 + 2|H[N-1]|^2(1-r)\mu^2\right), \\ N\text{even}. \end{cases} \quad (34)$$

Unifying both the cases into a parity-free expression, it follows that

$$\mathcal{R}(a,b,r) \geq r + (1-r)\frac{|H[N-(N+1) \bmod 2]|^2}{|H[1]|^2}, \quad ab > 0. \quad (35)$$

The SNR, used to measure the power of the target sinusoidal component $\bar{x}[n]$ in the observed signal $x[n]$, is defined as the ratio of the sinusoid power to the total noise power contained in the bins $\hat{X}[2], \ldots, \hat{X}[2N-2]$ and becomes

$$SNR = 10\log_{10}\frac{r}{1-r}, \quad (36)$$

which can be solved for r to give

$$r = \frac{10^{\frac{SNR}{10}}}{1+10^{\frac{SNR}{10}}}. \quad (37)$$

The two bounds derived in (33) and (35) are plotted versus the SNR in Fig. 1(a) for the case $a = b = 1$, and $N = 7$.

Now that we have obtained sharp lower and upper bounds on $\mathcal{R}(a,b,r)$, we are in a position to consider the inverse problem of relating the value of $\mathcal{R}(a,b,r)$ to that of r. In other words, suppose that, for a given signal, we have calculated the numerical value of the ratio $\mathcal{R}$; we are interested in obtaining an interval that contains all possible values of r or, equivalently, the SNR, that could have resulted in our calculated ratio. Since for very low SNR values (negative values with large magnitudes), the value of r is close to zero, it can be shown, by solving the inequalities (33) and (35) for r, that the ratio $\mathcal{R}$ is bounded as

$$\mathcal{R} \in \left(\frac{|H[N-(N+1) \bmod 2]|^2}{|H[1]|^2}, \frac{|H[3]|^2}{|H[1]|^2}\right] \quad (38)$$

Whenever the calculated value of $\mathcal{R}$ lies in the above half-open interval, we can show that

$$r \in \left(0, \frac{\mathcal{R} - \frac{|H[N-(N+1 \bmod 2)]|^2}{|H[1]|^2}}{1 - \frac{|H[N-(N+1 \bmod 2)]|^2}{|H[1]|^2}}\right]. \quad (39)$$

Otherwise, if

$$\mathcal{R} \in \left(\frac{|H[3]|^2}{|H[1]|^2}, 1\right), \quad (40)$$

the exact value of r lies in the closed interval

$$r \in \left[\frac{\mathcal{R} - \frac{|H[3]|^2}{|H[1]|^2}}{1 - \frac{|H[3]|^2}{|H[1]|^2}}, \frac{\mathcal{R} - \frac{|H[N-(N+1 \bmod 2)]|^2}{|H[1]|^2}}{1 - \frac{|H[N-(N+1 \bmod 2)]|^2}{|H[1]|^2}}\right]. \quad (41)$$

The region in the SNR-$\mathcal{R}$ plane specified by the above two pairs of intervals is shown in grey for $a = 1, b = 1, N = 7$ in Fig. 1(a). It can be seen that, for a given signal, if the calculated ratio is greater than $\frac{|H[3]|^2}{|H[1]|^2} \approx 0.64$, possible SNR values lie in a narrow range. The grey region specifies the corresponding SNR interval even when there is no information about the stochastic nature of the noise. However, for $\mathcal{R} < 0.64$, the inequality loses its sensitivity to the target sinusoid and the ratio cannot be used to provide a useful estimate for the SNR range.

3.2 Analysis of Inequality II under Noise of Unknown Characteristic

Under the same assumption on the distribution of the total signal power, a ratio for the inequality II can be defined by

$$\mathcal{R}' \overset{\text{def}}{=} \frac{(a^2+b^2+2ab\cos\frac{\pi}{N})\sum_{n=0}^{N-1}|x[n]|^2}{\sum_{n=0}^{N-2}(ax[n]+bx[n+1])^2 + (ax[N-1]-bx[0])^2}, \quad ab < 0, \quad (42)$$

which assumes a positive value that can never exceed unity. To obtain lower and upper bounds on $\mathcal{R}'$, it is not difficult to see that

$$\frac{1}{2N}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2|H[k]|^2 \geq \frac{1}{2N}\left(2r\mu^2|H[1]|^2 + 2|H[3]|^2(1-r)\mu^2\right) \quad (43)$$

