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A Synthesis of Variable IIR Digital Filters

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SUMMARY It is sometimes required to change the frequency characteristics of a digital filter during its operation. In this paper a new synthesis of variable even-order IIR digital filters is proposed. The cut-off frequency of the filter can be changed by a single parameter. The fundamental filter structure is a cascade of second-order sections. The multiplier coefficients of each section are determined by using the Taylor series expansion of the lowpass to lowpass frequency transformation. For this method any second-order section can be used as a prototype, but here in this paper only the direct form and the lattice form are described. Unlike the conventional method, any transfer functions can be used for the proposed method. Finally a designed example shows that the proposed filter has wider tuning range than the conventional filter, and the advantage of the proposed filters is confirmed.

key words: *variable filter, IIR filter, cut-off frequency, frequency transformation, direct form, lattice form*

1. Introduction

It is required to change the frequency characteristics of a digital filter during its operation, for applications of digital filters to digital audio and adaptive filters.

Constantinides gives the theoretical background for changing the frequency characteristics of an IIR digital filter(1). If $H(z)$ denotes the transfer function of a digital filter, its frequency characteristic is changed by a transformation replacing z^{-1} with a suitable function $T(z)$. In this method $T(z)$ is an allpass function, and the desired lowpass, highpass, bandpass, bandstop filters are obtained by transforming a lowpass prototype filter.

The following two methods to apply the transformation proposed by the Ref. (1) are considered.

The first method is to replace delay elements in a prototype lowpass filter with allpass sections directly. In most of IIR digital filters, however, such replacement causes delay-free loops. The only case where no delay-free-loops occur is when $T(z)$ has a factor of z^{-1} in its numerator. In this case the bandwidth of a bandpass filter and the stop-bandwidth of a bandstop filter can not be arbitrary but are equal to the bandwidth of the prototype lowpass filter.

The second method is to calculate the transfer function of a transformed filter, and re-synthesize its circuit. This requires complicated computations and operations. Accordingly it is difficult to use this method if real-time tuning is desired in an application.

Some methods of using frequency transformations for the design of variable IIR filters have been proposed(2)–(5). Originally the operations to compute the transformed transfer function from an Nth-order prototype transfer function are $O(N^2)$. The method of Johnson requires $O(N)$ operations to compute the coefficients for an Nth-order direct form filter(2), but still needs that operations. It also needs to update each multipliers. After all, it requires a lot of operations. The method of Steiglitz requires about one multiplication and addition per coefficient to be updated. However, the complexity of the filtering operation doubles essentially(3). Ahuja and Dutta Roy suggested the method to compute the coefficients simply using their expressions for filters designed as a cascade of second- and first-order sections(4), but the operations to compute half of the coefficients are still complicated and the operations to update the multipliers are needed. Tan has shown that for a low-sensitivity wave filter structure the coefficients can be updated by using short-wordlength formula(5). Similarly the operations to compute and to update the coefficients are needed. In addition, the methods by Refs. (2)-(5) limit the prototype filter structure.

For the purpose of solving this problem, Mitra et al. others suggest the method for changing the cut-off frequency with a single parameter(6). The method is quite efficient from both the tuning and filtering points of view. Only one multiplication and addition per filter coefficient is needed in the operations to update. The method has no effect on the complexity of the actual filtering. The tuning range is several octaves for narrowband filters. The fundamental filter structure is a parallel connection of allpass sections. They use the Taylor series expansion of the lowpass to lowpass frequency transformation.

However that method limits transfer functions to Butterworth, Chebyshev and elliptic functions. If the prototype transfer function has even-order, allpass sections have complex coefficients. A complex allpass

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section is essentially a two-port circuit and has many real coefficients. Thus when the cut-off frequency is changed, the frequency response is more degraded than odd-order transfer functions. Particularly in the case of audio-type applications the disordered frequency response is undesirable for high-fidelity.

In this paper a new synthesis of variable IIR digital filters with the only one parameter is proposed, when a realized transfer function has even-order[7]. A cascade connection of second-order sections is used as a fundamental structure. As any even-order transfer function can be factored into a product of second-order transfer functions, the proposed method has the advantage of dealing with any even-order transfer function. The Taylor series expansion of the lowpass to lowpass frequency transformation is also applied to the second-order direct form and lattice form[8]. This method is able to be applied to any other second-order sections besides the direct form and lattice form. Finally it is shown that the proposed filters have the wider tuning range than Ref. (6).

