Network capacity improvement with two dimensional MIMO network coding

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***Abstract*—A combination of MIMO and network coding for one dimensional (1D) topology in wireless mesh network has been proposed in recent literature. The technique supplies higher network capacity compared to that of conventional schemes. In this paper, the authors extend MIMO network coding to two dimensional (2D) topology. Owing to the efficient sharing of frequency of network coding and co-channel interference cancellation ability of MIMO, the proposed technique provides a significant gain to end-to-end network capacity. Furthermore, in a 2D mesh network where interferences from long distance nodes cannot be ignored, the authors proposed cooperative nulling algorithm by which interference signals can be suppressed. Simulation results show the good performance of the proposed methods.**

I. Introduction

Wireless mesh network (WMN) consisting of mesh routers and mesh clients has been achieving much more attention in recent years as there are more demands for WMN applications, namely wireless sensor networks, public wireless access networks, plant control systems, etc. [1]. The advantages of WMN are its ability to form a flexible network topology, robustness and wide area coverage owing to multi-hop relay property. In the mesh network which is used as a backbone network, nodes in the network are almost fixed, well power supplied and can be equipped with multiple antennas as shown in Fig. 1.

With increasing in applications of WMN, and in the number of mesh clients joining WMN, there is a requirement for a mesh network which can support high traffic and small packet delay. It means that there is demand for a network with higher efficiency in usage of frequency and time compared to traditional ones, e.g. Carrier Sense Multiple Access (CSMA), Time Division Multiple Access (TDMA) or Multi-channel mesh network. In [4], for a single channel one dimensional (1D) mesh network, the authors propose a method which combines Multiple-Input-Multiple-Output (MIMO) technique [2], [3] and network coding [5], [6] for multi-hop network. The advantage of this method is that MIMO is used as a multiple access method which reduces time required for transmissions of signals between mesh nodes and network coding supports two flows of information (two directions) to be simultaneously transported per transmission. In this way, the proposed method achieves superior capacity performance compared to conventional schemes.

The problem of the single channel 1D MIMO mesh network algorithm when it is applied in 2D mesh network is that when there are crossing routes in the network, each route has to be assigned with different frequencies or a time sharing algorithm is used at the intersection nodes to prevent co-channel interference. In this paper, the authors extend the technique used in [4] to a single frequency two dimensional (2D) MIMO mesh network with four antenna equipped mesh nodes. Using the properties of MIMO, the authors proposed an efficient way of network coding at the intersection node such that only a single frequency channel is required. Consequently, the network capacity can be significantly increased given that the total bandwidth of the network is restricted. Numerical analysis in this paper shows an 8-fold in capacity achieving compared to traditional Single Input Single Output (SISO) multichannel method.

In the above proposed technique, the authors assume that receivers in the mesh network do not receive any interference except for its desired signals sent from adjacent nodes to show the efficiency of the proposed network coding algorithm. In this paper, the authors also deal with the case when interference cannot be ignored. Using array antenna processing techniques, the authors propose a cooperative nulling algorithm to suppress the undesired interference signals in the network. Simulation results show the efficiency of the proposed algorithm in improving capacity performance when interference cannot be ignored. Consider a 2D network with total bandwidth 5MHz and an wireless environment with pathloss exponent 3.5, the proposed algorithm can provide a network capacity of 14.5 Mbps/flow, or a total capacity of 58 Mbps (four flows of information).

The rest of this paper is organized as follows. Section 2 defines the network topology and raises the problem. Section 3 presents the proposed network coding algorithm. Section 4 deals with cooperative nulling when interference cannot be ignored. Section 5 explains the calculation method of end-to-end capacity. Numerical results and discussion are in Section 6. Finally, Section 7 concludes the paper. Table I summarizes mathematical notations used in this paper.

II. Problem definition

One of the most general topologies of a mesh network is given in Fig. 2. Adjacent nodes are connectted to each other by wireless link and communication between mesh nodes can be performed through multi-hop. The bold line connecting nodes in the figure represents the virtual routes which are configured by the network layer at each node. Due to the broadcast characteristic of wireless channel, assignment of resource (frequency, time, etc.) is performed at the Media Access Control (MAC) layer of each mesh node to avoid co-channel interference and to maintain

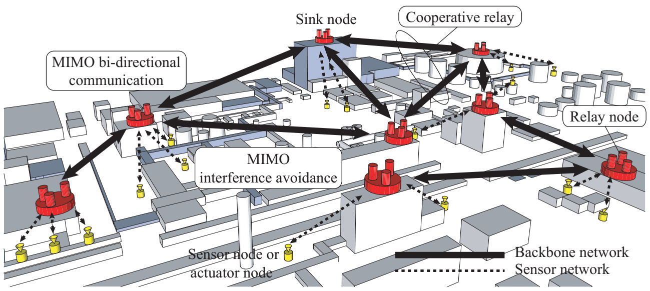


Fig. 1. An example of a backbone mesh network.

TABLE I
MATHEMATICAL NOTATIONS.

x	Scalar variable
$\boldsymbol{x}$	Vector variable
$\boldsymbol{X}$	Matrix variable
$[\bullet]^*$	Conjugate of vector and matrix
$[\bullet]^T$	Transpose of vector and matrix
$[\bullet]^H$	Hermitian transpose of vector and matrix
$\mathrm{E}[\bullet]$	Sample avarage of a random process
$\mathrm{null}\,(\boldsymbol{X})$	Orthonormal basis of the null space of $\boldsymbol{X}$
$\boldsymbol{x}^{(i)}$	ith column of $\boldsymbol{X}$
$x^{(i)}$	ith component of $\boldsymbol{x}$

continuos transportation of information. The authors define the network spectral efficiency coefficient β is as the reciprocal of the total number of frequencies used in the network at the same time. For resource sharing algorithm in e.g. Fig. 2(a), $\beta = \frac{1}{4}$ as two channels are used for the horizontal line and the other two are used for the vertical line. However, the resource assignment and sharing approach might reduce spectral efficiency of the network on the overall. In [4], the authors proposed an efficient way to use the same frequency resource to perform bi-directional communication in a 1D mesh network (a line of mesh nodes). Applying this approach in a 2D mesh network, the network spectral efficiency coefficient can be mostly reduced to $\beta = \frac{1}{2}$ if any adjacent parallel 1D mesh lines are seperated larger than an interference range (Fig. 2(b)). In this paper, the authors would like to increase the network spectral efficiency coefficient to $\beta = 1$ which means that only a single frequency is used in the network. In this way, the spectral efficiency of the network can be maximized.

This problem can be degenerated into solving the frequency sharing problem in a simple network topology of two crossing lines as shown in Fig. 2(c), especially at the intersection node. The authors solves the simplified problem by using MIMO algorithms, network coding and array antenna processing techniques as shown in later sections.

III. 2D MIMO NETWORK CODING

A. Review of 1D MIMO network coding

Details about 1D MIMO network coding are given in [4]. Here, the authors summarize the algorithm in Fig. 3. Consider an array of mesh nodes evenly located on a single line. Each node is equipped with m antennas ($m \geq 2$). Assume that there are two flows of information: the forward flow which transports information from the leftmost node to the rightmost node and the backward flow which transports information in the reverse

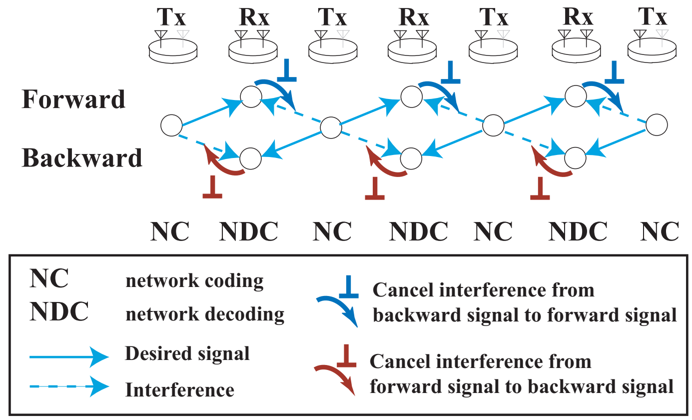


Fig. 3. 1D MIMO network coding.

direction. At a time slot, for any two nodes adjacent to each other, one node is the transmitter (Tx) and the other is the receiver (Rx). The Tx node uses one of its antennas to broadcast transmit signal to its two adjacent Rx nodes. Each Rx node listens to the two different signals sent from its two adjacent Tx nodes and uses MIMO multiple access algorithm to detect these signals. A network coding and network decoding are performed to the signals at the Tx node before transmission and at the Rx node after receiving respectively to ensure the forward and backward information is sent simultaneously at every time slot. In the next time slot, nodes switch their functions. Tx nodes become Rx node and vice versa. This cycle is repeated every two time slots.