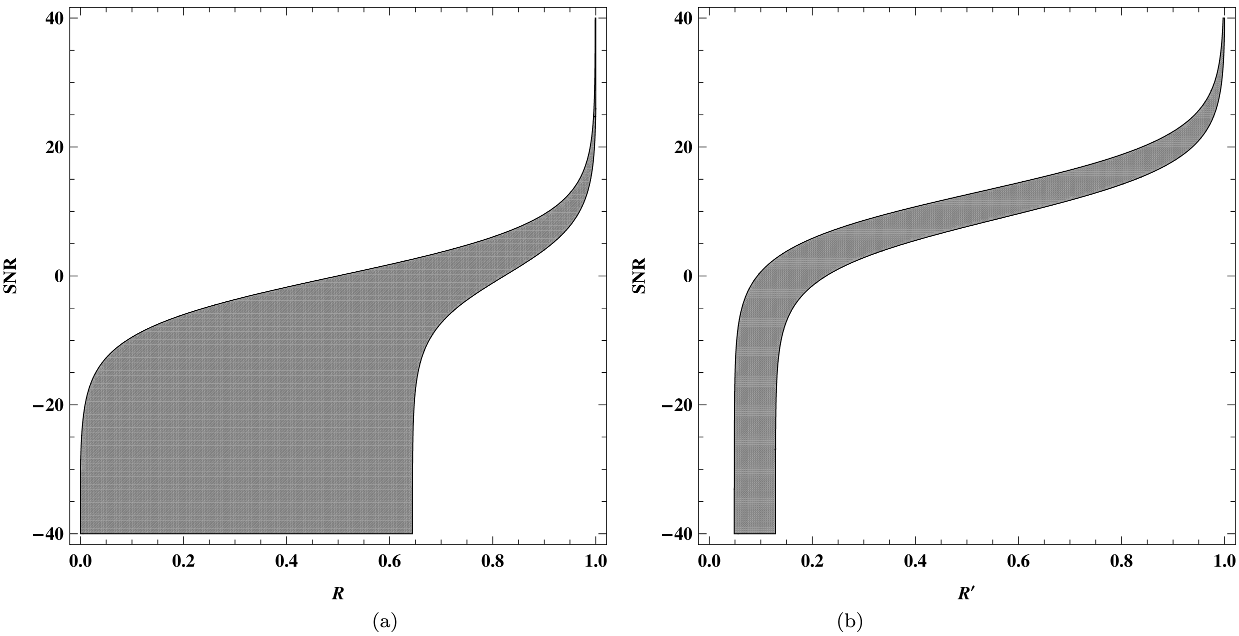


Fig. 1 Upper and lower bounds on $\mathcal{R}$ and $\mathcal{R}'$ for a range of SNR values are the boundary curves surrounding the grey regions. The curves correspond to the cases where $a = b = 1$, $N = 7$ in inequality I, Fig. 1(a), and $a = 1$, $b = -1$, $N = 7$ in inequality II, Fig. 1(b). Performance regions are shown in grey.

while

$$\frac{1}{2N}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2|H[k]|^2 \leq \frac{1}{2N}\Big(2r\mu^2|H[1]|^2+2|H[N-(N+1) \bmod 2]|^2(1-r)\mu^2\Big). \tag{44}$$

We conclude that

$$\frac{1}{r+(1-r)\dfrac{|H[N-(N+1) \bmod 2]|^2}{|H[1]|^2}} \leq \mathcal{R}'(a,b,r) \leq \frac{1}{r+(1-r)\dfrac{|H[3]|^2}{|H[1]|^2}}, \quad ab<0. \tag{45}$$

As an example, the bounds corresponding to $a = 1, b = -1, N = 7$, are shown in 1(b).