2. Frequency Transformation

When the transfer function of the prototype lowpass filter is $H(z)$, the transformation replacing z^{-1} in $H(z)$ with a suitable function $T(z)$ provides the transfer functions of desired lowpass, highpass, bandpass or bandstop filters, where $T(z)$ is first-order or second-order allpass functions[1]. The transformed transfer functions is denoted by $H'(z)$.

For the lowpass to lowpass transformation, $T(z)$ is a first-order allpass function given by

$$T_1(z)=\frac{z^{-1}-\beta}{1-\beta z^{-1}} \tag{1}$$

where

$$\beta=\frac{\sin\left(\frac{\theta_P-\omega_P}{2}\right)}{\sin\left(\frac{\theta_P+\omega_P}{2}\right)} \tag{2}$$

Here θ_P is the cut-off frequency of the prototype lowpass filter and ω_P is the desired cut-off frequency of the transformed lowpass filter.

If the transformed filter is made by replacing all delay elements with a first-order allpass section of Eq. (1) in a prototype IIR filter, the resultant filter structure has generally delay-free loops.

For the lowpass to bandpass transformation which retains the bandwidth, $T(z)$ is given by

$$T_2(z)=-z^{-1}\left(\frac{z^{-1}-\alpha}{1-\alpha z^{-1}}\right) \tag{3}$$

where

$$\alpha=\frac{\cos\left(\frac{\omega_U+\omega_L}{2}\right)}{\cos\left(\frac{\theta_P}{2}\right)} \tag{4}$$

Here ω_U and ω_L are the desired upper and lower cut-off frequencies of the transformed bandpass filter, and they satisfy $\omega_U-\omega_L=\theta_P$. So the transformation of Eq. (3) retains the bandwidth and only changes the center frequency. As Eq. (3) is the product of a delay and a first-order allpass function, the transformed filter is realized by replacing each delay element with a cascade of a delay element and a first-order allpass section. Then the resultant filter has no delay-free-loops[9],[10].

A general bandpass transformed filter is realized by using both the lowpass to lowpass transformation for bandwidth tuning and the lowpass to bandpass transformation for center frequency tuning together. Since this latter transformation is simple as mentioned above, the problem of realizing a lowpass to bandpass transformed filter results in that of realizing a lowpass to lowpass transformed filter. Similarly the lowpass to highpass and the lowpass to bandstop transformations are solved.

For the above reason we describe about realizing the lowpass to lowpass transformed filter in this paper.

3. Variable Cut-Off Frequency

3. 1 Direct Form

We consider a second-order transfer function

$$H(z)=\frac{1+b_1z^{-1}+z^{-2}}{1+a_1z^{-1}+a_2z^{-2}} \tag{5}$$

which has zeros on the unit circle. By substituting the lowpass to lowpass transformation of Eq. (1) for z^{-1} in Eq. (5) we arrive at

$$H'(z)=\gamma\frac{1+\hat{b}_1z^{-1}+z^{-2}}{1+\hat{a}_1z^{-1}+\hat{a}_2z^{-2}} \tag{6}$$

where

$$\hat{a}_1=\frac{a_1-2\beta(1+a_2)+\beta^2a_1}{1-\beta a_1+\beta^2a_2} \tag{7}$$

$$\hat{a}_2=\frac{a_2-\beta a_1+\beta^2}{1-\beta a_1+\beta^2a_2} \tag{8}$$

$$\hat{b}_1=\frac{b_1-4\beta+\beta^2b_1}{1-\beta b_1+\beta^2} \tag{9}$$

$$\gamma=\frac{1-\beta b_1+\beta^2}{1-\beta a_1+\beta^2a_2} \tag{10}$$

Here Eqs. (7)-(10) are expanded by the Taylor series with respect to β. When $\beta\ll 1$ is assumed and the expansion after the β term is neglected, this leads to

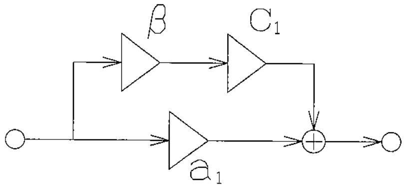


Fig. 1 Transformed a_1 branch.