Transmit signal s_{i-1} and s_{i+1} at node $i-1$ and $i+1$ in time slot n is coded using network coding as follows,

$$s_{i-1} = {}^{n}_{\mathrm{F}}s_{i-1} + {}^{n}_{\mathrm{B}}s_{i-1} \mod q \quad (1)$$

$$s_{i+1} = {}^{n}_{\mathrm{F}}s_{i+1} + {}^{n}_{\mathrm{B}}s_{i+1} \mod q, \quad (2)$$

where ${}^{n}_{\mathrm{F}}s_{i-1}$ and ${}^{n}_{\mathrm{F}}s_{i+1}$ represent the forward information at node $i-1$ and node $i+1$ respectively. Similarly, ${}^{n}_{\mathrm{B}}s_{i-1}$ and ${}^{n}_{\mathrm{B}}s_{i+1}$ represent the backward information at node $i-1$ and node $i+1$, and q denotes the lattice size used in network coding to restrict the transmit power. This results in a shaping loss of $\rho = \frac{1}{2}$.

The receive signal $\boldsymbol{y}_i$ at node i is given by

$$\boldsymbol{y}_i = \boldsymbol{h}_{i,i-1}s_{i-1} + \boldsymbol{h}_{i,i+1}s_{i+1} + \boldsymbol{n}_i \quad (3)$$

$$= \boldsymbol{H}_i^{\mathrm{eff}}\boldsymbol{s}_i^{\mathrm{eff}} + \boldsymbol{n}_i \quad (4)$$

$$\boldsymbol{H}_i^{\mathrm{eff}} = [\boldsymbol{h}_{i,i-1}\ \boldsymbol{h}_{i,i+1}] \quad (5)$$

$$\boldsymbol{s}_i^{\mathrm{eff}} = [s_{i-1}\ s_{i+1}]^T, \quad (6)$$

where $\boldsymbol{h}_{i,i-1} \in C^m$ and $\boldsymbol{h}_{i,i+1} \in C^m$ represent the channel vectors from node $i-1$ and node $i+1$ to node i respectively, $\boldsymbol{n}_i \in C^m$ is an additive white Gaussian noise vector with zero mean and $\sigma^2\boldsymbol{I}$ covariance matrix.

Owing to the multiple antennas at the Rx, Eq. (4) is a system of linear equations with the number of equations larger or equal to the number of variables. Using linear decoding algorithm, $\boldsymbol{s}_i^{\mathrm{eff}}$ can be estimated as,

$$\hat{\boldsymbol{s}}_i^{\mathrm{eff}} = [\hat{s}_{i-1}\ \hat{s}_{i+1}]^T = \boldsymbol{W}^{\mathrm{r}T}\boldsymbol{y}_i, \quad (7)$$

where $\boldsymbol{W}^{\mathrm{r}*} = \left(\boldsymbol{H}_i^{\mathrm{eff}}\boldsymbol{H}_i^{\mathrm{eff}H} + \sigma^2\boldsymbol{I}\right)^{-1}\boldsymbol{H}_i^{\mathrm{eff}}$ for a Minimum Mean Square Error (MMSE) decoder and σ^2 denotes the variance of noise at node i.

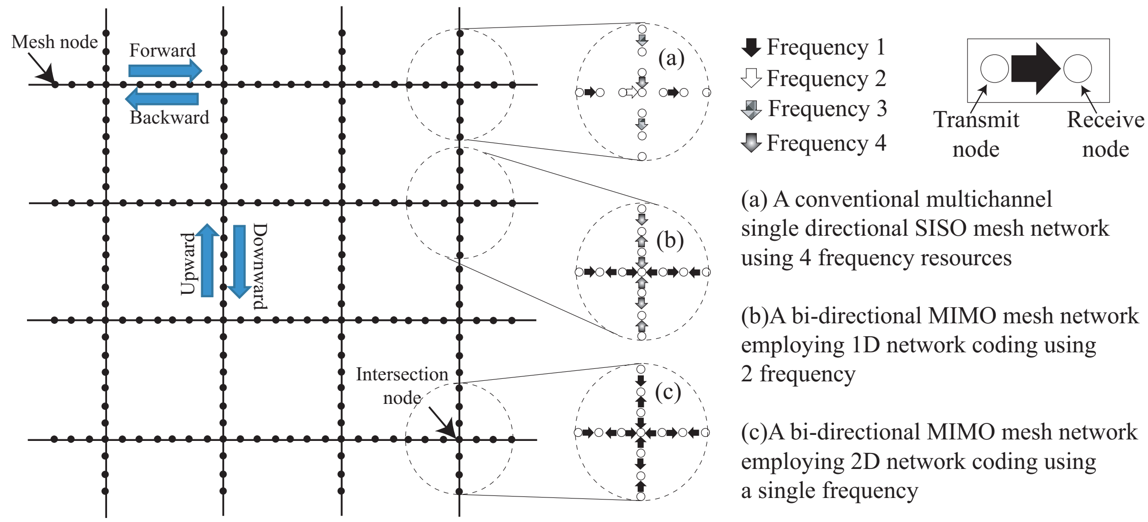


Fig. 2. An example of communication and resource assignment in mesh network.

Using the fact that node i knows the forward information ${}^{n}_{\mathrm{F}}s_{i+1}$ of node $i+1$ and the backward information ${}^{n}_{\mathrm{B}}s_{i-1}$ of node $i-1$ and the property of network coding, the two unknown signals to node i: ${}^{n}_{\mathrm{F}}s_{i-1}$ and ${}^{n}_{\mathrm{B}}s_{i+1}$ can be estimated as,

$$\begin{aligned} {}^{n+1}_{\mathrm{F}}s_i \triangleq {}^{n}_{\mathrm{F}}\hat{s}_{i-1} &= \hat{s}_{i-1} - {}^{n}_{\mathrm{B}}s_{i-1} \mod q && (8)\\ &= \hat{s}_{i-1} - {}^{n-1}_{\mathrm{B}}s_{i} \mod q && (9)\\ {}^{n+1}_{\mathrm{B}}s_i \triangleq {}^{n}_{\mathrm{B}}\hat{s}_{i+1} &= \hat{s}_{i+1} - {}^{n}_{\mathrm{F}}s_{i+1} \mod q && (10)\\ &= \hat{s}_{i+1} - {}^{n-1}_{\mathrm{F}}s_{i} \mod q. && (11) \end{aligned}$$

In conclusion, at any time slot, Rx node receives two flows of information: the forward information sent from its left node and the backward information sent from its right node. In the $n+1$ time slot, node i becomes Tx node and transmit $s_i = {}^{n+1}_{\mathrm{F}}s_i + {}^{n+1}_{\mathrm{B}}s_i \mod q$. This cycle is repeated to maintain a bidirectional 1D mesh network.

B. 2D MIMO network coding

Consider a network topology of two crossing lines with node C_0 at the intersection and nodes H_i ($i \in \{-K, \ldots, -1, 1, \ldots, K\}$), $\mathrm{V}_j, j \in \{-K, \ldots, -1, 1, \ldots, K\}$ on the horizontal and vertical lines respectively as shown in Fig. 4. These nodes are evenly located with distance d and are assumed to be equipped with $m = 4$ antennas. There are four flows of information in the network: the *forward* flow from H_{-K} to H_K, the *backward* flow from H_K to H_{-K}, the *downward* flow from V_K to V_{-K} and the *upward* flow from V_{-K} to V_K. In this paper, the authors propose a communication scheme which allows these four flows to be transported through the network simultaneously using only single frequency.

As in the case of 1D MIMO network coding, at a time slot, for any pair of adjacent nodes, one is Tx node and the other is Rx node. Here, the adjacent node is defined as nodes in the range of distance d. In the next time slot, Rx node becomes Tx node and vice versa. Particularly, for the topology shown in Fig. 4, assume that at time slot n, C_0, $\mathrm{H}_{2k(k\neq0\in\mathcal{Z})}$ and $\mathrm{V}_{2k(k\neq0\in\mathcal{Z})}$ are Rx node and the others are Tx node and in the next time slot $n+1$, $\mathrm{H}_{2k+1(k\in\mathcal{Z})}$ and $\mathrm{V}_{2k+1(k\neq0\in\mathcal{Z})}$ will become Rx node and the others become Tx node (Fig. 5).

The MIMO network coding scheme performed at nodes on each line of the network topology follows exactly what explained in the previous part of 1D MIMO network coding except for at the intersection node C_0 and its adjacent nodes $\mathrm{H}_{\pm1}$ and $\mathrm{V}_{\pm1}$. Here, we focus on the algorithm for these nodes.

