PAPER

Magnitude and Phase Approximation of IIR Digital Filters Using Linear Programming

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SUMMARY This paper discusses a new simultaneous design method of both magnitude and phase of IIR digital filters. It is inherently a nonlinear problem to approximate the magnitude and phase of an IIR digital filter simultaneously. In this paper, however, such a nonlinear problem is converted into a linear one by vector rotation method[(1),(2)] and solved by the linear programming (LP) technique. As a result our method requires no initial guesses for any type of filter specifications. A stable digital filter is designed which approximately satisfies the linearity of the phase in the pass band and the given attenuation in the stop band. The design examples given in this paper shows the usefulness of the proposing method.

1. Introduction

The increasing availability of digital signal processing technology has generated much interest in designing a digital filter which satisfies given specifications. It is well known that the delay ripples as well as the amplitude ripples of a filter circuit cause the waveform distortion. Thus, filters used in communication systems need not only the good amplitude characteristics but also the flat delay characteristics (i. e. the linearity of the phase) in order to avoid the waveform distortion. From this reason, it is important to approximate both magnitude and phase simultaneously when we design a filter for the communication systems.

As such a filter, a linear phase FIR filter has been mainly used because it is able to realize the complete linearity of the phase and designed easily by Remez algorithm[(3)]. On the other hand, a linear phase FIR filter requires a large numger of taps in order to realize a highly-selective filter, and the group delay becomes large in proportional to the number of taps. Therefore, we cannot expect the high throughput when a linear phase FIR filter is implemented on a DSP (Digital Signal Processor). From this reason, a request arises to design an IIR digital filter which has satisfactory linear phase and sharp cut off characteristics with lower order as compared with that of the FIR filter.

Many approaches have been proposed to solve this problem. For example, Inukai successfully applied a nonlinear programming method to this problem[(4)]. But the most serious problem associated with the nonlinear programming method is the requirement of a very good starting point (i. e. initial coefficients) as used in Ref. (4). Thus, it is very difficult to apply this method to a filter design with complicated specifications such as the multipassband filters. On the other hand, Chottera and Jullien proposed a linear programming approach[(5)]. Although their method does not require any initial guesses and a global optimum solution can be obtained, it is difficult to specify an arbitrary stop band attenuation. Thus, their method is not suitable for a practical use.

This paper proposes a new simultaneous design method of an IIR digital filter. The proposing method does not need any initial guesses because it uses a linear programming technique. The stop band attenuation can be specified by a constraining inequality of LP[(6)].

We first explain the proposing method. Then we will show some design examples and the results will be compared with a linear phase FIR filter.

2. Approximation Procedure

Let $H(z)$ be the transfer function of an IIR digital filter with the following form;

$$H(z)=\frac{Q(z)}{P(z)}=\frac{\sum_{i=0}^{N_N} a_i z^{-i}}{1+\sum_{i=1}^{N_D} b_i z^{-i}}, \qquad (1)$$

where N_N and N_D are the orders of the numerator polynomial $Q(z)$ and the denominator polynomial $P(z)$, respectively, and a_i, b_i are the coefficients of $Q(z)$ and $P(z)$, respectively. The frequency response of $H(z)$ can be calculated by substituting $z=e^{j\omega T}$, where T is a sampling period. In a practical use, a filter with the linear phase property in the pass band is especially important. Thus, let $H_T(\omega)$ be a desired frequency response (i. e. target function) for $H(z)$ as follows;

$$H_T(\omega)=A_P(\omega)\cdot e^{-j\tau\omega}, \qquad (2)$$

where $A_P(\omega)$ is an amplitude target function and τ is a value of the group delay. $e^{-j\tau\omega}$ represents the linear phase property. Using l_∞ norm (Chebyshev norm), the

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simultaneous design problem is defined as follows;

$$\min_{a_i,b_i}\|H_T(\omega)-H(e^{j\omega T})\|_\infty$$
$$=\min_{a_i,b_i}\max_{\omega}|H_T(\omega)-H(e^{j\omega T})| \quad (3)$$
$$\omega\in\{\text{Pass band}\}$$
s. t. constraints

