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A Formal Theory on Decision Making with Interperception

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Preface

In this paper we deal with situations of decision making with *interperception.* That is, each decision maker in a situation has perceptions of what the others perceive about the situation, perceptions of what the others perceive about the others' perceptions of the situation, and so on. In spite of the existence of the aspect of interperception in the real world, the aspect has not been satisfactorily coped with in existing theories for decision making such as game theory, metagame theory, and hypergame theory. Of course, those theories have greatly contributed in the field of analyses of complex situations of decision making, but it is also the fact that there are contradictions between those theories and the real world. We believe that the contradictions are caused by the restricted treatments of the aspect of interperception in those theories, and that overcoming the contradictions leads us to a fruitful theory on decision making. We believe, moreover, that the theory provides us detailed and exact descriptions of situations of decision making and great insight about behaviors of decision makers in the situations. To construct such a formal theory for decision making with interperception is the purpose of this paper.

The elements that make the treatments of the aspect of interperception restricted in the existing theories are the assumption of *completeness of information* and that of *economic rationality.* The former requires that the components of a situations of decision making, the set of all decision makers in the situation, that is, the set of all possible outcomes of the situation, and the preferences of the decision makers for the possible outcomes of the situations, are *common knowledge* among all decision makers in the situation. In other words, decision makers in the situation of decision making correctly perceive the components, and they believe that the components are correctly perceived by all decision makers, and they think that all decision makers believe that the components are correctly perceived by all decision makers, and so on. The latter implies that the preferences of a decision maker for the possible outcomes are consistent with the profit that the decision maker obtains in each outcome. That is, each decision maker prefers a more profitable outcome to a less profitable one.

Our construction of a formal theory for decision making with interperception begins from the elimination of these assumptions. In this paper we use the concepts, *schemes* and *emotions,* to treat the properties, *incompleteness of information* and *economic irrationality*, of decision making, respectively. We provide formal frameworks for dealing with not only those properties but also *exchanges of information and changes of perceptions caused by the exchanges* for satisfactory understanding of the situations of decision making with interperception.

Then, using the frameworks, we propose concepts such as *inside common knowledge*, *integration of perceptions*, and *stability of emotions*, that are required to examine the situations. The concept of inside common knowledge describes the individual part of the concept of common knowledge, and it enable us to express more detailed structure of decision makers' perceptions. The concept of integration of perceptions provides us a way to merge many schemes into one, and thus it enlarges the extent of application of our theory. Moreover, a realistic structure of schemes of emotions is given by the concept of stability of emotions.

Employing the frameworks and the concepts, furthermore, we analyze *competitive* situations and *cooperative* situations of decision making. There are four topics that we cope with in this paper: *outcomes of competitive situations, information in decision making, a solution concept involving emotions,* and *deadlock in a meeting.* In terms of the first topic, we show that positive emotions play important roles when decision makers try to achieve the outcome that is hoped by all decision makers to be realized. We focus on *deception* by decision makers regarding the second topic, and provide sufficient conditions for strategic information exchanges to be failed. The third topic relates to a solution concept, called *emotional equilibrium.* We define an emotional equilibrium as an outcome that does not cause any changes of decision makers' perceptions, and we give a sufficient condition for an outcome to be an emotional equilibrium. In the last topic, we deal with *deadlock* in a cooperative situation of decision making, in which no decision maker conveys information to the others. It is shown that stability of decision makers' emotions are essential for a meeting *not* to reach a deadlock. These analyses in this paper show the contrasts between decision making with interperception and that without interperception.

This paper, entitled by *"A Formal Theory for Decision Making with Interperception,"* is supervised by Professor Bunpei Nakano, and the frameworks, concepts, and propositions provided in this paper are developed in cooperation with Dr. Shingo Takahashi. I am most grateful to them, whose advice on the issues in this paper is apt and telling. I would also like to thank Dr. Tadashi Yamamoto, Dr. Toshitami Matsumoto and Mr. Tetsuya Abe for helpful discussions on the topic of this paper. Moreover, a special note of gratitude is extended to Ms. Michiko Urata for her supports on clerical works. Finally, I wish to express my gratitude to my parents, Kenzou Inohara and Atsuko Inohara, and my brother, Shigekazu Inohara for their unstinting, financial and mental, support.

Contents

Chapter 1

Introduction

Decision Making with Interperception

In this paper we deal with situations of decision making. When a person takes part in a situation of decision making as a decision maker, he/she frequently finds that other decision makers also participate in the situation and that actions of the decision makers are mutually interactive: a decision maker gets profits depending on not only his/her action but also the others' actions. For example, the profit of a company competing with another in the market of a good is determined by the price, the quality, the output, and so on, of the opponent's product as well as those of its own. Another example is a situation of group decision making by using a vote. The decision is usually made referring to not only the selections of a special voter but also the selection of all of the other voters. In such a situation a decision maker cares for the actions of the other decision makers to make the outcome of the situation preferable.

We can classify the situations of decision making into two categories in terms of characteristics of decision makers involved in the situations. One is the set of situations in which decision makers are mutually *competitive*. The example of competing companies above is included in this category. In a situation in this category each decision maker tries to get a bigger profit for him/herself. We can often express a situation in this category with a normal form game [23, 73]: a class of the set of all decision makers in the situation, for each decision maker the set of all actions that he/she can can select in the situation, and for each decision maker the list of the profits that he/she obtains in the possible outcomes in the situation. The other category is the set of *cooperative* situations. Decision makers involved in a situation in this category intend to make the outcome of the situation preferable as a whole. This category includes, for example, situations of group decision making by using a vote. The situations can often be expressed with simple games [72, 80]. A simple game is a pair of a set of all decision makers and a set of winning coalitions. The decision makers have a choice problem: a set of all possible alternatives, and try to select one out of the set referring to their preferences for the alternatives. A winning coalition can totally control the selection if the decision makers in the coalition properly cooperate. In this paper we treat only the situations that can be described by using normal form games or simple games in spite of the existence of other various types of situations: situations that can be expressed by games in extensive form, games in coalitional form, frameworks for bargaining problems, frameworks for cost sharing problems, frameworks for surplus sharing problems, and so on [23, 72].

When actions of decision makers in a situation are mutually interactive, whether the situation is competitive or cooperative, each decision maker often tries to know what the others perceive in terms of the decision situation, how the others select actions by using the perceptions, what the actual actions of the others are, and so on, in order to choose an appropriate action. If a company is competing in the market of a good, to make a big hit in the market, the company tries to know which companies are competing in the market, and their perceptions of the performance of their products sold in the market, the products that they will be able to manufacture in the future, the way that they select the products to be sold, the products that they actually choose, and so on. A voter, similarly, tries to know who are the other voters and their perceptions of which coalitions are winning. He/she, moreover, should try to know correctly their perceptions of the possible alternatives and their preferences for the alternatives, in order to make a preferable choice for the group of the voters. We should notice that each decision maker not only tries to know those components of the situation but also knows that the others also try to know the components, that another decision maker knows that the others try to know the components, and so on. In other word, that each decision maker in a situation is trying to know the components of the situation is common knowledge [1, 24, 62, 63] among all of the decision makers in the situation. They, moreover, often misperceive in spite of their efforts to perceive correctly decision makers' perceptions of the situation and of each other. In this paper we call this aspect of decision makers *interperception* by the decision makers.

Interperception and Existing Theories

In spite of the existence of the aspect of interperception by decision makers in the real world, existing theories for decision making deal with the aspect only in limited extent. Game theory [23, 73] usually treat situations with complete information [23], that is, the situations whose all components are common knowledge among the participants to the situation. In a competitive context, for example, game theory regards companies in a market as decision makers who correctly perceive which companies are taking part in the market, all possible actions of the companies, and the profits of the companies in the possible outcomes. Similarly, in a cooperative context in game theory, all of the voters in the situation, all of the winning coalitions, all of the possible alternatives, and all of the preferences of the voters for the alternatives are correctly perceived by the voters. The assumption of complete information means that decision makers in a situation never misperceive the components of the situation. In game theory another element that restricts the treatments of the aspect of interperception by decision makers is the assumption of economic rationality. Under the assumption each decision maker is treated as to try to achieve an outcome as profitable for him/herself as possible. For instance, in game theory each company in a market is assumed to be trying to get a bigger profit, and each voter is dealt with as to intend to make the selection more profitable for him/herself. The assumption of economic rationality means that game theory treats only the situations in which decision makers never misperceive the way that each decision maker selects an action.

Metagame theory [3, 41, 42] also has the assumptions of complete information and economic rationality. A difference between game theory and metagame theory is what decision makers select. While each decision maker in game theory selects an action out of the set of all possible ones, each decision maker in metagame theory chooses a correspondence that indicates an action of the decision maker for each possible list of actions of the others. Although decision makers in a situation in metagame theory select a correspondence instead of an action, they never misperceive the components of the situation and the ways of selection as in game theory. In hypergame theory [4, 5, 6, 22, 100] decision makers in a situation may misperceive the components of the situation, that is, the decision makers may incorrectly perceive of who are involved in the situation, which actions are available for each decision maker, and how each decision maker estimates the outcomes of the situation. Though decision makers in hypergame theory may misperceive the components of the situation, they never misperceive the ways that the decision makers choose their actions because of the assumption of economic rationality. Even in a hypergame each decision maker tries to make the outcome of the situation more profitable for him/herself, and that each of the decision makers in the situation tries to achieve such an outcome is assumed to be common knowledge among the decision makers.

It is true that the assumption of economic rationality is one of the most important and contributing concepts in the existing theories for decision making. Thanks to the assumption, various solution concepts, especially, the Nash equilibrium [23] in game theory, the metagame equilibrium [41] in metagame theory, and the sequential stability [100] in hypergame theory are proposed and justified. It is also true, however, that validity of the assumption is a controversial issue. According to an experiment, many persons involved in the situation known as "prisoners' dilemma" choose an action different from the one suggested in game theory by using the assumption of economic rationality [23]. This contradiction between the real world and game theory cannot have been overcome even in metagame theory and hypergame theory. In metagame theory they try to interpret the selection by the persons in the experiment by using the concept of metagame strategies and metagame equilibrium. They, however, have not reached a satisfactory explanation of the selection, because the definition of metagames needs to fix an ordering of decision makers that cannot be justified in the real world. In hypergame theory the selection in the experiment is explained by using the misperception by the persons of the preferences for the possible outcomes. This explanation, however, is not consistent with the settings of the experiment: the situation of "prisoners' dilemma" is so clear that the persons in the situation cannot misperceive the components of the situation.

Another difficulty for accepting the assumption of economic rationality in a decision theory is the existence of economically irrational behavior in the real world. Helping behavior [27] and aggression [8] of one person to another are examples of economically irrational actions. Helping behavior of a decision maker is an action that raises others' profits sacrificing his/her own profit. Aggression of a decision maker is an action that causes some damages to others sacrificing his/her profit. Decision makers never select such actions under the assumption of economic rationality, but persons in the real world often select. Independence of preferences of a decision maker from others' in existing theories is also a difficulty for approving of the assumption of economic rationality. The assumption implies the independence: under the assumption, preferences of each decision maker for possible outcomes in a situation depend on only his/her profits in the outcomes, and do not depend on the others'. Helping behavior and aggression, however, imply dependence of preferences of decision makers: a decision maker who takes helping behavior or aggression prefers one outcome to another referring to not only his/her own profit in the outcomes but also the others'.

In order to overcome the contrast between the theories for decision making and decision making in the real world, we need to eliminate the assumption of economic rationality from the theories, because the assumption seems to be a cause of the difficulties above. Without the assumption, decision makers select actions either economically rationally or economically irrationally. They know that each decision maker has a way to choose an action, but do not know whether the way is economically rational or economically irrational. Thus they may misperceive the ways of selection of decision makers. We should notice that treating such decision makers, we have to distinguish profits of a decision maker in possible outcomes and preferences of the decision maker for the possible outcomes. Under the assumption of economic rationality the profits and the preferences can be identified, but they can be inconsistent with each other without the assumption. Even when one outcome is more profitable for a decision maker than another, the latter can be preferable for the decision maker to the former.

Discriminating between the profits and the preferences, we can consider various types of relations between them. Coincidence of them, that is, the case that a decision maker prefers an outcome to another if and only if he/she can get more profit in the former outcome than that in the latter, is one type of the relations. Another type is the case of no-relation: any profit of a decision maker in a possible outcome can be associated with any preference for the outcome, and vice versa. Neither of the types can be accepted to express realistic relations between the profits and the preferences. We think of a type that exists between these extremes by using the concept of *emotions* [43, 44, 45]. Consider two decision makers participating in a situation of decision making. If one of them has positive emotions toward another, then helping behavior by the former to the latter is likely to happen, whereas aggression by the former to the latter is not likely to occur. Oppositely, if one of them has negative emotions toward another, then aggression by the former to the latter seems to be able to happen, but the helping behavior by the former to the latter does not seems to be realized. This consideration leads us to assumptions regarding emotions and irrational actions of decision makers: helping behavior of one decision maker to another is induced only if the former has positive emotions toward the latter, and aggression of one decision maker to another is induced only if the former has negative emotions toward the latter. These assumptions give a type of relations between the profits and the preferences of decision makers for any fixed state of emotions among decision makers. The assumptions are based on the axioms in 'soft' game theory [44, 45]. The assumptions refer to the emotional aspect as well as the economic aspect of decision making, and the relations between the profit and the preferences of decision makers provided by the assumptions are convincing and realistic. Thus we also treat them as axioms in this paper as in 'soft' game theory.

Frameworks for Treatments of Interperception

As mentioned above, the concept of emotions is introduced in 'soft' game theory. Drama theory [46] also involves the concept in it. These existing theories, however, provide only a verbal definition of functions of emotions and explanations of decision makers' economically irrational behavior caused by the emotions. Thus arguments about the way of selection of decision makers in these theories sometimes have some extent of vagueness. For example, we cannot specify the choices of actions of decision makers in a 'soft' game. We should obtain a strict and formal framework for dealing with emotions, and consequently, decision makers' economically irrational actions in order to understand satisfactorily situations of decision making with economic and emotional aspects. In such a framework we can analyze situations more strictly than in existing ones. Such a framework, furthermore, allows us to deal with economic and emotional effects on decision making separately. In analyses of situations with economic and emotional effects, to specify the economic effects on decision makers and their emotions separately is much easier than to describe exactly their preferences made from the economic and emotional effects. Thus a framework that can deal with the economic and emotional effects separately is useful for the analyses. One of the purposes of this paper is, therefore, to provide such a framework.

When we treat the concept of emotions, irrational actions of decision makers, and their preferences discriminated from their profits in a framework, it is often required to deal with incompleteness of information in terms of preferences. Consider, for example, two decision makers involved in a situation. Under the assumption of linearity of preferences of decision makers, the fact that one of them has positive emotions toward the other and the other has negative emotions toward the one can contradict the assumption of completeness of information regarding preferences: if they try to choose helping behavior and aggression, respectively, their preferences for possible outcomes is never consistent with the assumption. On the one hand, the most preferable outcome for the decision maker who has positive emotions to the other is the least preferable one for the other because of the negative emotions to the decision maker. On the other hand, the least preferable outcome for the decision maker that has negative emotions to the other is the least preferable outcome for the other because of the positive emotions to the decision maker. Thus the most preferable outcome for a decision maker coincides with the least preferable outcome for him/her under the assumption of completeness of information in terms of preferences, that fact contradicts the assumption of linearity of preferences. This consideration implies that intending to deal with the concept of emotions, we should eliminate the assumption of completeness of information in terms of preferences.

In spite of our requirement for a framework that can treat situations with incompleteness of information and the concept of emotion, existing frameworks have difficulties to be adopted. The framework for hypergames [5, 99, 100] is formal and general, and can be used to cope with situations with incompleteness of information in terms of who are involved in the situation, which actions are available for each decision maker, and how each decision maker estimates the outcomes of the situation, that is, the components of a situation, but we cannot deal with situations with incompleteness of information in terms of emotions in the framework. The framework for information structure [1, 24, 62, 63] is strict and general, but usually involves the hypothesis of mutual rationality in it. That is, the assumption that each decision maker correctly perceives what each of the others perceives and each decision maker correctly perceives that each of the others perceives what each of the others perceives, and so on. We need a framework that is more general than the existing ones, that is, a formal framework that can be used without special hypotheses to deal with situations with incompleteness of information in terms of not only the components of the situations but also emotions. To provide such a formal and general framework is another purpose in this paper.

Eliminating the assumption of completeness of information in terms of an element of a situation causes us to consider exchanges of information among decision makers regarding the element, and changes of their perceptions of the element. For example, if a company obtains information about a new product of an opponent, then its perceptions of the way of decision making of the opponent can be changed. New information about voters' preferences for alternatives of a voting can change their perceptions of preferences. We need a framework for treating the exchanges of information and the changes of perceptions caused by the exchanges in order to examine situations with incompleteness of information. Existing frameworks, however, do not meet our requirement. The framework for hypergames [5, 99] is not satisfying for dealing with exchanges of information because it is used for static analyses of situations and the aspect of changes of perceptions is not focused. The framework for information structure [1, 24] fully focuses the aspect, but the hypothesis mentioned above is still assumed. We must construct a formal framework for treating exchanges of information and changes of perceptions caused by the exchanges, eliminating special hypotheses. Giving such a framework is also another purpose in this paper.

Analyses on Decision Making with Interperception

Not only do we provide the frameworks mentioned above but also we examine situations by using the frameworks in this paper. We describe a possible state of a decision makers' perceptions of an element of a situation by *schemes* of the element. Examining situations requires some concepts for schemes to classify possible structures of decision makers' perceptions. One of the well known concepts is *common knowledge* [24]. The concept describes the states that each decision maker perceives the correct information, each decision maker perceives that each of the others perceives the correct information, each decision maker perceives that each of the others perceives that the others know the correct information, and so on. We propose the concept of *inside common knowledge* in this paper to describe individual part of the concept of common knowledge: the concept of inside common knowledge expresses the states that a decision maker believes that a piece of information is common knowledge. We also provide the concepts that are generalization of the concepts of common knowledge and inside common knowledge. Another concept that we propose in this paper is *integration of perceptions.* We can often regard decision makers as involved in many situations each of which interacts with the others. For example, a company with several sorts of products may be in competition with rivals in each of the markets of the products, while not all of the products may be allowed to be fully developed because of financial constraints. When we model and analyze such a whole situation, we often use a method that separates the whole situation into several parts, whose models we combined into one considering interactions of them. We can utilize the concept of integration of perceptions to make an appropriate model of the whole situation. The concept of *stability of emotions* is also a concept that we provide in this paper. This concept for schemes of emotions is emerged from the concept of balancedness of sentiments in psychology [13, 35], and give us structures of perceptions of emotions. We also propose the concept of *complete stability of emotions* by applying the concept of stability of emotions to higher degree of perceptions. These concepts are useful for examining situations with incompleteness of information, because these express realistic incomplete structures of perceptions.

There are four main topics that are examined in this paper by using the frameworks and the concepts above. The first one is relations among economic and emotional aspects of decision making, interperception by decision makers, and actions chosen by the decision makers. Examinations of the relations make it possible to specify decision makers' choices in a situation with economic and emotional effects. We propose in this paper the concepts of *honesty*, *confidence*, and *partial confidence* to examine the relations. A decision maker with honesty does not deceive the others, and a decision maker with confidence does not doubt the others' word. A decision maker with partial confidence believes the others depending on emotions that the others have toward the decision maker. We treat decision makers with honesty and confidence, and show that not all of the decision makers involved in the situation of "prisoners' dilemma" correctly perceive the ways of selection of them. We also show that if an outcome is hoped by all decision makers to be realized, and if each decision maker believes that the others think that the decision maker will select the action that is required to realize the outcome, then the outcome will be selected. We deal with, moreover, decision makers with honesty and partial confidence, and prove that a sufficient condition for the outcome that is hoped to be realized by all decision makers to be chosen is that they have positive emotions to one another and they believe that they are thought to select the actions that are needed to realize the outcome. Regarding interperception by decision makers, we propose the concept of *generation of perceptions* by using *integration of perceptions*, and show analyses of examples of situations generated from a class of situations.

The second topic relates to exchanges of information, especially, deception by decision makers and credibility of information. In order to examine the aspect of deception we propose in this paper two concepts for impossibility of deception: *inside strategyproofness* and *outside strategyproofness*. The former describes situations that deception of a decision maker causes changes of his/her preferences, thus the deception cannot be effective. The latter expresses situations that decision makers' attempts to change the others' preferences for possible outcomes by deception end in failure. We provide, moreover, the concepts of *separability* and *extremeness* of confidence for examinations of this aspect. Separability of confidence implies that whether a decision maker trusts another or not is independent of information about the others, and extremeness of confidence means that getting new information, a decision maker believes either the information or what he/she had believed when the information was conveyed. In this paper we examine deception under incompleteness of information in terms of preference, and show two theorems that mean senselessness of deception: these give sufficient conditions for inside strategyproofness and outside strategyproofness, respectively. Examinations of credibility of information requires the concepts of *credibility* and *complete credibility* of information for a decision maker. We treat credibility of information as depending on each decision maker's perceptions, and propose the concept of credibility of information for a decision maker. The concept of complete credibility of information for a decision maker is also provided by applying the idea to higher degree of perceptions. We examine relations between the aspect of interperception by decision makers regarding emotions, and the concepts of credibility and complete credibility. We give a sufficient condition for information to be completely credible for a decision maker. This condition means that if a decision maker's perceptions of emotions is inside common knowledge, and a piece of information is credible for him/her, then the information is completely credible for him/her.

The third topic concerns a solution concept that is defined referring to economic and emotional aspects of decision making. We call the solution *emotional equilibrium* in this paper. We examine relations between the solution concept and modification of perceptions. A decision maker in a situation selects an action referring to his/her perceptions of the situation. After the selections of actions by all decision makers, each decision maker modifies his/her perceptions depending on the differences between his/her expectations about the selections and the actual selections. If there is no difference, then decision makers may not modify their perceptions. We provide conditions under which it is satisfied that if the actual selections of actions coincide with the expectations, then the selections form an emotional equilibrium.

The last one is about *deadlock* in a meeting. The concept of deadlock describes situations that no decision maker conveys information to the other decision makers despite of no decision is made as a whole. Such a situation is not favorable especially when the decision makers have to reach a consensus in the meeting. We provide, moreover, the concept of *complete deadlock* by applying the concept of deadlock to higher degree of perceptions. We examine relations among emotions of decision makers, the voting rule of the meeting, and deadlock. We treat meetings with odd number of members and majority rule, and give a proposition that shows that it is impossible for a meeting with completely stable emotions to reach a complete deadlock. Since we have an example of a meeting at a complete deadlock with not completely stable emotions, the property implies important roles of completely stable emotions of the decision makers in a meeting. We also give a proposition that shows impossibility of reaching a deadlock for a meeting with stable emotions. We also show that there is a meeting at a deadlock with not stable emotions, thus the importance of stable emotions is implied by the property.

Structure of this Paper

In the next chapter, Chapter 2, we provide the frameworks mentioned above: a framework for describing situations of decision making, that for dealing with economic and emotional effects on decision making separately, that for treating the aspect of interperception by decision makers and incompleteness of information, and that for coping with exchanges of information and changes of perceptions caused by the exchanges of information. Particularly, in order to express decision makers' perceptions of elements of situations, we first specify the elements of situations of decision making, and define the concepts of *strings of decision makers*, *perceptions*, *schemes*, and *views* of the elements. Then we show a proposition that implies equivalence of giving a scheme of decision makers to providing a set of all strings of decision makers. We also prove decomposability of a view into perceptions of schemes.

In Chapter 3 we formally define the concepts that are fundamental for examining situations: the concept of *inside common knowledge*, that of *integration of perceptions*, that of *stability of emotions*, and so on. In terms of the concept of inside common knowledge, we prove that the feature of inside common knowledge is implied by that of common knowledge. The concept of integration of perceptions is expressed by using the concept of integration of schemes. We show that schemes integrated into one form another scheme. Concerning the concept of stability of emotions, we give propositions that show that it is possible that emotions of decision makers become stable and completely stable. We show, moreover, a separation theorem for completely stable emotions, that is a generalization of the separation theorem for balanced graphs [13, 14, 28]. We also examine relations among stable emotions, completely stable emotions, and the concept of inside common knowledge, and equivalence of the concept of stability to that of complete stability under the condition of inside common knowledge is shown.

Chapter 4 is devoted to analyses of situations of decision making. First, we examine relations among economic and emotional aspects of decision making, interperception by decision makers, and outcomes that are determined by actions of decision makers. We deal with *honest* and *confident* decision makers, and show a proposition that indicates impossibility of correct perceptions of the ways of selection of the decision makers in the situation of "prisoners' dilemma." This implies necessity of examinations of the aspect of interperception by decision makers. We also give a sufficient condition for a cooperative outcome to be the outcome of the situation. This assures the verbal suggestion by Howard [44]. Furthermore, we provide a similar proposition for situations with decision makers with honesty and *partial confidence.* This implies that 'naivety' of decision makers is not always necessary for achieving a cooperative outcome. Employing the concept of integration of perceptions, moreover, we deal with the issue of *generation of schemes*, and analyze examples of generated schemes. Secondly, we focus on exchanges of information. Brams's examinations on *deception* of decision makers shows that a deceiver can get a more profitable outcome than the outcome that can be realized through honest offers [9]. In this paper, in contrast to the examinations, we prove two propositions that mean senselessness of deception by introducing interdependent preferences of decision makers instead of independent ones: sufficient conditions for *inside strategyproofness* and *outside strategyproofness* respectively. The issue of *credibility* of information is also examined in this paper. In 'soft' game theory [44] positive emotions and negative emotions are considered to be able to make unwilling promises and unwilling threats credible, respectively. The concept of credibility of information, however, is defined only under the assumption of complete information in terms of emotions. We provide a definition of the concepts of *credibility* and *complete credibility* without the assumption, and show a proposition that implies equivalence of the concept of credibility to that of complete credibility under the condition of inside common knowledge. Thirdly, we deal with a solution concept, called *emotional equilibrium.* Existing solution concepts such as Nash equilibrium [23] in game theory, metagame equilibrium [41] in metagame theory, and sequential stability [100] in hypergame theory are defined not referring to emotional aspects of decision makers. Thus we propose a solution concept involving the aspect of emotions as well as the aspect of interperception by decision makers, and examine relations between the solution concept, and coincidence of decision makers' expectations about the outcome of a situation and the actual outcome of the situation. We give conditions under which we have that if the actual outcome is not an emotional equilibrium, then there exists a decision maker whose inferences of the others' selections are incorrect. Fourthly, we deal with cooperative situations of decision making. Giving sufficient conditions for meetings not to reach a *deadlock* and not to reach a *complete deadlock* respectively, we show that it is important for progression of a meeting to achieve stable emotions.

Summary and comments for possible directions of further researches are given in Chapter 5. It includes issues on unification of hypergame theory and 'soft' game theory, classification of situations of decision making, treatments of groups as decision makers, exchanges of information in cooperative situations of decision making, and so on.

Chapter 2

Models

In this chapter we give frameworks for examinations of situations of decision making with interperception. To begin with, classification of the situations into two categories, *competitive* and *cooperative*, is provided, and give a formal model for describing the situations in each category. Then, models for expressing the aspects of *emotions*, *interperception*, and *exchanges of information* in the situations are strictly defined.

2.1 Classification of Situations of Decision Making

Situations of decision making can be classified into two categories in terms of characteristics of decision makers in the situations. If each decision maker in a situation tries to obtain a bigger profit for him/herself, then the situation is said to be *competitive*. This category includes competition among companies in a market. The situations of "prisoners' dilemma" and "chicken" are also examples of the situations in this category. A situation whose participants intend to make the outcome of the situation preferable as a whole is called a *cooperative* situation. A selection of a car by a family is a typical example of the situations in this category. In this paper we deal with only competitive and cooperative situations that can be described as normal form games [23, 73] and simple games [80], respectively, in spite of the existence situations that are expressed by other types of frameworks: games in extensive form, games in coalitional form, frameworks for bargaining problems, frameworks for cost sharing problems, frameworks for surplus sharing problems, and so on [23, 73].

2.1.1 Competitive Situations

Suppose that two companies, A and B, are competing in a market. If both companies invest in developing a new product or neither do, they obtain equal profits. Each of them gets 15 million dollars in the first case, and 10 million dollars in the second. If one of them invests and the other does not, then the former obtains 35 million dollars and the latter does not get anything. We can regard this situation as a normal form game [23, 73], called a *base competition* in this paper. A base competition consists of three components: the set of all *decision makers* in the situation, for each decision maker the set of all *actions* that he/she can select in the situation, and for each decision maker the list of *profits* that he/she obtains in the possible *outcomes* of the situation. An individual or an organization can be considered as a decision maker of a base competition. Thus, two companies, A and B, can be seen as being involved in a competition. Each of them has two actions, 'invest' and 'not invest,' in the competition. If each of the decision makers makes a selection, then an outcome is determined. Thus there are four possible outcomes of the competition. Profits of a decision maker for the outcomes can be expressed by ordinal numbers or cardinal numbers. If we describe the profits of the companies by cardinal numbers, we can describe the competition as Table 1. In the table A chooses a row and B chooses a column. Each cell represents an outcome of the competition. Numbers in each cell show the profits that the decision makers gain in the outcome. A's profit is first. We should notice that the cardinal expression of profits can be converted into the ordinal expression, and thus the ordinal expression is more general than the cardinal expression.

		B	
		Invest	Not Invest
A	Invest	(15, 15)	(35, 0)
	Not Invest	(0, 35)	(10, 10)

Million Dollers

Table 1. A competitive situation between two companies.

We define a *base competition* strictly. Let $N = \{1, 2, \ldots, n\}$ be the set of all *decision makers* in the competition, and S_i the set of all *actions* that decision maker i can select in the competition. An *outcome* of the competition is a list of selections by all decision makers, that is, a list $s = (s_i)_{i \in N}$ of actions s_i in S_i for each i in N. The set of all *outcomes* is denoted by S. An outcome $s = (s_i)_{i \in N}$ is often denoted by (s_i, s_{-i}) for some i in N, where $s_{-i} = (s_j)_{j \in N \setminus \{i\}}$. For any i in N, the set of all s_{-i}s is denoted by S_{-i}. Decision maker i in N gets monetary profit and loss according to the outcome of the competition. We express profits and losses of decision maker i in the possible outcomes by a linear ordering F_i on S, called the *profits* of decision maker i. For any s and s' in S, sF_is' denotes that decision maker i's profit in outcome s is smaller than that in outcome s', and $s\not{F}_is'$ denotes that decision maker i's profit in outcome s is *not* smaller than that in outcome s'. F is the list $(F_i)_{i \in N}$ of the profits F_i for each i in N, called the *profits* of decision makers.