Through the rest of the paper, the following symbolic representation is used. ${}^{n}_{\mathrm{F}}s_{\mathrm{H}_i}$ and ${}^{n}_{\mathrm{B}}s_{\mathrm{H}_i}$ represent the forward and backward information of node H_i on the horizontal line respectively. Similarly, ${}^{n}_{\mathrm{D}}s_{\mathrm{V}_j}$ and ${}^{n}_{\mathrm{U}}s_{\mathrm{V}_j}$ represent the downward and upward information of node V_j on the vertical line. Also, ${}^{n}_{\mathrm{F}}s_{\mathrm{C}_0}$, ${}^{n}_{\mathrm{B}}s_{\mathrm{C}_0}$, ${}^{n}_{\mathrm{D}}s_{\mathrm{C}_0}$ and ${}^{n}_{\mathrm{U}}s_{\mathrm{C}_0}$ correspondingly denote the forward, backward, downward and upward information at node C_0. The left superscript n represents the time index. Due to the characteristic of flow, the following equations hold $\forall i, j$ and $\mathrm{C}_0 \triangleq \{\mathrm{H}_0, \mathrm{V}_0\}$.

$$\begin{aligned} {}^{n}_{\mathrm{F}}s_{\mathrm{H}_i} &= {}^{n-1}_{\mathrm{F}}s_{\mathrm{H}_{i-1}} && (12)\\ {}^{n}_{\mathrm{B}}s_{\mathrm{H}_i} &= {}^{n-1}_{\mathrm{B}}s_{\mathrm{H}_{i+1}} && (13)\\ {}^{n}_{\mathrm{D}}s_{\mathrm{V}_j} &= {}^{n-1}_{\mathrm{D}}s_{\mathrm{V}_{j+1}} && (14)\\ {}^{n}_{\mathrm{U}}s_{\mathrm{V}_j} &= {}^{n-1}_{\mathrm{U}}s_{\mathrm{V}_{j-1}} && (15) \end{aligned}$$

At time slot n, $\mathrm{H}_{\pm1}$ and $\mathrm{V}_{\pm1}$ are Tx nodes and their transmit signals $s_{\mathrm{H}_{\pm1}}$, $s_{\mathrm{V}_{\pm1}}$ given by,

$$\begin{aligned} s_{\mathrm{H}_{\pm1}} &= {}^{n}_{\mathrm{F}}s_{\mathrm{H}_{\pm1}} + {}^{n}_{\mathrm{B}}s_{\mathrm{H}_{\pm1}} \mod q && (16)\\ s_{\mathrm{V}_{\pm1}} &= {}^{n}_{\mathrm{D}}s_{\mathrm{V}_{\pm1}} + {}^{n}_{\mathrm{U}}s_{\mathrm{V}_{\pm1}} \mod q. && (17) \end{aligned}$$

These transmit signals are transmitted by using one of four antennas of the Tx node. The receive signal $\boldsymbol{y}_{\mathrm{C}_0}$ at node C_0 is given by,

$$\begin{aligned} \boldsymbol{y}_{\mathrm{C}_0} &= \sum \boldsymbol{h}_{\mathrm{C}_0,\mathrm{H}_{\pm1}} s_{\mathrm{H}_{\pm1}} + \sum \boldsymbol{h}_{\mathrm{C}_0,\mathrm{V}_{\pm1}} s_{\mathrm{V}_{\pm1}} + \boldsymbol{n}_{\mathrm{C}_0} && (18)\\ &= \boldsymbol{H}^{\mathrm{eff}}_{\mathrm{C}_0} \boldsymbol{s}^{\mathrm{eff}}_{\mathrm{C}_0} + \boldsymbol{n}_{\mathrm{C}_0} && (19)\\ \boldsymbol{H}^{\mathrm{eff}}_{\mathrm{C}_0} &= \left[\boldsymbol{h}_{\mathrm{C}_0,\mathrm{H}_{-1}}\ \boldsymbol{h}_{\mathrm{C}_0,\mathrm{H}_{1}}\ \boldsymbol{h}_{\mathrm{C}_0,\mathrm{V}_{-1}}\ \boldsymbol{h}_{\mathrm{C}_0,\mathrm{V}_{1}}\right] && (20)\\ \boldsymbol{s}^{\mathrm{eff}}_{\mathrm{C}_0} &= \left[s_{\mathrm{H}_{-1}}\ s_{\mathrm{H}_{1}}\ s_{\mathrm{V}_{-1}}\ s_{\mathrm{V}_{1}}\right]^T, && (21) \end{aligned}$$

where $\boldsymbol{h}_{\mathrm{C}_0,\mathrm{H}_{\pm1}} \in \mathcal{C}$ and $\boldsymbol{h}_{\mathrm{C}_0,\mathrm{V}_{\pm1}} \in \mathcal{C}^m$ represent the channel vectors from node $\mathrm{H}_{\pm1}$ and node $\mathrm{V}_{\pm1}$ to node C_0 respectively, $\boldsymbol{n}_{\mathrm{C}_0} \in \mathcal{C}^m$ is an additive white Gaussian noise vector at node C_0.

As node C_0 has four antennas, four unknown signals can be estimated by using linear decoding algorithm as follows,

$$\hat{\boldsymbol{s}}^{\mathrm{eff}}_{\mathrm{C}_0} = [\hat{s}_{\mathrm{H}_{-1}} \;\; \hat{s}_{\mathrm{H}_1} \;\; \hat{s}_{\mathrm{V}_{-1}} \;\; \hat{s}_{\mathrm{V}_1}]^T = \boldsymbol{W}^{\mathrm{r}T}\boldsymbol{y}_{\mathrm{C}_0}, \tag{22}$$

where $\boldsymbol{W}^{\mathrm{r}*} = \left(\boldsymbol{H}^{\mathrm{eff}}_{\mathrm{C}_0}\boldsymbol{H}^{\mathrm{eff}\,H}_{\mathrm{C}_0} + \sigma^2\boldsymbol{I}\right)^{-1}\boldsymbol{H}^{\mathrm{eff}}_{\mathrm{C}_0}$ for a MMSE decoder.

Using the fact that node C_0 knows the forward information of node H_1, backward information of node H_{-1}, downward information of node V_{-1} and upward information of node V_1, and the property of network coding, the four unknown signals to node C_0: ${}^{n}_{\mathrm{F}}s_{\mathrm{H}_{-1}}$, ${}^{n}_{\mathrm{B}}s_{\mathrm{H}_1}$, ${}^{n}_{\mathrm{D}}s_{\mathrm{V}_1}$ and ${}^{n}_{\mathrm{U}}s_{\mathrm{V}_{-1}}$ can be estimated as,

$$\begin{aligned}
{}^{n+1}_{\mathrm{F}}s_{\mathrm{C}_0} \triangleq {}^{n}_{\mathrm{F}}\hat{s}_{\mathrm{H}_{-1}} &= \hat{s}_{\mathrm{H}_{-1}} - {}^{n}_{\mathrm{B}}s_{\mathrm{H}_{-1}} \mod q && (23)\\
&= \hat{s}_{\mathrm{H}_{-1}} - {}^{n-1}_{\mathrm{B}}s_{\mathrm{C}_0} \mod q && (24)\\
{}^{n+1}_{\mathrm{B}}s_{\mathrm{C}_0} \triangleq {}^{n}_{\mathrm{B}}\hat{s}_{\mathrm{H}_1} &= \hat{s}_{\mathrm{H}_1} - {}^{n}_{\mathrm{F}}s_{\mathrm{H}_1} \mod q && (25)\\
&= \hat{s}_{\mathrm{H}_1} - {}^{n-1}_{\mathrm{F}}s_{\mathrm{C}_0} \mod q && (26)\\
{}^{n+1}_{\mathrm{D}}s_{\mathrm{C}_0} \triangleq {}^{n}_{\mathrm{D}}\hat{s}_{\mathrm{V}_1} &= \hat{s}_{\mathrm{V}_1} - {}^{n}_{\mathrm{U}}s_{\mathrm{V}_1} \mod q && (27)\\
&= \hat{s}_{\mathrm{V}_1} - {}^{n-1}_{\mathrm{U}}s_{\mathrm{C}_0} \mod q && (28)\\
{}^{n+1}_{\mathrm{U}}s_{\mathrm{C}_0} \triangleq {}^{n}_{\mathrm{U}}\hat{s}_{\mathrm{V}_{-1}} &= \hat{s}_{\mathrm{V}_{-1}} - {}^{n}_{\mathrm{D}}s_{\mathrm{V}_{-1}} \mod q && (29)\\
&= \hat{s}_{\mathrm{V}_1} - {}^{n-1}_{\mathrm{D}}s_{\mathrm{C}_0} \mod q. && (30)
\end{aligned}$$

At this point, node C_0 receives four flows of information sent from surrounding nodes: the forward information from node H_{-1}, backward information from node H_1, downward information from node V_1 and upward information from node V_{-1}.