"Constraints" means constraining condition inequalities (i. e. stability condition, stop band attenuation constraint and transition band constraint) which will be explained in the next section. The substitution of Eqs. (1) and (2) into Eq. (3) gives

$$\min_{a_i,b_i}\max_{\omega}\left|A_P(\omega)\cdot e^{-j\tau\omega}-\frac{Q(e^{j\omega T})}{P(e^{j\omega T})}\right| \quad (4)$$
$$\omega\in\{\text{Pass band}\}$$
s. t. constraints

Furthermore, multiplying Eq. (4) by the denominator for simplicity of calculation, we get

$$\min_{a_i,b_i}\max_{\omega}|P(e^{j\omega T})A_P(\omega)\cdot e^{-j\tau\omega}-Q(e^{j\omega T})| \quad (5)$$
$$\omega\in\{\text{Pass band}\}$$
s. t. constraints

Equation (5) is a nonlinear programming problem with respect to filter coefficients because the calculation of the absolute value of a complex number is required.

An application of the linearizing method used in Ref. (1), however, can convert this problem into a linear one with respect to filter coefficients.

The absolute value of a complex number $w=x+jy$ can be represented as follows;

$$|w|=\sqrt{x^2+y^2}$$
$$=\max_{-\pi\leqq s<\pi}\{\mathrm{Re}(w\cdot e^{js})\} \quad (6)$$

To discretize the variable s, by using an integer p, let

$$S_m=2\pi(m-p-1)/2p \qquad (m=1, 2, 3, \cdots, 2p) \quad (7)$$

From Eqs. (6) and (7), if we define

$$M=\max_{m}\{\mathrm{Re}(w\cdot e^{js_m})\} \quad (8)$$

then M is approximately considered as the l_∞ norm of w (i. e. $M\approx l_\infty$). An example for $p=4$ is shown in Fig. 1. In this case, a circle is approximated by a regular octagon. The complex vector ω is rotated along its apices and projected on the real axis. Thus, the choice of a sufficiently large p makes the error satisfactorily small.

Applying Eq. (6) to Eq. (5), we get

$$\min_{a_i,b_i}\max_{\omega}\max_{s}[\mathrm{Re}\{(P(e^{j\omega t})A_p(\omega)\cdot e^{-j\tau\omega}$$
$$-Q(e^{j\omega T}))e^{js}\}] \quad (9)$$
$$\omega\in\{\text{Pass band}\}$$

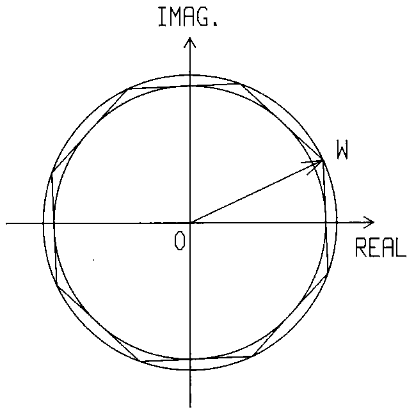


Fig. 1 Approximation of a circle with an octagon.

$$s\in[-\pi, \pi)$$
s. t. constraints

Furthermore, the substitution of Eq. (1) into Eq. (9) leads to

$$\min_{a_i,b_i}\max_{\omega}\max_{s}\Big[A_p(\omega)\cos(s-\tau\omega)$$
$$-\Big(\sum_{i=0}^{N_N}a_i\cos(s-i\omega T)$$
$$-A_p(\omega)\sum_{i=1}^{N_D}b_i\cos(s-\tau\omega-i\omega T)\Big)\Big] \quad (10)$$
$$\omega\in\{\text{Pass band}\}$$
$$s\in[-\pi, \pi)$$
s. t. constraints

Then Eq. (5) is converted to Eq. (10), which is a linear problem with respect to filter coefficients. Thus, this problem can be solved by the standard LP technique.