Definition 1 (Base Competitions) *A* base competition C *is a triple* (N, S, F).

Example 1 (Examples of Base Competitions) *Table 2 gives two examples of base competitions. (a) is the situation of "prisoners' dilemma," and (b) is the situation of "chicken." The following stories may make these situations clearer. In the situation (a) two murder suspects are separately questioned by a sheriff. They have agreed not to talk. If one of them 'defects' from this agreement and the other does not, then the prisoner who defects gets off free and the other gets death penalty. If they 'cooperate' with each other, then each of them gets a few years for armed robbery. If both 'defect,' then each gets a life sentence. In the situation (b) two teenagers are to drive toward a head-on collision. If one of them 'swerves,' then the one is shamed. If both 'keep on,' then both die. In each situation there are two decision makers, that is,* $N = \{1, 2\}$, *and in each table decision maker 1 chooses a row and decision maker 2 chooses a column. Thus,* $S_1 = \{c_1, d_1\}$ *and* $S_2 = \{c_2, d_2\}$ *in (a), and* $S_1 = \{w_1, k_1\}$ *and* $S_2 = \{w_2, k_2\}$ *in (b). The four outcomes are ranked as 4, 3, 2 and 1 for each decision maker, where 4 and 1 correspond to the most and least profitable outcomes, respectively. Decision maker 1's profit is* *first. These ranks represent the profits of decision maker 1, F_1, and those of decision maker 2, F_2, respectively.*

(a) Prisoners' dilemma.

		2: c_2 cooperate	2: d_2 defect
1	c_1 cooperate	(3, 3)	(1, 4)
1	d_1 defect	(4, 1)	(2, 2)

(b) Chicken.

		2: w_2 swerve	2: k_2 keep on
1	w_1 swerve	(3, 3)	(2, 4)
1	k_1 keep on	(4, 2)	(1, 1)

Table 2. Examples of base competitions.

2.1.2 Cooperative Situations

Suppose a family to buy a car. The family consists of five members: a husband and wife, two daughters, and one son. They always adopt the majority rule when they make a selection that relates to all members of the family. There are three alternatives in this case: a white sedan for 5 persons, a silver wagon for 7 persons, and a red convertible for 5 persons. The husband prefers the white one to the silver one, and the silver one to the red one, because he wants to use the car on business. The wife likes the silver one more than the white one, and the white one more than the red one, because she intends to go shopping by car. One of the daughters likes the red one best, and the silver one least, because she cares about only drives with a boyfriend. Another daughter cares about family trips, and likes the silver one best, and the white one least. The son loves the red convertible more than the silver one, and the silver one more than the white one, because of their colors.

The family may hold a meeting to make the selection as a whole. We can often see several stages in a meeting. At the first stage, members in the meeting have to learn the components of the meeting; the members in the meeting, the alternatives from which the members have to select just one, the favors of the members for the alternatives, the rule of final voting, and so on. At the next stage, the members interact each other. Each of the members tries to persuade others to agree on the alternative that the member most prefers. Some members may compromise and others not. The final stage is devoted to the actual voting, and an alternative is chosen by using a given voting rule. We can describe the situation of the meeting for the selection of a car by the family as a *base meeting.* A base meeting consists of four components: the set of all *decision makers* of the meeting, the *voting rule* of the meeting, the set of all *alternatives* of the meeting, and for each decision maker his/her *favors* for the alternatives. The decision makers have to select just one from the set of all alternatives. In the situation above there are five decision makers, and three alternatives. The voting rule is the majority rule, and the favors of the decision makers for the alternatives are expressed as in Table 3. In the table a column describe the favors of a decision maker for the alternatives. A decision maker likes the alternative placed at the top best, and the alternative placed at the bottom least.

husband	wife	daughter 1	daughter 2	son
w	s	r	s	r
s	w	w	r	s
r	r	s	w	w

w: white sedan, s: silver wagon, r: red convertible.

Table 3. Favors of decision makers for cars.

We provide a formal definition of a *base meeting*. Let $N = \{1, 2, \ldots, n\}$ be the set of all *decision makers* in the meeting. A base meeting has a *voting rule* to make a selection at the final stage of the meeting, denoted by W. W is the set of all *winning coalitions*, and a winning coalition is a non-empty subset of N. It is assumed that W satisfies that if S is an element of W, and T includes S, then T is also an element of W. Thus a pair of N and W forms a simple game [80]. W expresses a voting rule, because it is also assumed that if all of the decision makers in a winning coalition can agree on an alternative, then they have enough power to make the alternative be the selection of the final voting, and W is the set of all winning coalitions in terms of the voting rule. The decision makers have to select just one from the set A of all *alternatives*. For any i in N, decision maker i has the *favors* for the alternatives, denoted by F_i. The favors F_i of decision maker i is expressed by a linear ordering on A. For any a and a' in A, aF_ia' denotes that decision maker i likes alternative a' more than alternative a, and $a\not{F}_ia'$ denotes that decision maker i does not like alternative a' more than alternative a. The list $(F_i)_{i \in N}$ of the *favors* F_i for each i in N is denoted by F, called the *favors* of decision makers.

Definition 2 (Base Meetings) *A* base meeting *M is a 4-tuple (N, W, A, F).*

Example 2 (Examples of Voting Rules) *Consider a base meeting $M = (N, W, A, F)$. If the meeting adopts the* majority rule*, then we have that for any non-empty subset S of N, S is an element of W if and only if the cardinality of S is more than a half of the number of the decision makers. If decision maker i in N is a* vetoer *in the meeting, then decision maker i is an element of S for any winning coalition S in W. Moreover, if decision maker i is a* dictator *in the meeting, then we have that for any non-empty subset S of N, decision maker i is an element of S if and only if S is an element of W.*

2.2 Emotional Aspects of Decision Making

Not only in game theory [23, 73] but also in metagame theory [41] and in hypergame theory [5, 88], the assumption of economic rationality is adopted, and situations of decision making are examined under the assumption. Thanks to the assumption, various solution concepts, especially, the Nash equilibrium [23] in game theory, the metagame equilibrium [41] in metagame theory, and the sequential stability [100] in hypergame theory are proposed and justified. An experiment shows, however, that a person in the situation of "prisoners' dilemma" often selects an action different from the one that the theories suggest [41]. We can, furthermore, see helping behaviors [27] and aggression [8] in real situations. Helping behaviors of a decision maker are actions that raise others' profits or favors sacrificing his/her own profit or favor. Aggression of a decision maker is an action that causes others some damages sacrificing his/her profit or favor. Thus they are examples of economically irrational actions. Decision makers never select such actions under the assumption of economic rationality, but persons in the real world often select the actions. Therefore, we should deal with economically irrational behaviors as well as economically rational behaviors to examine comprehensively situations of decision making.

Treating economically irrational behaviors naturally requires discrimination between the profits or the favors, and the preferences of decision makers for possible outcomes. The discrimination induces various types of relations between the profits or the favors, and the preferences, whereas in this paper we focus on a type of the relations determined by using the concept of *emotions*. The concept is introduced in 'soft' game theory [44]. Drama theory [46] also involves the concept in it. These existing theories, however, provide definitions of functions of emotions and explanations of decision makers' economically irrational behavior caused by emotions only verbally. Then arguments about the way of selection by decision makers in these theories sometimes have some extent of vagueness. In order to make clear examinations of situations of decision making, we provide a formal framework for treating emotions and strict definitions of functions of emotions. The framework, that can deal with economic and emotional aspects of decision making separately, is useful for analyses of situations with those aspects, because in the analyses, to specify the economic and the emotional aspects separately is much easier than to determine decision makers' preferences made from those aspects.

While economically irrational behaviors can be seen in cooperative situations as well, existing theories treat them only in competitive situations. Thus we introduce the concept of emotions to cooperative contexts in order to describe economically irrational behavior in cooperative situations.

2.2.1 Functions of Emotions

Emotions are often classified into several groups and treated mathematically. Heider [35] classifies emotions into two groups: *positive* and *negative*. Heider [35], moreover, assigns '+' and '−' to positive and negative emotions, respectively, and allows to multiply them. For example, positive emotions, '+,' multiplied by negative emotions, '−,' is '−.' Howard [44] classifies emotions into three groups: *positive*, *negative*, and *mixed*. Howard [44], moreover, gives functions to the positive ones and the negative ones, and treats them as axioms of 'soft' game theory [44]. Howard employs the concepts of unwilling promises, unwilling threats to define the functions of the emotions. An unwilling promise of a decision maker to another is information that the decision maker selects a helping behavior for the other, and an unwilling threat of a decision maker to another is information that the decision maker chooses an aggressive behavior for the other. Unwilling promises and unwilling threats are not credible for decision makers, because the promises and the threats are economically irrational, but it is assumed in 'soft' game theory that positive emotions from a decision maker to another can make an unwilling promise of the decision maker to the other credible, and that negative emotions from a decision maker to another can make an unwilling threat of the decision maker to the other credible.

In this paper we deal with two types of emotions: *positive* and *negative*, and employ the multiplication of emotions as Heider [35]. Furthermore, we adopt the functions of emotions by Howard [44], that is, the assumptions that positive emotions from a decision maker toward another can make information about the decision maker's helping behaviors for the other credible, and that negative emotions from a decision maker toward another can make information about the decision maker's aggressive behaviors for the other credible. Moreover, we expand the assumptions in this paper. The assumptions about credibility of information about helping and aggressive behaviors and emotions allows us to assume that a decision maker with positive emotions toward another tends to select a helping behavior for the other, and that a decision maker with negative emotions toward another tends to choose an aggressive behavior for the other. We should think that these tendencies of decision makers' behaviors cause the assumptions about credibility of promises and threats. Thus we assume in this paper that positive emotions from a decision maker to another can cause his/her helping behaviors to the other, and that negative emotions from a decision maker to another can cause his/her aggressive behaviors to the other.

We describe possible emotions and relations among them induced by multiplication with the *space of emotions.*

Definition 3 (Space of Emotions) *The* space of emotion, T, *is the set* $\{+, -\}$ *with the binary operation* '$\times$' *that satisfies* $(+ \times +) = (- \times -) = +$ *and* $(+ \times -) = (- \times +) = -$.

Table 4 shows the relations among the signs, '+' and '−,' and the binary operation '×.'

×	+	−
+	+	−
−	−	+

Table 4. Space of emotions

We assume that each decision maker has positive emotions or negative emotions toward each of the decision makers. Each decision maker has positive emotions toward him/herself. Positive emotions and negative emotions are identified to the signs, '+' and '−,' respectively, thus the emotions of a decision maker toward the decision makers are expressed by a list of the signs for each of the decision makers, called the *emotions* of the decision maker (Figure 1).

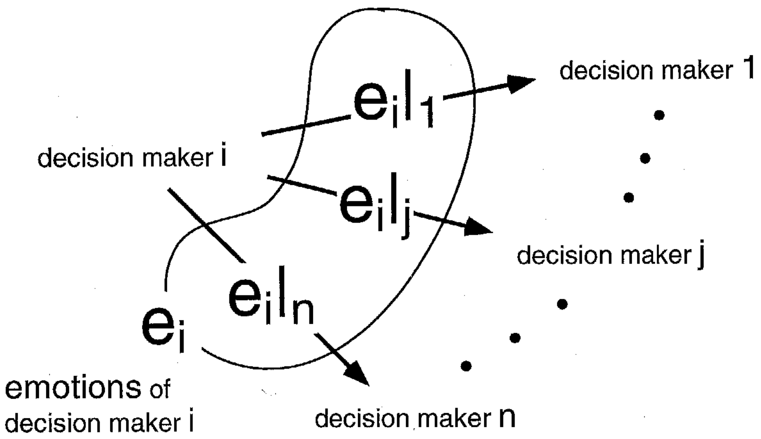

Figure 1. Emotions of a decision maker.

Definition 4 (Emotions) *Let N be a set of all decision makers. For any i in N, the* emotions *of decision maker i is an element of T^N, denoted by e_i. The emotions e_i of decision maker i is a list $(e_i|_j)_{j \in N}$, and $e_i|_j$ is the emotions of decision maker i toward decision maker j for any j in N. For any i in N, it is assumed that $e_i|_i = +$. e denotes the list $(e_i)_{i \in N}$ of the emotions e_i of decision maker i for each i in N, called the* emotions *of decision makers.*

2.2.2 Emotions in Competitive Situations

Helping behaviors [27] and aggression [8] of a decision maker in a competitive situation indicate that his/her preferences for possible outcomes of the situation depend on his/her profits in the outcomes, his/her emotions, and the others' preferences for the outcomes. We should think that the preferences of each of the others are also determined in the same manner. Thus in this paper we assume that the profits that the decision makers obtain in the outcomes, and the emotions of the decision makers determine the preferences of each decision maker for the outcomes. This assumption means that construction of the preferences of each decision maker consists of two steps. In the first step each decision maker infers the preferences of the other decision makers from the profits and the emotions of decision makers. In the second step each decision maker constructs his/her own preferences from his/her own profits, his/her own emotions, and the other decision makers preferences that he/she inferred in the first step.

To define formally the *preferences* of a decision maker for possible outcomes, consider a base competition $C = (N, S, F)$ and the emotions $e = (e_i)_{i \in N}$ of decision makers.

Definition 5 (Preferences) *For any i in N, the* preferences *of decision maker i is a linear ordering on S, denoted by P_i. It is assumed that P_i is determined from the profits F and the emotions e of decision makers. For any s and s' in S, sP_is' denotes that decision maker i prefers outcome s' to outcome s, and $s\not{P}_is'$ denotes that decision maker i does* not *prefer outcome s' to outcome s. P denotes the list $(P_i)_{i \in N}$ of the preferences P_i of decision maker i for each i in N, called the* preferences *of decision makers.*

Explaining functions of emotions requires the concept of *rules* of decision makers. A rule of a decision maker indicates his/her action corresponding to each possible list of the others' actions. The idea of rules of decision makers comes from the concept of metagame strategies in metagame theory [41]. While metagame theory deal with nth-level of metagame strategies for any positive integer n, in this paper we treat only what correspond to first-level metagame strategies. In 'soft' game theory information about behaviors of decision makers is conveyed in the form of inducement tactics [44, 49], that are a special type of rules of decision makers. Inducement tactics of a decision maker, however, do not always exist in the case that there are many decision makers and each of them has many possible actions. Each decision maker should be allowed to exchange information even if he/she does not have any inducement tactics. The expression of information about behaviors of decision makers by using rules of decision makers is adequate to deal with the decision makers equally.

Definition 6 (Rules) *Consider a base competition $C = (N, S, F)$. For any i in N, a function from S_{-i} to S_i is called a* rule *of decision maker i. A list of rules of decision maker i for each i in N is called a* rule *of decision makers.*

Example 3 (Examples of Rules) *Consider the situation of "prisoners' dilemma" in Table 2 (a). Decision maker 1 has four possible rules. Each rule of decision maker 1 is a function from $S_2 = \{c_2, d_2\}$ to $S_1 = \{c_1, d_1\}$. Each function indicates that c_2 corresponds to c_1 or d_1, and d_2 to c_1 or d_1. Similarly, consider the situation of "chicken" in Table 2 (b). Decision maker 1 in the situation also has four rules, and they are the possible functions from $S_2 = \{w_2, k_2\}$ to $S_1 = \{w_1, k_1\}$. Each of them indicates that w_2 corresponds to w_1 or k_1, and k_2 to w_1 or k_1.*

In this paper we assume that each decision maker in a base competition $C = (N, S, F)$ has just one rule to make a selection in the situation, and the *rule* of decision maker i in N is denoted by r_i. This means that if decision maker i infers that the others will select a list of actions $s_{-i} = (s_j)_{j \in N \setminus \{i\}}$ in S_{-i}, then decision maker i selects the action $r_i(s_i)$ in S_i. We assume, moreover, that the rule r_i of decision maker i must be consistent with the preferences P_i of decision maker i. That is, for any s_{-i} in S_{-i}, if $r_i(s_{-i}) = s_i$ if and only if $(s_i, s_{-i}) \not{P}_i (s'_i, s_{-i})$ for any s'_i in S_i. In other words, the rule of a decision maker is the best reply function determined by the preferences of the decision maker. The list $(r_i)_{i \in N}$ of the rules r_i of decision maker i for each i in N is called the *rules* of decision makers, and denoted by r.

Given a base competition $C = (N, S, F)$ and the preferences P of decision makers, we can define *rational, helping,* and *aggressive rules* of a decision maker, considering relations among the preferences P of decision makers and the profits F of the decision maker.

Definition 7 (Rational Rules) *For any s_{-i} in S_{-i}, if a rule r'_i of decision maker i in N satisfies that $(r'_i(s_{-i}), s_{-i})$ $\not{F}_i$ (s_i, s_{-i}) for any s_i in S_i, then r'_i is a* rational rule *of decision maker i at s_{-i}.*

Definition 8 (Helping rules) *Consider two decision makers, i and j, in N. For any s_{-i} in S_{-i}, if a rule r'_i of decision maker i satisfies that (s^*_i, s_{-i}) P_j $(r'_i(s_{-i}), s_{-i})$, then r'_i is a* helping rule *of decision maker i to decision maker j at s_{-i}, where s^*_i satisfies that (s^*_i, s_{-i}) $\not{F}_i$ (s_i, s_{-i}) for any s_i in S_i.*

Definition 9 (Aggressive Rules) *Consider two decision makers, i and j, in N. For any s_{-i} in S_{-i}, if a rule r'_i of decision maker i satisfies that $(r'_i(s_{-i}), s_{-i})$ P_j (s^*_i, s_{-i}), then r'_i is an* aggressive rule *of decision maker i to decision maker j at s_{-i}, where s^*_i satisfies that (s^*_i, s_{-i}) $\not{F}_i$ (s_i, s_{-i}) for any s_i in S_i.*

A rational rule of a decision maker chooses the most profitable action for the decision maker, receiving a list of actions of the other decision makers. A helping rule of a decision maker to another selects an action that causes an outcome that is preferable for the other to the outcome that is caused in the case that the decision maker chooses the most profitable action for him/her. An aggressive rule of a decision maker to another selects an action that causes an outcome that is not preferable for the other to the outcome that is caused in the case that the decision maker chooses the most profitable action for him/her.

Example 4 (Examples of Rational, Helping, and Aggressive Rules) *Consider the situation of "prisoners' dilemma" in Table 2 (a), the preferences P of decision makers, and the rules r of decision makers. If the rule r_1 of decision maker 1 satisfies that $r_1(c_2) = d_1$, then r_1 is a rational rule of decision maker 1 at c_2. Considering the case that the preferences P_2 of decision maker 2 coincide with the profits F_2 of decision maker 2, we have that if $r_1(c_2) = c_1$, then r_1 is a helping rule of decision maker 1 to decision maker 2 at c_2. In the situation of "chicken" in Table 2 (b) with the preferences P of decision makers and the rules r of decision makers, if $r_1(k_2) = w_2$, then r_1 is a rational rule of decision maker 1 at k_2. If the preferences P_2 of decision maker 2 coincide with the profits F_2 of decision maker 2, then we have that if $r_1(k_2) = k_1$, then r_1 is an aggressive rule of decision maker 1 to decision maker 2 at k_2.*

We have assumed that emotions have functions in terms of credibility of information about helping and aggressive behaviors, and the tendencies of decision makers' behaviors. We first describe the function in terms of credibility of information by defining *credible information.* In this paper, regarding competitive situations, we focus on information about the rules of decision makers. It is assumed that the information is conveyed in the form of a rule of a decision maker.

Consider a base competition $C = (N, S, F)$, the preferences P of decision makers, the emotions e_i of decision maker i in N, and information $\hat{r}_i$ about the rule r_i of decision maker i, where $\hat{r}_i$ is a function from S_{-i} to S_i.

Definition 10 (Credible Information) *For any s_{-i} in S_{-i}, the information $\hat{r}_i$ is* credible *at s_{-i}, if we have either*

1. *$\hat{r}_i$ is a rational rule of decision maker i at s_{-i},*
2. *there exists decision maker k in N such that $e_i|_k = +$ and $\hat{r}_i$ is a helping rule of decision maker i to decision maker k at s_{-i},*
   or
3. *there exists decision maker k in N such that $e_i|_k = -$ and $\hat{r}_i$ is an aggressive rule of decision maker i to decision maker k at s_{-i}.*

This definition means that information about economically rational behavior is always credible for decision makers, and that emotions have the functions to make information about economically irrational behavior credible for decision makers.

Example 5 (Credible Information) *Consider the situation of "prisoners' dilemma" in Table 2 (a), the preferences P, the rules r, and the emotions e of decision makers. If information $\hat{r}_1$ about the rule of decision maker 1 satisfies that $\hat{r}_1(c_2) = d_1$, then the information is credible at c_2, because it is a rational rule of decision maker 1. In the case that the preferences P_2 of decision maker 2 coincide with the profits F_2 of decision maker 2, if we have that $\hat{r}_1(c_2) = c_1$ and $e_1|_2 = +$, then the information is credible at c_2, because it is an helping rule of decision maker 1 to decision maker 2, and decision maker 1 has positive emotions toward decision maker 2. In the base competition of "chicken" in Table 2 (b) with the preferences P and the rules r of decision makers, if information $\hat{r}_1$ about the rule of decision maker 1 satisfies that $\hat{r}_1(k_2) = w_2$, then the information is credible, because it is a rational rule for decision maker 1. In the case that the preferences P_2 of decision maker 2 coincide with the profits F_2 of decision maker 2, if we have that $\hat{r}_1(k_2) = k_1$ and $e_1|_2 = -$, then the information is credible at k_2, because it is an aggressive rule of decision maker 1 to decision maker 2, and decision maker 1 has negative emotions toward decision maker 2.*

Given a base competition $C = (N, S, F)$, the preferences P, the emotions e, and the rule r of decision makers, we can describe the function of emotions in terms of tendencies of decision makers' behaviors by the assumption that for any i in N and any s_{-i} in S_{-i}, we have either that r_i is a rational rule of decision maker i at s_{-i}, that for some k in N such that $e_i|_k = +$, r_i is a helping rule of decision maker i to decision maker k at s_{-i}, or that for some k in N such that $e_i|_k = -$, r_i is an aggressive rule of decision maker i to decision maker k at s_{-i}. Thus the rule of a decision maker is also information that is credible for decision makers.

Example 6 (Functions of Emotions) *Consider the situation of "prisoners' dilemma" in Table 2 (a) with the preferences P, the rules r, and the emotions e of decision makers. In the case that the preferences P_2 of decision maker 2 coincide with the profits F_2 of decision maker 2, if $e_1|_2 = +$, then $r_1(c_2) = c_1$ or d_1 because of positive emotions of decision maker 1 toward decision maker 2. If $e_1|_2 = -$, then $r_1(c_2) = d_1$ because of negative emotions of decision maker 1 toward decision maker 2. In the situation of "chicken" in Table 2 (b) with the preferences P, the rules r, and the emotions e of decision makers, if the preferences P_2 of decision maker 2 coincide with the profits F_2 of decision maker 2, then we have that if $e_1|_2 = +$, then $r_1(k_2) = w_1$ because of positive emotions of decision maker 1 toward decision maker 2. If $e_1|_2 = -$, then $r_1(k_2) = w_1$ or k_1 because of negative emotions of decision maker 1 toward decision maker 2.*

2.2.3 Emotions in Cooperative Situations

Even in a cooperative situation as a meeting to make a selection of a car by a family as a whole, helping behavior and aggression of decision makers can be observed. For example, the son in the family in Table 3 may vote in favor of the white sedan to make his father happy. The daughter who loves the silver wagon may vote for the red convertible to make her mother sad. Thus, as in competitive situations, we should assume that the preferences of decision makers for possible alternatives in a cooperative situation are determined from the favors of decision makers and the emotions of decision makers. We define, therefore, the *preferences* of decision makers in a base meeting $M = (N, W, A, F)$ with the emotions $e = (e_i)_{i \in N}$ of decision makers as follows.

Definition 11 (Preferences) *For any i in N, the* preferences *of decision maker i is a linear ordering on A, denoted by P_i. It is assumed that P_i is determined from the favors F and the emotions e of decision makers. For any a and a' in A, aP_ia' denotes that decision maker i prefers alternative a' to alternative a, and $a\not{P}_ia'$ denotes that decision maker i does* not *prefer alternative a' to alternative a. P denotes the list $(P_i)_{i \in N}$ of the preferences P_i of decision maker i for each i in N, called the* preferences *of decision makers.*

To describe formally functions of emotions in cooperative situations, we need to define rational, helping, and aggressive behaviors of a decision maker in a meeting such as rational, helping, and aggressive rules in a competitive situation. Consider a base meeting with the majority rule as in the family in Table 3. We can easily think of a definition of rational behaviors of a decision maker. It is apparently rational for a decision maker to vote according to the favors of the decision maker. We can, moreover, think of two types of definitions of helping and aggressive behaviors. One is parallel to the definitions of helping and aggressive rules in the case of competitive situations. We regard possible patterns of the vote of a decision maker as actions of a decision maker in a competitive situation, and define a rule of a decision maker in a meeting as a function that indicates a pattern of his/her vote corresponding to each list of patterns of the others' votes. A vote of a decision maker can be identified with a helping rule toward another if the vote yields a preferable alternative for the other to the alternative that is caused when the decision maker votes according to his/her favors. Similarly, an aggressive rule of a decision maker to another can be defined as a vote that does not cause a preferable alternative for the other to the alternative that the vote according to the favors of decision maker results.

This way of definitions of helping and aggressive behaviors in a meeting has a problem that a vote can be labeled as neither rational, helping, nor aggressive, because a voting rule, especially the majority rule without any tie-braking rule, often does not yield just one alternative when a list of votes of decision makers is given: in this way, just one alternative has to be specified for each list of votes of decision makers in order to determine whether a vote of a decision maker is helping or aggressive. For example, if each member in the family in Table 3 votes according to his/her favors for the alternatives, then the majority rule cannot result the final selection of the family as a whole. Thus even if the son votes for the white sedan, the vote cannot be labeled as neither helping nor aggressive. The difficulty makes us adopt the other way of definitions of helping and aggressive behaviors in a meeting. This way employs the concept of *one-rank improvements* of the position of an alternative, and a metric defined on the set of all possible patterns of the vote of a decision maker. Consider a set A of all alternatives, and the set of all linear orderings on A, denoted by $L(A)$.

Definition 12 (One-Rank Improvements) *Consider two linear orderings, P and P', in $L(A)$, alternative a in A, and alternative a' in A such that aPa' and there is no alternative a'' such that $aPa''Pa'$. Ordering P' is obtained by a* one-rank improvement *of the position of alternative a from ordering P, if we have that bPa if and only if $bP'a$, and bPa' if and only if*

$bP'a'$ *for any alternatives* b *in* $A\backslash\{a, a'\}$*, and that* bPb' *if and only if* $bP'b'$ *for any alternatives* b, b' *in* $A\backslash\{a, a'\}$*, and that* $a'P'a$*: the exchange of the positions of* a *and* a' *remaining the other alternatives' positions.*

Example 7 (One-Rank Improvements) *In the case of the family in Table 3, the favors of the husband (red convertible F silver wagon F white sedan) is a one-rank improvement of the position of alternative 'white sedan' from the favors of the wife (red convertible F white sedan F silver wagon).*

Definition 13 (Distance between Two Orderings) *Consider distinct linear orderings,* P *and* P'*, in* $L(A)$*, and define the* distance *between ordering* P *and ordering* P' *as the minimal length* q *of the sequence* $P = P_0, P_1, \ldots, P_q = P'$ *of orderings in* $L(A)$ *such that ordering* P_r *is obtained by a one-rank improvement of the position of an alternative form ordering* P_{r-1} *for any* $r = 1, 2, \ldots, q$*. For any ordering* P *in* $L(A)$*, the distance between ordering* P *and ordering* P *is defined as 0. The distance between* P *and* P' *is denoted by* $d(P, P')$*.*

Example 8 (Distances between Two Orderings) *In the case of the family in Table 3, the distance between the favors of the husband and the favors of the wife is 1. The distance between the favors of the husband and the favors of the son is 3. The distance between the favors of the wife and the favors of the son is 2.*

We can easily confirm that the distance defined above satisfies the conditions to be a metric.

Proposition 1 *The distance defined above is a metric on* $L(A)$*.*

(proof) First, we have that $d(P, P') \geq 0$ for any two orderings, P and P', in $L(A)$ from the definition, and that $d(P, P') = 0$ if and only if $P = P'$, because an ordering needs positive numbers of one-rank improvements in order to reach a distinct ordering. Second, we have $d(P, P') = d(P', P)$ for any two orderings, P and P', in $L(A)$. If one of minimal sequences from ordering P to ordering P' by one-rank improvements is $P = P_0, P_1, \ldots, P_q = P'$, then the reversed sequence $P' = P_q, P_{q-1}, \ldots, P_0 = P$ is one of minimal sequence from ordering P' to ordering P by one-rank improvements, because if ordering P is obtained by a one-rank improvement of the position of an alternative from ordering P', then we can obtain ordering P' by a one-rank improvement of the position of another alternative from ordering P. Third, for any three orderings, P, P' and P'', in $L(A)$, we have that $d(P, P'') \leq d(P, P') + d(P', P'')$, because $d(P, P'') > d(P, P') + d(P', P'')$ implies that one of the shortest sequence from P to P'' is longer than the connection of a sequence from P to P' and a sequence from P' to P'', and this contradicts the definition of d. ■

Using the metric on $L(A)$, we can define *rational*, *helping*, and *aggressive* behaviors in a meeting. The idea of the definitions is as follows: given a base meeting and the preferences of decision makers, we define a *rational vote* of a decision maker as a linear ordering that is the same as the favors of the decision maker. A *helping vote* of a decision maker to another is a linear ordering whose distance from the preferences of the other is shorter than the distance between the favors of the decision maker and the preferences of the other. An *aggressive vote* of a decision maker to another is a linear ordering whose distance from the preferences of the other is longer than the distance between the favors of the decision maker and the preferences of the other. A linear ordering whose distance from the preferences of the other is equal to the distance between the favors of the decision maker and the preferences of the other is called an *irrelevant vote.* Consider a base meeting $M = (N, W, A, F)$ and the preferences P of decision makers.