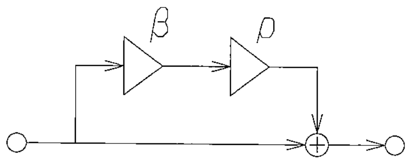


Fig. 2 Gain scaling branch.

$$\hat{a}_1 \approx a_1 + \beta c_1 \tag{11}$$

$$\hat{a}_2 \approx a_2 + \beta c_2 \tag{12}$$

$$\hat{b}_1 \approx b_1 + \beta d_1 \tag{13}$$

$$\gamma \approx 1 + \beta \rho \tag{14}$$

where

$$c_1 = -2 - 2a_2 + a_1^2 \tag{15}$$

$$c_2 = -a_1 + a_1 a_2 \tag{16}$$

$$d_1 = -4 + b_1^2 \tag{17}$$

$$\rho = -b_1 + a_1 \tag{18}$$

In a second-order transfer function having the above approximate values given by Eqs. (11)-(14) as its coefficients, its cut-off frequency can be changed by the only one parameter β, on condition that the tuning range is narrow. Positive values of β decrease the cut-off frequency whereas negative values increase it.

When an even-order prototype filter is realized by the cascade of second-order sections and each section is the direct form, the coefficients of Eq. (5) represent its multipliers. Consequently when second-order sections have multipliers whose coefficients are given by the approximate values of Eqs. (7)-(10), and are connected in cascade, an even-order filter with a variable cut-off frequency is realized. Each second-order section is constructed by connecting multipliers with the coefficients of βc_1, βc_2, βd_1 and $\beta \rho$ in parallel with the original multipliers in the prototype section. For example, a_1 branch is shown in Fig. 1, and the gain scaling branch is shown in Fig. 2.

3.2 Lattice Form

By using a second-order allpass section of the lattice form, a second-order transfer function which has its zeros on the unit circle is realized as shown in Fig. 3[8]. The realized transfer function is given by

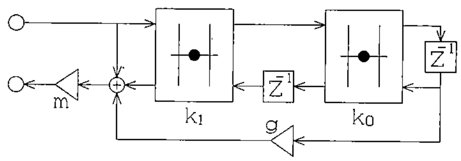


Fig. 3 Second-order lattice section.

$$H(z) = m(1+k_1)\frac{1+(2k_0+g(1+k_0))z^{-1}+z^{-2}}{1+k_0(1+k_1)z^{-1}+k_1 z^{-2}} \tag{19}$$

Equation (19) can have any zeros on the unit circle whose angle is controlled by the parameter g. To equalize $m(1+k_1)$ in Eq. (19) to 1, the value of m in Fig. 3 is determined as

$$m = \frac{1}{1+k_1} \tag{20}$$

Comparison between Eqs. (5) and (19) leads to the relation of coefficients given by

$$a_1 = k_0(1+k_1) \tag{21}$$

$$a_2 = k_1 \tag{22}$$

$$b_1 = 2k_0 + g(1+k_0) \tag{23}$$

To realize a filter with a variable cut-off frequency using this section, the following two methods are derived.

(1) Method 1

First the coefficients k_0 and k_1 in the denominator are determined, next the coefficient g in the numerator is determined, and finally m is determined.

If the coefficients k_0 and k_1 change into $k_0 + \beta l_0$ and $k_1 + \beta l_1$ respectively, the coefficients of z^{-1} and z^{-2} in the denominator of Eq. (19) are given by

$$\begin{aligned}\text{the coefficient of } z^{-1} &= k_0(1+k_1)\\ &\quad + \beta(k_0^2(k_1^2-1) + l_0(1+k_1))\\ &\quad + \beta^2 k_0 l_0 (k_1^2-1)\end{aligned} \tag{24}$$

$$\text{the coefficient of } z^{-2} = k_1 + \beta l_1 \tag{25}$$

If the values of l_0 and l_1 are determined so as to equate the coefficients of Eqs. (24) and (25) and the approximate values of Eqs. (11) and (12), the denominator of the second-order section becomes the desired one.