The comparison of Eq. (5) with the original Eq. (4), however, shows another problem that the denominator $P(e^{j\omega T})$ works as a weighting function for Eq. (5) because Eq. (4) is multiplied by the denominator. This degrades the overall characteristics of the designed filter.

Iterative solution of the weighted approximation problem can cancel this weighting function[7]. Let $w_n(\omega)$ ($n=1, 2, 3, \cdots$) be a new weighting function for canceling the influence of the denominator. Then we solve the following problem iteratively for $n=1, 2, 3, \cdots$ until the denominator $P_n(e^{j\omega T})$ converges.

$$P_0(e^{j\omega T})\equiv 1 \quad (11)$$

$$w_n(\omega)=\frac{1}{|P_{n-1}(e^{j\omega T})|} \quad (12)$$

$$\min_{a_{ni},b_{ni}}\max_{\omega}\max_{s} w_n(\omega)[\mathrm{Re}\{(P_n(e^{j\omega T})A_p(\omega)\cdot e^{-j\tau\omega}$$
$$-Q_n(e^{j\omega T}))e^{js}\}] \quad (13)$$
$$\omega\in\{\text{Pass band}\}$$

$s \in [-\pi, \pi)$

s. t. constraints,

Once the denominator has converged after several iterations, i. e.

$$w_n(\omega)=\frac{1}{|P_n(e^{j\omega T})|}\approx\frac{1}{|P_{n-1}(e^{j\omega T})|} \tag{14}$$

the influence of the denominator is canceled, and the designed transfer function is given as

$$H(z)=\frac{Q_n(z)}{P_n(z)} \tag{15}$$

3. Constraints

Since the proposing method uses the LP technique, the three constraints mentioned above (i. e. (a) stability constraint, (b) stop band attenuation, (c) transition band constraint) must be also linear with respect to filter coefficients.

(a) Stability Constraint

Since the approximating function is an IIR transfer function, the stability must be guaranteed. In this paper, we use a linear stability condition shown in Ref. (5) for the stability constraint, i. e.

$$-\sum_{i=1}^{N_D} b_i\cos(i\omega T)\leqq 1-\varepsilon, \quad 0\leqq\omega T\leqq\pi, \tag{16}$$

where ε is a small positive number. Since this is a sufficient condition for the stability as shown in Ref. (5), only a subclass of a stable filter can be designed.

(b) Stop Band Attenuation Constraint

The stop band attenuation constraint is given by

$$|H(e^{j\omega T})|\leqq A_s. \quad \omega\in\{\text{Stop band}\}, \tag{17}$$

where A_s is the minimum attenuation in the stop band given in the specification. Substituting Eq. (15) into Eq. (17) and multiplying by the denominator, we get

$$|Q_n(e^{j\omega T})|\leqq A_s\cdot|P_n(e^{j\omega T})| \tag{18}$$

Eq. (18) is a nonlinear inequality with respect to filter coefficients. The linearizing method of Eq. (6) can be applied to the left hand side of Eq. (18), but unfortunately cannot to the right hand side. This is because of the function "max" and the direction of the inequality sign in Eq. (6). Thus, we replace right hand side of Eq. (18) (i. e. $|P_n(e^{j\omega T})|$) with $|P_{n-1}(e^{j\omega T})|$ because $|P_{n-1}(e^{j\omega T})|$ is easily calculated from $P_{n-1}(e^{j\omega T})$ in the last iteration of Eq. (12). Furthermore, applying Eq. (6) to the left hand side of Eq. (18), we get

$$\sum_{i=0}^{N_N} a_i\cos(s-i\omega T)\leqq A_s\cdot|P_{n-1}(e^{j\omega T})| \tag{19}$$

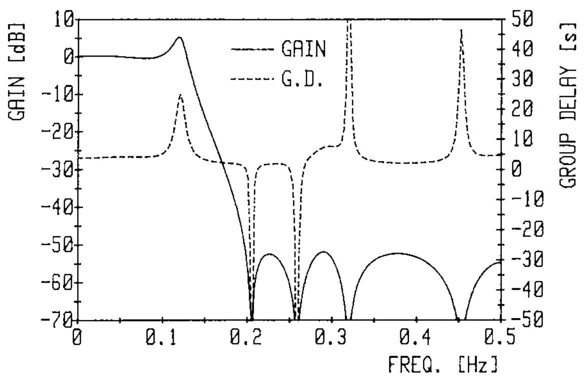


Fig. 2 A design example without the transition band constraint.