Definition 14 (Rational Votes) *A linear ordering P' in $L(A)$ is a* rational vote *of decision maker i in N, if ordering P' is the same as the favors F_i of decision maker i, that is, $P' = F_i$.*

Definition 15 (Helping Votes) *Consider two decision makers, i and j, in N. A linear ordering P' in $L(A)$ is a* helping vote *of decision maker i to decision maker j, if the distance between the preferences P_j of decision maker j and the ordering P' is shorter than the distance between the preferences P_j of decision maker j and the favors F_i of decision maker i, that is, $d(P_j, P') < d(P_j, F_i)$.*

Definition 16 (Aggressive Votes) *Consider two decision makers, i and j, in N. A linear ordering P' in $L(A)$ is an* aggressive vote *of decision maker i to decision maker j, if the distance between the preferences P_j of decision maker j and the ordering P' is longer than the distance between the preferences P_j of decision maker j and the favors F_i of decision maker i, that is, $d(P_j, P') > d(P_j, F_i)$.*

Definition 17 (Irrelevant Votes) *Consider two decision makers, i and j, in N. A linear ordering P' in $L(A)$ is an* irrelevant vote *of decision maker i to decision maker j, if the ordering P' is different from the favors F_i of decision maker i, and the distance between the preferences P_j of decision maker j and the ordering P' is equal to the distance between the preferences P_j of decision maker j and the favors F_i of decision maker i, that is, $P' \neq F_i$ and $d(P_j, P') = d(P_j, F_i)$.*

Example 9 (Rational, Helping, Aggressive, and Irrelevant Votes) *In the case of the family in Table 3, assume that each decision maker has the preferences that is equal to his/her favors. Then we have apparently that the preferences of each decision maker is a rational vote of the decision maker. The preferences of the husband is a helping vote of the son to the wife, because the distance between the preferences of the wife and the preferences of the husband is 1 and the distance between the preferences of the wife and the preferences of the son is 2. The preferences of the son is an aggressive vote of the wife to the husband, because the distance between the preferences of the husband and the preferences of the son is 3 and the distance between the preferences of the husband and the preferences of the wife is 1. The preferences of one of the daughters is an irrelevant vote of the other daughter to the son, because the distance between the preferences of the son and one of the daughters is the same as the distance between the preferences of the son and the other daughter.*

The functions of emotions in terms of credibility of information about helping and aggressive behaviors in cooperative situations can be described by defining *credible information* as in the case of competitive situations. In this paper, regarding cooperative situations, we focus on information about the preferences of decision makers, and assume that the information is conveyed in the form of a linear ordering on the set of all alternatives. Consider a base meeting $M = (N, W, A, F)$, the preferences P and the emotions e_i of decision maker i in N, and information $\hat{P}_i$ about the preferences P_i of decision maker i.

Definition 18 (Credible Information) *The information $\hat{P}_i$ is* credible, *if we have either*

1. *$\hat{P}_i$ is a rational vote of decision maker i,*
2. *there exists decision maker k in N such that $e_i|_k = +$ and $\hat{P}_i$ is a helping vote of decision maker i to decision maker k,*
3. *there exists decision maker k in N such that $e_i|_k = -$ and $\hat{P}_i$ is an aggressive vote of decision maker i to decision maker k,*
   or

4. $\hat{P}_i$ is an irrelevant vote of decision maker i to decision maker k for any k in $N \backslash \{i\}$.

Example 10 (Functions of Emotions) *In the case of the family in Table 3 with the emotions e of decision makers, assume that each decision maker has the preferences that is equal to his/her favors. If information about the preferences of the son is equal to the preferences of the son, then the information is credible, because it is a rational vote of the son. If the information is the same as the preferences of the wife, and the son has positive emotions toward the husband, then the information is credible, because it is a helping vote of the son to the husband. If information about the preferences of the wife is equal to the preferences of the son, and the wife has negative emotions toward the husband, then the information is credible, because it is an aggressive vote of the wife to the husband.*

The functions of emotions in terms of tendencies of decision makers' behaviors can be expressed as follows: in a base meeting $M = (N, W, A, F)$ with the preferences P of decision makers and the emotions e_i of decision maker i in N, we have either that P_i is a rational vote of decision maker i, that for some k in N such that $e_i|_k = +$, P_i is a helping vote of decision maker i to decision maker k, that for some k in N such that $e_i|_k = -$, P_i is an aggressive vote of decision maker i to decision maker k, or that P_i is an irrelevant vote of decision maker i to decision maker k for any k in $N \backslash \{i\}$. Thus the preferences of a decision maker is also information that is credible for decision makers.

Example 11 (Credible Information) *In the case of the family in Table 3 with the emotions e and the preferences P of decision makers. Assume that the preferences of the husband is equal to the favors of the husband. If the son has positive emotions toward the husband, then the son can have the preferences that are the same as, for example, the favors of the son and the favors of the wife. If the wife has negative emotions toward the husband, then the wife can have the preferences that is equal to, for example, the favors of the wife and the favors of the son.*

2.3 Interperception by Decision Makers

Introducing emotions of decision makers into situations of decision making and dealing with economically irrational behaviors of decision makers require us to treat the aspect of interperception by decision makers. Considering two decision makers; one has positive emotions toward the other, and the other has negative emotions toward the one, we have that the one may try to have the same preferences as the other, and that the other may intend to have the preferences that are equal to the reversed preferences of the one. This situation contradicts the assumption of complete information in terms of preferences; at least one of them has to have incorrect perceptions of the preferences of the other. The most preferable outcome or alternative for the decision maker with positive emotions toward the other is least preferable for the decision maker with negative emotions toward the other, and the most preferable outcome or alternative for the decision maker with negative emotions toward the other is most preferable for the decision maker with positive emotions toward the other. Thus the most preferable outcome or alternative coincide with the least preferable outcome or alternative under the assumption of complete information in terms of preferences, and this contradicts the assumption of linearity of the preferences of decision makers.

We need an appropriate framework to treat interperception by decision makers, but existing frameworks are not satisfactory. In the framework for hypergames [5, 100] we can deal with the interperception of components of a base competition, but cannot deal with that of the emotions of decision makers. The framework, moreover, does not treat strictly the interperception of the set of all decision makers. The framework for information structure [1, 24, 62, 63] employs the hypothesis of mutual rationality, thus we can apply the framework to only limited cases. Furthermore, interperception by decision makers in cooperative situations has not been treated in any framework. In this section, therefore, we provide a framework to treat interperception by decision makers without special hypotheses. In the framework we can appropriately deal with interperception in terms of any component of a situation, especially, the set of all decision makers, the emotions of decision makers, and the components of a cooperative situation.

Introducing interperception by decision makers to situations of decision making requires re-definition of functions of emotions. In this section we give the modified version of definition of *credibility* of information caused by the employment of interperception by decision makers.

2.3.1 Interperception of the Set of All Decision Makers

Each situation, whether it is competitive or cooperative, involves a set of decision makers in it. Each decision maker in a situation may misperceive who are participating in the situation. Thus we should treat interperception by decision makers in terms of the set of all decision makers. We employ the concepts of *schemes* and *strings* of decision makers to describe the interperception of the set of all decision makers. The concepts of schemes of decision makers and strings of decision makers are similar to those of hypermaps [12] and strings of players [100], respectively. In order to describe decision makers' perceptions of the set of all decision makers in a situation, we have to employ both of the concepts at the same time.

Consider the set of decision makers in a situation. Each decision maker may believe that a person who is not included in the set is also participating in the situation. In this paper we regard such a person as a decision maker in the situation, while the person does not influence the decision actually, and appears in only *actual* decision makers' perceptions. We call such a decision maker a *fake* decision maker. Let N be the set of all decision makers including all fake decision makers such as the person. Furthermore, for any i in N, let N^i be decision maker i's perception of N, that is, decision maker i believes that the set of all decision makers in the situation is N^i. N^i must be included in N, because N consists of the set of all actual and fake decision makers as well. Decision makers in N are classified into three groups. First group consists of all of the fake decision makers, and second group consists of the decision makers each of who is an actual decision maker, but does not believe that he/she is an actual decision maker. Third group is the set of the actual decision makers each of who believes that he/she is an actual decision maker. For any decision maker i in the first and the second group, we should have that $N^i = \{i\}$, because he/she does not believe that he/she is participating in some interactive situations of decision making. For any decision maker i in the third group, we should have that $i \in N$ and $N^i \neq \{i\}$, because he/she believes that he/she is a decision maker, and participating in an interactive situation. For any decision maker i in the third group, and any j in N^i such that $j \neq i$, decision maker i regards decision maker j as an actual decision maker. Since decision maker i knows that decision maker j also has a perception of N, decision maker i tries to perceive correctly the perception. Let N^{ji} be decision maker i's perception of decision maker j's perception of N, then we should have that N^{ji} is included in N^i, because decision maker i cannot perceive the persons whom he/she does not know. Oppositely, if decision maker i perceives a decision maker j's perception of N, then we should have that j is included in N^i, because decision maker regards decision maker j as a decision maker. Applying this consideration to higher degree of perceptions, we obtain a definition of decision makers' *pairs of a scheme and a set of strings of decision makers.* For any i in N, let Σ_i^* be the set of all ordered strings of decision makers $\sigma = i_1 i_2 \cdots i_q$ $(q = 1, 2, \ldots)$ such that $i_1, i_2, \ldots, i_q$ are elements of N, i_q is equal to i, and i_r is not equal to i_{r+1} for any $r = 1, 2, \ldots, q-1$, that is, $\Sigma_i^* = \{\sigma = i_1 i_2 \cdots i_q \ (q = 1, 2, \ldots) \mid i_1, i_2, \ldots, i_q \in N,\ i_q = i,\ i_r \neq i_{r+1} (r = 1, 2, \ldots q-1)\}$.

Definition 19 (Pairs of a Scheme and a Set of Strings) *For any i in N, decision maker i's* pair of a scheme and a set of strings of decision makers *is a pair* $(\mathbf{N}_i, \Sigma_i)$*, where* $\mathbf{N}_i = (N^\sigma \mid N^\sigma \subset N)_{\sigma \in \Sigma_i}$ *and* $\Sigma_i \subset \Sigma_i^*$*, which satisfies the following conditions:*

1. *the string i is an element of Σ_i,*

2. *for any string $\sigma = i_1 i_2 \cdots i_q$ in Σ_i $(q = 1, 2, 3, \ldots)$, decision maker i_1 is an element of N^σ,*

3. *for any string $\sigma = i_1 i_2 \cdots i_q$ in Σ_i $(q = 1, 2, 3, \ldots)$, and any decision maker j in $N^\sigma \backslash \{i_1\}$, string $j\sigma$ is an element of Σ_i, and $N^{j\sigma}$ is included in N^σ,*
   and

4. *for any $q = 2, 3, \ldots$, if string $\sigma = i_1 i_2 \cdots i_q$ is an element of Σ_i, then string $i_2 i_3 \cdots i_q$ is an element of Σ_i, and decision maker i_1 is an element of $N^{i_2 i_3 \cdots i_q}$.*

For any i in N and any decision maker i's pair $(\mathbf{N}_i, \Sigma_i)$ of a scheme and a set of strings of decision makers, $\mathbf{N}_i$ and Σ_i are called decision maker i's *scheme* of decision makers and decision maker i's *set of strings* of decision makers, respectively. For any σ in Σ_i, N^σ is called σ's *perception* of decision makers. A list $(\mathbf{N}_i, \Sigma_i)_{i \in N}$ of pairs $(\mathbf{N}_i, \Sigma_i)$ of a scheme and a set of strings of decision makers for each i in N is called decision makers' *pair of a scheme and a set of strings of decision makers*, denoted by $(\mathbf{N}, \Sigma)$.

For any i in N, any $\sigma = i_1 i_2 \cdots i_q$ in Σ_i, and any j in N^σ, $j\sigma$ denotes the string $j i_1 i_2 \cdots i_q$ if $j \neq i_1$, and the string $i_1 i_2 \cdots i_q$ if $j = i_1$. Similarly, σj denotes the string $i_1 i_2 \cdots i_q j$ if $j \neq i_q$, and the string $i_1 i_2 \cdots i_q$ if $j = i_q$.

For any i in N, decision maker i's pair of a scheme and a set of strings of decision makers in a situation expresses decision maker i's perceptions of the set of all decision makers in the situation. If decision maker i in N believes that all of the decision makers in N are involved in the situation, and that it is common knowledge among them, then his/her scheme of decision makers, $\mathbf{N}_i = (N^\sigma)_{\sigma \in \Sigma_i}$, satisfies that $N^\sigma = N$ for any σ in Σ_i, and his/her set Σ_i of strings of decision makers is equal to Σ_i^*. If decision maker i does not think that he/she is participating in the situation, then $\mathbf{N}_i = (N^i)$, where $N^i = \{i\}$, and $\Sigma_i = \{i\}$. Between these extremes, we think of various states of decision makers' perceptions of the set of all decision makers. Each of the states is described as a pair of a scheme and a set of strings of decision makers.

We can show equivalence of giving a decision maker's scheme of decision makers to providing his/her set of strings of decision makers as follows.

Proposition 2 (Equivalence of a Scheme to a Set of Strings) *For any i in N, consider two decision maker i's pairs,* $(\mathbf{N}_i, \Sigma_i)$ *and* $(\mathbf{N}'_i, \Sigma'_i)$*, of a scheme and a set of strings of decision makers, where* $\mathbf{N}_i = (N^\sigma)_{\sigma \in \Sigma_i}$ *and* $\mathbf{N}'_i = (N'^{\sigma'})_{\sigma' \in \Sigma'_i}$*. If we have that $\Sigma_i = \Sigma'_i$, then it is satisfied that $N^\sigma = N'^\sigma$ for any σ in $\Sigma_i = \Sigma'_i$.*

(proof) Assume that $\Sigma_i = \Sigma'_i$, and suppose a string $\sigma = i_1 i_2 \ldots i_q$ in $\Sigma_i = \Sigma'_i$. From Condition 2, we have that i_1 is an element of $N^\sigma \cap N'^\sigma$. For any j in $N^\sigma \backslash \{i_1\}$, the string $j\sigma$ is an element of Σ_i because of Condition 3. Then, from Condition 4, j is an element of N'^σ, since $\Sigma_i = \Sigma'_i$. Thus N^σ is included in N'^σ. Replacing N^σ and N'^σ, we have that $N^\sigma = N'^\sigma$. ■

Thanks to this proposition, we can describe a decision maker's perceptions of the set of all decision makers by providing either his/her scheme of decision makers, or his/her set of strings of decision makers.

2.3.2 Interperception of Components of Situations

Consider a situation of decision making. A decision maker in the situation can perceive, and may misperceive, any component of the situation. In a competitive situation the the set N of all decision makers, the set S of all outcomes, the profits F, the emotions e, the preferences P, and the rules r of decision makers; and in a cooperative situation the set N of all decision makers, the set W of all winning coalitions, the set A of all alternatives, the favors F, the emotions e, and the preferences P of decision makers; are regarded as the components of the situation so far. We express decision makers' perceptions of the set of all decision makers by using decision makers' pairs of a scheme and a set of strings of decision makers. For any component of the situation, we can describe decision makers' perceptions of the component by employing decision makers' pairs of a scheme of decision makers and a set of strings of decision makers, and decision makers' *schemes* of the component.

For example, suppose the set N of all decision makers in a situation, decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, and the emotions e of decision makers. Decision maker i in N believes that all of the decision makers in N^i are participating in the situation, and that each decision maker in N^i has emotions. We express decision maker i's perceptions of the emotions by defining decision maker i's *perception of emotions* as a list $(e^i_j)_{j\in N^i}$ of decision maker i's perceptions e^i_j of the emotions e_j of decision maker j for each j in N^i, denoted by e^i. Decision maker i, moreover, believes that each decision maker j in N^i thinks that N^{ji} is the set of all decision makers in the situation, and each of them has emotions. We describe decision maker i's perceptions of decision maker j's perceptions of the emotions by defining ji's *perception of emotions* as a list $(e^{ji}_k)_{k\in N^{ji}}$ of decision maker i's perceptions e^{ji}_k of decision maker j's perceptions e^j_k of the emotions e_k of decision maker k in N^{ji}, denoted by e^{ji}. Generally, for any string σ of decision makers in Σ_i, σ's *perception of emotions* is defined as a list $(e^\sigma_l)_{l\in N^\sigma}$, denoted by e^σ. Then we obtain the following definition of decision maker i's *schemes of emotions*.

Definition 20 (Decision Makers' Schemes of Emotions) *Consider the set N of all decision makers in a situation and decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers. Decision maker i's* scheme of emotions *is a list $(e^\sigma)_{\sigma\in\Sigma_i}$ of σ's perceptions e^σ of emotions for each string σ in Σ_i, denoted by $\mathbf{e}_i$. A list $(\mathbf{e}_i)_{i\in N}$ of decision maker i's schemes of emotions $\mathbf{e}_i$ for each i in N is called decision makers'* scheme of emotions, *denoted by* $\mathbf{e}$.

We can apply the same consideration to any component of a situation. Given the set N of all decision makers in the situation, decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, and a component X of the situation, where X can be N, S, W, A, F, P, r, e, or even C or M, for any i in N and any σ in Σ_i, we can define σ's *perception* of X, denoted by X^σ. Thus we get N^σ, S^σ, W^σ, A^σ, F^σ, P^σ, r^σ, e^σ, C^σ, or M^σ. Then we define decision maker i's *scheme* of X as a list $(X^\sigma)_{\sigma\in\Sigma_i}$ of σ's perceptions X^σ of X for each σ in Σ_i, denoted by $\mathbf{X}_i$. Then we get $\mathbf{N}_i$, $\mathbf{S}_i$, $\mathbf{W}_i$, $\mathbf{A}_i$, $\mathbf{F}_i$, $\mathbf{P}_i$, $\mathbf{r}_i$, $\mathbf{e}_i$, $\mathbf{C}_i$ or $\mathbf{M}_i$. Decision maker i's scheme $\mathbf{X}_i$ of X expresses decision maker i's perceptions of X. A list $(\mathbf{X}_i)_{i\in N}$ of decision maker i's schemes $\mathbf{X}_i$ of X for each i in N is called decision makers' *scheme* of X, denoted by $\mathbf{X}$. Thus we have $\mathbf{N}$, $\mathbf{S}$, $\mathbf{W}$, $\mathbf{A}$, $\mathbf{F}$, $\mathbf{P}$, $\mathbf{r}$, $\mathbf{e}$, $\mathbf{C}$, or $\mathbf{M}$.

Adding to the concepts of perceptions and schemes of a component, we define the concept of decision makers' *views* on the component. Consider the set N of all decision makers in a situation and decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers.

Definition 21 (Decision Makers' Views on Components) *For any i in N and any component X of the situation, decision maker i's* view *on X is a list $(X^\sigma)_{\sigma\in\Sigma_i\setminus\{i\}}$, denoted by* $\mathbf{X}^i$.

For any i in N, decision maker i's view on X consists of all decision maker i's perceptions of the others' perceptions of X. In fact, regarding the same set of all decision makers N and the same decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, the difference between decision maker i's scheme $\mathbf{X}_i$ of X and a decision maker i's view $\mathbf{X}^i$ on X is only decision maker i's perception X^i of X, that is, $\mathbf{X}_i = (X^i, \mathbf{X}^i)$.

Applying the consideration above to higher degree of perceptions, for any i in N, any σ in Σ_i, and any component X of a situation, we have the concepts of σ's *schemes* of component X and σ's *views* on component X. Let Σ_σ be the set of all strings τ in Σ_i such that $\tau = i_1 i_2 \cdots i_q \sigma$ for some $i_1, i_2, \ldots, i_q$ in N $(q = 0, 1, 2, \cdots)$, that is, $\Sigma_\sigma = \{\tau = i_1 i_2 \cdots i_q i_{q+1} i_{q+2} \cdots i_p \ (q = 0, 1, 2, \ldots) \mid \tau \in \Sigma_i, i_{q+1} i_{q+2} \cdots i_p = \sigma\}$. Especially, the string σ is an element of Σ_σ.

Definition 22 (Strings' Schemes of Components) *For any σ in Σ_i, σ's* scheme *of X is a list $(X^\tau)_{\tau \in \Sigma_\sigma}$ of τ's perceptions X^τ of X for each string τ in Σ_σ, denoted by $\mathbf{X}_\sigma$. Particularly, for the string i in Σ_i, i's scheme of X, $\mathbf{X}_i$, is equal to decision maker i's scheme of X.*

Definition 23 (Strings' Views on Components) *For any σ in Σ_i, σ's* view *on X is a list $(X^\tau)_{\tau \in \Sigma_\sigma \setminus \{\sigma\}}$, denoted by $\mathbf{X}^\sigma$. Particularly, for the string i in Σ_i, i's view on X, $\mathbf{X}^i$, is equal to decision maker i's view on X.*

Regarding relations between views and schemes, we have the following proposition.

Proposition 3 (Decomposition of a View into Schemes) *Consider the set N of all decision makers in a situation, decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, and a component X of the situation. For any i in N, i's view $\mathbf{X}^i$ on X is decomposed into ji's schemes of X for each j in $N^i \setminus \{i\}$, that is, $\mathbf{X}^i = (\mathbf{X}_{ji})_{j \in N^i \setminus \{i\}}$. Generally, for any $\sigma = i_1 i_2 \cdots i_q$ in Σ_i, σ's view $\mathbf{X}^\sigma$ on X is decomposed into $j\sigma$'s schemes of X for each j in $N^\sigma \setminus \{i_1\}$, that is, $\mathbf{X}^\sigma = (\mathbf{X}_{j\sigma})_{j \in N^\sigma \setminus \{i_1\}}$.*

(proof) We show the general case. For any $\sigma = i_1 i_2 \cdots i_q$ in Σ_i, we have that $\mathbf{X}^\sigma = (X^\tau)_{\tau \in \Sigma_\sigma \setminus \{\sigma\}}$ by the definition. Classifying τs in terms of the decision maker immediately before σ, and considering the fact that the possible ones are included in N^σ, we have that $\mathbf{X}^\sigma = (X^{\mu j \sigma} \mid \mu j \sigma \in \Sigma_\sigma \setminus \{\sigma\})_{j \in N^\sigma \setminus \{i_1\}}$, where the string μ can be null-string. For any j in $N^\sigma \setminus \{i_1\}$, the set $\{\mu j \sigma \mid \mu j \sigma \in \Sigma_\sigma \setminus \{\sigma\}\}$ coincide with the set $\Sigma_{j\sigma}$, thus we have that $\mathbf{X}^\sigma = ((X^\nu)_{\nu \in \Sigma_{j\sigma}})_{j \in N^\sigma \setminus \{i_1\}}$. Since $(X^\nu)_{\nu \in \Sigma_{j\sigma}} = \mathbf{X}_{j\sigma}$ for any j in $N^\sigma \setminus \{i_1\}$, we have the result.
■

This proposition implies that a decision maker's view on a component can be interpreted as perceptions of the others' schemes of the component. From this proposition, moreover, we have that $\mathbf{X}_i = (X^i, (\mathbf{X}_{ji})_{j \in N^i \setminus \{i\}})$ for any i in N. This means that decision maker i's scheme of X can be expressed as the pair of decision maker i's perception X^i of X and decision maker i's perceptions of the others schemes of X. Figure 2 illustrates the structure of a decision maker's scheme of a component.

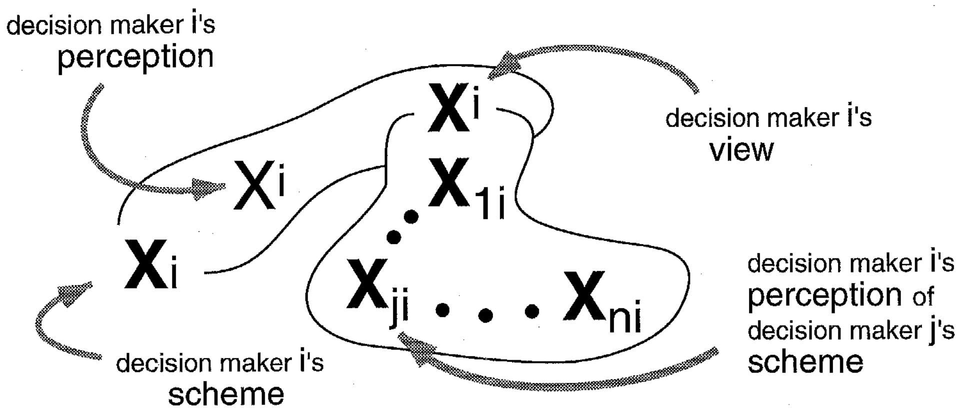

Figure 2. Structure of a scheme.

2.3.3 Interperception and Functions of Emotions

We need to re-define the functions of emotions when we introduce interperception by decision makers in situations of decision making. Each decision maker in a situation perceives the situation subjectively, and behaves referring to his/her subjective perceptions. Thus credible information for one decision maker can be incredible for another. We provide a revised version of definition of credibility of information, and consequently, functions of emotions considering decision makers' subjective point of view.

Emotions in competitive situations

Suppose the set N of all decision makers in a competitive situation and decision makers pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers. For any i in N, decision maker i regards the situation as $\mathbf{C}_i = (\mathbf{N}_i, \mathbf{S}_i, \mathbf{F}_i)$, where $\mathbf{N}_i = (N^\sigma)_{\sigma \in \Sigma_i}$, $\mathbf{S}_i = (S^\sigma)_{\sigma \in \Sigma_i}$, $\mathbf{F}_i = (F^\sigma)_{\sigma \in \Sigma_i}$, because of his/her subjective point of view. This means that decision maker i thinks that the situation is $C^i = (N^i, S^i, F^i)$, and that decision maker j believes that the situation is $C^{ji} = (N^{ji}, S^{ji}, F^{ji})$, and so on.

Decision maker i also regards the emotions, the preferences, and the rules of decision makers as $\mathbf{e}_i = (e^\sigma)_{\sigma \in \Sigma_i}$, $\mathbf{P}_i = (P^\sigma)_{\sigma \in \Sigma_i}$, and $\mathbf{r}_i = (r^\sigma)_{\sigma \in \Sigma_i}$, respectively. The assumption that the preferences of each decision maker are determined by the profits and the emotions of decision makers is still valid in this case, and we should describe the assumption more exactly. We assume that decision maker i's perception P_i^i of the preferences P_i of decision maker i is determined by decision maker i's scheme $\mathbf{F}_i$ of profits and decision maker i's scheme $\mathbf{e}_i$ of emotions. Decision maker i's scheme $\mathbf{r}_i$ of rules must be consistent with decision maker i's scheme $\mathbf{P}_i$ of preferences, that is, for any σ in Σ_i, r_i^σ must be consistent with P^σ.

For any i in N, if we have decision maker i's scheme $\mathbf{C}_i = (\mathbf{N}_i, \mathbf{S}_i, \mathbf{F}_i)$ of base competitions and decision maker i's scheme $\mathbf{P}_i$ of preferences, we can define *rational*, *helping*, and *aggressive* rules for decision maker i. Consider decision maker j in N^i.

Definition 24 (Rational Rules) *For any s_{-j} in S^{ji}_{-j}, if ji's perception r'^{ji}_j of the rule of decision maker j satisfies that $(r'^{ji}_j(s_{-j}), s_{-j}) \not{F}^{ji}_j (s_j, s_{-j})$ for any s_j in S^{ji}_j, then r'^{ji}_j is decision maker j's* rational rule *for decision maker i at s_{-j}.*

Definition 25 (Helping rules) *Consider decision maker k in N^{ji}. For any s_{-j} in S^{ji}_{-j}, if ji's perception r'^{ji}_j of rule of decision maker j satisfies that $(s^{*ji}_j, s_{-j}) P^{ji}_k (r'^{ji}_j(s_{-j}), s_{-j})$, then*

r'^{ji}_j *is decision maker* j*'s* helping rule *to decision maker* k *for decision maker* i *at* s_{-j}*, where* s^{*ji}_j *satisfies that* $(s^{*ji}_j, s_{-j})\not{F}^{ji}_j(s_j, s_{-j})$ *for any* s_j *in* S^{ji}_j*.*

Definition 26 (Aggressive Rules) *Consider decision maker* k *in* N^{ji}*. For any* s_{-j} *in* S^{ji}_{-j}*, if* ji*'s perception* r'^{ji}_j *of rule of decision maker* j *satisfies that* $(r'^{ji}_j(s_{-j}), s_{-j})P^{ji}_k(s^{*ji}_j, s_{-j})$*, then* r'^{ji}_j *is a decision maker* j*'s* aggressive rule *to decision maker* k *for decision maker* i *at* s_{-j}*, where* s^{*ji}_j *satisfies that* $(s^{*ji}_j, s_{-j})\not{F}^{ji}_j(s_j, s_{-j})$ *for any* s_j *in* S^{ji}_j*.*

Considering decision maker i's scheme $\mathbf{C}_i = (\mathbf{N}_i, \mathbf{S}_i, \mathbf{F}_i)$ of base competitions, decision maker i's scheme $\mathbf{P}_i$ of preferences, and decision maker i's scheme $\mathbf{e}_i$ of emotions, credibility of information $\hat{r}_j$ about the rule of decision maker j for decision maker i, where $\hat{r}_j$ is a function from S^{ji}_{-j} to S^{ji}_j, can be re-defined as follows.