Suppose that the value of $\mathcal{R}'$ has been determined for a given signal $x[n]$. It is of interest then to deduce the range of possible values of r based on this information. It can be verified that whenever

$$\mathcal{R}' \in \left(\frac{|H[1]|^2}{|H[N-(N+1 \bmod 2)]|^2}, \frac{|H[1]|^2}{|H[3]|^2}\right], \tag{46}$$

the range for the ratio is given by

$$r \in \left(0, \frac{\mathcal{R}'\dfrac{|H[N-(N+1 \bmod 2)]|^2}{|H[1]|^2}-1}{\mathcal{R}'\dfrac{|H[N-(N+1 \bmod 2)]|^2}{|H[1]|^2}-\mathcal{R}'}\right], \tag{47}$$

and whenever

$$\mathcal{R}' \in \left(\frac{|H[1]|^2}{|H[3]|^2}, 1\right), \tag{48}$$

the ratio lies in the closed interval

$$r \in \left[\frac{\mathcal{R}'\dfrac{|H[3]|^2}{|H[1]|^2}-1}{\mathcal{R}'\dfrac{|H[3]|^2}{|H[1]|^2}-\mathcal{R}'}, \frac{\mathcal{R}'\dfrac{|H[N-(N+1 \bmod 2)]|^2}{|H[1]|^2}-1}{\mathcal{R}'\dfrac{|H[N-(N+1 \bmod 2)]|^2}{|H[1]|^2}-\mathcal{R}'}\right]. \tag{49}$$

The region in the SNR-$\mathcal{R}'$ plane specified by the above two pairs of intervals is shown in grey for $a = 1, b = -1, N = 7$ in Fig. 1(b). Note that if the calculated value of $\mathcal{R}'$ is greater than $\frac{|H[1]|^2}{|H[3]|^2} \approx 0.13$, the signal can have SNR values in a narrow interval, depicted in grey, even when there is no information about the stochastic nature of the noise. But for $\mathcal{R}' < 0.13$, the ratio cannot be used to provide a useful estimate for the SNR. In comparison to Fig. 1(a), we observe that the dynamic range of the values of the ratio is higher when we use $\mathcal{R}'$, but at low SNR's the sensitivity of $\mathcal{R}'$ is lost faster than that of $\mathcal{R}$.

3.3 Analysis of Inequalities I and II for Sinusoid Embedded in White Noise

If $\bar{x}[n]$ is buried in additive zero-mean white noise and we desire to make an estimate of the ratio $\mathcal{R}_{\text{WN}}$, defined in the same way as (27), we can utilize the flatness property of the power spectrum of the noise. The goal is to derive a relation between $\mathcal{R}_{\text{WN}}$ and SNR to be used for gauging the power of the sinusoid. In the DFT domain, the additive noise model (28), when extended to a $2N$-point signal according to (3), can be written as

$$\hat{X}[k] = \hat{\hat{X}}[k] + \hat{E}[k], \tag{50}$$

where $\hat{E}[k]$ is the kth DFT coefficient of the extended noise signal produced by a zero-mean white noise process. Note that the coefficients $\hat{E}[1]$ and $\hat{E}[2N-1]$, which have equal moduli, sit in the equalizing signal's bin in the DFT domain. They may even contribute positively to the strength of that signal. Following the same signal and noise power assumptions as those used in the previous subsection, we estimate that of the total measured power, $(1-r)\frac{\mu^2}{N}$ has been evenly distributed among the moduli of the remaining $(N-2)$ non-vanishing coefficients of $\hat{E}[k]$. Therefore, following (6), we can write

$$\begin{aligned} |\hat{E}[k]|^2 &= 4\sin^2\left(\frac{k\pi}{2}\right)\left|E\left(\frac{k}{2}\right)\right|^2 \\ &\approx \begin{cases} \frac{2(1-r)\mu^2}{N-2}, & k \text{ odd}, \\ 0, & k \text{ even}. \end{cases} \end{aligned} \tag{51}$$

The total power in the DFT coefficients $H[k]\hat{X}[k]$ then becomes

$$\begin{aligned} &\frac{1}{2N}\sum_{k=0}^{2N-1}|\hat{X}[k]|^2|H[k]|^2 \\ &\approx \frac{1}{2N}\Bigg(r\mu^2|H[1]|^2 + r\mu^2|H[2N-1]|^2 \\ &\qquad + \frac{2(1-r)\mu^2}{N-2}\sum_{\substack{k=3 \\ k \text{ odd}}}^{2N-2}|H[k]|^2\Bigg), \end{aligned} \tag{52}$$

and thus the ratio is approximated as

$$\mathcal{R}_{\text{WN}} \approx \frac{\frac{1}{2N}\left(2r\mu^2|H[1]|^2 + \frac{2(1-r)\mu^2}{N-2}\sum_{k=1}^{N-2}|H[2k+1]|^2\right)}{|H[1]|^2\frac{\mu^2}{N}}. \tag{53}$$