First, the coefficient of z^{-2} is examined. To equalize Eqs. (25) and (12), both the coefficients of β must be identical. Hence

$$l_1 = -a_1 + a_1 a_2 \tag{26}$$

must hold. By substitution of Eqs. (21) and (22) for a_1 and a_2 in Eq. (26), l_1 is determined as

$$l_1 = k_0(k_1^2 - 1) \tag{27}$$

Second, the coefficient of z^{-1} is examined. The term of β^2 in Eq. (24) is neglected on the assumption that $\beta \ll 1$ holds. To equalize Eq. (24) to Eq. (11), both the coefficients of β must be identical. Hence

$$k_0^2(k_1^2 - 1) + l_0(1 + k_1) = -2 - 2a_2 + a_1^2 \tag{28}$$

must hold. By substitution of Eqs. (21) and (22) for a_1 and a_2 in Eq. (28), l_0 is determined as

$$l_0 = 2(k_0^2 - 1) \tag{29}$$

Next, the coefficient g in the numerator is examined. The way used in the denominator is similarly applied. Since k_0 changes into $k_0 + \beta l_0$ and l_0 is determined as Eq. (29), if the coefficient g changes into $g + \beta h$, the coefficients of z^{-1} in the numerator of Eq. (19) is given by

$$\begin{aligned} &\text{the coefficient of } z^{-1} \\ &= 2k_0 + g(1 + k_0) \\ &\quad + \beta(k_0 + 1)((k_0 - 1)(4 + 2g) + h) \\ &\quad + \beta^2 2h(k_0^2 - 1) \end{aligned} \tag{30}$$

The term of β^2 in Eq. (30) is neglected by the above assumption. To equalize Eq. (30) to Eq. (13), both the coefficients of β must be identical. Hence

$$(k_0 + 1)((k_0 - 1)(4 + 2g) + h) = -4 + b_1^2 \tag{31}$$

must hold. By substitution of Eq. (23) for b_1 in Eq. (31), h is determined as

$$h = g(k_0 + 1)(g + 2) \tag{32}$$

Finally, the coefficient m is examined. Since k_1 changes into $k_1 + \beta l_1$ and l_1 is determined as Eq. (26), if the coefficient m changes into $m + \beta n$, $m(1 + k_1)$ in Eq. (19) is given by

$$\begin{aligned} &(m + \beta n)(1 + k_1 + \beta l_1) \\ &= m(1 + k_1) + \beta(l_1 m + n(1 + k_1)) + \beta^2 l_1 n \quad (33) \\ &= 1 + \beta(l_1 m + n(1 + k_1)) + \beta^2 l_1 n \quad (34) \end{aligned}$$

where Eq. (20) is substituted for $1/(1 + k_1)$. To equalize Eq. (34) to γ of Eq. (14) on the assumption that the term of β^2 in Eq. (34) can be neglected, the coefficient of β in Eq. (34) and ρ of Eq. (18) must be identical. Hence

$$m l_1 + n(1 + k_1) = -b_1 + a_1 \tag{35}$$

must hold. By substitution of Eqs. (21), (23), (27) and (20) for a_1, b_1, l_1 and m in Eq. (35) respectively, n is determined as

$$n = -g\frac{(1 + k_0)}{(1 + k_1)} \tag{36}$$

$$= -gm(1 + k_0) \tag{37}$$

where Eq. (20) is substituted for $1/(1 + k_1)$.

(2) Method 2

Both $\hat{k}_0$ *and* $\hat{k}_1$ denote the transformed versions of k_0 and k_1 respectively, like Eq. (6). If follows from Eqs. (21) and (22) that

$$\hat{k}_0 = \frac{\hat{a}_1}{1 + \hat{a}_2} \tag{38}$$

$$\hat{k}_1 = \hat{a}_2 \tag{39}$$

are obtained. Substitution of Eqs. (38) and (39) for $\hat{a}_1$ and $\hat{a}_2$ in Eqs. (7) and (8) gives

$$\hat{k}_0 = \frac{a_1 - 2\beta(1 + a_2) + \beta^2 a_1}{1 + a_2 - 2\beta a_1 + \beta^2(1 + a_2)} \tag{40}$$

$$\hat{k}_1 = \frac{a_2 - \beta a_1 + \beta^2}{1 - \beta a_1 + \beta^2 a_2} \tag{41}$$