In the $n+1^{\mathrm{th}}$ time slot, node C_0 becomes Tx node. The node is required to perform network coding on signals received in the previous time slot such that the surrounding Rx nodes (which are now node $\mathrm{H}_{\pm1}$ and $\mathrm{V}_{\pm1}$), can perform network decoding on the receive signal. As nodes on the horizontal line do not have any information about signals on the vertical line (except for the intersection node) and vice versa, the network coding at node C_0 should perform on the forward and backward information, and on the downward and upward information independantly. Owing to the multiple antennas at node C_0, this approach can be done and the transmit signal of node C_0 is given by

$$\boldsymbol{s}_{\mathrm{C}_0} = \begin{bmatrix} {}^{n+1}_{\mathrm{F}}s_{\mathrm{C}_0} + {}^{n+1}_{\mathrm{B}}s_{\mathrm{C}_0} \mod q \\ {}^{n+1}_{\mathrm{D}}s_{\mathrm{C}_0} + {}^{n+1}_{\mathrm{U}}s_{\mathrm{C}_0} \mod q \end{bmatrix}. \tag{31}$$

It means that node C_0 uses two out of its four antennas to transmit two different signals: the horizontal line network coding signal and the vertical line network coding signal simultaneously.

At the same time, node $\mathrm{H}_{\pm2}$ and $\mathrm{V}_{\pm2}$ are Tx nodes and their transmit signals which have been network coded are given by,

$$\begin{aligned}
s_{\mathrm{H}_{\pm2}} &= {}^{n}_{\mathrm{F}}s_{\mathrm{H}_{\pm2}} + {}^{n}_{\mathrm{B}}s_{\mathrm{H}_{\pm2}} \mod q && (32)\\
s_{\mathrm{V}_{\pm2}} &= {}^{n}_{\mathrm{D}}s_{\mathrm{V}_{\pm2}} + {}^{n}_{\mathrm{U}}s_{\mathrm{V}_{\pm2}} \mod q. && (33)
\end{aligned}$$

The receive signal at node $\mathrm{H}_{\pm1}$ and $\mathrm{V}_{\pm1}$ are then given by,

$$\begin{aligned}
\boldsymbol{y}_{\Omega_{\pm1}} &= \boldsymbol{H}_{\Omega_{\pm1},\mathrm{C}_0}\boldsymbol{s}_{\mathrm{C}_0} + \boldsymbol{h}_{\Omega_{\pm1},\Omega_{\pm2}}s_{\Omega_{\pm2}} + \boldsymbol{n}_{\Omega_{\pm1}} && (34)\\
&= \boldsymbol{H}^{\mathrm{eff}}_{\Omega_{\pm1}}\boldsymbol{s}^{\mathrm{eff}}_{\Omega_{\pm1}} + \boldsymbol{n}_{\Omega_{\pm1}} && (35)\\
\boldsymbol{H}^{\mathrm{eff}}_{\Omega_{\pm1}} &= [\boldsymbol{H}_{\Omega_{\pm1},\mathrm{C}_0} \;\; \boldsymbol{h}_{\Omega_{\pm1},\Omega_{\pm2}}] && (36)\\
\boldsymbol{s}^{\mathrm{eff}}_{\Omega_{\pm1}} &= \left[\boldsymbol{s}^T_{\mathrm{C}_0} \;\; s_{\Omega_{\pm2}}\right]^T, && (37)
\end{aligned}$$

where Ω stands for H and V, $\boldsymbol{H}_{\Omega_{\pm1},\mathrm{C}_0}$ represents the channel matrix between node C_0 and node $\Omega_{\pm1}$; $\boldsymbol{h}_{\Omega_{\pm1},\Omega_{\pm2}}$ denotes the channel vector betweern node $\Omega_{\pm2}$ and node $\Omega_{\pm1}$ and $\boldsymbol{n}_{\Omega_{\pm1}}$ is the noise vector at node $\Omega_{\pm1}$.

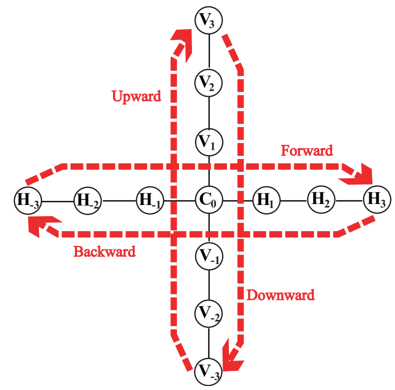
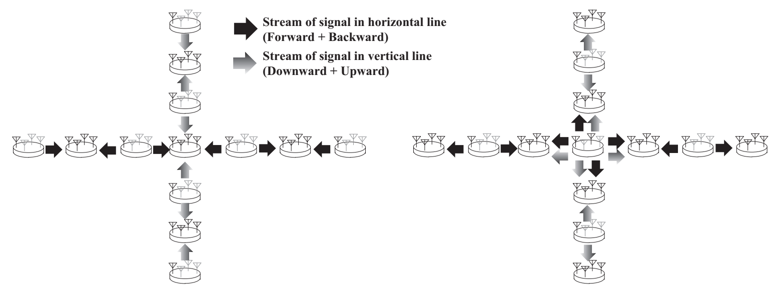


Fig. 4. A 2D mesh topology with $K = 3$.



(a) At time slot n **(b) At time slot** n+1

Fig. 5. Transmission with a 2D MIMO network coding.

In Eq. (35), the transmit signals can be decoded using linear decoding method as the number of linear equations is larger than the number of variables,

$$\hat{\boldsymbol{s}}^{\mathrm{eff}}_{\Omega_{\pm1}} = \left[\hat{\boldsymbol{s}}^T_{\mathrm{C}_0} \;\; \hat{s}_{\Omega_{\pm2}}\right]^T = \boldsymbol{W}^{\mathrm{r}T}\boldsymbol{y}_{\Omega_{\pm1}}, \tag{38}$$

where $\boldsymbol{W}^{\mathrm{r}*} = \left(\boldsymbol{H}^{\mathrm{eff}}_{\Omega_{\pm1}}\boldsymbol{H}^{\mathrm{eff}\;H}_{\Omega_{\pm1}} + \sigma^2\boldsymbol{I}\right)^{-1}\boldsymbol{H}^{\mathrm{eff}}_{\Omega_{\pm1}}$ for a MMSE decoder.

From $\hat{s}^{(1)}_{\mathrm{C}_0}$ and $\hat{s}_{\mathrm{H}_{\pm2}}$, node $\mathrm{H}_{\pm1}$ can decode the desired forward and backward information. Similarly, from $\hat{s}^{(2)}_{\mathrm{C}_0}$ and $\hat{s}_{\mathrm{V}_{\pm2}}$, node $\mathrm{V}_{\pm1}$ can decode the desired downward and upward information. The decoding algorithm which uses the property of network coding and properties shown in Eq. (12) - Eq. (15) is similar to what was explained in the previous part of this paper. (Eq. 9-11, Eq. 24-30)

IV. Interference Aware Cooperative Beamforming

Due to the broadcast characteristic of the wireless channel, a receiver does not only receive its desired signal but also interference signals from surrounding Tx nodes. The farther the interference source, the better the Signal-to-Interference-and-Noise Ratio (SINR). In [4], the architecture of 1D MIMO network coding system has the interference distance of $3d$ where d is the distance between two adjacent nodes. In the proposed architecture of 2D MIMO network coding, the interference distance decreases. Thus, further algorithms should be employed to maximally exploit the performance of 2D MIMO network coding. Owing to the multiple antennas equipped, this problem can be partially solved with array antenna processing algorithm..

In Fig. 6, desired signal and interference signal are categorized. At a receiver, only signals sent from adjacent Tx nodes are desired

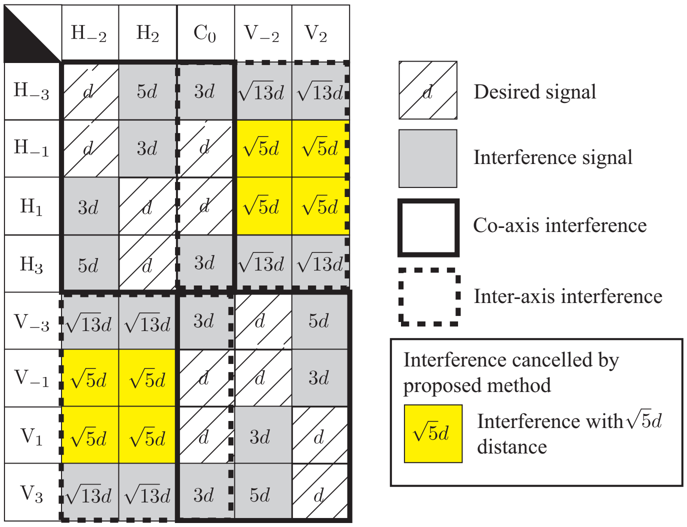


Fig. 6. Signal and interference category.