Definition 27 (Credible Information) *For any* s_{-j} *in* S^{ji}_{-j}*, the information* $\hat{r}_j$ *is* credible *for decision maker* i *at* s_{-j}*, if we have either*

1. $\hat{r}_j$ *is decision maker* j*'s rational rule for decision maker* i *at* s_{-j}*,*
2. *there exists decision maker* k *in* N^{ji} *such that* $e^{ji}_j|_k = +$ *and* $\hat{r}_j$ *is decision maker* j*'s helping rule to decision maker* k *for decision maker* i *at* s_{-j}*,*
   or
3. *there exists decision maker* k *in* N^{ji} *such that* $e^{ji}_j|_k = -$ *and* $\hat{r}_j$ *is decision maker* j*'s aggressive rule to decision maker* k *for decision maker* i *at* s_{-j}*.*

If for any s_{-j} *in* S^{ji}_{-j}*,* $\hat{r}_j$ *is credible for decision maker* i *at* s_{-j}*, then* $\hat{r}_j$ *is said to be* credible *for decision maker* i*.*

Given decision maker i's scheme $\mathbf{C}_i = (\mathbf{N}_i, \mathbf{S}_i, \mathbf{F}_i)$ of base competitions, decision maker i's scheme $\mathbf{P}_i$ of preferences, decision maker i's scheme $\mathbf{e}_i$ of emotions, and decision maker i's scheme $\mathbf{r}_i$ of rules, we can re-define the function of emotions in terms of tendencies of decision makers' behaviors by the assumption that for any s_{-i} in S^i_{-i}, we have either that r^i_i is decision maker i's rational rule for decision maker i at s_{-i}, that for some k in N^i such that $e^i_i|_k = +$, r^i_i is decision maker i's helping rule to decision maker k for decision maker i at s_{-i}, or that for some k in N^i such that $e^i_i|_k$, r^i_i is decision maker i's aggressive rule to decision maker k for decision maker i at s_{-i}.

Emotions in cooperative situations

Given the set N of all decision makers in a competitive situation and decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, as in the case of competitive situations, we can suppose for any i in N, decision maker i's scheme $\mathbf{M}_i = (\mathbf{N}_i, \mathbf{W}_i, \mathbf{A}_i, \mathbf{F}_i)$ of base meetings, decision maker i's scheme $\mathbf{e}_i$ of emotions, decision maker i's scheme $\mathbf{P}_i$ of preferences. For any σ in Σ_i, moreover, we can define a metric d^σ on the set $L(A^\sigma)$ of all linear orderings on A^σ as the metric d defined on $L(A)$ in a previous section. For any i in N and any j in N^i, we can define decision maker j's *rational, helping, aggressive,* and *irrelevant votes* for decision maker i by using these metrics.

Definition 28 (Rational Votes) *A linear ordering* P' *in* $L(A^{ji})$ *is decision maker* j*'s* rational vote *for decision maker* i*, if ordering* P' *is the same as* ji*'s perception* F^{ji}_j *of the favors of decision maker* j*, that is,* $P' = F^{ji}_j$*.*

Definition 29 (Helping Votes) *Consider decision maker k in N^{ji}. A linear ordering P' in $L(A^{ji})$ is decision maker j's* helping vote *to decision maker k for decision maker i, if the distance between ji's perception P_k^{ji} of the preferences of decision maker k and the ordering P' is shorter than the distance between ji's perceptions P_k^{ji} of the preferences of decision maker k and ji's perception F_j^{ji} of the favors of decision maker j, that is, $d^{ji}(P_k^{ji}, P') < d^{ji}(P_k^{ji}, F_j^{ji})$.*

Definition 30 (Aggressive Votes) *Consider decision maker k in N^{ji}. A linear ordering P' in $L(A^{ji})$ is decision maker j's* aggressive vote *to decision maker k for decision maker i, if the distance between ji's perception P_k^{ji} of the preferences of decision maker k and the ordering P' is longer than the distance between ji's perceptions P_k^{ji} of the preferences of decision maker k and ji's perception F_j^{ji} of the favors of decision maker j, that is, $d^{ji}(P_k^{ji}, P') > d^{ji}(P_k^{ji}, F_j^{ji})$.*

Definition 31 (Irrelevant Votes) *Consider decision maker k in N^{ji}. A linear ordering P' in $L(A^{ji})$ is decision maker j's* irrelevant vote *to decision maker k for decision maker i, if the distance between the ji's perception P_k^{ji} of the preferences of decision maker k and the ordering P' is equal to the distance between ji's perceptions P_k^{ji} of the preferences of decision maker k and ji's perception F_j^{ji} of the favors of decision maker j, that is, $d^{ji}(P_k^{ji}, P') = d^{ji}(P_k^{ji}, F_j^{ji})$.*

Considering decision maker i's scheme $\mathbf{M}_i = (\mathbf{N}_i, \mathbf{W}_i, \mathbf{A}_i, \mathbf{F}_i)$ of base meetings, decision maker i's scheme $\mathbf{P}_i$ of preferences, and decision maker i's scheme $\mathbf{e}_i$ of emotions, the credibility of information $\hat{P}_j$ about the preferences of decision maker j for decision maker i, where $\hat{P}_j$ is a linear ordering in $L(A^{ji})$ can be re-defined as follows.

Definition 32 (Credible Information) *The information $\hat{P}_j$ is* credible *for decision maker i, if we have either*

1. *$\hat{P}_j$ is decision maker j's rational vote for decision maker i,*
2. *there exists decision maker k in N^{ji} such that $e_j^{ji}|_k = +$ and $\hat{P}_j$ is decision maker j's helping vote to decision maker k for decision maker i,*
3. *there exists decision maker k in N^{ji} such that $e_j^{ji}|_k = -$ and $\hat{P}_j$ is decision maker j's aggressive vote to decision maker k for decision maker i,*
   or
4. *$\hat{P}_j$ is decision maker j's irrelevant vote to decision maker k for decision maker i for any k in N^{ji}.*

Given decision maker i's scheme $\mathbf{M}_i = (\mathbf{N}_i, \mathbf{W}_i, \mathbf{A}_i, \mathbf{F}_i)$ of base meetings, decision maker i's scheme $\mathbf{e}_i$ of emotions, and decision maker i's scheme $\mathbf{P}_i$ of preferences, we can re-define the function of emotions in terms of tendencies of decision makers' behaviors by the assumption that we have either that P_i^i is decision maker i's rational vote for decision maker i, that for some k in N^i such that $e_i^i|_k = +$, P_i^i is decision maker i's helping vote to decision maker k for decision maker i, that for some k in N^i such that $e_i^i|_k = -$, P_i^i is decision maker i's aggressive vote to decision maker k for decision maker i, or that P_i^i is decision maker i's irrelevant vote to decision maker k for decision maker i for any k in $N^i\backslash\{i\}$.

2.4 Information Exchanges

Under incompleteness of information about a component of a situation, information in terms of the component influences decision makers' perceptions of the component. Decision makers may or may not believe the information, and may or may not change their perceptions according to the information. Treating situations with incompleteness of information requires us to cope with exchanges of information and changes of decision makers' perceptions caused by the exchanges. We propose a framework for treating the changes of perceptions.

While exchanges of information in terms of any element of a situation can be occurred, we focus in this paper on the exchanges of information in terms of preferences, more precisely, information about the rules of decision makers in competitive situations, and information about the preferences of decision makers in cooperative situation, because one of the targets in the paper is their irrational behaviors that are induced by their preferences.

2.4.1 Exchanges of Information and Changes of Perceptions

In a competitive situation the rule of a decision maker is regarded as to be consistent with his/her preferences, and it is also assume that the preferences depend on the profits and the emotions of the decision makers in the situation. Under the assumption of completeness of information in terms of profits, information about the rules of decision makers may cause changes of decision makers' perceptions of their preferences, and then the changes may induce modification of decision makers' perceptions of their emotions. In a cooperative situation, similarly, it is assumed that the preferences of a decision maker are determined by the favors and the emotions of decision makers. If the favors of decision makers are common knowledge among them, then exchanges of information in terms of the preferences of decision makers may cause modification of their perceptions of the preferences, and consequently, alterations of decision makers' perceptions of their emotions.

Let us consider strictly. Consider, for example, a competitive situation. Suppose the set N of all decision makers in the situation and decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers. Decision maker i in N has decision maker i's scheme $\mathbf{C}_i = (\mathbf{N}_i, \mathbf{S}_i, \mathbf{F}_i)$ of base competitions, where $\mathbf{N}_i = (N^\sigma)_{\sigma\in\Sigma_i}$, $\mathbf{S}_i = (S^\sigma)_{\sigma\in\Sigma_i}$, $\mathbf{F}_i = (F^\sigma)_{\sigma\in\Sigma_i}$, and i's scheme $\mathbf{e}_i = (e^\sigma)_{\sigma\in\Sigma_i}$ of emotions. Then, moreover, decision maker i constructs his/her schemes, $\mathbf{P}_i = (P^\sigma)_{\sigma\in\Sigma_i}$ and $\mathbf{r}_i = (r^\sigma)_{\sigma\in\Sigma_i}$, of preferences and rules, respectively. We assume that we can describe relations among the scheme of profits, emotions, preferences, and rules by *rule functions* of decision maker i. A rule function f_i of decision maker i is a function from the product of the set of all decision maker i's schemes of profits and that of emotions to the set of all rules of decision maker i, that is, $f_i(\mathbf{F}_i, \mathbf{e}_i) = r_i$. Because we assume that the rule of a decision maker is the vest reply function determined by his/her preferences, f_i expresses composition of construction of his/her preferences from his/her schemes of profits and emotions, and association of his/her rule with the preferences.

If the competitive situation satisfies that the profits of decision makers are common knowledge among them, then we can regard the rule of a decision maker as depending on only the scheme of emotions, that is, $f_i(\mathbf{e}_i) = r_i$. Moreover, if a decision maker has a fixed rule function through the situation, changes of his/her rule induced by new information is reduced to changes of his/her scheme of emotions. Then, considering that decision maker i's scheme $\mathbf{X}_i$ is expressed as a pair of decision maker i's perception X^i and decision maker i's view $\mathbf{X}^i$, and decision maker i's view can be decomposed into ji's schemes $\mathbf{X}_{ji}$ for each j in $N^i\backslash\{i\}$, we employ three types of functions to describe changes of perceptions caused by exchanges of information: *scheme functions*, *perception functions*, and *view functions*. Suppose the case that information $\hat{r}$ about the rules of decision makers is conveyed to decision makers. We assume that the conveyance of the information becomes common knowledge among the decision makers in the situation. Changes of the scheme of a decision maker as a whole caused by $\hat{r}$ is described by the *scheme function* of the decision maker in terms of $\hat{r}$. A scheme function $g_i(\hat{r})$ of decision maker i in terms of $\hat{r}$ is a function from the set of all decision maker i's schemes of emotions to the set itself, that is, $g_i(\hat{r})(\mathbf{e}_i) = \mathbf{e}'_i$. A *perception function* describes only the change of the perception. A perception function $h_i(\hat{r})$ of decision maker i in terms of $\hat{r}$ is a function from the set of all decision maker i's schemes of emotions to the set of all decision maker i's perceptions of emotions, that is, $g_i(\hat{r})(\mathbf{e}_i) = e'^i$, where $\mathbf{e}_i = (e^i, \mathbf{e}^i)$. A *view function* $l_i(\hat{r})$ of decision maker i in terms of $\hat{r}$ is a function from the set of all decision maker i's schemes of emotions to the set of all decision maker i's views of emotions, that is, $l_i(\hat{r})(\mathbf{e}_i) = \mathbf{e}'^i$, where $\mathbf{e}_i = (e^i, \mathbf{e}^i)$.

2.4.2 Changes of Perceptions and Interperception

Since rule functions, scheme functions, perception functions, and view functions can be regarded as components of the situation, we can introduce interperception by the decision makers into the functions. Thus, given the set N of all decision makers in a situation and decision makers pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, we get, for any i in N and any information $\hat{r}$, decision maker i's schemes, $\mathbf{f}_i(\hat{r})$, $\mathbf{g}_i(\hat{r})$, $\mathbf{h}_i(\hat{r})$, and $\mathbf{l}_i(\hat{r})$, of rule functions, scheme functions, perception functions, and view functions in terms of $\hat{r}$, respectively.

On the one hand, for any i in N decision maker i knows his/her own perceptions of emotions, thus changes of his/her perceptions caused by new information can be expressed by his/her perception function in terms of the information. On the other hand, because he/she does not always perceive the exact perceptions of emotions by the others, modification of his/her view of emotions requires two steps (Figure 3). First, he/she create another view referring to the existing view, the information, and his/her scheme $\mathbf{f}_i$ of rule functions by using his/her view function. This step means that after the exchange of information each decision maker tries to reconstruct the states of the others' schemes before the exchange of information. Then, in the second step, each decision maker gets a view that is regarded as the present states of the others' scheme, applying the perceptions of the others' scheme functions to the view obtained in the first step. We assume, therefore, the following relations among the schemes, $\mathbf{g}_i(\hat{r})$ of scheme functions, $\mathbf{h}_i(\hat{r})$ of perception functions, and $\mathbf{l}_i(\hat{r})$ of view functions in terms of $\hat{r}$: for any σ in Σ_i and any j in N^σ,

$$g_j^\sigma(\hat{r}) = (h_j^\sigma(\hat{r}), (g_k^{j\sigma}(\hat{r}))_{k \in N^{j\sigma} \setminus \{j\}} \circ l_j^\sigma(\hat{r})),$$

and particularly, for any i in N,

$$g_j^i(\hat{r}) = (h_j^i(\hat{r}), (g_k^{ji}(\hat{r}))_{k \in N^{ji} \setminus \{j\}} \circ l_j^i(\hat{r})).$$

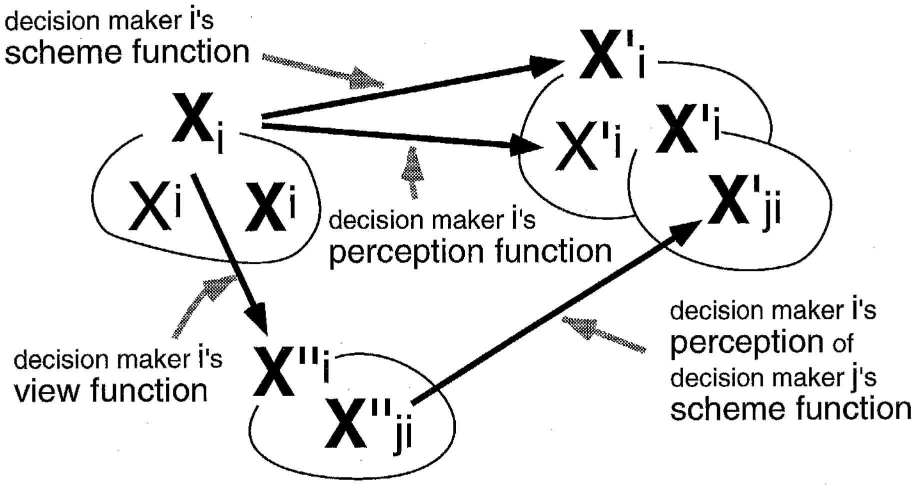

Figure 3. Relations among scheme functions, perception functions, and view functions.

This framework for expressing modification of decision makers' perceptions of emotions caused by information about the rules of decision makers can be applied to changes of perceptions of any component of a situation caused by information about any component of the situation, especially, perceptions and information of the components in a cooperative situation. In any case, we define scheme functions, perception functions, and view functions in terms of the information, and assume the relations among the functions above.

Chapter 3

Fundamental Concepts

In this chapter we provide definitions of concepts of *inside common knowledge*, *integration of perceptions*, *stability of emotions*, and so on, that are essential for examinations of situations of decision making. We also give fundamental properties of relations among the concepts.

3.1 Common Knowledge and Inside Common Knowledge

In this section we treat the concept of *common knowledge* [1, 24] and its generalizations. The concept of *inside common knowledge* describes individual part of the concept of common knowledge. Applying the idea of the concept of inside common knowledge to higher degree of perceptions, we provide the generalizations.

3.1.1 Common Knowledge

The concept of *common knowledge* expresses a type of structures of decision makers' perceptions. If each decision maker in a situation correctly perceives an event regarding the situation, and if each decision maker believes that each of the others correctly perceives the event, and if each decision maker thinks that each of the others believes that each of the others correctly perceives the event, and so on, then the event is said to be *common knowledge* among the decision makers. The concept of common knowledge is strictly defined and examined in the framework of information structure by Aumann [1, 24] with the concepts of *states of the world* and *knowledge operators*. The framework, however, requires the *hypothesis of mutual rationality*, that is, the assumption that each decision maker correctly perceives what each of the others perceives, and that each decision maker believes that each of the others correctly perceives what each of the others perceives, and so on. The hypothesis restricts the extent of application of the framework and the concept of common knowledge. In order to make it possible to use the concept of common knowledge in more general contexts, we define the concept in our framework for interperception. Consider the set N of all decision makers in a situation and decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers. We define decision makers' pairs *in common knowledge*, first.

Definition 33 (Pairs in Common Knowledge) *The pair* $(\mathbf{N}, \Sigma)$ *is* in common knowledge, *if we have that* $\Sigma_i = \Sigma_i^*$ *for any* i *in* N, *or equivalently, that* $N^\sigma = N$ *for any* σ *in* Σ_i *and any* i *in* N.

The equivalence can be shown as follows: assume that $\Sigma_i = \Sigma_i^*$ for any i in N. Then, by the definition of decision maker's pair, for any σ in Σ_i, we have i is an element of N^σ, and we also have that j is an element of N^σ for any j in $N \backslash \{i\}$, because the string $j\sigma$ is an element of Σ_i. Oppositely, assume that $N^\sigma = N$ for any σ in Σ_i and any i in N. Particularly, we have that $N^i = N$, because the string i is an element of Σ_i by the definition of decision makers' pair. Moreover, it is satisfied that for any j in $N \backslash \{i\}$, the string ji is an element of Σ_i because $N^i = N$. Thus if string σ in Σ_i^* consists of two decision makers, then σ is also an element of Σ_i. Assume that for any string $\sigma = i_1 i_2 \cdots i_q$ in Σ_i^* that consists of q decision makers, σ is an element of Σ_i. Then by the definition of decision maker's pair, for any j in $N \backslash \{i_1\}$, we have that $j\sigma$ is an element of Σ_i, because $N^\sigma = N$. Thus for any string in Σ_i^* that consists of $q+1$ decision makers is an element of Σ_i. Therefore, by mathematical induction, we have that $\Sigma_i = \Sigma_i^*$.

Next, consider decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ of emotions. We define decision makers' scheme of emotions *in common knowledge*.

Definition 34 (Schemes in Common Knowledge) *The scheme* $\mathbf{e}$ *of emotions is* in common knowledge, *if it is satisfied that for any i in N, any σ in Σ_i, any j in N^σ, and any k in $N^{j\sigma} \cap N^j$, $e_j^\sigma|_k = e_j^j|_k$.*

If decision makers' scheme of emotions is in common knowledge, then each decision maker correctly perceives the emotions of decision makers, and each decision maker believes that each of the others correctly perceives the emotions of decision makers, and each decision maker thinks that each of the others believes that each of the others correctly perceives the emotions of decision makers, and so on.

In the case that decision makers' pair $(\mathbf{N}, \Sigma)$ is in common knowledge, we can identify e_j^{ji} and e_j^i for any i and j in N, because decision maker j's perception of his/her emotions should be regarded as the true state of the emotions of decision maker j by decision maker i. Then we have an equivalent definition of decision makers' scheme of emotions in common knowledge to the above definition.

Definition 35 (An Equivalent of Common Knowledge) *The scheme* $\mathbf{e}$ *of emotions is* in common knowledge, *if we have that for any i and j in N, and any σ in Σ_i^*, $e^\sigma = e^{\sigma j}$.*

If decision makers' pair $(\mathbf{N}, \Sigma)$ is in common knowledge, the former definition says that decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ of emotions is in common knowledge if for any i, j, k in N, any σ in Σ_i^*, $e_j^\sigma|_k = e_j^j|_k$, that is, $e_j^\sigma = e_j^j$. Considering another decision maker l in N, we have that for any i and l in N, and any σ in Σ_i^*, $e_j^\sigma = e_j^j = e_j^{\sigma l}$ for any j in N, that is, $e^\sigma = e^{\sigma l}$. Therefore we obtain the latter definition. Oppositely, assume that for any i and j in N, and any σ in Σ_i^*, $e^\sigma = e^{\sigma j}$. Then we have that for any k in N, $e_k^\sigma = e_k^{\sigma j}$. Considering the case that $i = k$ and $\sigma = k$, we have that for any j in N, $e_k^k = e_k^{kj}$. Identifying e_k^{kj} and e_k^j, we obtains that $e_k^k = e_k^j$ for any j in N. Applying the assumption inductively, we have that for any i in N, any σ in Σ_i^*, any k in N, $e_k^\sigma = e_k^k$, that is, $e_k^\sigma|_l = e_k^k|_l$ for any l in N.

We can apply the idea of common knowledge to any component of a situation: for any component in a situation, we can define decision makers' scheme of the component *in common knowledge*.

3.1.2 Inside Common Knowledge

The concept of *common knowledge* involves decision makers' scheme of a component in its definition. It is difficult for a decision maker to check whether decision makers' scheme of a component is in common knowledge or not, because decision makers' scheme includes not only his/her own scheme but also the others' schemes, and perceiving correctly all of the elements in the others' schemes requires the decision maker to gather huge amount of information. On the contrary, whether decision makers' scheme is *in inside common knowledge* or not is easily checked. The concept of inside common knowledge describes individual part of the idea of common knowledge, and only a decision maker's scheme is involved in the definition of the concept. If decision makers' scheme is in inside common knowledge for a decision maker, then the decision maker believes that decision makers' scheme is in common knowledge (Figure 4).

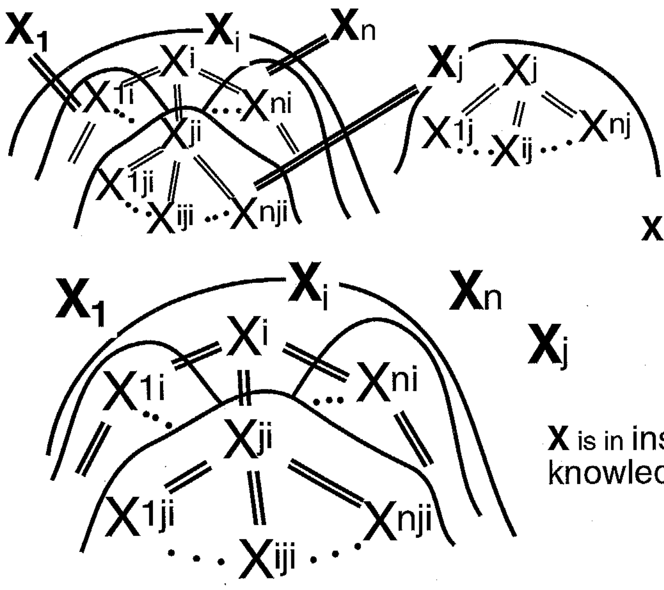

Figure 4. Common knowledge and inside common knowledge.

Consider the set N of all decision makers in a situation and decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers. We define decision makers' pairs *in inside common knowledge* for a decision maker i in N.

Definition 36 (Pairs in Inside Common Knowledge) *The pair* $(\mathbf{N}, \Sigma)$ *is* in inside common knowledge *for decision maker* i*, if we have that for any* σ *in* Σ_i*,* $N^\sigma = N^i$.

Decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ of emotions *in inside common knowledge* for a decision maker is defined as follows:

Definition 37 (Schemes in Inside Common Knowledge) *Decision makers' scheme* $\mathbf{e}$ *of emotions is* in inside common knowledge *for decision maker* i*, if it is satisfied that for any* σ *in* Σ_i*, any* j *in* N^σ*, and any* k *in* $N^{j\sigma}$*,* $e^\sigma_j|_k = e^i_j|_k$.

For any i in N, if decision makers' scheme of emotions is in inside common knowledge for decision maker i, then decision maker i believes that he/she correctly perceives the emotions of decision makers, and that what each of the others believes about the emotions is the same as decision maker i's perceptions of the emotions.

Under the condition that decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers is in common knowledge, decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ of emotions is in inside common knowledge for decision maker i in N if and only if for any σ in Σ^*_i, any j and any k in N, $e^\sigma_j|_k = e^i_j|_k$, that is, for any σ in Σ^*_i, $e^\sigma = e^i$. Then, considering another decision maker l in N, we have that for any l in N, and any σ in Σ^*_i, $e^\sigma = e^i = e^{l\sigma}$. Oppositely, if we assume that for any l in N, and any σ in Σ_i^*, $e^\sigma = e^{l\sigma}$, then we have, particularly, that $e^\sigma = e^i$ for any σ in Σ_i^*. Thus for any σ in Σ_i^*, and any j and k in N, $e_j^\sigma|_k = e_j^i|_k$. Therefore, we have an equivalent definition of in inside common knowledge for decision maker i in N under the condition of decision makers' pair $(\mathbf{N}, \Sigma)$ in common knowledge.

Definition 38 (An Equivalent of Inside Common Knowledge) *The scheme* $\mathbf{e}$ *of emotions is* in inside common knowledge *for decision maker* i*, if we have that for any* j *in* N *and any* σ *in* Σ_i^*, $e^\sigma = e^{j\sigma}$.

We can easily check the following relation between the concepts of common knowledge and inside common knowledge under the condition of decision makers' pair $(\mathbf{N}, \Sigma)$ in common knowledge. Suppose the set N of all decision makers in a situation, decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, and decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ of emotions.

Proposition 4 *Assume decision makers' pair* $(\mathbf{N}, \Sigma)$ *is in common knowledge. If* $\mathbf{e}$ *is in common knowledge, then for any* i *in* N*,* $\mathbf{e}$ *is also in inside common knowledge for decision maker* i*.*

(proof) Assume that $\mathbf{e}$ is in common knowledge. Then, we have that for any i and j in N and any σ in Σ_i^*, $e^\sigma = e^{\sigma j}$, because decision makers' pair $(\mathbf{N}, \Sigma)$ is in common knowledge. Thus for any k in N, $e_k^\sigma = e_k^{\sigma j}$. Considering the case that $i = k$ and $\sigma = k$, we have that $e_k^k = e_k^{kj}$ for any j in N. Since we can identify e_k^{kj} with e_k^j, it is satisfied that for any j in N, $e_k^k = e_k^j$. Given a string $\tau = i_1 i_2 \cdots i_q$ in Σ_i^*, we have that $e_k^k = e_k^{i_1} = e_k^{i_1 i_2} = \cdots = e_k^\tau$ for any k in N. Similarly, for the string $j\tau$, we have that $e_k^k = e_k^j = e_k^{ji_1} = e_k^{ji_1 i_2} \cdots = e_k^{j\tau}$ for any k in N. Therefore for any j in N and any τ in Σ_i^*, we obtain that $e_k^\tau = e_k^k = e_k^{j\tau}$ for any k in N, that is, $e^\tau = e^{j\tau}$. ■

As a corollary of this proposition, we have that if for some i in N, decision makers' scheme of emotions is *not* in inside common knowledge for decision maker i, then the scheme is *not* in common knowledge. This implies that if a decision maker does not believe that the scheme is in common knowledge, then the scheme is not in common knowledge, actually.

3.1.3 Generalizations of Inside Common Knowledge

We can generalize the concepts of common knowledge and inside common knowledge, applying the idea to higher degree of perception. Consider the set N of all decision makers, decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, and decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ of emotions.

Definition 39 (Inside Common Knowledge for Strings) *For any* i *in* N *and any* σ *in* Σ_i*, decision makers' scheme* $\mathbf{e}$ *of emotions is* in inside common knowledge *for* σ*, if it is satisfied that for any* τ *in* Σ_σ*, any* j *in* N^τ*, and any* k *in* $N^{j\tau}$*,* $e_j^\tau|_k = e_j^\sigma|_k$.

Particularly, if the string σ is equal to i, the the definition of decision makers' scheme of emotions in inside common knowledge for σ is equivalent to the definition of the scheme in inside common knowledge for decision maker i. Moreover, we can verify the following relation between schemes in inside common knowledge. Consider the set N of all decision makers in a situation, decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, and decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ of emotions.

Proposition 5 *For any i in N, any σ in Σ_i, and any τ in Σ_σ, if decision makers' scheme* $\mathbf{e}$ *of emotions is in inside common knowledge for σ, then the scheme is also in inside common knowledge for τ.*

(proof) Assume that the scheme is in common knowledge for σ. Because Σ_τ is included in Σ_σ, for any τ' in Σ_τ, we have that for any j in $N^{\tau'}$, and any k in $N^{j\tau'}$, $e_j^{\tau'}|_k = e_j^{\sigma}|_k$. Since the string τ is also an element in Σ_σ, we have that for any j in N^τ, and any k in $N^{j\tau}$, $e_j^{\tau}|_k = e_j^{\sigma}|_k$. Because $N^{\tau'}$ and $N^{j\tau'}$ are included in N^τ and $N^{j\tau}$, respectively, it is satisfied that for any j in $N^{\tau'}$, and any k in $N^{j\tau'}$, $e_j^{\tau'}|_k = e_j^{\tau}|_k$. Thus the scheme is in inside common knowledge for τ. ■

Examination of the aspect of interperception by decision makers requires classification of decision makers' schemes of any component of a situation. Such concepts of common knowledge, inside common knowledge, and their generalizations can be defined on the schemes, and are useful for the classification.

3.2 Integration of Perceptions

We can often regard decision makers as involved in many situations each of which interacts with the others. For example, a married business person may compete with his/her co-workers, and may attempt to make his/her spouse happy. A company with several sorts of products may be in competition with rivals in each of the markets of the products, while not all of the products may be allowed to be fully developed because of the financial constraints of the company. When we describe a situation as a base competition, the whole situation of a decision maker can be expressed by several base competitions and interactions of them. The business person may be regarded as being involved in a base competition of promotion on the one hand, and a base competition of a good husband and a good wife on the other hand. One of the interactions of the base competitions in this case is a constraint on "time." Similarly, competition among companies in the market of each product may be identified with a base competition, and the financial constraints can be interpreted as interactions of the base competitions. Without considering the interactions, we could not sufficiently understand the whole situation by analyzing the base competitions with frameworks given in this paper and standard solution concepts in game theory. We should construct a proper model of the whole situation considering not only the isolated base competitions but also the interactions of them. Thus we provide a formal method for making an appropriate model of the whole situation. The method integrates all the base situations into one taking the interactions of them into consideration so that the whole situation is described as a single base situation. Hence, in analyzing it we can take full advantage of frameworks provided in this paper and standard game theory.