Once the denominator $P_n(e^{j\omega T})$ converges in Eq. (13), the stop band attenuation constraint of Eq. (17) is guaranteed.

(c) Transition Band Constraint

The characteristics in the transition band are not generally specified in the approximation problem of a transfer function. Without the transition band constraint, however, a peak appears in both amplitude and group delay characteristics as shown in Fig. 2. This occurs because the poles in the transition band approach the unit circle in order to reduce the approximation error at the pass band edge. For the practical use, it is desirable to remove such a peak as in Fig. 2 because a signal in the transition band may generally exist.

In order to remove this peak, we include the following inequality in the constraints, i. e.

$$|H(e^{j\omega T})|\leqq A_T. \quad \omega\in\{\text{Transition band}\}, \tag{20}$$

where A_T is the maximum amplitude in the transition band. We set A_T the same value as the amplitude at the pass band edge. Since Eq. (20) has the same form as Eq. (17), we can make use of Eq. (19) for Eq. (20) by only replacing A_s with A_T. It is considered that Eq. (18) also works to reduce the peak of the group delay because it prevents the poles in the transition band from approaching the unit circle.

4. Computational Details

Now we consider discretization of Eq. (13) in order to solve it by the standard LP algorithm. For the frequency ω, we sample k_p, k_t, k_s points uniformly in the pass band, transition band and stop band, respectively. We also discretize variable s by Eq. (7). Since

$$\max_{-\pi\leqq s<\pi}\{\mathrm{Re}(\omega\cdot e^{js})\} \tag{21}$$

equals

$$\max_{-\pi\leq s<0}|\mathrm{Re}(w\cdot \mathrm{e}^{js})| \tag{22}$$

we only consider S_m in Eq. (7) for $m=1, 2, \cdots, p$.

From Eqs. (10), (11)-(13), (16), (19), (20) and (22), the approximation problem is formulated as Eqs. (23)-(28).

$$P_0(\mathrm{e}^{j\omega_k T})\equiv 1 \tag{23}$$

for $n=1, 2, 3, \cdots$, until converge

$$w_n(\omega_k)=\frac{1}{|P_{n-1}(\mathrm{e}^{j\omega_k})|} \tag{24}$$

$$\min_{a_{ni},b_{ni}}\max_{\omega_k}\max_{s_m} w_n(\omega_k)\cdot\Big|A_p(\omega_k)\cos(s_m-\tau\omega_k) -\Big(\sum_{i=0}^{N_N}a_{ni}\cos(s_m-i\omega_k T) -A_p(\omega_k)\sum_{i=1}^{N_D}b_{ni}\cos(s_m-\tau\omega_k-i\omega_k T)\Big)\Big| \tag{25}$$

$\omega_k\in\{\text{Pass band}\}\quad (k=1, 2, 3, \cdots, k_p)$

$s_m\in[-\pi, 0)\quad (m=1, 2, 3, \cdots, p)$

s. t.

$$-\sum_{i=1}^{N_n}b_{ni}\cos(i\omega_k T)\leqq 1-\varepsilon \tag{26}$$

$\omega_k\in\{\text{Pass band}\}\cup\{\text{Transition band}\}\cup\{\text{Stop band}\}$

$(k=1, 2, 3, \cdots, k_p+k_T+k_s)$

$$\Big|\sum_{i=0}^{N_N}a_{ni}\cos(s_m-i\omega_k T)\Big|\leqq A_s\cdot|P_{n-1}(\mathrm{e}^{j\omega_k T})| \tag{27}$$