It follows that

$$\mathcal{R}_{\text{WN}} \approx r + \frac{(1-r)}{N-2}\frac{\left(a^2+b^2\right)(N-2) - 4ab\cos\left(\frac{\pi}{N}\right)}{a^2+b^2+2ab\cos\left(\frac{\pi}{N}\right)}, \quad ab > 0. \tag{54}$$

The ratio is plotted in Fig. 2 versus SNR for $a = b = 1$ and two values of N. It is interesting to note that for very low SNR values in very long signals, the horizontal asymptote of the curve is given by

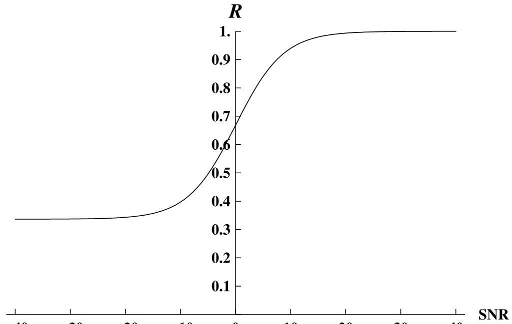
$$\mathcal{R}_{\text{WN}} = \frac{a^2+b^2}{(a+b)^2}. \tag{55}$$

For white noise, the ratio can be used to provide an estimate to the SNR even when the noise is strong. There is little dependence between the signal length N and the performance of the ratio for different SNR values.

For inequality II, under the same white noise condition, the ratio $\mathcal{R}'_{\text{WN}}$ is the reciprocal of $\mathcal{R}_{\text{WN}}$ and thus

$$\mathcal{R}'_{\text{WN}} \approx \frac{1}{r + \frac{(1-r)}{N-2}\frac{\left(a^2+b^2\right)(N-2)-4ab\cos\left(\frac{\pi}{N}\right)}{a^2+b^2+2ab\cos\left(\frac{\pi}{N}\right)}}, \quad ab < 0. \tag{56}$$

The ratio is plotted in Fig. 3 versus SNR for two values of N and the weights $a = 1, b = -1$. From these examples, it can be seen that for white noise, the dynamic range in inequality II is still higher than inequality I but for larger values of N, inequality II loses its sensitivity for SNR values of less than 10 dB. In contrast to that, inequality I has a lower dynamic range but can provide a meaningful measure of the target sinusoid's power even for SNR values of less than 0 dB without losing its sensitivity.



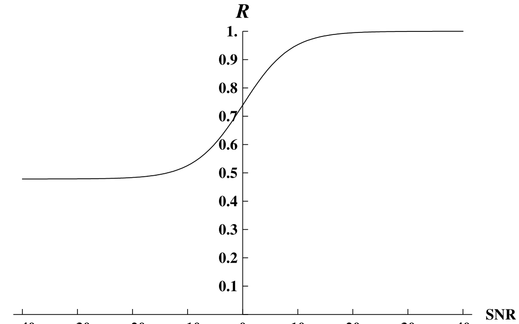


Fig. 2 Power ratio $\mathcal{R}_{\text{WN}}$ versus SNR for a sinusoid buried in white noise based on inequality I for $a = b = 1$, $N = 7$ (a), and $N = 47$ (b).