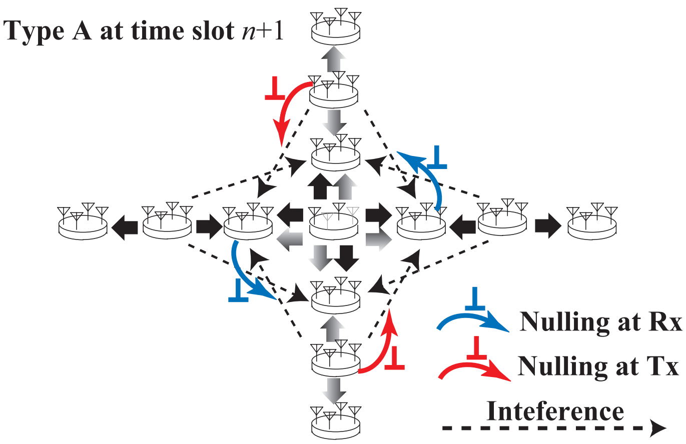


Fig. 7. Cooperative nulling for $\sqrt{5}d$ interference at time slot $n + 1$.

signals and all the others are interference signals. In this paper, the interference from nodes on the same axis (vertical or horizontal) is called as co-axis interference, otherwise the interference is called as inter-axis interference. Node C_0 belongs to both the axes. For co-axis interference, the mininum interference distance is $3d$ which is exactly the minimum interference distance of 1D MIMO network coding. For inter-axis interference, the minimum interference distance reduces to $\sqrt{5}d < 3d$. This kind of interference will severely degrade the performance of 2D MIMO network coding and should be eliminated.

In this paper, the authors deal with the $\sqrt{5}d$ interference at both time n and $n+1$. Also, from this section, Tx node and Rx node are assumed to use all of their four antennas in the transmission. In the following, $\boldsymbol{H}_{\mathcal{R},\mathcal{T}} \in C^{m\times m}$ $(m = 4)$ denotes the channel matrix between Tx node $\mathcal{T}$ and Rx node $\mathcal{R}$.

A. Interference with $\sqrt{5}$d distance ($n + 1^{th}$ time slot)-TypeA

Consider node H_{-1} in Fig. 7. As explained in Section III-B, node H_{-1} receives one stream of network coded data from node H_{-2}, two streams of network coded data from node C_0 (one for vertical line, one for horizontal line). Also, there are two strong interference sources: node $V_{\pm 2}$. The total number of signals (in the range of $\sqrt{5}d$) arriving at node H_{-1} is five. Node H_{-1} which is equipped with only four antennas cannot totally eliminate the intererence while receive its desired signals. To solve this problem, the authors propose a cooperative nulling algorithm that node H_{-1} uses Rx MMSE nulling to partially eliminate interference (strongest interference from node V_{-2}), and node V_2 uses Tx nulling to null out its interference to node H_{-1}. Similar approach is applied at node H_1. Node H_1 uses Rx MMSE nulling to partially eliminate interference (strongest interference from node V_2) and node V_{-2} uses Tx nulling to null out its interference to node H_1. However, Tx nulling at node $V_{\pm 2}$ and Rx MMSE nulling at node $H_{\pm 1}$ will affect each other. Here, an iterative solution to find the Tx weight and Rx weight can be found.

Assume that the Tx weight (for Tx nulling) at node V_{-2} is $\boldsymbol{w}^{\mathrm{t}}_{V_{-2}} \in C^m$, where the superscript t denotes the Tx weight. Interference signal from node V_{-2} to node H_{-1} are partially cancelled using Rx MMSE weight $\boldsymbol{W}^{\mathrm{r}}_{H_{-1}} \in C^{m\times(m-1)}$ at node H_{-1} as follows,

$$\boldsymbol{W}^{\mathrm{r}}_{H_{-1}}{}^{*} = \left(\mathrm{E}\left[\boldsymbol{y}_{H_{-1}}\boldsymbol{y}_{H_{-1}}{}^{H}\right]\right)^{-1}\left[\boldsymbol{h}^{(1)}_{H_{-1},C_0}\ \boldsymbol{h}^{(2)}_{H_{-1},C_0}\ \boldsymbol{h}^{(1)}_{H_{-1},H_{-2}}\right], \quad (39)$$

where $\boldsymbol{y}_{H_{-1}}$ denotes the receive signal vector at node H_{-1} which depends on $\boldsymbol{w}^{\mathrm{t}}_{V_{-2}}$. The Rx MMSE weight in Eq. (39) is the Wiener solution where $\mathrm{E}\left[\boldsymbol{y}_{H_{-1}}\boldsymbol{y}_{H_{-1}}{}^{H}\right]$ is the covariance matrix of receive signal and $\left[\boldsymbol{h}^{(1)}_{H_{-1},C_0}\ \boldsymbol{h}^{(2)}_{H_{-1},C_0}\ \boldsymbol{h}^{(1)}_{H_{-1},H_{-2}}\right]$ is the expectation of the desired signals at node H_{-1}.

Sequentially, the Tx weight $\boldsymbol{w}^{\mathrm{t}}_{V_2} \in C^m$ at node V_2 which is chosen to null its interference to node H_{-1} is given by,

$$\boldsymbol{w}^{\mathrm{t}}_{V_2} = \mathrm{null}\left(\boldsymbol{W}^{\mathrm{r}}_{H_{-1}}{}^{T}\boldsymbol{H}_{H_{-1},V_2}\right). \quad (40)$$

Now, with the Tx weight at node V_2, the Rx weight $\boldsymbol{W}^{\mathrm{r}}_{H_1} \in C^{m\times 3}$ at node H_1 and the Tx weight $\boldsymbol{w}^{\mathrm{t}}_{V_{-2}} \in C^m$ at node V_{-2} can be similarly derived as follows,

$$\boldsymbol{W}^{\mathrm{r}}_{H_1}{}^{*} = \left(\mathrm{E}\left[\boldsymbol{y}_{H_1}\boldsymbol{y}_{H_1}{}^{H}\right]\right)^{-1}\left[\boldsymbol{h}^{(1)}_{H_1,C_0}\ \boldsymbol{h}^{(2)}_{H_1,C_0}\ \boldsymbol{h}^{(1)}_{H_1,H_2}\right], \quad (41)$$

$$\boldsymbol{w}^{\mathrm{t}}_{V_{-2}} = \mathrm{null}\left(\boldsymbol{W}^{\mathrm{r}}_{H_1}{}^{T}\boldsymbol{H}_{H_1,V_{-2}}\right). \quad (42)$$

Here, it is easy to see that the weight calculation in Eq. (39) - Eq. (42) can be performed in an iterative manner as shown in Algorithm 1, where Δ denotes a certain threshold to stop the iterative loop.

Similarly, for $\sqrt{5}d$ interference cancellation at node $V_{\pm 1}$, the same cooperative nulling process is applied at four nodes $V_{\pm 1}$ and $H_{\pm 2}$.

B. Interference with $\sqrt{5}$d distance (n^{th} time slot)-Type B

This type of interference is plotted in Fig. 8. In this case, nodes suffered from the $\sqrt{5}d$ interference are $V_{\pm 2}$ and $H_{\pm 2}$. Consider node V_2 which receives two desired signals from node V_1 and V_3, and two interference signals from node $H_{\pm 1}$. Node V_2 which is equipped with four antennas can cancel the two interference signals itself. However, this way of interference cancellation would decrease the diversity order of desired signals at node V_2. Here, the cooperative nulling algorithm can be applied at four nodes $V_{\pm 2}$ and $H_{\pm 1}$ for interference cancellation at $V_{\pm 2}$ and at four nodes $H_{\pm 2}$ and $V_{\pm 1}$ for interference cancellation at $H_{\pm 2}$. Algorithm 2 summarizes the cooperative nulling process at $V_{\pm 2}$, where $\mathrm{w}^{\mathrm{t}}_{H_{\pm 1}} \in C^m$ denote the Tx weight at node $H_{\pm 1}$ and $\mathrm{W}^{\mathrm{r}}_{V_{\pm 2}} \in C^{m\times 2}$ denote the Rx weight at node $V_{\pm 2}$ respectively.