$\omega_k\in\{\text{Stop band}\}\quad (k=1, 2, 3, \cdots, k_s)$

$s_m\in[-\pi, 0)\quad (m=1, 2, 3, \cdots, p)$

$$\Big|\sum_{i=0}^{N_N}a_{ni}\cos(s_m-i\omega_k T)\Big|\leqq A_T\cdot|P_{n-1}(\mathrm{e}^{j\omega_k T})| \tag{28}$$

$\omega_k\in\{\text{Transition band}\}\quad (k=1, 2, 3, \cdots, k_T)$

$s_m\in[-\pi, 0)\quad (m=1, 2, 3, \cdots, p)$

Furthermore, an introduction of an auxiliary variable y_n in the $n-th$ iteration can convert Eqs. 25-28 into the standard LP problem as Eqs. (29)-(36).

$$\min_{a_{ni},b_{ni},y_n}[y_n] \tag{29}$$

s. t.

$$\frac{y_n}{w_n(\omega_k)}-A_p(\omega_k)\sum_{i=1}^{N_D}b_{ni}\cos(s_m-\tau\omega_k-i\omega_k T) +\sum_{i=0}^{N_N}a_{ni}\cos(s_m-\omega_k T)\geqq -A_p(\omega_k)\cos(s_m-\tau\omega_k) \tag{30}$$

$$\frac{y_n}{w_n(\omega_k)}+A_p(\omega_k)\sum_{i=1}^{N_D}b_{ni}\cos(s_m-\tau\omega_k-i\omega_k T) -\sum_{i=0}^{N_N}a_{ni}\cos(s_m-\omega_k T)\geqq +A_p(\omega_k)\cos(s_m-\tau\omega_k)\ ; \tag{31}$$

$\omega_k\in\{\text{Pass band}\}\quad (k=1, 2, 3, \cdots, k_p)$

$s_m\in[-\pi, 0)\quad (m=1, 2, 3, \cdots, p)$

$$+\sum_{i=1}^{N_D}b_{ni}\cos(i\omega_k T)\geqq \varepsilon-1\ ; \tag{32}$$

$\omega_k\in\{\text{Pass band}\}\cup\{\text{Transition band}\}\cup\{\text{Stop band}\}$

$(k=1, 2, 3, \cdots, k_p+k_T+k_s)$

$$-\sum_{i=0}^{N_N}a_{ni}\cos(s_m-i\omega_k T)\geqq -A_s\cdot|P_{n-1}(\mathrm{e}^{j\omega_k T})| \tag{33}$$

$$+\sum_{i=0}^{N_N}a_{ni}\cos(s_m-i\omega_k T)\geqq -A_s\cdot|P_{n-1}(\mathrm{e}^{j\omega_k T})|\ ; \tag{34}$$

$\omega_k\in\{\text{Stop band}\}\quad (k=1, 2, 3, \cdots, k_s)$

$s_m\in[-\pi, 0)\quad (m=1, 2, 3, \cdots, p)$

$$-\sum_{i=0}^{N_N}a_{ni}\cos(s_m-i\omega_k T)\geqq -A_T\cdot|P_{n-1}(\mathrm{e}^{j\omega_k T})| \tag{35}$$

$$+\sum_{i=0}^{N_N}a_{ni}\cos(s_m-i\omega_k T)\geqq -A_T\cdot|P_{n-1}(\mathrm{e}^{j\omega_k T})|\ ; \tag{36}$$

$\omega_k\in\{\text{Transition band}\}\quad (k=1, 2, 3, \cdots, k_T)$

$s_m\in[-\pi, 0)\quad (m=1, 2, 3, \cdots, p)$

Eqs. (29)-(36) are solved iteratively until the denominator converges. The convergence is tested by

$$\left|\frac{y_n-y_{n-1}}{y_{n-1}}\right|\leqq\delta, \tag{37}$$

where δ is the upper bound which should be given in the specifications. Once Eq. (37) is satisfied after several iterations, we can get the optimum solution from Eq. (15), and y_n represents the Chebyshev error in the pass band of the designed filter. In the practical calculation, the dual problem of Eqs. (29)-(36) is more efficient to solve because the number of the inequalities is much larger than that of the variables[6]. We can easily solve the dual problem by applying well known LP algorithm, i. e. Simplex Method[6].