In this paper we focus on formal treatments of the interactions that affect only possible actions open to decision makers, but there exist various types of interactions among base competitions. Actually, mutually interactive base competitions and integration of them have been theoretically studied under the headings such as *linkages between games* [82], *two-level games* [20, 81], *composition of games* [96, 98], and so on. Radford comprehensively, but verbally, discusses the issue of linkages between games [82]. He deals with several forms of linkages, and especially points out that the linkages may affect the possible strategies open to the players in the games and the preferences of the players for possible outcomes of the games. Putnam et.al. conceives of the connection between diplomacy and domestic politics as a two-level game [20, 81]. Since they focus on analyzing the entanglements of domestic and international politics, only two games — an international negotiation game and a domestic politics game — are engaged in their discussion. Srikant and Başar study only two groups such that there are strong interactions within each group and a weak interaction between the two groups [96]. Vilkov considers composition of games whose members are disjoint [98]. Our method can deal with finitely many base competitions whose members may mutually intersect.

Practically, the idea of *integration of base competitions* relates to procedures for modeling a complex situation. We can recognize at least two types of methods to make a model. One is the class of methods with which we construct a model of the whole situation directly gathering all information about it [3, 41, 85]. These methods are used to analyze relatively simple situations. The other type of methods separates the whole situation into several parts, whose models we combined into one considering interactions of them [82]. This type of methods is more efficient to make a model of an extremely complex situation than the methods in the former class. Although the integration of base competitions has been implicitly introduced into the methods even in the latter class so far, we deal with them in an explicit manner. We can also utilize the concept of integration of base competitions when we modify the model of the whole situation in practice. The modification of a model is required when we face a new situation different from the situation for which the model is constructed. If some of the components of the original model are recyclable, in making a modified version of the model it is much more efficient to reuse the recyclable components than repeating all procedures for modeling from the scratch.

When we introduce the aspect of interperception by decision makers to the concept of integration of base competitions, the whole situation of a decision maker can be expressed by several schemes of base competitions and interactions of them, and it is required to construct a proper model of the whole situation considering the interactions as well as the schemes of base competitions. Replacing base competitions with schemes of base competitions in the discussions above, we reach the idea of *integration of schemes*. We give, therefore, a formal method to integrate schemes of base competitions into one as well as a method for integration of base competitions.

3.2.1 Relations among Actions

In order to deal with decision makers involved in many competitive situations, we describe each of the situations as a base competition, and interactions of the situations as *relations among actions*. Then, the whole situation of the decision makers is represented as a single base competition through a method of *integration of base competitions*. We provide a definition of *relations among actions*, first.

Suppose that two companies, A_1 and A_2, are competing in a market (Table 5). If both companies invest in developing a new product or both do not, they will obtain equal profits. Each of them will get bigger profit in the former case than that in the latter case. If one of them invests and the other does not, the former will be extremely successful, whereas the latter will not get anything. This situation can be expressed by a base competition $C = (N, S, F)$, where in this case we adopt *cardinal* profits F of decision makers. That is, for any i in N, the profits of decision maker i is a function F_i from the set S of all outcomes to the set $\mathbf{R}$ of all real numbers, and F is the list $(F_i)_{i \in N}$ of the profits F_i of decision maker i for each i in N.

		A_2 Invest	A_2 Not Invest
A_1	Invest	(15, 15)	(35, 0)
	Not Invest	(0, 35)	(10, 10)

Million Dollers

Table 5. Competition in a market.

If A_1 in Table 5 is diversified, it may participate in another market described as in Table 6 at the same time. In the market it is expected that the investment of A_1 will be very efficient, since the existing facilities of the company can be utilized into production of a new product, whereas A_3's investment will lead the company to poor result because of the high cost of introducing new facilities. When we analyze the whole situation of A_1, we have to take account of interactions of all situations in which A_1 is participating. Generally speaking, however, if many competitive situations are mutually interactive, it is often difficult to express separately each of the situations as a base competitions, since we frequently cannot determine the components of a base competition without referring to the others. In particular, the profits of decision makers in one base competition often depend on the consequences of the other competitions. In this paper, however, we analyze only competitive situations that can be described as base situations the components of each of which are completely determined not depending on the other games. Particularly, the profits of decision makers are cardinally and commensurably assessed, e.g., as in Table 5 and 6.

		A_3 Invest	A_3 Not Invest
A_1	Invest	(20, 5)	(25, 0)
	Not Invest	(0, 0)	(5, 5)

Million Dollers

Table 6. Competition in another market.

There are various kinds of interactions between base competitions [82]. In particular, the interactions may affect the possible actions and the profits of decision makers. If a decision maker is involved in several base competitions, he/she has to make a selection in each of them. It is often impossible to take a particular combination of actions because of his/her constraints of finance, time, and so on. For example, it may be impossible for A_1 to invest in two markets because of the financial constraints of the company. On the other hand, the profits of a decision maker for a combination of outcomes may not be consistent with the profits in the individual outcomes, even if they are cardinally assessed. A company may have a big profit even combining poorly assessed outcomes of investment, since the cost of facilities can be reduced. While the both kinds of interactions have great importance, we treat only the interactions that affect the possible actions, the interactions that are mathematically more tractable than those affecting profits. Thus we assume that the profits of a decision maker in a combination of outcomes is additively determined by the profits in the outcomes, that is, the profits in the combination of the outcomes is given by the sum of all cardinal profits in the outcomes.

In order to deal with the base competitions whose interactions affect the possible actions of decision makers, we define *relations among actions* of each decision maker. The relations among actions of a decision maker is the set of all possible combinations of actions of the decision maker. An outline of the relations among actions is as follows. Suppose that a decision maker is engaged in just two base competitions, α and β, simultaneously. The decision maker has to select a combination of two actions. One of them is for α, and the other is for β. If the decision maker selects action s for α, and if action s is also one of the alternatives for β, then the decision maker must select the action for β, since each decision maker has to select exactly one action for each base competitions in which he/she is involved. Thus the combination (s, s) of actions must be an element of the relations among actions of the decision maker. Moreover, for any action of the decision maker in α, e.g., t, there must exist at least one action of the decision maker in β, e.g., t', such that the combination (t, t') of actions is possible to take, since each action in α must have possibility to be selected. The symmetric condition has to be satisfied for each action in β. Generalizing these considerations, we obtain two conditions that have to be satisfied by the relations among actions of a decision maker in a class of finitely many base competitions. Let I be a finite index set, and consider a class $\mathbf{c} = (c^\mu)_{\mu \in I}$ of base competitions, where $c^\mu = (N^\mu, S^\mu, F^\mu)$ is a base competition for each μ in I, called base situation μ. For any i in $\cup_{\mu \in I} N^\mu$, let I_i be the set of all μ such that decision maker i is involved in c^μ, that is, $M_i = \{\mu \in I \mid i \in N^\mu\}$.

Definition 40 (Relations among Actions) *The* relations among actions $\theta_i(\mathbf{c})$ *of decision maker i in $\mathbf{c}$ is a subset of $\prod_{\mu \in I_i} S_i^\mu$ such that;*

1. *for any $(s_i^\mu)_{\mu \in I_i}$ in $\theta_i(\mathbf{c})$ and any μ in I_i, if there exists μ' in I_i such that s_i^μ in $S_i^{\mu'}$, then $s_i^{\mu'} = s_i^\mu$,*
2. *for any subset I' of I_i and any s_i in $\cap_{\mu' \in I'} S_i^{\mu'}$, there exists an element $(s_i^\mu)_{\mu \in I_i}$ of $\theta_i(\mathbf{c})$ such that for any μ' in I', $s_i^{\mu'} = s_i$.*

The relations among actions $\theta_i(\mathbf{c})$ of decision maker i in $\mathbf{c}$ is often denoted simply by θ_i if it does not cause confusion. Moreover, the list $(\theta_i(\mathbf{c}))_{i \in \cup_{\mu \in I} N^\mu}$ of the relations among actions $\theta_i(\mathbf{c})$ in $\mathbf{c}$ for each player i in $\cup_{\mu \in I} N^\mu$ is denoted by θ, called the *relations among actions* of decision makers in $\mathbf{c}$.

For understanding the whole situation of a decision maker participates in many base competition at once, it is often insufficient, and sometimes even inappropriate, to analyze only the base competitions without considering the interactions of them. For example, examining each of the base competitions in Table 5 and 6 separately by using the concept of dominant strategies, we would suggest that A_1 should invest in both markets. The analyses, however, are not useful for making decisions if financial constraints of the company do not allow to invest both markets. In order to make an appropriate model of the whole situation for analyzing it more properly, we propose the concept of *integration of base competitions*, that takes not only the base competitions but also the interactions of the base competitions into account. By using the concept, the whole situation is described as a base competition.

Consider a class of base competitions and the relations among actions of decision makers in the class. A decision maker involved in the class has to select just one action for each of the base competition in which he/she participates. That is, he/she must select a combination of actions from the possible combinations listed in the relations among actions of the decision maker. If each decision maker selects a combination, a list of outcomes for each of the base competitions in the class is determined. We can assess the profit of a decision maker for each list of outcomes, since we assume that the profit is given by the sum of all profits of the decision maker for each of the outcomes. Thus we can express the whole situation as a base competition, and obtain the definition of *integration of base competitions* as follows.

Definition 41 (Integration of Base Competitions) *Suppose a class* $\mathbf{c} = (c^\mu)_{\mu \in I}$ *of base competitions, and the relations among actions* θ *of decision makers in* $\mathbf{c}$*, where for any* μ *in* I*,* $c^\mu = (N^\mu, S^\mu, F^\mu)$*. An* integration $\sum_{\mu \in I} c^\mu$ *of* $\mathbf{c}$ *in* θ *is a base competition* $\hat{c} = (\hat{N}, \hat{S}, \hat{F})$*, where* $\hat{N} = \cup_{\mu \in I} N^\mu$*,* $\hat{S} = \prod_{i \in \hat{N}} \hat{S}_i$*,* $\hat{S}_i = \theta_i$ *for each* $i \in \hat{N}$ *and* $\hat{F} = (\hat{F}_i)_{i \in \hat{N}}$ *where* $\hat{F}_i$ *is defined by* $\hat{F}_i(\hat{s}) = \sum_{\mu \in I_i} F_i^\mu((s_j^\mu)_{j \in N^\mu})$ *for any* $i \in \hat{N}$ *and any* $\hat{s} \in \hat{S}$*.*

Example 12 (Integration of Base Competitions) *Suppose the base competitions described as in Table 5 and 6, denoted by* c^α *and* c^β*, respectively. Considering the financial constraints of* A_1*, we can give the relations among actions of each decision maker in* (c^α, c^β) *as follows:*

$$\theta_{A_1} = \{(\mathbf{I},\mathbf{N}),(\mathbf{N},\mathbf{I}),(\mathbf{N},\mathbf{N})\}, \theta_{A_2} = \{\mathbf{I},\mathbf{N}\}, \theta_{A_3} = \{\mathbf{I},\mathbf{N}\}.$$

If each decision maker selects an element in the relations among actions of the decision maker, a list of outcomes for each base competition is determined. For instance, if A_1*,* A_2*, and* A_3 *choose* $(\mathbf{I},\mathbf{N})$*,* $\mathbf{I}$*, and* $\mathbf{N}$*, respectively, the outcomes of* c^α *and* c^β *are* $(\mathbf{I},\mathbf{I})$ *and* $(\mathbf{N},\mathbf{N})$*, respectively. The profit of each decision maker in the outcome is also determined. The profit of* A_1 *is 20, since the it gains 15 from* c^α *and 5 from* c^β*.* A_2 *gets 15 from* c^α *and* A_3 *gains 5 from* c^β*. Consequently, the integration of* (c^α, c^β) *in* θ *is depicted as in Table 7.*

A_3 = I		A_2 = I	A_2 = N
A_1	(I, N)	(15, 15, 0)	(35, 0, 0)
	(N, I)	(20, 35, 5)	(30, 10, 5)
	(N, N)	(0, 35, 0)	(10, 10, 0)

A_3 = N		A_2 = I	A_2 = N
A_1	(I, N)	(20, 15, 5)	(40, 0, 5)
	(N, I)	(25, 35, 0)	(35, 10, 0)
	(N, N)	(5, 35, 5)	(15, 10, 5)

Million Dollers

Table 7. Integration of base competitions.

3.2.2 Integration of Schemes

A scheme of base competitions, as well as a base competition, is a model of a competitive situation. Thus we can regard decision makers in many competitive situations as engaged in many schemes of base competitions. As in the case of integration of base competitions, we need to integrate schemes to understand the whole situation of the decision makers. Since a scheme is defined on a pair of a scheme and a set of strings of decision makers, we have to integrate several pairs into one in order to obtain a way of integration of schemes.

Suppose that a decision maker is involved in several schemes of base competitions, then the decision maker has the corresponding pairs of a scheme and a set of strings of decision makers for each of the schemes. As in the case of integration of base competitions, integrated schemes should compose one scheme so as to express the whole situation, and to be analyzed by using frameworks in this paper and various solution concepts of hypergame theory [100]. It requires that integrated pairs must be one pair of a scheme and a set of strings of decision makers. We provide a definition of *integration of pairs* of a scheme and a set of strings of decision makers, and give a proposition which shows that the integration of pairs satisfies the conditions to be a pair of a scheme and a set of strings of decision makers. Suppose the set N of all decision makers in a situation and decision maker i in N.

Definition 42 (Integration of Pairs) *Given a class of decision maker i's pairs $(\mathbf{N}_i^\mu, \Sigma_i^\mu)_{\mu\in I}$ of a scheme and a set of strings of decision makers, where $\mathbf{N}_i^\mu = ((N^\mu)^\sigma)_{\sigma\in\Sigma_i^\mu}$, an* integration *$\sum_{\mu\in I}(\mathbf{N}_i^\mu, \Sigma_i^\mu)$ of the pairs is the pair $(\hat{\mathbf{N}}_i, \hat{\Sigma}_i)$ which satisfies that $\hat{\Sigma}_i = \cup_{\mu\in I}\Sigma_i^\mu$, $\hat{\mathbf{N}}_i = (\hat{N}^\sigma)_{\sigma\in\hat{\Sigma}_i}$, and $\hat{N}^\sigma = \cup_{\mu\in I_\sigma}(N^\mu)^\sigma$, where $I_\sigma = \{\mu \in I \mid \sigma \in \Sigma_i^\mu\}$.*

Proposition 6 (Integration of Pairs is Another Pair) *The integration $\sum_{\mu\in I}(\mathbf{N}_i^\mu, \Sigma_i^\mu)$ of the pairs $(\mathbf{N}_i^\mu, \Sigma_i^\mu)_{\mu\in I}$ of a scheme and a set of strings of decision makers is also decision maker i's pair of a scheme and a set of strings of decision makers.*

(proof) Firstly, $\hat{\Sigma}_i$ is a subset of Σ_i^*, since $\hat{\Sigma}_i = \cup_{\mu\in I}\Sigma_i^\mu$ and Σ_i^μ is a subset of Σ_i^* for each μ in I. Secondly, the string i is an element of $\hat{\Sigma}_i$, since $\hat{\Sigma}_i = \cup_{\mu\in I}\Sigma_i^\mu$ and the string i is in Σ_i^μ for any μ in I. Thirdly, for any string $\sigma = i_1 i_2 \cdots i_q$ in $\hat{\Sigma}_i$, i_1 is an element of $(N^\mu)^\sigma$ for any μ in I_σ. Thus i_1 is an element of $\hat{N}^\sigma = \cup_{\mu\in I_\sigma}(N^\mu)^\sigma$. Fourthly, since for any string $\sigma = i_1 i_2 \cdots i_q$ in $\hat{\Sigma}_i$ and any decision maker j in $\hat{N}^\sigma\backslash\{i_1\}$, player j is an element of $(N^\mu)^\sigma\backslash\{i_1\}$ for some μ in I_σ, the string $j\sigma$ is an element of Σ_i^μ. Thus we have that the string $j\sigma$ is an element of $\hat{\Sigma}_i = \cup_{\mu\in I}\Sigma_i^\mu$. We have, moreover, that $\hat{N}^{j\sigma} = \cup_{\mu'\in I_{j\sigma}}(N^{\mu'})^{j\sigma}$ is included in $\hat{N}^\sigma = \cup_{\mu\in I_\sigma}(N^\mu)^\sigma$, since $I_{j\sigma}$ is included in I_σ and $(N^\mu)^{j\sigma}$ are included in $(N^\mu)^\sigma$ for any $\mu \in I_{j\sigma}$. Fifthly, for any string $\sigma = i_1 i_2 \cdots i_q$ in $\hat{\Sigma}_i$ $(q = 2, 3, \ldots)$ and any μ in I_σ, the string $\tau = i_2 i_3 \cdots i_q$ is in Σ_i^μ, and decision maker i_1 is in $(N^\mu)^\tau$. Thus, we have that the string τ is an element of $\hat{\Sigma}_i = \cup_{\mu\in I}\Sigma_i^\mu$, and decision maker i_1 is in $\hat{N}^\tau = \cup_{\mu\in I_\tau}(N^\mu)^\tau$. ■

We are now able to integrate schemes into one. Consider a finite number of decision makers' schemes $(\mathbf{C}^\mu)_{\mu\in I}$ of base competitions. Assume that for each μ in I, $\mathbf{C}^\mu = (\mathbf{C}_i^\mu)_{i\in N^\mu}$, that is, the set of all decision makers involved in the scheme $\mathbf{C}^\mu$ is N^μ. For any μ in I and any i in N^μ, decision maker i has decision maker i's pair of a scheme and a set of strings of decision makers, denoted by $(\mathbf{N}_i^\mu, \Sigma_i^\mu)$. Thus, $\mathbf{C}_i^\mu = ((c^\mu)^\sigma)_{\sigma\in\Sigma_i^\mu}$ and $(c^\mu)^\sigma = ((N^\mu)^\sigma, (S^\mu)^\sigma, (F^\mu)^\sigma)$ for any σ in Σ_i^μ. For any i in $\hat{N} = \cup_{\mu\in I}N^\mu$, moreover, decision maker i has decision maker i's pairs of a scheme and a set of strings of decision makers for each of the schemes in which he/she participates. Using the definition of integration of pairs, we have the pair $(\hat{\mathbf{N}}_i, \hat{\Sigma}_i)$ for any i in $\hat{N}$. We assume, furthermore, that for any class $\mathbf{c}$ of base competitions which are components of the schemes, decision makers' relation among actions θ in $\mathbf{c}$ is given. Let I_σ be the set of all μs in I such that the string σ is an element of Σ_i^μ for some i in $\hat{N}$.

Definition 43 (Integration of Schemes) *Suppose a class $(\mathbf{C}^\mu)_{\mu\in I}$ of schemes of base situations. An* integration *$\sum_{\mu\in I}\mathbf{C}^\mu$ of the schemes is the scheme $\hat{\mathbf{C}} = (\hat{\mathbf{C}}_i)_{i\in\hat{N}}$, where $\hat{\mathbf{C}}_i = (\hat{c}^\sigma)_{\sigma\in\hat{\Sigma}_i}$ for any i in $\hat{N}$, and $\hat{c}^\sigma$ is the integration $((c^\mu)^\sigma)_{\mu\in I_\sigma}$ of base competitions for any σ in $\hat{\Sigma}_i$.*

As in the case of integration of base competitions, integration of schemes is defined as to compose one scheme. Thus the whole situation can be expressed by a scheme, and can be analyzed by using frameworks in this paper and various solution concepts of hypergame theory [100].

3.3 Stability and Complete Stability of Emotions

The concept of *balancedness* in psychology by Heider [13, 35] gives us a type of structures of decision makers' perceptions of emotions. Heider's theory on the concept have been considered as about structural balance of actual relations among participants in a situation [13]. The original idea of Heider, however, should be regarded as about stability of the participants' perceptions of the relations, because in the theory Heider employs the emotions of a decision maker toward the others, the decision maker's perceptions of the emotions of the others, and so on. Thus, we transfer the concept of balancedness to our framework for describing interperception by decision makers in terms of emotions, and propose the concept of *stability* of emotions. Applying the Heider's idea to higher degrees of perceptions, we also propose the concept of *complete stability* of emotions. Furthermore, we show the equivalence between the stability and the complete stability under the condition of inside common knowledge.

3.3.1 Stable Emotions

The original claim by Heider [35] is as follows: consider three participants, P, Q and O, in a situation, and multiply the emotions of P toward Q, the emotions of P toward O, and P's perception of the emotions of Q toward O. If the consequence of multiplication is $+$, then P's perception of the structure of the relations among the participants is *balanced*, otherwise it is not balanced (Figure 5).

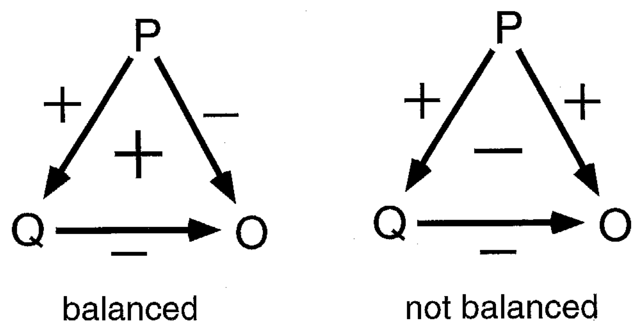

Figure 5. Heider's balancedness of emotions.

We should notice that all of the elements appeared in the multiplication are included in participant P's scheme of emotions. Thus, *balancedness* is not a concept about the structure of the relations among the participants but a concept about participants' perceptions of the structure of the relations. The concept of *stable schemes* defined below represents the idea. Consider the set N of all decision makers in a situation, decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, and decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ of emotions.

Definition 44 (Stable Schemes of Emotions) *For any i in N, decision makers' scheme* $\mathbf{e}$ *of emotions is* stable *for decision maker i, if for any j in N^i and any k in N^{ji}, $e^i_i|_j \times e^i_j|_k = e^i_i|_k$.*

Decision makers' scheme $\mathbf{e}$ *of emotions is* stable, *if decision makers' scheme* $\mathbf{e}$ *of emotions is stable for decision maker* i *for any* i *in* N.

We should check whether stable schemes exist. The following proposition shows the existence.

Proposition 7 (Existence of Stable Schemes) *Consider decision maker* i *in* N *and the emotions* e_i^i *of decision maker* i. *There are decision maker* i*'s perceptions* e_j^i *of the emotions of decision maker* j *for any* j *in* $N^i\backslash\{i\}$ *such that* $e_i^i|_j \times e_j^i|_k = e_i^i|_k$ *for any* k *in* N^{ji}. *Moreover, if the pair* $(\mathbf{N}, \Sigma)$ *is in inside common knowledge for decision maker* i, *then the perceptions* e_j^i *is determined uniquely.*

(proof) Let $e_j^i|_k$ be $e_i^i|_j \times e_i^i|_k$ for any k in N^{ji}. Then, we have that for any k in N^{ji},

$$\begin{aligned}
e_i^i|_j \times e_j^i|_k &= e_i^i|_j \times (e_i^i|_j \times e_i^i|_k) \\
&= (e_i^i|_j \times e_i^i|_j) \times e_i^i|_k \\
&= + \times e_i^i|_k \\
&= e_i^i|_k.
\end{aligned}$$

If the pair $(\mathbf{N}, \Sigma)$ is in inside common knowledge for decision maker i, and if e'^i_j also satisfies the condition, then we have that $e_i^i|_j \times e'^i_j|_k = e_i^i|_k = e_i^i|_j \times e_j^i|_k$ for any k in $N^{ji} = N^i$. This means that $e'^i_j = e_j^i$, because it is satisfied that for any k in $N^{ji} = N^i$,

$$\begin{aligned}
e'^i_j|_k &= + \times e'^i_j|_k \\
&= (e_i^i|_j \times e_i^i|_j) \times e'^i_j|_k \\
&= e_i^i|_j \times (e_i^i|_j \times e'^i_j|_k) \\
&= e_i^i|_j \times (e_i^i|_j \times e_j^i|_k) \\
&= (e_i^i|_j \times e_i^i|_j) \times e_j^i|_k \\
&= + \times e_i^i|_k \\
&= e_i^i|_k.
\end{aligned}$$

Thus the uniqueness of the perceptions e_j^i is shown. ■

This proposition implies that each decision maker can establish decision makers' scheme that is stable for him/herself through appropriate modification of *only* his/her perceptions of the emotions of the others, whatever emotions he/she has.

3.3.2 Completely Stable Emotions

According to Heider [35], if decision makers' scheme of emotions is stable for a decision maker in a situation, then the decision maker feels balanced in terms of his/her perceptions of the relations among the decision makers in the situation. It is natural to think that each decision maker feels more balanced if decision makers' scheme is stable for the decision maker, and if the decision maker believes that each of the others feels balanced. This consideration leads us to the concept of *completely stable schemes.* Completely stable schemes are defined as being stable in each degree of decision makers' perceptions of emotions.

Definition 45 (Completely Stable Schemes) *For any* i *in* N, *decision makers' scheme* $\mathbf{e}$ *of emotions is* completely stable *for decision maker* i, *if for any* σ *in* Σ_i, *any* j *in* N^σ, *any*

k *in* $N^{j\sigma}$, *and any* l *in* $N^{kj\sigma}$, $e^{\sigma}_{j}|_k \times e^{j\sigma}_{k}|_l = e^{\sigma}_{j}|_l$. *Decision makers' scheme* $\mathbf{e}$ *of emotions is* completely stable, *if decision makers' scheme* $\mathbf{e}$ *of emotions is completely stable for decision maker* i *for any* i *in* N.

For any i in N, if decision makers' scheme of emotions is completely stable for decision maker i, then considering the case that $\sigma = i$ and $j = i$, we have that it is also stable for decision maker i. Moreover decision maker i believes that decision makers' scheme of emotions is stable for each of the others, that each of the others thinks that decision makers' scheme of emotions is stable for each of the others, and so on. We can also check the existence of completely stable schemes.

Proposition 8 (Existence of Completely Stable Schemes) *Consider decision maker* i *in* N *and the emotions* e^i_i *of decision maker* i. *There are decision maker* i*'s perceptions* e^i_j *of the emotions of decision maker* j *for any* j *in* $N^i \backslash \{i\}$, *and decision maker* i*'s view* $\mathbf{e}^i = (e^{\sigma})_{\sigma \in \Sigma_i \backslash \{i\}}$ *on emotions such that* $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$, *where* $\mathbf{e}_i = ((e^i_i, (e^i_j)_{i \in N^i \backslash \{i\}}, \mathbf{e}^i)$, *becomes a completely stable scheme of emotions for decision maker* i. *Moreover, if the pair* $(\mathbf{N}, \Sigma)$ *is in inside common knowledge for decision makers* i, *then the perceptions* $(e^i_j)_{i \in N^i \backslash \{i\}}$ *and the view* $\mathbf{e}^i$ *is uniquely determined.*

(proof) Let $e^i_j|_k$ be $e^i_i|_j \times e^i_i|_k$ for any k in N^{ji}. Moreover, for any $\sigma = i_1 i_2 \cdots i_q$ in $\Sigma_i \backslash \{i\}$, any j in N^{σ}, and any k in $N^{j\sigma}$, let $e^{\sigma}_j|_k$ be $e^i_i|_j \times e^i_i|_k$. Then for any σ in Σ_i, any j in N^{σ}, any k in $N^{j\sigma}$, and any l in $N^{kj\sigma}$,

$$
\begin{aligned}
e^{\sigma}_j|_k \times e^{j\sigma}_k|_l &= (e^i_i|_j \times e^i_i|_k) \times (e^i_i|_k \times e^i_i|_l) \\
&= e^i_i|_j \times (e^i_i|_k \times e^i_i|_k) \times e^i_i|_l \\
&= e^i_i|_j \times + \times e^i_i|_l \\
&= e^i_i|_j \times e^i_i|_l \\
&= e^i_j|_l \\
&= e^{\sigma}_j|_l.
\end{aligned}
$$

Suppose the case that the pair $(\mathbf{N}, \Sigma)$ is in inside common knowledge for decision maker i and $\mathbf{e}'$, where $\mathbf{e}'_i = ((e^i_i, (e'^i_j)_{i \in N^i \backslash \{i\}}, \mathbf{e}'^i)$ and $\mathbf{e}'^i = (e'^{\sigma})_{\sigma \in \Sigma_i \backslash \{i\}}$, is also completely stable for decision maker i. Because both $\mathbf{e}'$ and $\mathbf{e}$ are stable for decision maker i, we have that $e'^i_j = e^i_j$ for any j in $N^i \backslash \{i\}$. From the complete stability of $\mathbf{e}'$ and $\mathbf{e}$ for decision maker i, we have that for any j in $N^i \backslash \{i\}$, any k in $N^{ji} = N^i$, and any l in $N^{kji} = N^i$, $e'^i_j|_k \times e'^{ji}_k|_l = e'^i_j|_l$ and $e^i_j|_k \times e^{ji}_k|_l = e^i_j|_l$. Since $e'^i_j|_k = e^i_j|_k$ and $e'^i_j|_l = e^i_j|_l$ for any k in $N^{ji} = N^i$ and any l in $N^{kji} = N^i$, it is satisfied that for any j in $N^i \backslash \{i\}$, $e'^{ji}_k = e^{ji}_k$ for any k in $N^{ji} = N^i$, that is, $e'^{ji} = e^{ji}$. Assume that for any $\sigma = i_1 i_2 \cdots i_q$ in Σ_i that consists q decision makers, $e'^{\sigma} = e^{\sigma}$. By the complete stability of $\mathbf{e}'$ and $\mathbf{e}$ for decision maker i, we have that for any j in $N^{\sigma} = N^i$, any k in $N^{j\sigma} = N^i$, and any l in $N^{kj\sigma} = N^i$, $e'^{\sigma}_j|_k \times e'^{j\sigma}_k|_l = e'^{\sigma}_j|_l$ and $e^{\sigma}_j|_k \times e^{j\sigma}_k|_l = e^{\sigma}_j|_l$. Since $e'^{\sigma}_j|_k = e^{\sigma}_j|_k$ and $e'^{\sigma}_j|_l = e^{\sigma}_j|_l$ for any k in $N^{j\sigma} = N^i$ and any l in $N^{kj\sigma} = N^i$, we have that for any j in $N^{\sigma} = N^i$, $e'^{j\sigma}_k|_l = e^{j\sigma}_k|_l$ for any k in $N^{j\sigma} = N^i$ and any l in $N^{kj\sigma} = N^i$, that is, $e'^{j\sigma} = e^{j\sigma}$. Therefore for any string $\tau = i_1 i_2 \cdots i_q i_{q+1}$ in Σ_i that consists $q+1$ decision makers, $e'^{\tau} = e^{\tau}$. By mathematical induction, we have that for any σ in Σ_i, $e'^{\sigma} = e^{\sigma}$. ■

As in the case of stable schemes of emotions, this proposition implies that each decision maker can establish decision makers' scheme that is completely stable for him/herself through proper modification of his/her perceptions of emotion and his/her view on emotions, whatever emotions he/she has.