Algorithm 1 Nulling interference at $\mathrm{H}_{\pm1}$ (time $n+1$)

Initialisation

$$\boldsymbol{w}^{\mathrm{t}}_{\mathrm{V}_{-2}} \tag{43}$$
$$\Delta \tag{44}$$

repeat

Cooperative nulling

$$\begin{aligned}
{\boldsymbol{W}^{\mathrm{r}}_{\mathrm{H}_{-1}}}^{*} &= \left(\mathrm{E}\left[\boldsymbol{y}_{\mathrm{H}_{-1}}{\boldsymbol{y}_{\mathrm{H}_{-1}}}^{H}\right]\right)^{-1}\left[\boldsymbol{h}^{(1)}_{\mathrm{H}_{-1},\mathrm{C}_0}\ \boldsymbol{h}^{(2)}_{\mathrm{H}_{-1},\mathrm{C}_0}\ \boldsymbol{h}^{(1)}_{\mathrm{H}_{-1},\mathrm{H}_{-2}}\right]\\
\boldsymbol{w}^{\mathrm{t}}_{\mathrm{V}_2} &= \mathrm{null}\left({\boldsymbol{W}^{\mathrm{r}}_{\mathrm{H}_{-1}}}^{T}\boldsymbol{H}_{\mathrm{H}_{-1},\mathrm{V}_2}\right)\\
{\boldsymbol{W}^{\mathrm{r}}_{\mathrm{H}_{1}}}^{*} &= \left(\mathrm{E}\left[\boldsymbol{y}_{\mathrm{H}_{1}}{\boldsymbol{y}_{\mathrm{H}_{1}}}^{H}\right]\right)^{-1}\left[\boldsymbol{h}^{(1)}_{\mathrm{H}_{1},\mathrm{C}_0}\ \boldsymbol{h}^{(2)}_{\mathrm{H}_{1},\mathrm{C}_0}\ \boldsymbol{h}^{(1)}_{\mathrm{H}_{1},\mathrm{H}_{2}}\right]\\
\boldsymbol{w}^{\mathrm{t}}_{\mathrm{V}_{-2}} &= \mathrm{null}\left({\boldsymbol{W}^{\mathrm{r}}_{\mathrm{H}_{1}}}^{T}\boldsymbol{H}_{\mathrm{H}_{1},\mathrm{V}_{-2}}\right)
\end{aligned}$$

Interference power

$$\delta_{\mathrm{H}_{-1}} = \left\|{\boldsymbol{W}^{\mathrm{r}}_{\mathrm{H}_{-1}}}^{T}\sum \boldsymbol{H}_{\mathrm{H}_{-1},\mathrm{V}_{\pm2}}\boldsymbol{w}^{\mathrm{t}}_{\mathrm{V}_{\pm2}}\right\|_2^2 \tag{45}$$
$$\delta_{\mathrm{H}_{1}} = \left\|{\boldsymbol{W}^{\mathrm{r}}_{\mathrm{H}_{1}}}^{T}\sum \boldsymbol{H}_{\mathrm{H}_{1},\mathrm{V}_{\pm2}}\boldsymbol{w}^{\mathrm{t}}_{\mathrm{V}_{\pm2}}\right\|_2^2 \tag{46}$$

until $\delta_{\mathrm{H}_{\pm1}} \le \Delta$

Algorithm 2 Nulling interference at $\mathrm{V}_{\pm2}$ (time n)

Initialisation

$$\boldsymbol{w}^{\mathrm{t}}_{\mathrm{H}_{-1}} \tag{47}$$
$$\Delta \tag{48}$$

repeat

Cooperative nulling

$$\begin{aligned}
{\boldsymbol{W}^{\mathrm{r}}_{\mathrm{V}_{2}}}^{*} &= \left(\mathrm{E}\left[\boldsymbol{y}_{\mathrm{V}_{2}}{\boldsymbol{y}_{\mathrm{V}_{2}}}^{H}\right]\right)^{-1}\left[\boldsymbol{h}^{(1)}_{\mathrm{V}_{2},\mathrm{V}_1}\ \boldsymbol{h}^{(1)}_{\mathrm{V}_{2},\mathrm{V}_3}\right]\\
\boldsymbol{w}^{\mathrm{t}}_{\mathrm{H}_1} &= \mathrm{null}\left({\boldsymbol{W}^{\mathrm{r}}_{\mathrm{V}_{2}}}^{T}\boldsymbol{H}_{\mathrm{V}_{2},\mathrm{H}_1}\right)\\
{\boldsymbol{W}^{\mathrm{r}}_{\mathrm{V}_{-2}}}^{*} &= \left(\mathrm{E}\left[\boldsymbol{y}_{\mathrm{V}_{-2}}{\boldsymbol{y}_{\mathrm{V}_{-2}}}^{H}\right]\right)^{-1}\left[\boldsymbol{h}^{(1)}_{\mathrm{V}_{-2},\mathrm{V}_{-1}}\ \boldsymbol{h}^{(1)}_{\mathrm{V}_{-2},\mathrm{V}_{-3}}\right]\\
\boldsymbol{w}^{\mathrm{t}}_{\mathrm{H}_{-1}} &= \mathrm{null}\left({\boldsymbol{W}^{\mathrm{r}}_{\mathrm{V}_{-2}}}^{T}\boldsymbol{H}_{\mathrm{V}_{-2},\mathrm{H}_{-1}}\right)
\end{aligned}$$

Interference power

$$\delta_{\mathrm{V}_{2}} = \left\|{\boldsymbol{W}^{\mathrm{r}}_{\mathrm{V}_{2}}}^{T}\sum \boldsymbol{H}_{\mathrm{V}_{2},\mathrm{H}_{\pm1}}\boldsymbol{w}^{\mathrm{t}}_{\mathrm{H}_{\pm1}}\right\|_2^2 \tag{49}$$
$$\delta_{\mathrm{V}_{-2}} = \left\|{\boldsymbol{W}^{\mathrm{r}}_{\mathrm{V}_{-2}}}^{T}\sum \boldsymbol{H}_{\mathrm{V}_{-2},\mathrm{H}_{\pm1}}\boldsymbol{w}^{\mathrm{t}}_{\mathrm{H}_{\pm1}}\right\|_2^2 \tag{50}$$

until $\delta_{\mathrm{V}_{\pm2}} \le \Delta$

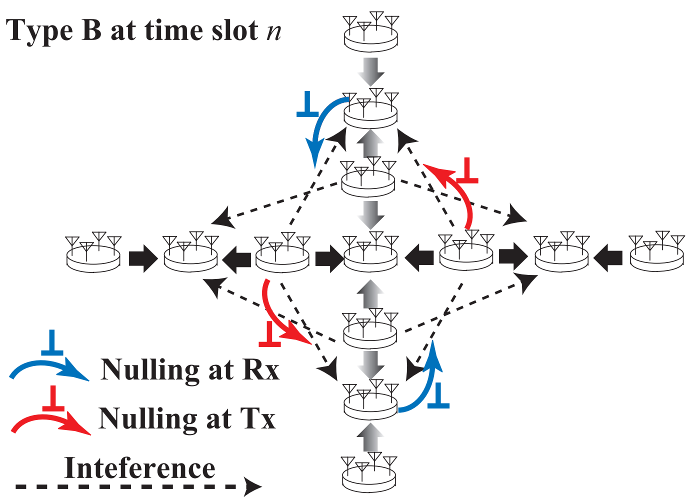


Fig. 8. Cooperative nulling for $\sqrt{5}d$ interference at time slot n.

C. Discussion

In this paper, the cooperative nulling algorithm requires cooperative nodes to exchange information about the channel state and beamforming matrix. It can be done by transmission of preambles at Tx nodes. Numerical results reveal that solutions of the optimal weight converge very fast. Thus, the authors recommend that the exchange of optimal weight and channel matrix can be done in several initial time slots. Assuming an environment where the variation of the channels occur not so fast, after an optimal solution is found, nodes can employ an adaptive filtering algorithm for tracking of the optimal solution at every time slot.

V. End-to-End Capacity Evaluation

The method for calculation of end-to-end capacity is given in this section. Here, the authors generalize the system model in Sec. III and Sec. IV for the simplification of notation. Assume that at a specific time slot, there are a set $\Omega = \{\mathcal{T}_1, \ldots, \mathcal{T}_N\}$ of N sources of transmit signals (STSs) in the network. If a Tx node transmits two signals by using two antennas e.g. node C_0, it is considered as two STSs. STS i transmits signal s_i by using its Tx weight $\boldsymbol{w}^{\mathrm{t}}_i \in C^m$. At the receiver $\mathcal{R}$ which has Rx weight $\boldsymbol{W}^{\mathrm{r}}$, the receive signal can be given by,

$$\boldsymbol{y} = \sum_i \sqrt{\alpha_i}\boldsymbol{H}_{\mathcal{R},\mathcal{T}_i}\boldsymbol{w}^{\mathrm{t}}_i s_i + \boldsymbol{n}, \tag{51}$$

where α_i is the transmit power of signal s_i, $\boldsymbol{H}_{\mathcal{R},\mathcal{T}_i}$ represents the channel matrix between STS i and receiver $\mathcal{R}$, $\boldsymbol{n}$ denotes the noise vector at the receiver $\mathcal{R}$. If the Tx node of the STS i performs Tx beamforming, $\boldsymbol{w}^{\mathrm{t}}_i$ is given in Sec. IV, otherwise it is the first column of the identity matrix $\boldsymbol{I}_m$. The only special case is node C_0. As explained above, when node C_0 is a Tx node, it can be seen as two STSs. The Tx weights of this two STSs are respectively the first and second column of the identity matrix $\boldsymbol{I}_m$.