5. Optimization of Group Delay

In general, it is desirable to have the value of the group delay optimized rather than given in the specification of the filter. But it is impossible to optimize the group delay τ by this LP algorithm because Eqs. (29)-(36) are highly nonlinear with respect to τ. Thus, we apply a univariant search in the specified range so as to obtain the optimum value of the group delay[1],[5].

Since the input signal is discrete, we prefer the group delay to be an integer multiple of the sampling period. From this reason, we restrict the group delay to an integer multiple of the sampling period. We solve Eqs. (23)-(28) for all the values of the group delay in the specified range. Then, we search a group delay which minimizes the Chebyshev error, i.e. y_n which satisfies Eq. (37). We can get the optimum transfer function if we select the solution for this optimum group delay. Many designn examples by this method show that the suitable range of the group delay for the univariant search is from $N/2$ to $N-1$, where N is the order of the transfer function. This range is the same as shown in Ref. (5).

6. Desigs Examples

In this section, we show two design examples by the proposing method. In all the examples given here, p, δ, ε are set as follows ;

$$p=12$$

$$\delta=5.0\times10^{-3}$$

$$\varepsilon=1.0\times10^{-6}$$

[Example 1] This example is a low-pass filter with the following specifications

Order	$N_N=N_D=12$
Pass band	0.00−0.25 Hz
Stop band	0.30−0.50 Hz
Sampling Period	1 sec
Pass band gain	0 dB
Stop band gain	−32 dB

From these specifications, A_p, A_s, A_T are set as follows ;

$$A_p(\omega)=1.00 \qquad (0.00 \leq\omega\leq0.50\pi)$$

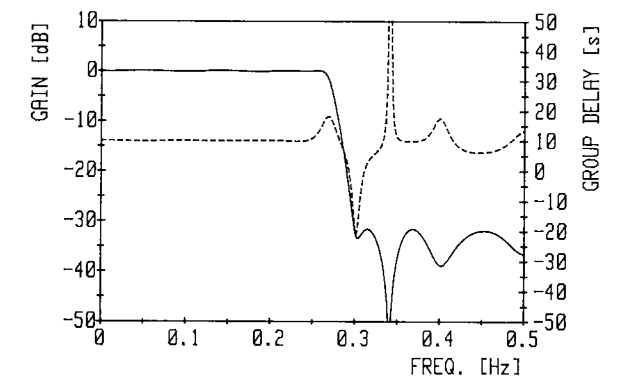
$$A_s=2.51\times10^{-2} \qquad (0.60\pi \leq\omega\leq\pi)$$

$$A_T=1.00 \qquad (0.50\pi <\omega<0.60\pi)$$

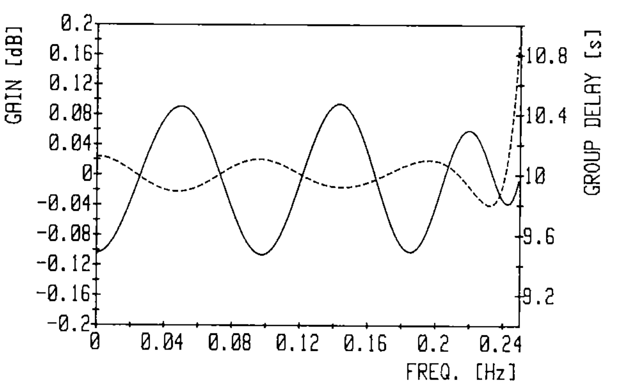
k_p, k_t, k_s are set equal to 25, 10, 12, respectively.