3.3.3 Propositions on Stability of Emotions

Suppose the set N of all decision makers in a situation, decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, and decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ of emotions. Concerning stable schemes of emotions, Cartwright and Harary [13, 28] have revailed the following proposition. Assume that the pair $(\mathbf{N}, \Sigma)$ is in common knowledge.

Proposition 9 (Separation Theorem for Stable Schemes) *For any i in N, we have that decision makers' scheme $\mathbf{e}$ of emotions is stable for decision maker i if and only if $N^i = N$ is partitioned into two subsets N_1 and N_2 such that for any j in $N^i = N$ and k in $N^{ji} = N$, $e^i_j|_k = +$ if j and k belong to the same subset, and $e^i_j|_k = -$ if i and j belong to different subsets.*

A proof of this proposition is shown in [28].

We can show a similar property regarding completely stable schemes of emotions.

Proposition 10 (Separation Theorem for Completely Stable Schemes) *For any i in N, we have that decision makers' scheme $\mathbf{e}$ of emotions is completely stable for decision maker i if and only if $N^i = N$ is partitioned into two subsets N_1 and N_2 such that for any σ in $\Sigma_i = \Sigma_i^*$, any j in $N^\sigma = N$, and k in $N^{j\sigma} = N$, $e^\sigma_j|_k = +$ if j and k belong to the same subset, and $e^\sigma_j|_k = -$ if j and k belong to different subsets.*

(proof) Assume that $\mathbf{e}$ is completely stable for decision maker i. Because $\mathbf{e}$ is also stable for decision maker i, we have two subsets N_1 and N_2 that partition the set N such that for any j in $N^i = N$ and k in $N^{ji} = N$, $e^i_j|_k = +$ if j and k belong to the same subset, and $e^i_j|_k = -$ if i and j belong to different subsets. Because of the uniqueness of the scheme $\mathbf{e}_i$, it is satisfied that for any σ in $\Sigma_i = \Sigma_i^*$, any j in $N^\sigma = N$, and any k in $N^{j\sigma} = N$, $e^\sigma_j|_k = e^i_j|_k$. Therefore, we have that $e^\sigma_j|_k = e^i_j|_k = +$ if j and k belong to the same subset, and $e^\sigma_j|_k = e^i_j|_k = -$ if j and k belong to different subsets. Oppositely, assume that $N^i = N$ is partitioned into two subsets N_1 and N_2 such that for any σ in $\Sigma_i = \Sigma_i^*$, any j in $N^\sigma = N$, and k in $N^{j\sigma} = N$, $e^\sigma_j|_k = +$ if j and k belong to the same subset, and $e^\sigma_j|_k = -$ if j and k belong to different subsets. Particularly, we have that for any j in $N^i = N$ and k in $N^{ji} = N$, $e^i_j|_k = +$ if j and k belong to the same subset, and $e^i_j|_k = -$ if i and j belong to different subsets. This leads the stability of the scheme $\mathbf{e}$ for decision maker i. Therefore, it is satisfied that for any j in $N^i = N$, and any k in $N^{ji} = N$, $e^i_i|_j \times e^i_j|_k = e^i_i|_k$. Considering another l in N and the equations: $e^i_i|_j \times e^i_j|_k = e^i_i|_k$, $e^i_i|_k \times e^i_k|_l = e^i_i|_l$, and $e^i_i|_j \times e^i_j|_l = e^i_i|_l$, we have that for any j, k, and l in N, $e^i_j|_k \times e^i_k|_l = e^i_j|_l$. Since for any σ in $\Sigma_i = \Sigma_i^*$, any j in $N^\sigma = N$, and k in $N^{j\sigma} = N$, $e^\sigma_j|_k = e^i_j|_k$, we have that for any σ in $\Sigma_i = \Sigma_i^*$, any j in $N^\sigma = N$, any k in $N^{j\sigma} = N$, and any l in $N^{kj\sigma} = N$,

$$\begin{aligned} e^\sigma_j|_k \times e^{j\sigma}_k|_l &= e^i_j|_k \times e^i_k|_l \\ &= e^i_j|_l \\ &= e^\sigma_j|_l \end{aligned}$$

Thus, $\mathbf{e}$ is completely stable for decision maker i. ■

We examine relations among the concepts of stability, complete stability and inside common knowledge. In particular, we explore the conditions for schemes of emotions to be completely stable, because completely stable schemes correspond to the most balanced structure of decision makers' perceptions of the relations among the decision makers. Suppose the set N of all decision makers in a situation, decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers, and decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ of emotions. Assume, moreover, that the pair $(\mathbf{N}, \Sigma)$ is in common knowledge. First, we obtain a necessary condition for schemes to be completely stable.

Proposition 11 (A Necessary Condition for Complete Stability) *For any $i \in N$, if decision makers' scheme* $\mathbf{e}$ *of emotions is completely stable for decision maker i, then the scheme is in inside common knowledge for decision maker i.*

(proof) Assume that decision makers' scheme $\mathbf{e}$ of emotions is completely stable for decision maker i. From the uniqueness of the scheme $\mathbf{e}_i$, it is satisfied that for any σ in $\Sigma_i = \Sigma_i^*$, any j in $N^\sigma = N$, and any k in $N^{j\sigma} = N$, $e_j^\sigma|_k = e_j^i|_k$. Thus the scheme $\mathbf{e}_i$ is in inside common knowledge for decision maker i. ■

From this proposition, we can imply that if a decision maker does not perceive decision makers' scheme as being in inside common knowledge for the decision maker, then the scheme of emotions is not completely stable for the decision maker.

We have already pointed out that for any i in N, if decision makers' scheme of emotions is completely stable for decision maker i, then it is also stable for decision maker i. Employing the fact, we can provide a necessary and sufficient condition for the scheme to be completely stable.

Proposition 12 (An Equivalent Condition for Complete Stability) *For any $i \in N$, decision makers' scheme* $\mathbf{e}$ *of emotions is completely stable for decision maker i if and only if the scheme is stable and in inside common knowledge for decision maker i.*

(proof) We have already shown that if decision makers' scheme of emotions is completely stable for a decision maker, then the scheme is in inside common knowledge for the decision maker. We have, moreover, already pointed out that if decision makers' scheme of emotions is completely stable for a decision maker, then the scheme is stable for the decision maker. Thus, the necessity has been proven. To prove the sufficiency, assume that decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i \in N}$ is stable and in inside common knowledge for decision maker i. Because the scheme is stable for decision make i, we have that for any j in $N^i = N$, and any k in $N^{ji} = N$, $e_i^i|_j \times e_j^i|_k = e_i^i|_k$. Moreover, since the scheme is in inside common knowledge for decision maker i, it is satisfied that for any σ in $\Sigma_i = \Sigma_i^*$, any j in $N^\sigma = N$, and any k in $N^{j\sigma} = N$, $e_j^\sigma|_k = e_j^i|_k$. From the stability of the scheme, we have that for any j, k, and l in N, $e_j^i|_k \times e_k^i|_l = e_j^i|_l$ because of the equations of $e_i^i|_j \times e_j^i|_k = e_i^i|_k$, $e_i^i|_k \times e_k^i|_l = e_i^i|_l$, and $e_i^i|_j \times e_j^i|_l = e_i^i|_l$. Thus, we have that for any σ in $\Sigma_i = \Sigma_i^*$, any j in $N^\sigma = N$, any k in $N^{j\sigma} = N$, and any l in $N^{kj\sigma} = N$,

$$\begin{aligned} e_j^\sigma|_k \times e_k^{j\sigma}|_l &= e_j^i|_k \times e_k^i|_l \\ &= e_j^i|_l \\ &= e_j^\sigma|_l \end{aligned}$$

Thus, $\mathbf{e}$ is completely stable for decision maker i. ■

This proposition implies that if a decision maker perceives decision makers' scheme as being in inside common knowledge for the decision maker, then the scheme of emotions is completely stable for the decision maker if and only if it is stable for the decision maker.

Each of the last two propositions shows that it is important for a decision maker to perceive decision makers' scheme of emotions as being in common knowledge in order to have balanced perceptions of relations among decision makers.

Chapter 4

Analyses

We treat in this chapter analyses of situations of decision making. First, dealing with *honest*, *confident*, and *partially confident* decision makers, we specify decision makers' selections in a situation and outcomes of the situation. Moreover, we cope with issue of *generation of schemes*, and analyze decision makers' selections in generated schemes. Secondly, we focus on exchanges of information. In the issue of *deception* we give sufficient conditions for *inside strategyproofness* and *outside strategyproofness*, respectively, that mean senselessness of deception. Furthermore, we define *complete credibility* of information, and prove the equivalence of the concept of credibility to that of complete credibility under the condition of inside common knowledge. Thirdly, we propose a *solution concept*, whose definition involves the emotional and interperceptional aspects of decision making as well as the economic aspect. Then we show that if the actual outcome of a situation is not an emotional equilibrium, then decision makers in the situation modify their perceptions of the situation. Fourthly, we deal with cooperative situations of decision making. Giving sufficient conditions for not to reach a *deadlock* and not to reach a *complete deadlock* respectively, we show that it is important for progression of a meeting to built up stable schemes of emotions.

4.1 Outcomes of Competitive situations

In this section we first examine 2×2 *base competitions*, that is, the situations that involve just two decision makers each of who has just two possible actions. Subsequently, the concept of *generation* of schemes of base competitions is defined by employing the concept of *integration* of schemes of base competitions. Analyses of examples of integrated and generated schemes are provided.

4.1.1 2×2 Base Competitions and 'Soft' Games

2×2 *base competitions* are the most simple and important group in base competitions. A 2×2 base competition involves just two decision makers, and each of the decision makers has just two actions. Thus the situations of "prisoners' dilemma" (Table 8 (a)) and "chicken" (Table 8 (b)) are examples of 2×2 base competitions.

		2	
		c_2	d_2
1	c_1	(3, 3)	(1, 4)
	d_1	(4, 1)	(2, 2)

(a) Prisoners' dilemma.

		2	
		s_2	k_2
1	s_1	(3, 3)	(2, 4)
	k_1	(4, 2)	(1, 1)

(b) Chicken.

Table 8. 2 × 2 base competitions.

Definition 46 **(2 × 2 Base Competitions)** *A base competition $C = (N, S, F)$ is said to be a* 2 × 2 base competition *if N consists of just two decision makers, and if S_i includes just two actions for any i in N.*

Adopting the assumptions about functions of emotions in 'soft' game theory [44, 49], and employing the frameworks and the concepts provided in the preceding chapters in this paper, we can analyze *'soft' games*. Consider a 2 × 2 base competition C, and assume that the base competition is common knowledge among the decision makers. That is, supposing decision makers' pair $(\mathbf{N}, \Sigma) = (\mathbf{N}_i, \Sigma_i)_{i\in N}$ that is in common knowledge, and decision makers' schemes $\mathbf{C} = (\mathbf{C}_i)_{i\in N}$ of base competitions, where $\mathbf{C}_i = (C^\sigma)_{\sigma\in\Sigma_i}$, we have for any i in N and any σ in $\Sigma_i = \Sigma_i^*$, $C^\sigma = C$. We also assume that decision makers' scheme $\mathbf{e} = (\mathbf{e}_i)_{i\in N}$ of emotions is in common knowledge. A *'soft' game* is defined as a pair of decision makers' scheme $\mathbf{C}$ of base competitions decision makers' scheme $\mathbf{e}$ of emotions.

Definition 47 **('Soft' Games)** *A* 'soft' game *is a pair $(\mathbf{C}, \mathbf{e})$, where for any i in N and any σ in Σ_i, C^σ is equal to a 2 × 2 base competition C, and $\mathbf{e}$ is in common knowledge.*

Let $\mathbf{r} = (\mathbf{r}_i)_{i\in N}$ be decision makers' scheme of rules. In 'soft' game theory [44, 49] a type of information about rules of decision makers, called *inducement tactics*, is treated.

Definition 48 **(Inducement Tactics)** *An* inducement tactic ι_i *of decision maker i in N is a tuple (p^i, t_i), where $p^i = (p^i_j)_{j\in N}$ in S and t_i in S_i, such that for any j in $N\backslash\{i\}$, $p^i \not{F}_j (t_i, s_{-i})$ for any s_{-i} such that $s_{-i} \neq p^i_{-i}$, where $p^i_{-i} = (p^i_j)_{j\in N\backslash\{i\}}$.*

Inducement tactic ι_i of decision maker i in N specifies rule r_i of decision maker i such that $r_i(s_{-i})$ is p^i_i if $s_{-i} = (p^i_j)_{j\in N\backslash\{i\}}$, otherwise t_i. We call the outcome $p^i = (p^i_j)_{j\in N}$ a *promise* of decision maker i, and the action t_i a *threat* for the promise p^i of decision maker i. A list $(\iota_i)_{i\in N}$ of inducement tactics of decision makers is denoted by ι, called decision makers' *inducement tactic*. For any inducement tactic $\iota_i = (p^i, t_i)$ of decision maker i in N, the promise p^i of decision maker i, is the outcome that decision maker i intends to realize in the 'soft' game that he/she involved in. We assume that a decision maker selects an outcome that he/she intends to realize referring to the economic aspect $\mathbf{F}$ and the emotional aspect $\mathbf{e}$ of the 'soft' game. Thus any outcome can be a promise of a decision maker as long as there is a threat for it. For example, a decision maker can choose the least profitable outcome for him/herself as his/her promise if there is a threat for the outcome. Similarly, any action can be a threat for an outcome if the pair of the outcome and the action satisfies the condition in the definition of inducement tactics. The condition means that if decision maker i chooses the action t_i, that is, the threat for the promise p^i, then other decision makers cannot reach the outcomes that are preferred to p^i by them. Thus the threat t_i for the promise p^i of decision maker i is conveyed to make other decision makers obey the promise p^i of decision maker i. We assume that each decision maker conveys an inducement tactic to the others, and that by inducement tactic $\iota_i = (p^i, t_i)$, decision maker i tries to inform the others, "I will choose p^i_i, if it is convincing that decision maker j will choose p^i_j for any j in $N \backslash \{i\}$. Otherwise, that is, if I am convinced that there is decision maker j in $N \backslash \{i\}$ who will choose an action different from p^i_j, then I will choose t_i." Especially, if inducement tactic $\iota = (p^i, t_i)$ satisfies that $p^i_i = t_i$, then it means, "I will choose p^i_i independently of the inference about others' selections." Furthermore, we assume that inducement tactics are exchanged once and simultaneously, and that each decision maker chooses an action independently and simultaneously after the exchange. Then the list of actions chosen by all decision makers determines a final outcome.

The following property shows that in any 2×2 base competition each decision maker has inducement tactics to convey to the others.

Proposition 13 (Existence of Inducement Tactics) *For any 2×2 base competition, each decision maker in the competition has three or four inducement tactics.*

(proof) It is suffice to prove in the case for decision maker 1 in N. We classify the proof in terms of the profits F_2 of decision maker 2. Let $s^* = (s^*_1, s^*_2)$ in S be the most profitable outcome for decision maker 2, and $s^{\#} = (s^{\#}_1, s^{\#}_2)$ in S the second.

1. In the case that $s^*_1 = s^{\#}_1$. Let $S_1 = \{s_1, s'_1\}$. Since $s^* \not{F}_2 s$ for any s in S, both (s^*, s_1) and (s^*, s'_1) are inducement tactics. If s_1 in S_1 satisfies $s_1 \neq s^*_1$, then $(s^{\#}, s_1)$ is an inducement tactic, but $(s^{\#}, s^*_1)$ does not satisfy the condition to be an inducement tactic, since $s^{\#} F_2 (s^*_1, s^*_2) = s^*$. If $s = (s_1, s_2)$ and $s' = (s_1, s'_2)$ for $\{s_2, s'_2\} = S_2$, then $s, s' F_2 s^{\#}, s^*$. Thus neither (s, s^*_1) nor (s', s^*_1) is an inducement tactic. If $s F_2 s'$, then (s, s_1) does not satisfy the condition to be an inducement tactic. As a result, the number of inducement tactics are four, that is, (s^*, s_1), (s^*, s'_1), $(s^{\#}, s_1)$, and (s, s_1) or (s', s_1).

2. In the case that $s^*_1 \neq s^{\#}_1$. By the same reason as in the case that $s^*_1 = s^{\#}_1$, both (s^*, s_1) and (s^*, s'_1) are inducement tactics. If $s^*_1 = s_1$, then $(s^{\#}, s_1)$ does not satisfy the condition to be an inducement tactic, since $s^{\#} F_2 (s1, s^*_2) = s^*$. $(s^{\#}, s'_1)$ satisfies the condition to be an inducement tactic. If $S_2 = \{s_2, s'_2\}$ and $s_2 \neq s^*_2$, then none of (s, s_1), (s, s'_1), (s', s_1) and (s', s'_1) is inducement tactics, since $s, s' F_2 s^*, s^{\#}$ for $s = (s_1, s_2)$ and $s' = (s'_1, s'_2)$. Thus only (s^*, s_1), (s^*, s'_1) and $(s^{\#}, s'_1)$ are inducement tactics. ■

Example 13 (Examples of Inducement Tactics) *In Table 8 (a), all inducement tactics of decision maker 1 are $((c_1, c_2), d_1)$, $((c_1, d_2), c_1)$, $((c_1, d_2), d_1)$, and $((d_1, d_2), d_1)$. In Table 8 (b), all inducement tactics of decision maker 2 are $((w_1, w_2), k_2)$, $((k_1, w_2), w_2)$, $((k_1, w_2), k_2)$, and $((w_1, k_2), k_2)$.*

In terms of the exchanges of inducement tactics, Howard [44] focuses primarily on the unilateral transmission of inducement tactics, that implies that a specific decision maker has the right of conveying information to the others. We consider the mutual exchanges of inducement tactics so as to give equal opportunity of conveying information to each decision maker.

Suppose that decision maker i in N conveys inducement tactic $\iota_i = (p^i, t_i)$, where $p^i = (p^i_j)_{j \in N}$, to the others. If the rule induced by the inducement tactics is rational at $(p^i_j)_{j \in N \backslash \{i\}}$, then the other decision makers will believe the promise that decision maker i will choose p^i_i if it is convincing for decision maker i that decision maker j will choose p^i_j for all j in $N\backslash\{i\}$. On the contrary, if there is an action s^*_i in S_i such that the outcome (s^*_i, p^i_{-i}) is more profitable for decision maker i that p^i, then it is natural for the other decision makers to doubt the promise of decision maker i. The outcome (s^*_i, p^i_{-i}) is called a *temptation* besetting the promise p^i of decision maker i, and if a promise has a temptation, then the credibility of the promise drops.

Definition 49 (Temptations Besetting Promises) *Suppose inducement tactic $\iota_i = (p^i, t_i)$ of decision maker i in N. A* temptation *besetting the promise p^i is an outcome $s^* = (s^*_i, p^i_{-i})$ such that $s^*_i \neq p^i_i$ and $s^* \not F_i p^i$.*

Similarly, we can define a *temptation* besetting the threat t_i for the promise p^i of decision maker i. The temptation reduces the credibility of the threat.

Definition 50 (Temptations Besetting Threats) *Suppose inducement tactic $\iota_i = (p^i, t_i)$ of decision maker i in N. A* temptation *besetting the threat t_i for the promise p^i is an outcome $s^* = (s^*_j)_{j\in N}$ such that $s^*_i \neq t_i$, $s^*_j \neq p^i_j$ for some j in $N\backslash\{i\}$, and $s^* \not F_i (t_i, s^*_{-i})$.*

By the definitions of the temptations, we can immediately get the following properties about them. Consider a 2×2 base competition $C = (N, S, F)$, a decision maker i in N, and an inducement tactic $\iota_i = (p^i, t_i)$ of decision maker i.

Proposition 14 (Unique Temptation Besetting Promises) *It is satisfied that if there exists a temptation besetting the promise p^i, then it is unique temptation besetting the promise.*

(proof) It is suffice to prove the case of decision maker 1 in N. If there exists a temptation besetting the promise, and both s and s' satisfy the condition to be a temptation besetting the promise, then we have that $s = (s_1, p^1_2)$ and $s' = (s'_1, p^1_2)$ for some s_1 and s'_1 in S_1, because of the definition of temptations. Since $s_1 \neq p^1_1$, $s'_1 \neq p^1_1$ and $|S_1| = 2$, $s_1 = s'_1$; hence $s = s'$. ■

Proposition 15 (Unique Temptation Besetting Threats) *It is satisfied that if there exists a temptation besetting the threat t_i for the promise p^i, then it is unique temptation besetting the threat for the promise.*

(proof) It is suffice to verify the case of decision maker 1 in N. Suppose that both s and s' satisfy the condition to be a temptation besetting the threat for the promise. Assume that $s = (s_1, s_2)$ and $s' = (s'_1, s'_2)$. It is suffice to show $s_1 = s'_1$ and $s_2 = s'_2$. Since we have that $s_1 \neq t_1$ and $s'_1 \neq t_1$ and $|S_1| = 2$, it is satisfied that $s_1 = s'_1$. Moreover, since $s_2 \neq p^1_2$ and $s'_2 \neq p^1_2$ and $|S_2| = 2$, we have that $s_2 = s'_2$. ■

Example 14 (Examples of Temptations) *In Table 8 (a), regarding decision maker 1's inducement tactic $((c_1, c_2), d_1)$, there is just one temptation (d_1, c_2) besetting the promise (c_1, c_2) of decision maker 1, and there is no temptation besetting the threat d_1 for the promise (c_1, c_2) of decision maker 1. In Table 8 (b), for inducement tactic $((w_1, k_2), k_2)$ of decision maker 2, there is no temptation besetting the promise (w_1, k_2) of decision maker 2, and there is just one temptation (k_1, w_2) besetting the threat k_2 for the promise (w_1, k_2) of decision maker 2.*

When we adopt the assumptions about functions of emotions as in 'soft' game theory [44, 45], we have that positive emotions of decision maker i make decision maker i's promise with a temptation credible for the others, and that negative emotions of decision maker i makes decision maker i's threat with a temptation credible for the others. We can strictly define the *credible* promises and threats. Suppose a 'soft' game $(\mathbf{C}, \mathbf{e})$, and inducement tactic $\iota_i = (p^i, t_i)$ of decision maker i in N.

Definition 51 (Credible Promises and Threats) *The promise p^i is said to be* credible *for decision maker j in $N \backslash \{i\}$, if there is no temptation besetting the promise and/or $e_i^i|_j = +$. The threat t_i for the promise p^i is* credible *for decision maker j in $N \backslash \{i\}$, if there is no temptation besetting the threat for the promise and/or $e_i^i|_j = -$.*

Example 15 (Examples of Credible Promises and Threats) *In Table 8 (a), for the inducement tactic $((c_1, c_2), d_1)$ of decision maker 1, if decision maker 1 has positive emotions toward decision maker 2, that is, $e_1^1|_2 = +$, then the promise (c_1, c_2) is credible for decision maker 2 by the function of positive emotions in spite of the existence of the temptation (d_1, c_2) besetting the promise. The threat d_1 for the promise is credible for decision maker 2, because there is no temptation besetting the threat. In Table 8 (b), for the inducement tactic $((w_1, k_2), k_2)$ of decision maker 2, the promise (w_1, k_2) is credible for decision maker 1, because there is no temptation besetting the promise. If decision maker 2 has negative emotions toward decision maker 1, that is, $e_2^2|_1 = -$, then the threat k_2 for the promise is credible for decision maker 1 by the function of negative emotions in spite of the existence of the temptation (k_1, w_2) besetting the threat.*

In a competitive situation each decision maker has to choose just one actions referring to his/her schemes of components of the situation: the base competition, the emotions of decision makers, the rules of decision makers, and so on. Given a 'soft' game, we express a way of the selection by a decision maker with a mapping that indicates an action of the decision maker corresponding to each possible pattern of the decision maker's scheme of rules. The mapping is called a *decision function* of the decision maker. Suppose a 'soft' game $(\mathbf{C}, \mathbf{e})$ and decision makers' scheme $\mathbf{r}$ of rules.

Definition 52 (Decision Functions) *For any i in N, a* decision function *d_i of decision maker i is a mapping that indicates an action s_i in S_i of decision maker i corresponding to decision maker i's scheme $\mathbf{r}_i$ of rules, that is, $d_i(\mathbf{r}_i) = s_i$. A list $(d_i)_{i \in N}$ of decision functions d_i of decision maker i for each i in N is called decision makers' decision function, denoted by d.*

We can regard decision function of decision maker i as a component of the situation. Thus given decision maker i's pair $(\mathbf{N}_i, \Sigma_i)$ of a scheme and a set of strings of decision makers, we can define decision maker i's scheme of decision functions, denoted by $\mathbf{d}_i$, and decision makers' scheme of decision functions, denoted by $\mathbf{d}$. For any i in N, decision maker i's scheme $\mathbf{d}_i$ must be consistent with decision maker i's scheme $\mathbf{r}_i$ of rules. That is, for any decision maker i's scheme $\mathbf{r}_i$ of rules, it must be satisfied that $d_i^i(\mathbf{r}_i) = r_i^i((d_j^i(\mathbf{r}_{ji}))_{j \in N^i \backslash \{i\}})$, and generally, for any σ in Σ_i, and any j in N^σ, $d_j^\sigma(\mathbf{r}_{j\sigma}) = r_j^\sigma((d_k^{j\sigma}(\mathbf{r}_{kj\sigma}))_{k \in N^{j\sigma} \backslash \{j\}})$. These relations between $\mathbf{d}_i$ and $\mathbf{r}_i$ are depicted as in Figure 6.

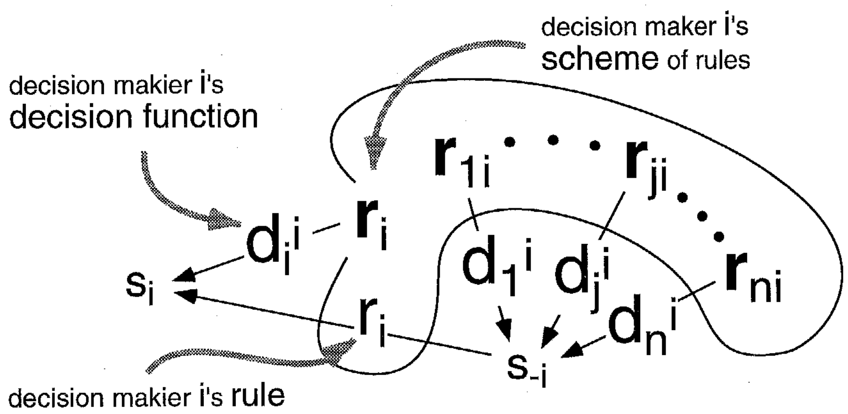

Figure 6. Relations between decison functions and rules.

Given a 'soft' game $(\mathbf{C}, \mathbf{e})$, for each i in N, decision maker i constructs decision maker i's schemes $\mathbf{r}_i$ and $\mathbf{d}_i$ of rules and decision functions. At the same time, decision maker i chooses an inducement tactic ι_i of decision maker i to be conveyed. After decision makers' inducement tactic ι is exchanged among the decision makers, decision maker i changes his/her schemes of rules and decision functions from $\mathbf{r}_i$ and $\mathbf{d}_i$ to $\mathbf{r}'_i$ and $\mathbf{d}'_i$, respectively, corresponding to the inducement tactic. Then each decision maker chooses just one action according to decision function $\mathbf{d}'_i$ of the decision maker, and the final outcome is determined. These moves in a 'soft' game can be described as in Figure 7.

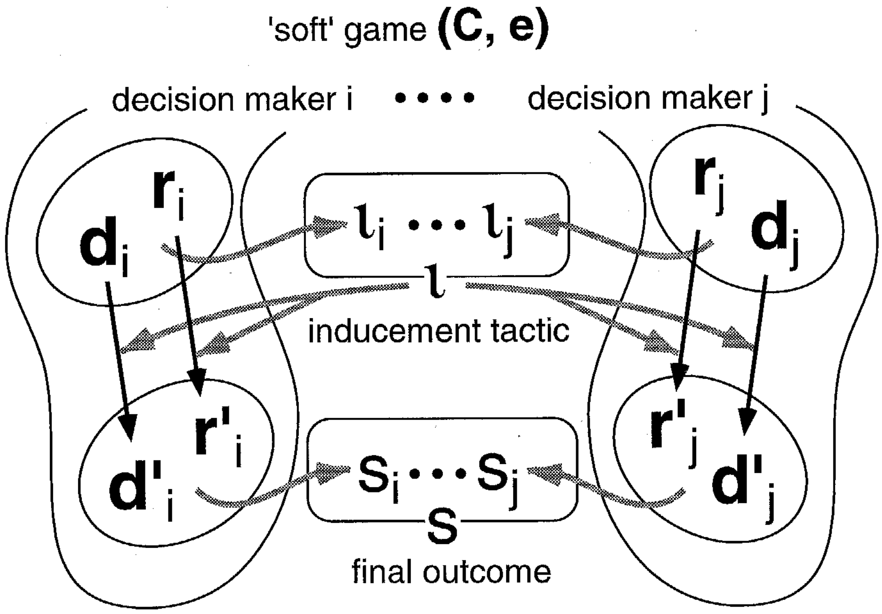

Figure 7. Moves in a 'soft' game.