To detect a desired signal s_p ($p \in 1, \ldots, N$), Rx node multiplies a linear weight vector ${\boldsymbol{w}^{\mathrm{r}}_p}^T$ to the receive signal. This Wiener solution of this weight is given as follows,

$$\begin{aligned}
{\boldsymbol{w}^{\mathrm{r}}_p}^{*} &= \left(\mathrm{E}\left[\boldsymbol{y}\boldsymbol{y}^H\right]\right)^{-1}\sqrt{\alpha_p}\boldsymbol{H}_{\mathcal{R},\mathcal{T}_p}\boldsymbol{w}^{\mathrm{t}}_p && (52)\\
&= \left(\sum_i \alpha_i \boldsymbol{h}_{\mathcal{R},\mathcal{T}_i}\boldsymbol{h}^H_{\mathcal{R},\mathcal{T}_i} + \sigma^2\boldsymbol{I}_m\right)^{-1}\sqrt{\alpha_p}\boldsymbol{H}_{\mathcal{R},\mathcal{T}_p}\boldsymbol{w}^{\mathrm{t}}_p. && (53)
\end{aligned}$$

Consequently, the estimate of s_p ($p \in \{1, \ldots, N\}$) is given by,

$$\begin{aligned}
\hat{s}_p &= {\boldsymbol{w}^{\mathrm{r}}_p}^T\boldsymbol{y} && (54)\\
&= \sqrt{\alpha_p}{\boldsymbol{w}^{\mathrm{r}}_p}^T\boldsymbol{H}_{\mathcal{R},\mathcal{T}_p}\boldsymbol{w}^{\mathrm{t}}_p s_p + \sqrt{\alpha_i}{\boldsymbol{w}^{\mathrm{r}}_p}^T\sum_{i\neq p}\boldsymbol{H}_{\mathcal{R},\mathcal{T}_i}\boldsymbol{w}^{\mathrm{t}}_i s_i + {\boldsymbol{w}^{\mathrm{r}}_p}^T\boldsymbol{n}.
\end{aligned}$$

A. End-to-end capacity

The capacity of the signal s_p is given by,

$$C^p_{\mathcal{R}} = \log_2\left(1 + \frac{\alpha_p\left\|{\boldsymbol{w}^{\mathrm{r}}_p}^T\boldsymbol{H}_{\mathcal{R},\mathcal{T}_p}\boldsymbol{w}^{\mathrm{t}}_p\right\|_2^2}{\sum_{i\neq p}\alpha_i\left\|{\boldsymbol{w}^{\mathrm{r}}_p}^T\boldsymbol{H}_{\mathcal{R},\mathcal{T}_i}\boldsymbol{w}^{\mathrm{t}}_i\right\|_2^2 + \alpha_{\mathrm{NC}}\sigma^2\left\|\boldsymbol{w}_p\right\|_2^2}\right), \tag{55}$$

where $\alpha_{\mathrm{NC}} = \frac{1}{\rho} = 2$ is a factor which compensates for the shaping loss of the network coding.

If the Rx node $\mathcal{R}$ is on the horizontal line, there exist p_{F} and $p_{\mathrm{B}} \in \{1, \ldots, N\}$ such that $s_{p_{\mathrm{F}}}$ and $s_{p_{\mathrm{B}}}$ are the two desired signals which contain the desired forward and backward information respectively. Therefore, the forward and backward capacity at node $\mathcal{R}$ are given by,

$$C_{\mathcal{R}}^{p_{\mathrm{F}}} = \log_2\left(1 + \frac{\alpha_{p_{\mathrm{F}}} \left\| {\boldsymbol{w}_{p_{\mathrm{F}}}^{\mathrm{r}}}^{T} \boldsymbol{H}_{\mathcal{R},\mathcal{T}_{p_{\mathrm{F}}}} \boldsymbol{w}_{p_{\mathrm{F}}}^{\mathrm{t}} \right\|_2^2}{\sum_{i \neq p_{\mathrm{F}}} \alpha_i \left\| {\boldsymbol{w}_{p_{\mathrm{F}}}^{\mathrm{r}}}^{T} \boldsymbol{H}_{\mathcal{R},\mathcal{T}_i} \boldsymbol{w}_i^{\mathrm{t}} \right\|_2^2 + 2\sigma^2 \left\| \boldsymbol{w}_{p_{\mathrm{F}}}^{\mathrm{r}} \right\|_2^2}\right) \quad (56)$$

$$C_{\mathcal{R}}^{p_{\mathrm{B}}} = \log_2\left(1 + \frac{\alpha_{p_{\mathrm{B}}} \left\| {\boldsymbol{w}_{p_{\mathrm{B}}}^{\mathrm{r}}}^{T} \boldsymbol{H}_{\mathcal{R},\mathcal{T}_{p_{\mathrm{B}}}} \boldsymbol{w}_{p_{\mathrm{B}}}^{\mathrm{t}} \right\|_2^2}{\sum_{i \neq p_{\mathrm{B}}} \alpha_i \left\| {\boldsymbol{w}_{p_{\mathrm{B}}}^{\mathrm{r}}}^{T} \boldsymbol{H}_{\mathcal{R},\mathcal{T}_i} \boldsymbol{w}_i^{\mathrm{t}} \right\|_2^2 + 2\sigma^2 \left\| \boldsymbol{w}_{p_{\mathrm{B}}}^{\mathrm{r}} \right\|_2^2}\right) , \quad (57)$$

Similarly, if the Rx node $\mathcal{R}$ is on the vertical line, there exist p_{D} and $p_{\mathrm{U}} \in \{1, \ldots, N\}$ such that $s_{p_{\mathrm{D}}}$ and $s_{p_{\mathrm{U}}}$ are the two desired signals which contain the desired downward and upward information respectively. Therefore, the downward and upward capacity at node $\mathcal{R}$ are given by,

$$C_{\mathcal{R}}^{p_{\mathrm{D}}} = \log_2\left(1 + \frac{\alpha_{p_{\mathrm{D}}} \left\| {\boldsymbol{w}_{p_{\mathrm{D}}}^{\mathrm{r}}}^{T} \boldsymbol{H}_{\mathcal{R},\mathcal{T}_{p_{\mathrm{D}}}} \boldsymbol{w}_{p_{\mathrm{D}}}^{\mathrm{t}} \right\|_2^2}{\sum_{i \neq p_{\mathrm{D}}} \alpha_i \left\| {\boldsymbol{w}_{p_{\mathrm{D}}}^{\mathrm{r}}}^{T} \boldsymbol{H}_{\mathcal{R},\mathcal{T}_i} \boldsymbol{w}_i^{\mathrm{t}} \right\|_2^2 + 2\sigma^2 \left\| \boldsymbol{w}_{p_{\mathrm{D}}}^{\mathrm{r}} \right\|_2^2}\right) \quad (58)$$

$$C_{\mathcal{R}}^{p_{\mathrm{U}}} = \log_2\left(1 + \frac{\alpha_{p_{\mathrm{U}}} \left\| {\boldsymbol{w}_{p_{\mathrm{U}}}^{\mathrm{r}}}^{T} \boldsymbol{H}_{\mathcal{R},\mathcal{T}_{p_{\mathrm{U}}}} \boldsymbol{w}_{p_{\mathrm{U}}}^{\mathrm{t}} \right\|_2^2}{\sum_{i \neq p_{\mathrm{U}}} \alpha_i \left\| {\boldsymbol{w}_{p_{\mathrm{U}}}^{\mathrm{r}}}^{T} \boldsymbol{H}_{\mathcal{R},\mathcal{T}_i} \boldsymbol{w}_i^{\mathrm{t}} \right\|_2^2 + 2\sigma^2 \left\| \boldsymbol{w}_{p_{\mathrm{U}}}^{\mathrm{r}} \right\|_2^2}\right) , \quad (59)$$

Therefore, the end-to-end capacity of the forward, backward, downward and upward flows of information can be calculated by the following formulas,

$$C_{\mathrm{e2e}}^{\mathrm{F}} = \min_{\mathcal{R} \in \Omega_{\mathrm{H}}} \mathrm{E}\left[C_{\mathcal{R}}^{p_{\mathrm{F}}}\right] \quad (60)$$

$$C_{\mathrm{e2e}}^{\mathrm{B}} = \min_{\mathcal{R} \in \Omega_{\mathrm{H}}} \mathrm{E}\left[C_{\mathcal{R}}^{p_{\mathrm{B}}}\right] \quad (61)$$

$$C_{\mathrm{e2e}}^{\mathrm{D}} = \min_{\mathcal{R} \in \Omega_{\mathrm{V}}} \mathrm{E}\left[C_{\mathcal{R}}^{p_{\mathrm{D}}}\right] \quad (62)$$

$$C_{\mathrm{e2e}}^{\mathrm{U}} = \min_{\mathcal{R} \in \Omega_{\mathrm{V}}} \mathrm{E}\left[C_{\mathcal{R}}^{p_{\mathrm{U}}}\right], \quad (63)$$

where Ω_{H}, Ω_{V} are two set of nodes on the horizontal and vertical lines respectively. Finally, the average end-to-end capacity per flow can be defined as,

$$C_{\mathrm{e2e}} = \beta \frac{C_{\mathrm{e2e}}^{\mathrm{F}} + C_{\mathrm{e2e}}^{\mathrm{B}} + C_{\mathrm{e2e}}^{\mathrm{D}} + C_{\mathrm{e2e}}^{\mathrm{U}}}{4} \ [\mathrm{bps/Hz/flow}], \quad (64)$$

where β is the network spectral efficiency coefficient explained in Section II.