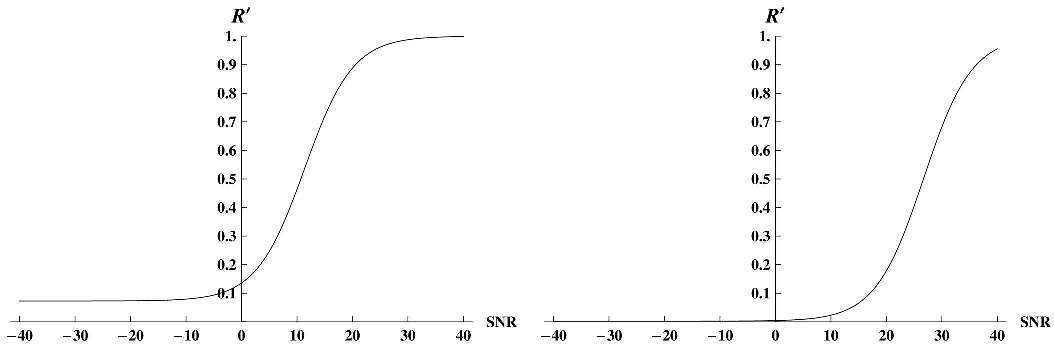


Fig. 3 Power ratio $\mathcal{R}'_{\mathrm{WN}}$ versus SNR for a sinusoid buried in white noise based on inequality II for $a = 1$, $b = -1$, $N = 7$ (a), and $N = 47$ (b).

4. Adaptation to Sinusoids of Higher Frequencies

A question concerning the generalized inequalities of the previous section is in regard to their applicability to the target sinusoids whose angular frequencies are not equal to $\frac{\pi}{N}$. In general, if a sinusoid of the form

$$C\cos(n\omega + \gamma), \tag{57}$$

where ω is a real number in the interval $(0, \pi)$, is the target signal in $x[n]$, it is always possible to obtain a rational approximation to $\frac{\omega}{\pi}$ by using the covergents of its continued fraction expansion. Such rational approximations to $\frac{\omega}{\pi}$ are optimal in the sense that there does not exist any other fraction with a denominator less that of the convergent rendering a smaller approximation error. Let us assume that the convergent is given by $\frac{M}{N}$ where M and N ($N > M$) are positive integers. If $M \neq 1$, then such sinusoids can not be the equalizing signals for the inequalities derived in the previous section. In this section, a method based on permutation and subsequent sign modulation of $x[n]$ is developed to circumvent this problem.

If a sinusoidal signal of the form

$$C\cos\left(\frac{M}{N}\pi n + \gamma\right) \tag{58}$$

is the target signal, the assumption henceforth is that the fraction $\frac{M}{N}$ has been reduced to the simplest form so that M and N are mutually prime. Euclid's algorithm can be used to generate two integers, p and q, that satisfy the relation

$$Mp + Nq = 1. \tag{59}$$

It can be shown that the sequence

$$(x[0], x[p \bmod N], x[2p \bmod N], \ldots, \\ x[(N-1)p \bmod N]) \tag{60}$$

contains the same samples as $x[n]$, $n = 0, 1, \ldots, N-1$, but in different order [9]. The goal is to find a simple sequence $f[n]$ to modulate the permuted version of $x[n]$ and obtain the signal

$$x'[n] = f[n]x[pn \bmod N] \tag{61}$$

in a way that the original target sinusoid of angular frequency $\frac{M}{N}\pi$ contained in $x[n]$ is down-converted to a sinusoid of frequency $\frac{\pi}{N}$. After the down-conversion, the signal $x'[n]$ can be used in the inequalities and the power ratios will indicate the SNR for a sinusoid of frequency $\frac{\pi}{N}$ which has a power identical to the original sinusoid. If such $f[n]$ exists, then the above argument leads us to the requirement that the relation

$$f[n]\cos\left(\frac{M(pn \bmod N)}{N}\pi + \gamma\right) = \cos\left(\frac{n}{N}\pi + \gamma\right). \tag{62}$$

must hold as an identity for all integers n. Writing

$$pn \bmod N = pn - QN, \tag{63}$$

where Q, the quotient in division of pn by N, is an integer, and using (59) we get

$$\begin{aligned}\frac{M(pn \bmod N)}{N} &= \frac{Mp}{N}n - MQ \\ &= \frac{1 - Nq}{N}n - MQ = \frac{n}{N} - qn - MQ.\end{aligned} \tag{64}$$

Consequently, by setting

$$f[n] = (-1)^{qn+MQ} \tag{65}$$

the relation (62) becomes an identity. To summerize, the signal given in (61) should be used instead of $x[n]$ in the calculation of $\mathcal{R}$ or $\mathcal{R}'$ if the target sinusoid has a frequency M times higher than that of the fundamental sinusoid.