We can describe a few types of decision makers depending on which inducement tactics the decision makers convey, and which inducement tactics the decision makers believe. If a decision maker conveys true information about his/her own rules, then the decision maker is said to be *honest*, and if a decision maker believes any information conveyed by the others, then the decision maker is said to be *confident.* Moreover, if a decision maker believes only information that is credible for the decision maker, then the decision maker is said to be *partially confident.* Consider a 'soft' game $(\mathbf{C}, \mathbf{e})$, decision makers' scheme $\mathbf{r}$ of rules, decision makers' inducement tactic ι, and decision makers' scheme $\mathbf{r}'$ of rules after the exchanges of the inducement tactic ι.

Definition 53 (Honest Decision Makers) *For any i in N, if inducement tactic $\iota_i = (p^i, t_i)$ of decision maker i, the rule r^i_i of decision maker i, and the rule r'^i_i of decision maker i after the exchanges of inducement tactic satisfy that for any s_{-i} in S^i_{-i},*

$$r^i_i(s_{-i}) = r'^i_i(s_{-i}) = \begin{cases} p^i_i & \text{if} \quad s_{-i} = (p^i_j)_{j \in N\setminus\{i\}} \\ t_i & \text{if} \quad s_{-i} \neq (p^i_j)_{j \in N\setminus\{i\}}, \end{cases}$$

then decision maker i is said to be honest.

Definition 54 (Confident Decision Makers) *For any i in N and j in $N\setminus\{i\}$, if inducement tactic $\iota_j = (p^j, t_j)$ of decision maker j and decision maker i's perception r'^i_j of the rule of decision maker j after the exchanges of inducement tactic ι satisfy that for any s_{-j} in S^i_{-j},*

$$r'^i_j(s_{-j}) = \begin{cases} p^j_j & \text{if} \quad s_{-j} = (p^j_k)_{k \in N\setminus\{j\}} \\ t_j & \text{if} \quad s_{-j} \neq (p^j_k)_{k \in N\setminus\{j\}} \end{cases}$$

then decision maker i is said to be confident *of inducement tactic ι_j of decision maker j. If decision maker i is confident of any inducement tactic of any decision maker, then decision maker i is said to be* confident.

Definition 55 (Partially Confident Decision Makers) *For any i in N and j in $N\setminus\{i\}$, if inducement tactic $\iota_j = (p^j, t_j)$ of decision maker j and decision maker i's perception r'^i_j of the rule of decision maker j after the exchanges of inducement tactic ι satisfy that for any s_{-j} in S^i_{-j},*

$$r'^i_j(s_{-j}) = \begin{cases} p^j_j & \text{if} \quad s_{-j} = (p^j_k)_{k \in N\setminus\{j\}} \text{ and } p^j \text{ is credible for decision maker } i \\ p'_j & \text{if} \quad s_{-j} = (p^j_k)_{k \in N\setminus\{j\}} \text{ and } p^j \text{ is not credible for decision maker } i \\ t_j & \text{if} \quad s_{-j} \neq (p^j_k)_{k \in N\setminus\{j\}} \text{ and } t_j \text{ is credible for decision maker } i \\ t'_j & \text{if} \quad s_{-j} \neq (p^j_k)_{k \in N\setminus\{j\}} \text{ and } t_j \text{ is not credible for decision maker } i, \end{cases}$$

then decision maker i is said to be partially confident *of inducement tactic ι_j of decision maker j, where p'_j and t'_j are the actions such that (p'_j, s_{-j}) and (t'_j, s_{-j}) compose temptations besetting p^j and t_j, respectively. If decision maker i is partially confident of any inducement tactic of any decision maker, then decision maker i is said to be* partially confident.

Thanks to the uniqueness of temptations, we can say that the function r'^i_j above is well-defined.

Example 16 (Partially Confident Decision Makers) *In Table 8 (a), let decision maker 2 be a partially confident of inducement tactic of decision maker 1, and consider that decision maker 1 conveys inducement tactic $((c_1, c_2), d_1)$ to decision maker 2. If decision maker 1 has positive emotions toward decision maker 2, then decision maker 2 believes decision maker 1's promise (c_1, c_2) in spite of the existence of temptation (d_1, c_2) besetting the promise. If decision maker 1 does not have positive emotions toward decision maker 2, then decision maker 2 believes that decision make 1 will choose d_1 instead of c_1 when decision maker 1 is convinced that decision maker 2 will choose c_2. In Table 8 (b), let decision maker 1 be a partially confident of* *inducement tactic of decision maker 2, and consider that decision maker 2 conveys inducement tactic $((w_1, k_2), k_2)$ to decision maker 1. If decision maker 2 has negative emotions toward decision maker 1, then decision maker 1 believes decision maker 2's threat k_2 in spite of the existence of temptation (k_1, w_2) besetting the threat k_2. If decision maker 2 does not have negative emotions toward decision maker 1, then decision maker 1 believes that decision maker 2 will choose w_2 instead of k_2 when decision maker 2 is convinced that decision maker 1 will choose k_1.*

Consider a 'soft' game $(\mathbf{C}, \mathbf{e})$, decision makers' schemes, $\mathbf{r}$ and $\mathbf{d}$, of rules and decision functions, decision makers' inducement tactic ι, and decision makers' schemes, $\mathbf{r}'$ and $\mathbf{d}'$, of rules and decision functions after the exchanges of the inducement tactic ι. In the situation each decision maker tries to perceive the correct action of the others by using the inducement tactic ι in order to select an appropriate action. Thus, for any i in N, the most desirable condition for decision maker i is the state that $d'^i_j(\mathbf{r}'_{ji}) = d'^j_j(\mathbf{r}'_j)$. We can show, however, that in the situations of "prisoners' dilemma" and "chicken," there is a case that a decision maker misperceives the correct action of the others even if we consider only honest and confident decision makers. Let N be the set $\{1, 2\}$.

Proposition 16 (Possibility of Misperception) *Let $(\mathbf{C}, \mathbf{e})$ be a 'soft' game with decision makers' inducement tactic $\iota = (\iota_i)_{i \in N} = ((p^1, t_1), (p^2, t_2))$ such that $p^1_1 = p^2_1$, $p^2_2 \neq p^1_2$, $t_1 \neq p^2_1$, and $t_2 = p^1_2$, and assume each decision maker is honest and confident. Then decision makers' schemes, $\mathbf{r}'$ and $\mathbf{d}'$, of rules and decision functions after the exchanges of inducement tactic ι do not satisfy either $d'^1_2(\mathbf{r}'_{21}) = d'^2_2(\mathbf{r}'_2)$ or $d'^2_1(\mathbf{r}'_{12}) = d'^1_1(\mathbf{r}'_1)$.*

(proof) Assume that $d'^1_2(\mathbf{r}'_{21}) = d'^2_2(\mathbf{r}'_2)$ and $d'^2_1(\mathbf{r}'_{12}) = d'^1_1(\mathbf{r}'_1)$. Then we have that

$$\begin{aligned} d'^1_1(\mathbf{r}'_1) &= r'^1_1(d'^1_2(\mathbf{r}'_{21})) \\ &= r'^1_1(d'^2_2(\mathbf{r}'_2)) \\ &= (r'^1_1 \circ r'^2_2)(d'^2_1(\mathbf{r}'_{12})) \\ &= (r'^1_1 \circ r'^2_2)(d'^1_1(\mathbf{r}'_1)). \end{aligned}$$

If we assume that $d'^1_1(\mathbf{r}'_1) = s_1$, then it is satisfied that

$$s_1 = r'^1_1 \circ r'^2_2(s_1) = \begin{cases} p^1_1 \quad \text{if} \quad \left(\begin{array}{c} s_1 = p^2_1 \text{ and } p^2_2 = p^1_2 \\ \text{or} \\ s_1 \neq p^2_1 \text{ and } t_2 = p^1_2 \end{array}\right. \\ t_1 \quad \text{if} \quad \left(\begin{array}{c} s_1 = p^2_1 \text{ and } p^2_2 \neq p^1_2 \\ \text{or} \\ s_1 \neq p^2_1 \text{ and } t_2 \neq p^1_2. \end{array}\right. \end{cases}$$

Thus, we have that

$$\begin{array}{ccc} s_1 = p^2_1 \text{ and } p^2_2 = p^1_2 & \Rightarrow & p^1_1 = s_1 = p^2_1, \\ s_1 \neq p^2_1 \text{ and } t_2 = p^1_2 & \Rightarrow & p^1_1 = s_1 \neq p^2_1, \\ s_1 = p^2_1 \text{ and } p^2_2 \neq p^1_2 & \Rightarrow & t_1 = s_1 = p^2_1, \\ & \text{and} & \\ s_1 \neq p^2_1 \text{ and } t_2 \neq p^1_2 & \Rightarrow & t_1 = s_1 \neq p^2_1. \end{array}$$

If we have that $s_1 = p^2_1$, then since it is assumed that ι satisfies that $p^2_2 \neq p^1_2$, we have that $t_1 = p^2_1$, but it contradicts with the assumption that ι satisfies that $t_1 \neq p^2_1$. If we have that $s_1 \neq p^2_1$, then since it is assumed that ι satisfies that $t_2 = p^1_2$, we have that $p^1_1 \neq p^2_1$, but it contradicts with the assumption that ι satisfies that $p^1_1 = p^2_1$. Therefore, either $d'^1_2(\mathbf{r}'_{21}) \neq d'^2_2(\mathbf{r}'_2)$ or $d'^2_1(\mathbf{r}'_{12}) \neq d'^1_1(\mathbf{r}'_1)$ is satisfied. ■

Because the situations of "prisoners' dilemma" and "chicken" have decision makers' inducement tactics that satisfy the conditions in this proposition, in those situations it can be occurred that a decision maker misperceives the correct action of the others even if we consider only honest and confident decision makers. Actually, in the situation of "prisoners' dilemma," decision makers' inducement tactic $(((c_1, d_2), d_1), ((c_1, c_2), d_2))$ satisfies the condition, and in the situation of "chicken," $(((w_1, k_2), k_1), ((w_1, w_2), k_2))$ satisfies the condition.

In the case that decision makers' inducement tactic $\iota = (\iota_i)_{i \in N} = ((p^1, t_1), (p^2, t_2))$ satisfies that $p^1 = p^2$, the outcome $p = p^1 = p^2$ should be selected as the final outcome, because the promise of a decision maker is the outcome that the decision maker intends to achieve. Howard [44] implies that if all decision makers are honest and confident, then p will be chosen as the final outcome, but he does not give any proofs of this property. We first deal with honest and confident decision makers, and show that if each decision maker believes that the others think that he/she will select the action that is required to realize p, then p will be selected as the final outcome. Furthermore, treating honest and partially confident decision makers, we prove that a sufficient condition for p to be chosen is that decision makers have positive emotions toward one another and they believe that they are thought to select the actions that are needed to realize p.

Proposition 17 (Confident Decision Makers Select p) *Suppose a 'soft' game $(\mathbf{C}, \mathbf{e})$ with honest and confident decision makers, decision makers' schemes, $\mathbf{r}$ and $\mathbf{d}$, of rules and decision functions, and decision makers' inducement tactic $\iota = ((p^1, t_1), (p^2, t_2))$ such that $p = p^1 = p^2$. Regarding decision makers' schemes, $\mathbf{r}'$ and $\mathbf{d}'$, of rules and decision functions after exchanges of the inducement tactic ι, if we have that $d'^{21}_1(\mathbf{r}'_{121}) = p^1_1$ and $d'^{12}_2(\mathbf{r}'_{212}) = p^2_2$ then $d'^1_1(\mathbf{r}'_1) = p^1_1$ and $d'^2_2(\mathbf{r}'_2) = p^2_2$.*

(proof) From the definition of decision functions, we have that

$$\begin{aligned} d'^1_1(\mathbf{r}'_1) &= r'^1_1(d'^1_2(\mathbf{r}_{21})) \\ &= (r'^1_1 \circ r'^1_2)(d'^{21}_1(\mathbf{r}_{121})). \end{aligned}$$

From the assumption that decision makers are honest and confident, we have that

$$r'^1_1(s_2) = \begin{cases} p^1_1 & \text{if} \quad s_2 = p^1_2 \\ t_1 & \text{if} \quad s_2 \neq p^1_2, \end{cases}$$

and that

$$r'^1_2(s_1) = \begin{cases} p^2_2 & \text{if} \quad s_1 = p^2_1 \\ t_2 & \text{if} \quad s_1 \neq p^2_1. \end{cases}$$

From the assumption that $d'^{21}_1(\mathbf{r}'_{121}) = p^1_1$, and that $p^1 = p^2$, we have that

$$\begin{aligned} (r'^1_1 \circ r'^1_2)(d'^{21}_1(\mathbf{r}_{121})) &= (r'^1_1 \circ r'^1_2)(p^1_1) \\ &= (r'^1_1 \circ r'^1_2)(p^2_1) \\ &= r'^1_1(p^2_2) \\ &= r'^1_1(p^1_2) \\ &= p^1_1. \end{aligned}$$

Similarly we also have that $d'^2_2(\mathbf{r}'_2) = p^2_2$. ■

Example 17 (**(c_1, c_2) in Table 8 (a) is Selected**) *In Table 8 (a), let decision maker 1 and 2 be honest and confident. Consider the case that decision maker 1 conveys inducement tactic $((c_1, c_2), d_1)$ to decision maker 2, and decision maker 2 conveys inducement tactic $((c_1, c_2), d_2)$ to decision maker 1. Thus decision makers' inducement tactic satisfies that $p^1 = p^2 = (c_1, c_2)$. The proposition above implies that if decision maker 1 believes that decision maker 2 thinks that decision maker 1 will select c_1, and if decision maker 2 believes that decision maker 1 thinks that decision maker 2 will select c_2, then the outcome (c_1, c_2) becomes the final outcome.*

Proposition 18 (**Partially Confident Decision Makers Select p**) *Consider a 'soft' game $(\mathbf{C}, \mathbf{e})$ with honest and partially confident decision makers, decision makers' schemes, $\mathbf{r}$ and $\mathbf{d}$, of rules and decision functions, and decision makers' inducement tactic ι such that $p = p^1 = p^2$. Regarding decision makers' schemes, $\mathbf{r}'$ and $\mathbf{d}'$, of rules and decision functions after exchanges of the inducement tactic ι, if we have that $d'^{21}_1(\mathbf{r}'_{121}) = p^1_1$ and $d'^{12}_2(\mathbf{r}'_{212}) = p^2_2$, and that $e^1_1|_2 = +$ and $e^2_2|_1 = +$, then $d'^1_1(\mathbf{r}'_1) = p^1_1$ and $d'^2_2(\mathbf{r}'_2) = p^2_2$.*

(proof) From the definition of decision functions, we have that

$$\begin{aligned} d'^1_1(\mathbf{r}'_1) &= r'^1_1(d'^1_2(\mathbf{r}_{21})) \\ &= (r'^1_1 \circ r'^1_2)(d'^{21}_1(\mathbf{r}_{121})). \end{aligned}$$

From the assumption that decision makers are honest and partially confident, we have that

$$r'^1_1(s_2) = \begin{cases} p^1_1 & \text{if} \quad s_2 = p^1_2 \\ t_1 & \text{if} \quad s_2 \neq p^1_2, \end{cases}$$

and that

$$r'^1_2(s_1) = \begin{cases} p^2_2 & \text{if} \quad s_1 = p^2_1 \text{ and } p^2 \text{ is credible for decision maker 1} \\ p'_2 & \text{if} \quad s_1 = p^2_1 \text{ and } p^2 \text{ is not credible for decision maker 1} \\ t_2 & \text{if} \quad s_1 \neq p^2_1 \text{ and } t_2 \text{ is credible for decision maker 1} \\ t'_2 & \text{if} \quad s_1 \neq p^2_1 \text{ and } t_2 \text{ is not credible for decision maker 1,} \end{cases}$$

where p'_2 and t'_2 are the actions such that (p'_2, s_1) and (t'_2, s_1) compose temptations besetting p^2 and t_2, respectively. Because $e^2_2|_1 = +$, the promise p^2 is credible for decision maker i. From the assumption that $d'^{21}_1(\mathbf{r}'_{121}) = p^1_1$, and that $p^1 = p^2$, we have that

$$\begin{aligned} (r'^1_1 \circ r'^1_2)(d'^{21}_1(\mathbf{r}_{121})) &= (r'^1_1 \circ r'^1_2)(p^1_1) \\ &= (r'^1_1 \circ r'^1_2)(p^2_1) \\ &= r'^1_1(p^2_2) \\ &= r'^1_1(p^1_2) \\ &= p^1_1. \end{aligned}$$

Similarly we also have that $d'^2_2(\mathbf{r}'_2) = p^2_2$. ■

We can imply from this proposition that it is not always necessary for each decision maker to make all part of the inducement tactic of the decision maker credible for the others in order that p is selected as the final outcome. Thus the 'naivety' of decision maker is not always necessary for p to be chosen as the final outcome.

Example 18 ((w_1, w_2) in Table 8 (b) is Selected) *In Table 8 (b), let decision maker 1 and 2 be honest and partially confident. Consider the case that decision maker 1 conveys inducement tactic* $((w_1, w_2), k_1)$ *to decision maker 2, and decision maker 2 conveys inducement tactic* $((w_1, w_2), k_2)$ *to decision maker 1. Thus decision makers' inducement tactic satisfies that* $p^1 = p^2 = (w_1, w_2)$*. The proposition above implies that if decision maker 1 and 2 have positive emotions toward each other, and if decision maker 1 believes that decision maker 2 thinks that decision maker 1 will select* w_1*, and decision maker 2 believes that decision maker 1 thinks that decision maker 2 will select* w_2*, then the outcome* (w_1, w_2) *is selected as the final outcome.*

4.1.2 Integrated and Generated Schemes

Generally, a competitive situation involves many decision makers, and each of the decision makers has many actions. The concepts of *integration*, that have been proposed in a preceding chapter in this paper, is useful when we analyze such an extremely complicated situation. We furthermore provide a definition of *generation of schemes*, employing the concept of integration of base competitions. The concept of generation is useful to classify schemes of base competitions. We analyze an example of competitive situations to clarify the concepts of integration and generation. In the example four companies are competing in three markets. We describe the markets as base competitions, which constitute the schemes which are models of the whole situation of the companies. We also show that the rational actions for the companies are changed depending on their perceptions of the whole situation.

If we integrate a finite number of schemes of base competitions each of which is made of finitely various base competitions, then the resulting scheme consists of finitely various base competitions. There is an interesting relationship between the class of base competitions in the original schemes and the class of base competitions which constitute the resulting scheme. Each base competition in the latter class is an integration of a subset of the former class. We call a scheme each element of which is an integration of a subset of a finite class of base competitions a *scheme generated from the class.* Let I be a finite index set, and consider a class $\mathbf{c} = (c^{\mu})_{\mu \in I}$ of base competitions and the relations among actions θ of decision makers in $\mathbf{c}$. Then we have a integration $\hat{c} = (\hat{N}, \hat{S}, \hat{F})$ of $\mathbf{c}$. Regarding the set $\hat{N}$ of all decision makers, moreover, suppose decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers.

Definition 56 (Generation of Schemes of Base Competitions) *A scheme* $\mathbf{C} = (\mathbf{C}_i)_{i \in \hat{N}}$ *of base competitions, where* $\mathbf{C}_i = (C^{\sigma})_{\sigma \in \Sigma_i}$ *for any* i *in* $\hat{N}$*, is a* scheme generated from $\mathbf{c}$, *if for any* i *in* $\hat{N}$ *and any* σ *in* Σ_i*, there exists a non-empty subset* I' *of* I *such that* C^{σ} *is an integration of* $\mathbf{c}' = (c'^{\mu'})_{\mu' \in I'}$ *in* θ*. If* $\mathbf{C}$ *is generated from* $\mathbf{c}$ *but not from any proper subset of* $\mathbf{c}$*, then* $\mathbf{c}$ *is called a* basis *of* $\mathbf{C}$*, and each member of* $\mathbf{c}$ *is called a* base *of* $\mathbf{C}$*.*

We give an example of competitive situations in order to make the concepts of integration and generation clear. In the example four companies are competing in three markets, but they do not always correctly perceive the situation. In general, we should treat the situations that decision makers have definite but incorrect beliefs about decision makers, possible actions, and profits. We should, furthermore, deal with emotional aspects as well as economic aspects of the situation. In the example, however, we deal with only economically rational decision makers who either perceive the relevant situation or have no view about it at all, for simplicity.

Consider that four companies, A_1, A_2, A_3, and A_4, are competing in three markets, α, β, and γ. A_1 is involved in α and β, and A_2 is engaged in α and γ. A_3 and A_4 participate in β and γ, respectively. Each company has two actions, invest (=**I**) and not invest (=**N**), for each markets. α, β, and γ are described as base competitions in Table 9 (a), (b), and (c), respectively. The base competitions are denoted by c^{α}, c^{β}, and c^{γ}, respectively.

c^α		A_2: I	A_2: N
A_1	I	(15, 15)	(35, 0)
	N	(0, 35)	(10, 10)

(a) Market α

c^β		A_3: I	A_3: N
A_1	I	(20, 5)	(25, 0)
	N	(0, 0)	(5, 5)

(b) Market β

c^γ		A_2: I	A_2: N
A_4	I	(0, 20)	(0, 0)
	N	(5, 25)	(5, 5)

(c) Market γ

Table 9. Base competitions of markets.

Assume that any company cannot invest in two markets at the same time. Then the relations among actions θ of the companies in the class $\mathbf{c} = (c^\alpha, c^\beta, c^\gamma)$ are as follows;

$$\theta_{A_1} = \{(\mathbf{I},\mathbf{N}),(\mathbf{N},\mathbf{I}),(\mathbf{N},\mathbf{N})\}, \theta_{A_2} = \{(\mathbf{I},\mathbf{N}),(\mathbf{N},\mathbf{I}),(\mathbf{N},\mathbf{N})\},$$

$$\theta_{A_3} = \{\mathbf{I},\mathbf{N}\}, \theta_{A_4} = \{\mathbf{I},\mathbf{N}\}.$$

Table 10 (a), (b), (c) describe integrations of (c^α, c^β), (c^α, c^γ), and $(c^\alpha, c^\beta, c^\gamma)$ in θ, respectively. The integration of (c^β, c^γ) is meaningless, since no decision maker engages in both games. Outcomes underlined are Nash equilibria in the base competitions.

(a) Integration of (c^α, c^β)

A_3: I		A_2: I	A_2: N
A_1	(I, N)	(15, 15, 0)	(35, 0, 0)
	(N, I)	(20, 35, 5)	(30, 10, 5)
	(N, N)	(0, 35, 0)	(10, 10, 0)

A_3: N		A_2: I	A_2: N
A_1	(I, N)	(20, 15, 5)	(40, 0, 5)
	(N, I)	(25, 35, 0)	(35, 10, 0)
	(N, N)	(5, 35, 5)	(15, 10, 5)

(b) Integration of (c^α, c^γ)

A_4: I		A_2: (I, N)	A_2: (N, I)	A_2: (N, N)
A_1	I	(15, 15, 0)	(35, 20, 0)	(35, 0, 0)
	N	(0, 35, 0)	(10, 30, 0)	(10, 10, 0)

A_4: N		A_2: (I, N)	A_2: (N, I)	A_2: (N, N)
A_1	I	(15, 20, 5)	(35, 25, 5)	(35, 5, 5)
	N	(0, 40, 5)	(10, 35, 5)	(10, 15, 5)

(c) Integration of (c^α, c^β, c^γ)

A_3 I, A_4 I		A_2 (I, N)	A_2 (N, I)	A_2 (N, N)
	(I, N)	(15, 15, 0, 0)	(35, 20, 0, 0)	(35, 0, 0, 0)
A_1	(N, I)	(20, 35, 5, 0)	(30, 30, 5, 0)	(30, 10, 5, 0)
	(N, N)	(0, 35, 0, 0)	(10, 35, 0, 0)	(10, 10, 0, 0)

A_3 N, A_4 I		A_2 (I, N)	A_2 (N, I)	A_2 (N, N)
	(I, N)	(20, 15, 5, 0)	(40, 20, 5, 0)	(40, 0, 5, 0)
A_1	(N, I)	(25, 35, 0, 0)	(35, 30, 0, 0)	(35, 10, 0, 0)
	(N, N)	(5, 35, 5, 0)	(15, 30, 5, 0)	(15, 10, 5, 0)

A_3 I, A_4 N		A_2 (I, N)	A_2 (N, I)	A_2 (N, N)
	(I, N)	(15, 20, 0, 5)	(35, 25, 0, 5)	(35, 5, 0, 5)
A_1	(N, I)	(20, 40, 5, 5)	(30, 35, 5, 5)	(30, 15, 5, 5)
	(N, N)	(0, 40, 0, 5)	(10, 35, 0, 5)	(10, 15, 0, 5)

A_3 N, A_4 N		A_2 (I, N)	A_2 (N, I)	A_2 (N, N)
	(I, N)	(20, 20, 5, 5)	(40, 25, 5, 5)	(40, 5, 5, 5)
A_1	(N, I)	(25, 40, 0, 5)	(35, 35, 0, 5)	(35, 15, 0, 5)
	(N, N)	(5, 40, 5, 5)	(15, 35, 5, 5)	(15, 15, 5, 5)

Table 10. Integrations of base competitions.

The integrations of (c^α, c^β), (c^α, c^γ), and $(c^\alpha, c^\beta, c^\gamma)$ are denoted by $c^\alpha + c^\beta$, $c^\alpha + c^\gamma$, and $c^\alpha + c^\beta + c^\gamma$, respectively. We can construct schemes of base competitions generated from the class $\mathbf{c} = (c^\alpha, c^\beta, c^\gamma)$. We need to describe two types of A_1's perceptions of the situation.

- **Type 1** expresses the case that A_1 correctly perceives all base competitions, and thinks that A_2 and A_4 perceive only c^α and c^γ, that is, $C^{A_1} = c^\alpha + c^\beta + c^\gamma$ and $C^{A_2 A_1} = C^{A_4 A_1} = c^\alpha + c^\gamma$. A_1 also believes that A_2 and A_4 think that $c^\alpha + c^\gamma$ is common knowledge, that is, $C^{\sigma A_2 A_1} = C^{\tau A_4 A_1} = c^\alpha + c^\gamma$ for any possible σ and τ. We assume in this case, moreover, that A_1 thinks that A_3 believes the same things that A_1 believes, that is, $C^{A_3 A_1} = C^{A_1}$, $C^{\sigma A_3 A_1} = C^{\sigma A_1}$ for any possible σ.

- **Type 2** describes that A_1 perceives only c^α and c^β, and believes that $c^\alpha + c^\beta$ is common knowledge, that is, $C^\sigma = c^\alpha + c^\beta$ for any σ in Σ_{A_1}.

Replacing A_1 with A_2, A_3 with A_4, and c^β with c^γ in the definitions above, respectively, we obtain the two types of A_2's perceptions of the situation. If we combine a type of A_1's perceptions and a type of A_2's perceptions, and assume that A_3's perceptions and A_4's perceptions are correctly perceived by A_1 and A_2, then we have four schemes of base competitions as follows:

- **Scheme 1** — A_1 with **Type 1** perceptions and A_2 with **Type 1** perceptions.
- **Scheme 2** — A_1 with **Type 1** perceptions and A_2 with **Type 2** perceptions.
- **Scheme 3** — A_1 with **Type 2** perceptions and A_2 with **Type 1** perceptions.
- **Scheme 4** — A_1 with **Type 2** perceptions and A_2 with **Type 2** perceptions.

In order to analyze the schemes, we determine the procedure which each company obeys in selecting an action as follows. We assume that the procedure is common knowledge among decision makers.

1. If a decision maker believes that the base competition he/she is participating is common knowledge among the decision makers in the base competition, and if the base competition has just one Nash equilibrium, then the decision maker selects the action which leads the equilibrium.

2. If a decision maker does not believe the base competition he/she is participating is common knowledge, but if the action selections of all of the others can be inferred from the decision maker's perceptions of the others and the procedure, then the decision maker selects the best reply action to these selections in the base competition he/she is participating.

3. If a decision maker does not believe the base competition he/she is participating is common knowledge, and if the selections of only some of the others can be inferred as in 2, and if the decision maker has a dominant action in the restriction of the base competition he/she is participating in terms of the selections, then the decision maker selects the dominant action.

Using the rules, we can consider only the class of schemes in which any base competitions believed to be common knowledge have just one Nash equilibrium, and where any decision maker who believes that he/she is participating a base competition which is not common knowledge and is able to infer the selections of only some of the others has a dominant action in the restriction of the base competition given by the selections. Before our analyses of examples, we should notice that they satisfy the conditions. We show that in spite of the fact that each of the schemes is generated from the same basis, they come to different outcomes from each other.

- **Scheme 1**. From the procedure, A_1 infers that A_2 will select $(\mathbf{N}, \mathbf{I})$, since A_1 thinks that A_2 believes that $c^\alpha + c^\gamma$ is common knowledge, and the unique Nash equilibrium of $c^\alpha + c^\gamma$ is $(\mathbf{I}, (\mathbf{N}, \mathbf{I}), \mathbf{N})$. Similarly, A_1 infers that A_4 will select $\mathbf{N}$. Since $(\mathbf{I}, \mathbf{N})$ is a dominant action of A_1 in the restriction of $c^\alpha + c^\beta + c^\gamma$ in terms of the list of $(\mathbf{N}, \mathbf{I})$ and $\mathbf{N}$, A_1 selects $(\mathbf{I}, \mathbf{N})$. A_3 believes the same things that A_1 believes, and the best reply of A_3 to $(\mathbf{I}, \mathbf{N})$, $(\mathbf{N}, \mathbf{I})$ and $\mathbf{N}$ is $\mathbf{N}$. Similar consideration concludes that A_2 selects $(\mathbf{I}, \mathbf{N})$, and that A_4 selects $\mathbf{N}$. Thus the final outcome of **Scheme 1** is $((\mathbf{I}, \mathbf{N}), (\mathbf{I}, \mathbf{N}), \mathbf{N}, \mathbf{N})$.