The univariant search of τ from 7(sec) to 11(sec) shows that $\tau=10$ minimizes the Chebyshev error in the pass band. At the calculation for $\tau=10$, canceling the influence of the denominator requires 9 iterations for $\delta=0.005$. Fig. 3 shows the designed filter characteristics. In this figure, the solid line represents the amplitude characteristics (left scale) and the dashed line represents the group delay characteristics (right scale). (This notation is also used in Figs. 4 and 5.)

[Example 2] Example 2 is a band-pass filter with the

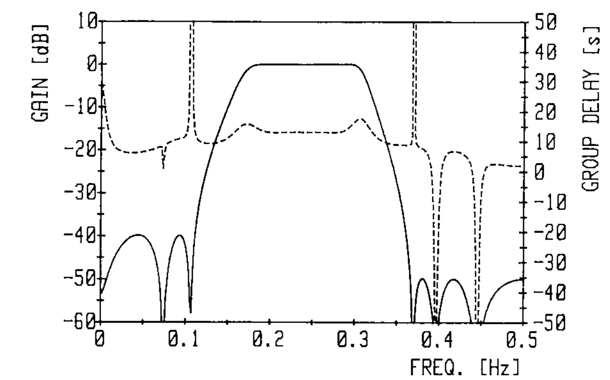


(a) Wide range characteristics.

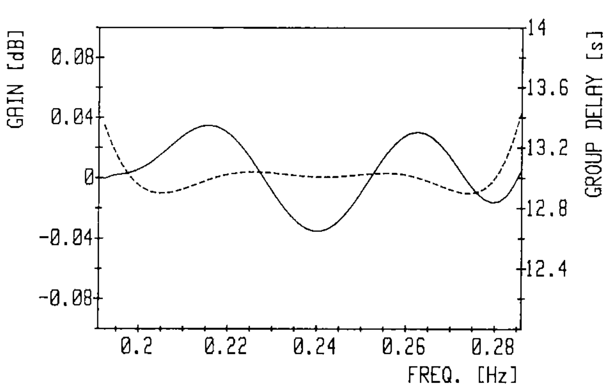


(b) Pass band characteristics.

Fig. 3 Designed filter characteristics of Example 1.

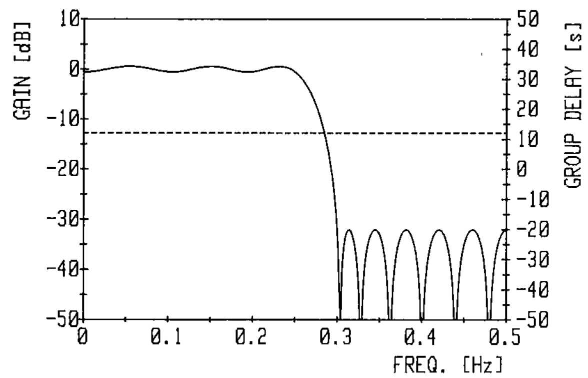


(a) Wide range characteristics.

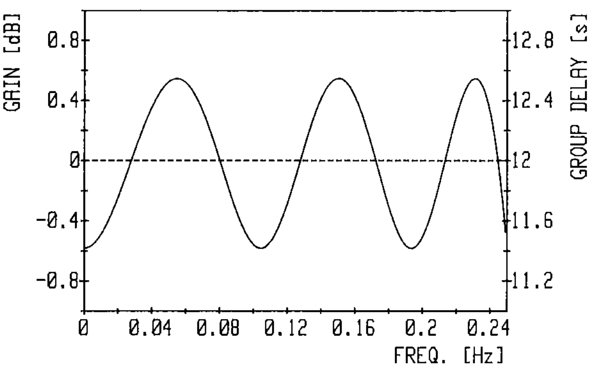


(b) Pass band characteristics.

Fig. 4 Designed filter characteristics of Example 2.



(a) Wide range characteristics.



(b) Pass band characteristics.