B. Transmit power

Normally, the transmit power is normalized such that each node transmits with unit power ($\alpha_i = 1$). The total transmit power of a mesh network given in Fig. 4 is $4K + 1$. However, in 2D MIMO network coding, as node C_0 has two STSs, the number of STSs in the 2D MIMO mesh network is $4K + 2$. Hence, for a fair comparison with other algorithms, in 2D MIMO network coding, each STS is assumed to be transmitted with equal power

$$\alpha_i = \frac{4K + 1}{4K + 2} \ \forall i. \quad (65)$$

VI. Numerical Analysis

Numerical simulations are conducted to validate the proposed two dimensional MIMO network coding. A path between any two adjacent nodes is called a link. In the simulation the average Signal-to-Noise Ratio (SNR) per receive antenna is the same for every links. The channel fading is assumed to be Rayleigh and the environment is quasi-static such that the channel matrix does not change during a period of one time slot.

To compare with conventional methods, we conducted simulation with multichannel SISO using 2 channels in each axis ($\beta = \frac{1}{4}$, Fig. 2(a), one antenna per node), link-by-link 4×4 multichannel MIMO using 2 channels in each axis ($\beta = \frac{1}{4}$, Fig. 2(a), four antennas per node), one dimensional MIMO network coding with single channel in each axis ($\beta = \frac{1}{2}$, Fig. 2(b), four antenna per node) and the propsed two dimensional MIMO network coding ($\beta = 1$, Fig. 2(c), four antennas per node).

Numerical analysis is divided into two parts. In the first part, it is assumed that there is no interference from Tx nodes with distances apart from the Rx larger than d . The main purpose of this part is to show the effectiveness of the network coding algorithm in combination with MIMO in a 2D mesh network. In the second part, the authors considered a more realistic environment with pathloss exponent γ and all the inteferences were included to the receive signal. The second part is to show the benefit of the proposed cooperative nulling method.

A. Without interference from long distance nodes

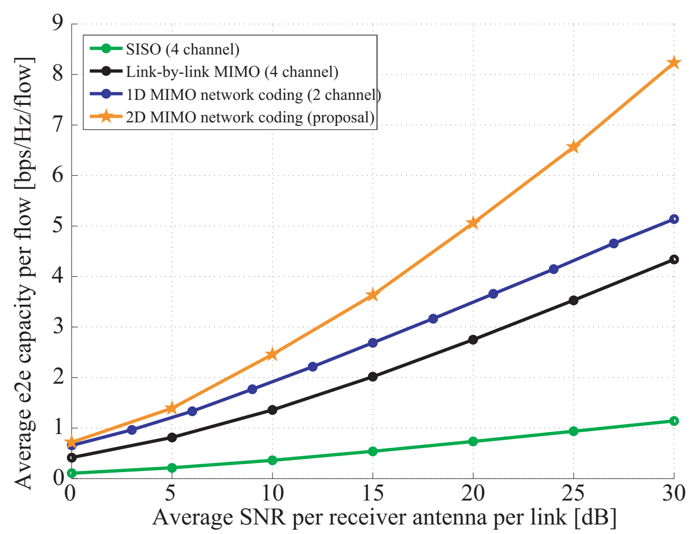


Fig. 9. End-to-end capacity in the case interference from long distance nodes are not included.

Figure 9 shows the average end-to-end capacity per flow with respect to average SNR per receive antenna per link of conventional and proposed methods in a 2D mesh network. The performance of multichannel SISO is bad due to the inefficiency in usage of time and frequency resources. This inefficiency can be improved by the introduction of MIMO into each link due to the effect of spatial multiplexing. It can be seen from Fig. 9 that 4×4 link-by-link MIMO goes beyond the four-fold increase in end-to-end capacity compared to that of SISO. The inefficiency in usage of time and frequency can be improved by introduction of network coding. 1D MIMO network coding in 2D network has a better performance compared to link by link MIMO by 1 bps/Hz/flow. However, the scheme which has the best performance is the proposed 2D MIMO network coding scheme. The proposed algorithm achieves an 8-fold gain in end-to-end capacity with respect to conventional multichannel SISO. This significant gain is due to the spatially efficient usage of frequency and time of this scheme as explained in previous sections. Consider a system with total bandwidth 5MHz at 30dB SNR, the equivalent capacity per flow of these schemes are: 6Mbps (multichannel SISO), 22Mbps (multichannel MIMO), 26Mbps (1D MIMO network coding) and 41Mbps (2D network coding).

B. With interference from long distance nodes

In a realistic environment, it is hard to ignore the interference signals. Consider an environment with atttenuation coefficient $\gamma = 3.5$, the distance dependant average Signal-to-Interference Ratio (SIR) is given in Table II and the end-to-end capacity is shown in Fig. 10. It is easy to see that interference signals severely degrade capacity performance of all schemes. Especially, the proposed 2D network coding algorithm has worse capacity performance than that of the 1D MIMO network coding at high SNR area. It is due to the strong interference from nodes $\sqrt{5}d$ apart.

TABLE II
DISTANCE DEPENDANT AVERAGE SIR ($\gamma = 3.5$).

SIR	$\sqrt{5}d$	$3d$	$\sqrt{13}d$	5d
[dB]	12	16	19	24

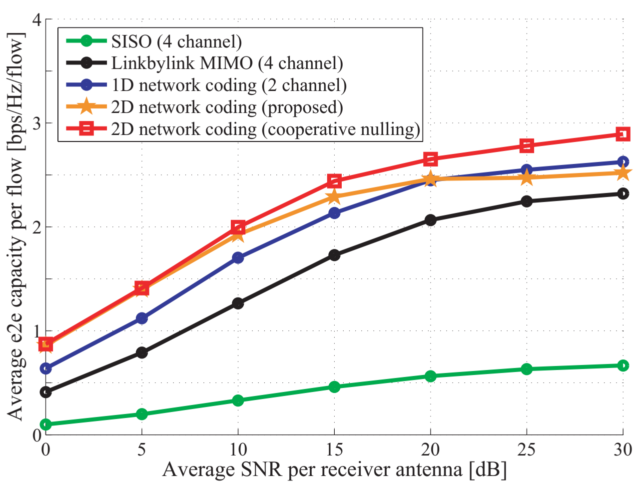


Fig. 10. End-to-end capacity ($\gamma = 3.5$) in the case interference from long distance nodes are included.

Capacity performance after the application of proposed cooperative nulling algorithms is also shown in Fig. 10. At high SNR area, the cooperative nulling algorithm can improve the capacity performance by almost 0.5 bps/Hz/flow. In low SNR area, due to the fact that the noise is dominant over the interference, the introduction of cooperative nulling does not improve capacity performance. Consider a system with total bandwidth 5MHz at 30dB SNR, the equivalent capacity per flow of all schemes in inteference environment are: 3Mbps (multichannel SISO), 11.5Mbps (multichannel MIMO), 13Mbps (1D network coding) and 14.5Mbps (2D network coding with cooperative nulling).

VII. CONCLUSION

The combination of MIMO and network coding for one dimensional (1D) topology in wireless mesh network has been proposed in recent literatures. The technique supplies higher network capacity compared to that of conventional schemes. In this paper, the authors extend MIMO network coding to two dimensional (2D) topology. Owing to the efficient sharing of frequency of network coding and co-channel interference cancellation ability of MIMO, the proposed technique provides a significant gain to end-to-end network capacity. Furthermore, in a 2D mesh network where inter-axis interferences from long distance nodes cannot be ignored, the authors proposed cooperative MIMO beamforming algorithm to suppress interference signals can be suppressed. Simulation results show the good performance of the proposed methods.

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