5. Core Inequality

The generalized inequalities proposed in this paper are parameterized by two real weights whose relative magnitude $\frac{a}{b}$ governs the performance ratios $\mathcal{R}$ and $\mathcal{R}'$. Examining the inequalities with greater care, one notices that there are a number of terms on the two sides of the expanded form of the inequalities that can be canceled. Specifically, expansion of the left side of (21) gives

$$(a^2+b^2)\sum_{n=0}^{N-1} x[n]^2 + 2ab\left(\sum_{n=0}^{N-2} x[n]x[n+1] - x[0]x[N-1]\right). \quad (66)$$

Substituting the above expression into the inequality, after cancelation of the identical terms on the left and right, we obtain

$$\sum_{n=0}^{N-2} x[n]x[n+1] - x[0]x[N-1] \le \cos\frac{\pi}{N}\sum_{n=0}^{N-1} x[n]^2. \quad (67)$$

In the same fashion, we can cancel the identical terms from both sides of (25). Taking into account that $ab < 0$ in this case, the result is again the same inequality as above. Clearly, the equality holds for the same equalizing signal $x[n] = \bar{x}[n] = C\cos(\frac{\pi}{N} + \gamma)$ as before. We call the above inequality the core inequality for $\bar{x}[n]$. The left side of the core inequality involves the sample autocorrelation of lag 1 while the right side involves the sample autocorrelation of lag 0.

There is a subtle difference in the behaviour of the core inequality compared to those that contain the weights a and b. In the latter, the two sides of the inequality are guaranteed to be positive while in the former there is no such constraint on the value of the left side. As a result, when the signal $x[n]$ deviates largely from $\bar{x}[n]$, for instance in the case where

$$x[0] > 0, \quad x[N-1] > 0, \quad x[1] = x[2] = \ldots = x[N-2] = 0, \quad (68)$$

the left side becomes negative.

6. Inequalities of Higher Orders

One may develop other double-weight inequalities by relaxing the constraint on the nonzero coefficients in $h[n]$, specified in (8), and allowing them to occur at arbitrary relative gaps with respect to each other. Further generalization of the two inequalities (21) and (25) is possible by replacing the sequence $h[n]$ with one that has three or more nonzero weights. To facilitate the generalization process, the selection of the weights can be carried out in a fashion that the moduli of the DFT coefficients $H[k]$ behave analogous to the two-weight cases proposed earlier in this paper. The derivation process and analysis results of inequalities I and II can then be applied with only slight modification. Specifically, for the generalization of (21) in a manner that is consistent with the analysis results, the requirement is that the new weights be chosen so that

$$|H[1]| > |H[2]| > \cdots > |H[N]|, \quad (69)$$

while, if inequality II is to be generalized, any real sequence with the property

$$|H[1]| < |H[2]| < \cdots < |H[N]| \quad (70)$$

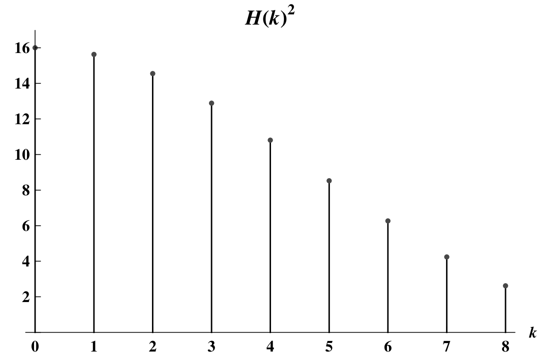


Fig. 4 Discrete values of $|H[k]|^2 = \left(a^2+b^2+c^2+2(a+c)b\cos(\frac{\pi}{N}) + 2ac\cos\left(\frac{2\pi}{N}\right)\right)$ for $a = c = 1$, $b = 2$, and $N = 17$.

can be used. When the above conditions are satisfied, it is not difficult to see that the only steps in the derivation process of the inequalities that require modification are the calculation of the cyclic convolution in the time domain and the subsequent calculation of the signal power.