Fig. 5 Designed characteristics of a linear phase FIR filter with the same specifications as Example 1.

following specifications.

Order	$N_N = N_D = 16$
Pass band	0.191-0.286 Hz
Lower stop band	0.000-0.111 Hz
Upper stop band	0.366-0.500 Hz
Sampling Period	1 sec
Pass band gain	0 dB
Lower stop band gain	-40 dB
Upper stop band gain	-50 dB

A_P, A_T, A_S are set as follows;

$$A_p(\omega) = 1.00 \qquad (1.20 \leq \omega \leq 1.80)$$

$$A_s \; . = \begin{cases} 1.00 \times 10^{-2} & (0.00 \leq \omega \leq 0.70) \\ 3.16 \times 10^{-3} & (2.30 \leq \omega \leq \pi) \end{cases}$$

$$A_T \;\; = 1.00 \qquad (0.70 < \omega < 1.20, 1.80 < \omega < 2.30)$$

k_P, k_t, k_s are set equal to 22, 20, 42, respectively.

The minimum error is obtained for $\tau = 13$. 17 iterations are required to cancel the influence of the denominator. The designed filter characteristics are given in

Table 1 Results of Comparison.

	Chebyshev Error E	Group Delay [s]
Proposing Method (Fig. 3)	0.011	10
Linear Phase FIR Filter (Fig. 5)	0.065	12

Fig. 4.

7. Comparison with an FIR Filter and Discussion

The characteristics of the filter designed by the proposing method is compared with those of a linear phase FIR filter[(3)]. An FIR filter is designed which has the same specifications as Example. 1 with length=25. This is just twice the order of the filter in Example 1. These two filters require the same number of instruction steps when implemented on a DSP, because the symmetry of the coefficients of an FIR filter cannot reduce the number of instruction steps on a DSP implementation. Fig. 5 shows the characteristics of the FIR filter.

From Figs. 3 and 5, it is obvious that the pass-band ripple of the amplitude of our method is smaller than that of the linear phase FIR filter while the pass-band ripple of the group delay is, of course, larger. In order to evaluate this tradeoff totally, we use the Chebyshev error E between the target and designed filter in the pass band as follows;

$$E = \|H_T(\omega) - H(e^{j\omega T})\|_\infty = \max_{\omega} |H_T(\omega) - H(e^{j\omega T})|$$

$$\omega \in \{\text{Pass band}\} \qquad (38)$$

The Chebyshev errors of the filters are summarized in Table 1 together with the value of the group delay. Table 1 indicate that much lower absolute error is attained by our method by allowing a ripple of the group delay. Furthermore, the group delay of our method is also smaller than that of the linear phase FIR filter.

Figure 3 depicts that the deviation of the group delay from 10 sec becomes large at the pass band edge. The main reason is that the constant group delay is approximated indirectly through the linear phase property. The reduction of the group delay ripple is left for future works.

20th or higher order filters can be designed without any difficulties of the calculation using double precision arithmetic. However, it requires a great deal of calculation costs (i. e. the memory space and CPU time) to design a higher order filter. For example, about 20 minutes are required to calculate the filter of Example 1 for the case $\tau = 10$ (sec) by FORTRAN77 on SONY NWS-821 whose throughput is 2. 3 MIPS. Therefore, a suitable selection of parameters such as k_P, k_t, k_s, p, and δ is important to reduce the calculation costs.

8. Conclusions

A new simultaneous approximating method of both magnitude and phase for IIR digital filters are presented. This method has a great advantage that it does not require any starting guesses because it uses the linear programming technique. A stable filter is designed which has the approximately linear phase property in the pass band and the prescribed attenuation in the stop band. The two design examples given in this paper indicate the usefulness of the proposed method. It is left for future works to reduce the peak of the group delay in the transition band and to save the calculation costs.

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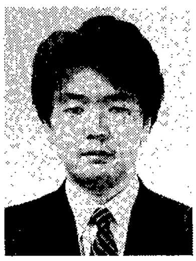

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