Here expressions of $\hat{k}_0$ and $\hat{k}_1$ are expanded by Taylor series with respect to β, and the expansions after the β terms are neglected. Then

$$\hat{k}_0 \approx k_0 + 2\beta(k_0^2 - 1) \tag{42}$$

$$\hat{k}_1 \approx k_1 + \beta k_0(k_1^2 - 1) \tag{43}$$

are obtained, where Eqs. (21) and (22) are substituted for a_1 and a_2 respectively.

By the similar way, the approximate values of $\hat{g}$ in the numerator and $\hat{m}$, which are the transformed versions of g and m, are obtained as

$$\hat{g} \approx g + \beta g(2k_0 + g(1 + k_0) + 2) \tag{44}$$

$$\hat{m} \approx m - \beta mg(1 + k_0) \tag{45}$$

The β terms of Eqs. (42)-(45) are identical to the result of the first method viz. Eqs. (27), (29), (32) and (37). This fact shows that these two methods are distinguished on the derivation but result in the same approximate values of the coefficients in the transfer function of Eq. (19). Accordingly either of two methods may be used.

From the above Eqs. (42)-(45), the transformed lattice coefficients $\hat{k}_0$, $\hat{k}_1$, $\hat{g}$, $\hat{m}$ are the sum of the original coefficients and the β terms. Consequently by connecting multipliers with respect to β in parallel with the original multipliers in the section of Fig. 3, like Fig. 1, a second-order section with a variable cut-off frequency is obtained.

The methods which is described in this section are applicable to any other second-order sections.

4. Stability Conditions

4.1 Direct Form

The stability condition of the second-order transfer function given by Eq. (5) are described in terms of its denominator coefficients a_1 and a_2 by

$$a_2 < 1 \tag{46}$$

$$a_2 > a_1 - 1 \tag{47}$$

$$a_2 > -a_1 - 1 \tag{48}$$

As a_1 and a_2 are replaced with $a_1+\beta c_1$ and $a_2+\beta c_2$ in a direct form filter with a variable cut-off frequency, the conditions of Eqs.(46)-(48) are modified as

$$a_2+\beta c_2 < 1 \tag{49}$$

$$a_2+\beta c_2 > a_1+\beta c_1 - 1 \tag{50}$$

$$a_2+\beta c_2 > -a_1-\beta c_1 - 1 \tag{51}$$

Equations(49)-(51) are solved for β. Then the following conditions are obtained as

$$\begin{cases} \beta > -\dfrac{1}{a_1} & \text{for } a_1>0 \\ \beta < -\dfrac{1}{a_1} & \text{for } a_1<0 \\ \beta\text{: unconditional} & \text{for } a_1=0 \end{cases} \tag{52}$$

$$\begin{cases} \beta > \dfrac{a_1-a_2-1}{c_2-c_1} & \text{for } c_2-c_1>0 \\ \beta < \dfrac{a_1-a_2-1}{c_2-c_1} & \text{for } c_2-c_1<0 \\ \beta\text{: unconditional} & \text{for } c_2-c_1=0 \end{cases} \tag{53}$$

$$\begin{cases} \beta > -\dfrac{a_1+a_2+1}{c_1+c_2} & \text{for } c_1+c_2>0 \\ \beta < -\dfrac{a_1-a_2-1}{c_1+c_2} & \text{for } c_1+c_2<0 \\ \beta\text{: unconditional} & \text{for } c_1+c_2=0 \end{cases} \tag{54}$$

If β satisfies the conditions of Eqs.(52), (53) and (54) simultaneously, the second-order section is stable. Therefore the possible range of β must be limited owing to these conditions.