As an example, consider the sequence

$$h[n] = a\delta[n] + b\delta[n-2N+1] + c\delta[n-2N+2], \quad n = 0, 1, \ldots, 2N-1, \quad (71)$$

whose cyclic convolution with $\hat{x}[n]$ is given by

$$y[n] = a\hat{x}[n] + b\hat{x}[(n+1) \bmod 2N] + c\hat{x}[(n+2) \bmod 2N], \; n = 0, 1, \ldots, 2N-1. \quad (72)$$

If the monotonicity requirements are met, the inequality associated with (71) and (72) becomes

$$\begin{aligned}&\sum_{n=0}^{N-3}(ax[n] + bx[n+1] + cx[n+2])^2 \\ &\quad + (ax[N-2] + bx[N-1] - cx[0])^2 \\ &\quad + (ax[N-1] - bx[0] - cx[1])^2 \\ &\le \left(a^2+b^2+c^2+2(a+c)b\cos\left(\frac{\pi}{N}\right)\right. \\ &\quad \left. + 2ac\cos\left(\frac{2\pi}{N}\right)\right)\sum_{n=0}^{N-1}|x[n]|^2. \end{aligned} \quad (73)$$

The simplest choice for the weights so that the conditions in (69) are satisfied is furnished by the coefficients of the powers of z^{-1} in the expansion of $(1+z^{-1})^2$, i.e.,

$$a = c = 1, \quad b = 2. \quad (74)$$

See Fig. 4 for a discrete plot of all the moduli $|H[k]|^2$ for $k = 0, 1, \ldots, 8$, and $N = 17$ in the above case. A rich source of weights for derivation of higher order inequalities is furnished by the monotone maximally flat lowpass and highpass filters.

Another possible scheme for generalization to higher order inequalities is to select the weights so that

$$\max_{1 \le k \le N} |H[k]| = |H[1]| \quad (75)$$

for the generalization of (21), and

$$\min_{1 \le k \le N} |H[k]| = |H[1]| \quad (76)$$

for the generalization of (25). It is obvious that the analysis results with respect to additive noise signal of unknown characteristic do not hold for the weaker conditions above and should be modified accordingly.

7. Conclusion

The main contributions of this paper are as follows. First, two Wirtinger-type inequalities relating the power of a finite-length signal to that of its circularly-convolved version have been proposed. The inequalities are free from the boundary conditions that restrict the applicability of the existing discrete Wirtinger inequalities and become equalities when the signal is a pure sinusoid of fixed frequency but arbitrary phase and amplitude. Another distinction of the proposed inequalities is the freedom in the selection of the underlying convolution weights. Specifically, unlike the existing discrete Wirtinger inequalities, which are formed using simple differences, they are valid for arbitrary real-valued weights.

Second, for signal processing applications, a simple ratio for gauging the power of the equalizing sinusoid contained in an observed signal has been proposed. It has been shown that the ratio can provide a viable solution to the problem of gauging the power of a target sinusoid of frequency $\frac{\pi}{N}$. The application of the proposed ratio in the evaluation of the power of the target sinusoid using N samples of the observed signal has been analyzed under additive noise of arbitrary statistical characteristic. The case of additive white noise has been studied as well and the ability of the proposed ratio to gauge the power of the target sinusoids in low SNR's with a small amount of computation has been confirmed. Formulas for plotting performance curves useful for finding the SNR range of the applicability of the inequalities have been derived. To enlarge the class of equalizing sinusoids to frequencies $\frac{M\pi}{N}$, a simple permutation scheme, followed by sign modulation, has been proposed. Tandem application of the proposed permutation and modulation with the subsequent calculation of one of the two proposed ratios offers a simple method to gauge the power of a wide range of sinusoids buried in additive white noise.

Finally, the generalization of the inequalities to higher orders involving three terms of the signal as well as simplification of the proposed inequalities have been examined.

Compared to the simple but popular periodogram technique [6] for sinusoid estimation which is based on the discrete Fourier transform, the proposed ratio has numerous advantages. Unlike the DFT coefficients, there is no need to update the weights when the length of the signal changes. As a result, the method can be implemented adaptively by changing the value of $|H[1]|^2$ which is the only length-dependent factor in the inequality. Only real arithmetic operations are needed in the calculation of the proposed ratio. The performance of the inequalities can be enhanced by introducing higher order filtering coefficients whereas the DFT has a fixed resolution. This provides a trade-off between accuracy and computational complexity. The ratio can be calculated directly for any integer length, whether it is a prime, composite, power of two, or any other special integer, with no requirement for zero-padding which is still a common practice in order to fit a given signal into non-flexible FFT-based routines. While calculating the ratio, we are performing a filtering operation that can have noise reduction or signal enhancement effects. Accordingly, the calculation of the ratio may be regarded as a minor extra computational load if simultaneous signal filtering is also of interest.

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