4.2 Lattice Form

The stability condition of a second-order transfer function given by Eq.(19) are given in terms of lattice coefficients k_0 and k_1 by

$$-1 < k_0 < 1 \tag{55}$$

$$-1 < k_1 < 1 \tag{56}$$

As k_0 and k_1 are replaced with $k_0+2\beta(k_0^2-1)$ and $k_1+\beta k_0(k_1^2-1)$ in a second-order lattice filter with a variable cut-off frequency, Eqs.(55) and (56) are modified, and the conditions with respect to β are obtained as

$$-\frac{1}{2(k_0-1)} > \beta > -\frac{1}{2(k_0+1)} \tag{57}$$

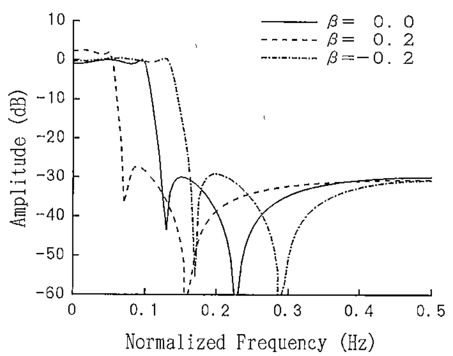
$$\begin{cases} -\dfrac{1}{k_0(k_1-1)} > \beta > -\dfrac{1}{k_0(k_1+1)} & \text{for } k_0>0 \\ -\dfrac{1}{k_0(k_1-1)} < \beta < -\dfrac{1}{k_0(k_1+1)} & \text{for } k_0<0 \\ \beta\text{: unconditional} & \text{for } k_0=0 \end{cases} \tag{58}$$

The range of β of a second-order lattice section must be limited owing to the condition of Eqs.(57) and (58).

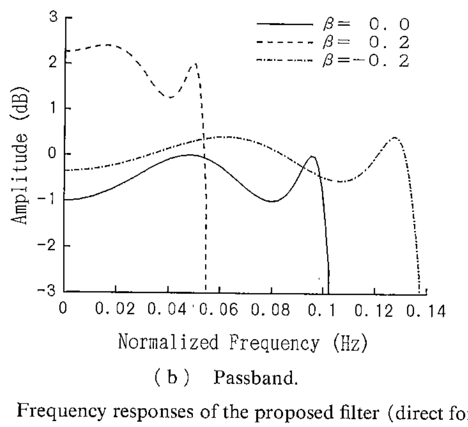
5. Ilustrative Examples

In this section a variable digital filter is designed by the proposed method, and is compared with that of Ref.(6) to show that the proposed method gives the wider tuning range.

A forth-order elliptic lowpass filter with a variable cut-off frequency is examined. A prototype transfer function is

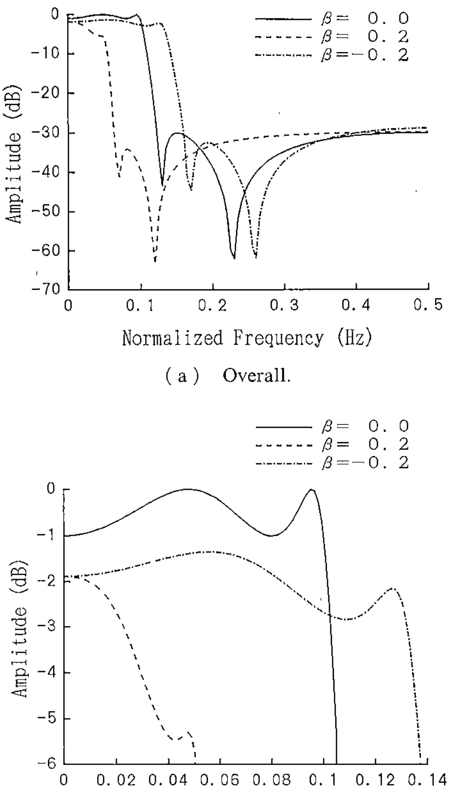


(a) Overall.



(b) Passband.

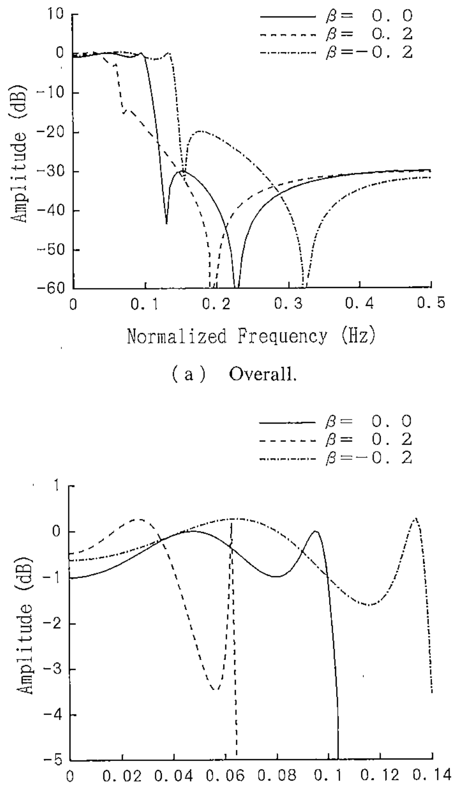
Fig. 4 Frequency responses of the proposed filter (direct form).



(a) Overall.

(b) Passband.

Fig. 5 Frequency responses of the proposed filter (lattice form).



(a) Overall.

(b) Passband.

Fig. 6 Frequency responses of the conventional filter.

$$H(z)=0.043715465 \times \frac{1-0.2779717807z^{-1}+z^{-2}}{1-1.474579525z^{-1}+0.616601493z^{-2}} \times \frac{1-1.38502411785z^{-1}+z^{-2}}{1-1.541340190z^{-1}+0.907084746z^{-2}} \quad (59)$$

which has 0.1 dB passband ripple, 30 dB minimum stopband attenuation and the normalized cut-off frequency of 0.1 Hz.

The frequency responses of the proposed filter using directand lattice form sections are shown in Figs. 4 and 5 respectively. The parameter β varies between -0.2 and 0.2. These values of β satisfy the stability conditions in both cases. For comparison the frequency response of Ref. (6) is shown in Fig. 6. The passband and stopband performance for Fig. 4 is found to be superior to Fig. 6 except the shifting of the amplitude level in the case of $\beta=0.2$. The passband performance for Fig. 5 seems to be worse than Fig. 6 in the case of $\beta=0.2$, but the stopband performance of Fig. 5 is better. In the case of $\beta=-0.2$, the passband performance for Fig. 5 seems to be similar to Fig. 6, whereas the stopband performance of Fig. 5 is better. From the above it is clear that the proposed method gives wider tuning range than Ref. (6).

By the way, Fig. 4 is superior to Fig. 5 in both the passband and stopband. The reason is described as follows. The lattice form realizes the coefficients in a second-order transfer function of Eq. (5) as Eqs. (21) -(23), which include the products of two multiplier coefficients such as k_0k_1 and gk_0. Accordingly when the lattice form is used, the coefficients in the transfor med transfer function are realized by using $(k_0+\beta l_0)(k_1+\beta l_1)$ and $(g+\beta h)(k_0+\beta l_0)$, and the β^2 terms of $\beta^2 l_0 l_1$ and $\beta^2 h l_0$ are generated. In the derivation described in the Sect. 3. 2, the β^2 terms are neglected. Therefore in comparison with the transformed coefficients by using the direct form, those by using the lattice form are more deviated from the true values which aren't approximated. Especially in this example, when β is 0.2, term of the coefficients of z^{-1} in the transformed numerator of the first section in Eq. (59) is large. The true value of the coefficient of z^{-1} in the transformed numerator, which isn't approximated, is -0.994064. The value of the same coefficient realized by using the direct form is -1.062518. The one by the lattice form is -1.46985.

6. Concluding Remarks

In this paper a new synthesis of variable even-order IIR digital filters with only one parameter has been proposed. The synthesized structure is a cascade of second-order sections, and their multiplier coefficients are determined by Taylor series expansion of the lowpass to lowpass frequency transformation. The direct form and the lattice form are used for the circuits of the second-order sections, and a design formula for changing the cut-off frequency is derived for each circuit. Finally examples show that the proposed filter has wider tuning range than the conventional filter.

Future assignments are to suppress the shifting of the amplitude level, and to extend the tuning range.

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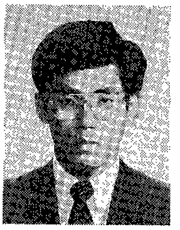

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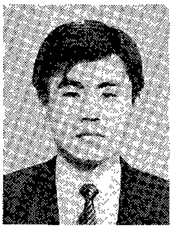

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