- **Scheme 2**. Since A_2 believes that $c^\alpha + c^\gamma$ that he/she is participating is common knowledge, A_2 selects $(\mathbf{N}, \mathbf{I})$ that is required to lead the unique Nash equilibrium of $c^\alpha + c^\gamma$. Similarly, A_4 selects $\mathbf{N}$. Since A_1 and A_3 have the same perceptions as in **Scheme 1**, they select $(\mathbf{I}, \mathbf{N})$ and $\mathbf{N}$, respectively. Thus, the final outcome of **Scheme 2** is $((\mathbf{I}, \mathbf{N}), (\mathbf{N}, \mathbf{I}), \mathbf{N}, \mathbf{N})$.

- **Scheme 3**. A_1 selects $(\mathbf{N}, \mathbf{I})$ that is needed to lead the unique Nash equilibrium of $c^\alpha + c^\beta$. A_3 also selects $\mathbf{I}$ that is required to lead the unique Nash equilibrium of $c^\alpha + c^\beta$. Since A_2 and A_4 has the same perceptions as in **Scheme 1**, they select $(\mathbf{I}, \mathbf{N})$ and $\mathbf{N}$, respectively. Thus, the final outcome of this scheme is $((\mathbf{N}, \mathbf{I}), (\mathbf{I}, \mathbf{N}), \mathbf{I}, \mathbf{N})$.

- **Scheme 4**. The final outcome in this case is the combination of the actions of A_1 and A_3 in **Scheme 3** and the actions of A_2 and A_4 in **Scheme 2**, that is, $((\mathbf{N}, \mathbf{I}), (\mathbf{N}, \mathbf{I}), \mathbf{I}, \mathbf{N})$, since the perceptions of A_1 and A_3 are the same as in **Scheme 3** and these of A_2 and A_4 are the same as in **Scheme 2**.

The final outcomes, $((\mathbf{I}, \mathbf{N}), (\mathbf{N}, \mathbf{I}), \mathbf{N}, \mathbf{N})$ in **Scheme 2** and $((\mathbf{N}, \mathbf{I}), (\mathbf{I}, \mathbf{N}), \mathbf{I}, \mathbf{N})$ in **Scheme 3**, are Nash equilibria of $c^\alpha + c^\beta + c^\gamma$, that is the true base competition in which the decision makers are involved. In **Scheme 1** and **Scheme 4**, on the other hand, each of A_1 and A_2 can improve their profit through changing their selection of action.

Through this example, we can see that even though a decision maker correctly perceives the true base competition in which the decision maker is involved, the selection of the decision maker is not always the best reply to the actions of the others. We can see, moreover, that if there is a decision maker who perceives the true base competition and the others' perceptions correctly, the selection of the decision maker is the best reply to the selections of the others. This example shows us that interperception by decision makers play important roles in competitive situations of decision making.

4.2 Information in Decision Making

In this section we focus on exchanges of information in situations of decision making. First we deal with strategic information exchanges, called *deception*, by decision makers in competitive situations. Treating interperception and interdependence in terms of decision makers' preferences, we show two properties that indicate senselessness of deception. One gives a sufficient condition for *inside strategyproofness*, where deception by a decision maker causes changes of preferences of the decision maker, thus the deception cannot be effective. The other provide a sufficient condition for *outside strategyproofness*, where any attempt of a decision maker to change another decision maker' preferences by deception will end in failure. Next, we raise the issue of *credibility* of information. Giving a definition of the concept of *complete credibility* of information, we examine relations between decision makers' perceptions of emotions, and credibility and complete credibility of information. We show the equivalence of credibility and complete credibility under schemes of emotions in inside common knowledge.

4.2.1 Deception by Decision Makers

In a situation of decision making with incompleteness of information, decision makers may deceive to induce a preferable outcome for him/her to the outcome that is obtained through sincere information exchanges. In fact, Brams [9] examines situations with three decision makers under incompleteness of information in terms of preferences of the decision makers, and shows that a profitable outcome for a deceiver to the outcome realized through honest offers can be achieved by deception. Brams [9] treats only decision makers with preferences independent of the others' preferences, but preferences of a decision maker often depend on the others' in real situations. Helping behaviors [27] and aggression [8] are examples that indicate the existence of interdependence of decision makers' preferences. Thus, we examine deception by decision makers with interdependent preferences under incompleteness of information in terms of the preferences.

Consider a competitive situation, the set N of all decision makers in the situation, and decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers. Suppose, moreover, decision makers' schemes, $\mathbf{C}$ and $\mathbf{e}$, of base competitions and emotions, respectively, where $\mathbf{C} = (\mathbf{C}_i)_{i\in N}$, $\mathbf{C}_i = (\mathbf{N}_i, \mathbf{S}_i, \mathbf{F}_i)$ for any i in N, and $\mathbf{e} = (\mathbf{e}_i)_{i\in N}$. Referring to the schemes $\mathbf{F}_i$ and $\mathbf{e}_i$, decision maker i in N constructs his/her scheme $\mathbf{r}_i$ of rules. Then, assuming that the scheme $\mathbf{C}$ is in common knowledge, we can regard the rule r_i of decision maker i in N as depending on only the scheme $\mathbf{e}_i$ and relations between the schemes $\mathbf{e}_i$ and $\mathbf{r}_i$ by decision maker i's scheme $\mathbf{f}_i$ of rule functions. That is, for any i in N, $f_i^i(\mathbf{e}_i) = r_i^i$, and generally, for any σ in Σ_i and any j in N^σ, $f_j^\sigma(\mathbf{e}_{j\sigma}) = r_j^\sigma$.

We treat information about decision makers' preferences is conveyed in the form $\hat{r} = (\hat{r}_i)_{i\in N}$ of a rule of decision makers. More precisely, for any i in N, decision maker i conveys the information $\hat{r}_i$ to the others. Each decision maker can convey false information to the others, but it is assumes that it becomes common knowledge among the decision makers that the information $\hat{r}$ is conveyed. The information $\hat{r}$ may or may not cause changes of decision makers' schemes of emotions and rules. For example, if decision maker i's information $\hat{r}_i$ implies a helping rule to decision maker j, then decision makers may modify their schemes of emotions so that decision maker i has positive emotions toward decision maker j, and consequently their schemes of rules may be changed. Oppositely, if decision maker i believes decision maker j's information $\hat{r}_i$ that means decision maker j's aggressive rule to decision maker k, then decision maker i may change his/her scheme $\mathbf{e}_i$ of emotions so as to satisfy that $e_j^i|_k = -$. These changes of schemes in terms of the information $\hat{r}$ are expressed by decision makers' schemes, $\mathbf{g}(\hat{r})$, $\mathbf{h}(\hat{r})$, and $\mathbf{l}(\hat{r})$, of scheme functions, perception functions, and view functions in terms of $\hat{r}$.

Completing these changes of the schemes, decision maker i constructs new schemes, $\mathbf{e}'_i$ and $\mathbf{r}'_i$, of emotions and rules for each i in N. In this case decision makers' preferences are interdependent, because the preferences of each decision maker may change depending on information about the others' preferences.

Each decision maker tries to manipulate the others' actions by deception in order to achieve a preferable outcome for the decision maker to that led by sincere information exchanges, but he/she conveys true information when the deception cannot be effective. We propose two states in which the deception will end in failure, *inside strategyproofness* and *outside strategyproofness.*

Definition 57 (Inside Strategyproofness) *A competitive situation satisfies* inside strategyproofness, *if for any i in N, any $\mathbf{e}_i$, and any $\hat{r}$, $f_i^i(\mathbf{e}_i) \neq \hat{r}_i$ implies $f_i^i \circ g_i^i(\hat{r})(\mathbf{e}_i) \neq f_i^i(\mathbf{e}_i)$.*

If a situation satisfies inside strategyproofness, then for any i in N, if decision maker i conveys the falsified information $\hat{r}_i$ in order to manipulate the others' actions, then the information causes changes of decision maker i's true rule, thus the falsification of information cannot be effective.

Definition 58 (Outside Strategyproofness) *A competitive situation satisfies* outside strategyproofness, *if for any i in N, any $\mathbf{e}_i$, any j in N^i, and any $\hat{r}_i$, where $\mathbf{e}_i = (e^i, \mathbf{e}^i)$ and $\mathbf{e}^i = (\mathbf{e}_{ji})_{j \in N^i}$, there exists $\hat{r}_{-i}$ such that $f_j^i \circ g_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) = f_j^i(\mathbf{e}_{ji})$.*

In a situation that satisfies outside strategyproofness, then for any i in N, decision maker i's attempts to manipulate the others' actions by the deception $\hat{r}_i$ will end in failure because of the information $\hat{r}_{-i}$ that is conveyed by the other decision makers.

If a situation is in one of the states, then decision makers convey information sincerely, because strategic information exchanges cannot be effective. We explore sufficient conditions for a situation to satisfy one of the states. First we provide definitions of several states of a competitive situation. For any i in N, π_i denotes a projection map to the ith component.

Definition 59 (Separability of Beliefs) *A situation is* separable, *if for any i in N, any $\mathbf{e}_i$, any σ in Σ_i, any k in N^σ, any j in $N^{k\sigma} \backslash \{k\}$, any $\hat{r}_j$, and any $\hat{r}_{-j}$ and $\hat{r}'_{-j}$, $f_j^{k\sigma} \circ \pi_j \circ l_k^\sigma(\hat{r}_i, \hat{r}_{-j})(\mathbf{e}_{k\sigma}) = f_j^{k\sigma} \circ \pi_j \circ l_k^\sigma(\hat{r}_i, \hat{r}'_{-j})(\mathbf{e}_{k\sigma})$.*

Considering the case that $\sigma = i$ and $k = i$, we have that if a situation is separable, then if for any i in N, any $\mathbf{e}_i$, any j in $N^i \backslash \{i\}$, any $\hat{r}_j$, and any $\hat{r}_{-j}$ and $\hat{r}'_{-j}$, $f_j^i \circ \pi_j \circ l_i^i(\hat{r}_i, \hat{r}_{-j})(\mathbf{e}_i) = f_j^i \circ \pi_j \circ l_i^i(\hat{r}_i, \hat{r}'_{-j})(\mathbf{e}_i)$. Thus, if a situation satisfies separability, then decision maker i infers decision maker j's rule referring to only the information $\hat{r}_j$ about decision maker j's rule: decision maker j's rule inferred by decision maker i is independent of the information $\hat{r}_{-j}$ about the other decision makers' rules.

Definition 60 (Extremeness of Beliefs) *A situation is said to satisfy* extremeness, *if for any i in N, any $\mathbf{e}_i$, any σ in Σ_i, any k in N^σ, any j in $N^{k\sigma} \backslash \{k\}$, any $\hat{r}$, and any s_{-j} in S_{-j}^i,*

$$f_j^{k\sigma} \circ \pi_j \circ l_k^\sigma(\hat{r})(\mathbf{e}_{k\sigma})(s_{-j}) = \left\{ \begin{array}{c} \hat{r}_j(s_{-j}) \\ or \\ f_j^{k\sigma}(\mathbf{e}_{jk\sigma})(s_{-j}). \end{array} \right.$$

Considering the case that $\sigma = i$ and $k = i$, we have that if a situation is extremeness, if for any i in N, any $\mathbf{e}_i$, any j in $N^i \backslash \{i\}$, any $\hat{r}$, and any s_{-j} in S_{-j}^i, where $\mathbf{e}_i = (e^i, \mathbf{e}^i)$ and $\mathbf{e}^i = (\mathbf{e}_{ji})_{j \in N^i}$,

$$f_j^i \circ \pi_j \circ l_i^i(\hat{r})(\mathbf{e}_i)(s_{-j}) = \left\{ \begin{array}{c} \hat{r}_j(s_{-j}) \\ \text{or} \\ f_j^i(\mathbf{e}_{ji})(s_{-j}). \end{array} \right.$$

Thus, if a situation satisfies extremeness, then decision maker i infers that decision maker j's rule is either the same as the information $\hat{r}_j$ implies, or equal to decision maker i's perception $f_j^i(\mathbf{e}_{ji})$ of decision maker j's rule before the exchanges of information. Especially, if the perception $f_j^i(\mathbf{e}_{ji})$ is the same as the information $\hat{r}_j$, then decision maker i believes that decision maker j's rule is $f_j^i(\mathbf{e}_{ji}) = \hat{r}_j$.

Definition 61 (Incredibility) *A situation is said to satisfy* incredibility, *if for any i in N, any $\mathbf{e}_i$, any j in $N^i \backslash \{i\}$, there exists $\hat{r}_j$ such that for any $\hat{r}_{-j}$, $f_j^i \circ \pi_j \circ l_i^i(\hat{r})(\mathbf{e}_i) \neq \hat{r}_j$.*

If a situation satisfies incredibility, then there exists decision maker j's information that is not credible for decision maker i.

Definition 62 (Scheme Stability) *A situation satisfies* view stability, *if for any i in N, any $\mathbf{e}_i$, any σ in Σ_i, any k in N^σ, any j in $N^{k\sigma} \backslash \{k\}$, any $\hat{r}$, $f_j^{k\sigma} \circ \pi_j \circ l_k^\sigma(\hat{r})(\mathbf{e}_{k\sigma}) = \hat{r}_j$ implies $\pi_j \circ l_k^\sigma(\hat{r})(\mathbf{e}_{k\sigma}) = \mathbf{e}_{jk\sigma}$.*

Considering the case that $\sigma = i$ and $k = i$, we have particularly that if a situation satisfies scheme stability, then for any i in N, any $\mathbf{e}_i$, any j in $N^i \backslash \{i\}$, and any $\hat{r}$, where $\mathbf{e}_i = (e^i, \mathbf{e}^i)$ and $\mathbf{e}^i = (\mathbf{e}_{ji})_{j \in N^i}$, $f_j^i \circ \pi_j \circ l_i^i(\hat{r})(\mathbf{e}_i) = \hat{r}_j$ implies $\pi_j \circ l_i^i(\hat{r})(\mathbf{e}_i) = \mathbf{e}_{ji}$. Thus in such a situation decision maker i's perception $\mathbf{e}_{ji}$ of decision maker j's scheme of emotions do not changed when decision maker i believes the information $\hat{r}_j$ about decision maker j's rule. Moreover, if decision maker i thinks that decision maker k believes the information $\hat{r}_j$ about rule of decision maker j, then decision maker i's perception $\mathbf{e}_{jki}$ of decision maker k's perception of decision maker j's scheme of emotions is not modified.

Definition 63 (Perception Stability) *A situation is said to satisfy* perception stability, *if for any i in N, any $\mathbf{e}_i$, any σ in Σ_i, k in N^σ, and any $\hat{r}$, if either for any j in $N^{k\sigma} \backslash \{k\}$, $f_j^{k\sigma} \circ \pi_j \circ l_k^\sigma(\hat{r})(\mathbf{e}_{k\sigma}) = \hat{r}_j$, or for any j in $N^{k\sigma} \backslash \{k\}$, $f_j^{k\sigma} \circ \pi_j \circ l_k^\sigma(\hat{r})(\mathbf{e}_{k\sigma}) \neq \hat{r}_j$, then $\pi_k \circ h_k^\sigma(\hat{r})(\mathbf{e}_{k\sigma}) = e_k^\sigma$.*

Considering the case that $\sigma = i$ and $k = i$, we have particularly that if a situation satisfies perception stability, for any i in N, any $\mathbf{e}_i$, and any $\hat{r}$, if either for any j in $N^i \backslash \{i\}$, $f_j^i \circ \pi_j \circ l_i^i(\hat{r})(\mathbf{e}_i) = \hat{r}_j$, or for any j in $N^i \backslash \{i\}$, $f_j^i \circ \pi_j \circ l_i^i(\hat{r})(\mathbf{e}_i) \neq \hat{r}_j$, then $\pi_i \circ h_i^i(\hat{r})(\mathbf{e}_i) = e_i^i$. Thus, when a situation satisfies perception stability, decision maker i's emotions is not changed if either decision maker i believes all information $\hat{r}_{-i}$ about the other decision makers' rule, or decision i doubts all of the information.

Definition 64 (Confidence from Stability) *A situation is said to satisfy* confidence from stability, *if for any i in N, any $\mathbf{e}_i$, any σ in Σ_i, any k in N^σ, and any $\hat{r}$, $\pi_k \circ h_k^\sigma(\hat{r})(\mathbf{e}_{k\sigma}) = e_k^\sigma$, implies that for any j in $N^{k\sigma} \backslash \{k\}$, $f_j^{k\sigma} \circ \pi_j \circ l_k^\sigma(\hat{r})(\mathbf{e}_{k\sigma}) = \hat{r}_j$.*

Supposing the case that $\sigma = i$ and $k = i$, we have that if a situation satisfies confidence from stability, then for any i in N, any $\mathbf{e}_i$, and any $\hat{r}$, $\pi_i \circ h_i^i(\hat{r})(\mathbf{e}_i) = e_i^i$, implies that for any j in $N^i \backslash \{i\}$, $f_j^i \circ \pi_j \circ l_i^i(\hat{r})(\mathbf{e}_i) = \hat{r}_j$. Thus, in this situation decision maker i believe the information $\hat{r}_j$ about decision maker j's rule, when no changes in decision maker i's emotion are caused by the information $\hat{r}$. Moreover, it is satisfied that if decision maker i believes that decision maker k's emotion is not changed by the information $\hat{r}$, then decision maker i thinks that decision maker k believes the information $\hat{r}_j$ about decision maker j's rule.

Definition 65 (Inside Commonality of Rules) *A situation if said to satisfy* inside commonality of rules, *if decision makers' scheme of rules is always in inside common knowledge, that is, for any i in N, any $\mathbf{e}_i$, any σ in Σ_i, and any j in N^σ, it is satisfied that $f_j^\sigma(\mathbf{e}_{j\sigma}) = f_j^i(\mathbf{e}_i)$.*

Considering the case that $\sigma = ki$ and $j = i$, we have that if a situation satisfies inside commonality of rules, then decision maker i believes, not only before the exchanges of information $\hat{r}$ but also after the exchanges, that decision maker k perceives the correct rule of decision maker i.

The following conditions are technical, but are required for verifying properties about inside and outside strategyproofness.

Definition 66 (0th Degree Equivalence relation) *For any i in N, any σ in Σ_i, and any j in N^σ, the* 0th degree equivalence relation $\sim^0_{j\sigma}$ *on $j\sigma$'s schemes of emotions is defined as follows: for any $j\sigma$'s schemes, $\mathbf{e}_{j\sigma}$ and $\mathbf{e}'_{j\sigma}$, of emotions,*

$$\mathbf{e}_{j\sigma} \sim^0_{j\sigma} \mathbf{e}'_{j\sigma} \Leftrightarrow e^{j\sigma}_j = e'^{j\sigma}_j.$$

Definition 67 (Well-Definedness of Schemes of Decision Functions) *For any i in N, decision maker i's scheme $\mathbf{f}_i$ of decision functions is said to be* well-defined in 0th degree, *if for any σ in Σ_i and any j in N^σ, f^σ_j satisfies that for any $j\sigma$'s schemes, $\mathbf{e}_{j\sigma}$ and $\mathbf{e}'_{j\sigma}$, of emotions, if $\mathbf{e}_{j\sigma} \sim^0_{j\sigma} \mathbf{e}'_{j\sigma}$, then $f^\sigma_j(\mathbf{e}_{j\sigma}) = f_j(\mathbf{e}'_{j\sigma})$.*

If decision maker i's scheme of decision functions is well-defined in 0th degree, then we have particularly that decision maker i's rule is determined on only the emotions e^i_i of decision maker i.

Definition 68 (1st Degree Equivalence relation) *For any i in N, any σ in Σ_i, and any j in N^σ, the* 1st degree equivalence relation $\sim^1_{j\sigma}$ *on $j\sigma$'s schemes of emotions is defined as follows: for any $j\sigma$'s schemes, $\mathbf{e}_{j\sigma}$ and $\mathbf{e}'_{j\sigma}$, of emotions,*

$$\mathbf{e}_{j\sigma} \sim^1_{j\sigma} \mathbf{e}'_{j\sigma} \Leftrightarrow e^{j\sigma} = e'^{j\sigma}.$$

Definition 69 (Injectivity of Schemes of Decision Functions) *For any i in N, decision maker i's scheme $\mathbf{f}_i$ of decision functions is said to be* injective in 1st degree, *if for any σ in Σ_i, and any j in N^σ, f^σ_j satisfies that for any $j\sigma$'s schemes, $\mathbf{e}_{j\sigma}$ and $\mathbf{e}'_{j\sigma}$, of emotions, if $f^\sigma_j(\mathbf{e}_{j\sigma}) = f_j(\mathbf{e}'_{j\sigma})$, then $\mathbf{e}_{j\sigma} \sim^1_{j\sigma} \mathbf{e}'_{j\sigma}$.*

If decision maker i's scheme of decision functions is injective in 1st degree, then we have particularly that if two distinct decision maker i's schemes, $\mathbf{e}_i$ and $\mathbf{e}'_i$, of emotions induces the same rule of decision maker i, then decision maker i's perceptions, e^i and e'^i, of emotions coincide.

Now, we can provide sufficient conditions for a situation to satisfy *inside strategyproofness* and *outside strategyproofness*, respectively.

Proposition 19 (A Condition for Inside Strategyproofness) *Consider a situation that satisfies the following conditions:*

1. *inside commonality of rules,*
2. *scheme stability,*
   and
3. *confidence from stability.*

Then, if for any $i \in N$, decision maker i's scheme $\mathbf{f}_i$ of decision functions is injective in 1st degree, then the situation satisfies inside strategyproofness.

(proof) For any i in N, any $\mathbf{e}_i$, and any $\hat{r}$, assume that $f_i^i \circ g_i^i(\hat{r})(\mathbf{e}_i) = f_i^i(\mathbf{e}_i)$. Because decision maker i's scheme $\mathbf{f}_i$ of decision functions is injective in 1st degree, it is satisfied that $g_i^i(\hat{r})(\mathbf{e}_i) \sim_i^1 (\mathbf{e}_i)$. By the definition of the equivalence relation $\sim_i^1$, we see that $h_i^i(\hat{r})(\mathbf{e}_i) = e^i$. On the one hand, because we have that $\pi_i \circ h_i^i(\hat{r})(\mathbf{e}_i) = e_i^i$, it is satisfied that for any j in $N^i \backslash \{i\}$, $f_j^i \circ p_j \circ l_i^i(\hat{r})(\mathbf{e}_i) = \hat{r}_j$, by confidence from stability. By scheme stability, moreover, it is satisfied that $\pi_j \circ l_i^i(\hat{r})(\mathbf{e}_i) = (\mathbf{e})_{ji})$ for any j in $N^i \backslash \{i\}$. On the other hand, because we have that $\pi_j \circ h_i^i(\hat{r})(\mathbf{e}_i) = e_j^i$ for any j in $N^i \backslash \{i\}$, it is satisfied that for any j in $N^i \backslash \{i\}$, $\pi_j \circ h_j^i(\hat{r}) \circ \pi_j \circ l_i^i(r\hat{}_N)(\mathbf{e}_i) = e_j^i$ identifying e_j^{ji} with e_j^i. Thus we have that for any j in $N^i \backslash \{i\}$, $\pi_j \circ h_j^i(\hat{r})(\mathbf{e}_{ji}) = e_j^i$. By confidence from stability, for any j in $N^i \backslash \{i\}$ and any k in $N^{ji} \backslash \{j\}$, $f_k^{ji} \circ \pi_k \circ l_j^i(\hat{r})(\mathbf{e}_{ji}) = \hat{r}_k$. By scheme stability, moreover, $\pi_k \circ l_j^i(\hat{r})(\mathbf{e}_{ji}) = (\mathbf{e}_{kji})$. Thus, for any j in $N^i \backslash \{i\}$ and any k in $N^{ji} \backslash \{j\}$, $f_k^{ji}(\mathbf{e}_{kji}) = \hat{r}_k$. Especially, for any j in $N^i \backslash \{i\}$, $f_i^{ji}(\mathbf{e}_{iji}) = \hat{r}_i$. Then, by inside commonality of rules, $f_i^i(\mathbf{e}_i) = \hat{r}_i$. For any i in N, therefore, if $f_i^i(\mathbf{e}_i) \neq \hat{r}_i$, then $f_i^i \circ g_i^i(\hat{r})(\mathbf{e}_i) \neq f_i^i(\mathbf{e}_i)$. ■

Proposition 20 (A Condition for Outside Strategyproofness) *Suppose a situation that satisfies the following conditions:*

1. *separability of beliefs,*
2. *extremeness of beliefs,*
3. *incredibility,*
   and
4. *perception stability.*

Then, if for any i in N, decision maker i's scheme $\mathbf{f}_i$ of decision functions is well-defined in 0th degree, then the situation satisfies outside strategyproofness.

(proof) For any i in N, any $\mathbf{e}_i$, any j in $N^i \backslash \{i\}$, and any $\hat{r}_i$, where $\mathbf{e}_i = (e^i, \mathbf{e}^i)$ and $\mathbf{e}^i = (\mathbf{e}_{ji})_{j \in N^i}$, we have that either $f_i^{ji} \circ \pi_i \circ l_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) = \hat{r}_i$ for each $\hat{r}_{-i}$, or $f_i^{ji} \circ \pi_i \circ l_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) \neq \hat{r}_i$ for each $\hat{r}_{-i}$ by separability of Beliefs.

1. In the case that $f_i^{ji} \circ \pi_i \circ l_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) = \hat{r}_i$ for each $\hat{r}_{-i}$. For any k in $N^{ji} \backslash \{i, j\}$, let $\hat{r}_k$ be $f_k^{ji}(\mathbf{e}_{kji})$. Because for any r_{-k}, $f_k^{ji} \circ p_k \circ l_j^i(\hat{r}_k, r_{-k})(\mathbf{e}_{ji}) = \hat{r}_k$ by extremeness of beliefs, we have that for any k in $N^{ji} \backslash \{j\}$, $f_k^{ji} \circ p_k \circ l_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) = \hat{r}_k$. By perception stability, we have that $h_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) = e_j^i$. Then, we have that $g_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) \sim_{ji}^0 \mathbf{e}_{ji}$, because $g_j^i(\hat{r}) = (h_j^i(\hat{r}), (g_k^{ji}(\hat{r}))_{k \in N^{ji} \backslash \{j\}} \circ l_j^i(\hat{r}))$ for any $\hat{r}$. Thus, by the assumption that decision maker j's scheme of decision functions is well-defined in 0th degree, we see that $f_j^i \circ g_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) = f_j^i(\mathbf{e}_{ji})$.

2. In the case that $f_i^{ji} \circ \pi_i \circ l_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) \neq \hat{r}_i$ for each $\hat{r}_{-i}$. For any k in $N^{ji} \backslash \{i, j\}$, there exists $\hat{r}_k$ such that for any r_{-k}, $f_k^{ji} \circ p_k \circ l_j^i(\hat{r}_k, r_{-k})(\mathbf{e}_{ji}) \neq \hat{r}_k$ by incredibility. Then for any k in $N^{ji} \backslash \{j\}$, $f_k^{ji} \circ p_k \circ l_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) \neq \hat{r}_k$. Thus, by perception stability, we have that $h_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) = e_j^i$. Then, similarly to the previous case, we see that $f_j^i \circ g_j^i(\hat{r}_i, \hat{r}_{-i})(\mathbf{e}_{ji}) = f_j^i(eji)$. ■

Treating decision makers with preferences independent of the others', Brams [9] shows that a profitable outcome for a deceiver to the outcome realized through honest offers can be achieved by deception. On the contrary, the propositions above imply that strategic information exchanges cannot be effective, and thus manipulation of actions of decision makers by deception can be impossible, when decision makers' preferences are interdependent. Interdependence of decision makers' preferences crucially affect decision makers' behaviors in a situation of decision making.

4.2.2 Credibility and Complete Credibility of Information

Exchanges of information about decision makers' actions in a competitive situation and *credibility* of the information are important for decision makers to make proper decisions in the situation. Because we assume that the actions depend on the emotions of the decision makers, and that the decision makers know the assumption about the dependence of the actions on the emotions, the credibility of information about the actions is also affected by the emotions. Actually, in 'soft' game theory [44], positive and negative emotions are considered to be able to make unwilling promises and unwilling threats credible, respectively. The concept of credibility of information in [44], however, is defined under completeness of information in terms of the emotions. In order to examine relations between the emotions and the credibility of information about decision makers' actions, we should deal with the credibility of information under incompleteness of information in terms of the emotions. We employ the framework given in this paper for describing interperception by decision makers in terms of emotions, and two concepts, *credibility* and *complete credibility* of information, to analyze relations between decision makers' emotions and credibility of information about decision makers' actions. *Credibility* has already defined in a preceding chapter in this paper, and *complete credibility* is obtained applying the idea of credibility to each degree of perceptions of decision makers.

Consider the set N of all decision makers in a situation, and decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers. For any i in N, moreover, suppose decision maker i's schemes, $\mathbf{C}_i$, $\mathbf{e}_i$, $\mathbf{P}_i$, and $\mathbf{r}_i$, of base competitions, emotions, preferences, and rules, respectively. Given string σ in Σ_i, we can define *rational, helping,* and *aggressive* rules for σ. Consider decision maker j in N^σ.

Definition 70 (Rational Rules for Strings) *For any s_{-j} in $S^{j\sigma}_{-j}$, if $j\sigma$'s perception $r'^{j\sigma}_j$ of the rule of decision maker j satisfies that $(r'^{j\sigma}_j(s_{-j}), s_{-j}) \not{P}^{j\sigma}_j (s_j, s_{-j})$ for any s_j in $S^{j\sigma}_j$, then $r'^{j\sigma}_j$ is decision maker j's* rational rule *for σ at s_{-j}.*

Definition 71 (Helping rules for Strings) *Consider decision maker k in $N^{j\sigma}$. For any s_{-j} in $S^{j\sigma}_{-j}$, if $j\sigma$'s perception $r'^{j\sigma}_j$ of rule of decision maker j satisfies that $(s^{*j\sigma}_j, s_{-j})\ P^{j\sigma}_k\ (r'^{j\sigma}_j(s_{-j}), s_{-j})$, then $r'^{j\sigma}_j$ is decision maker j's* helping rule *to decision maker k for σ at s_{-j}, where $s^{*j\sigma}_j$ satisfies that $(s^{*j\sigma}_j, s_{-j})\ \not{P}^{j\sigma}_j\ (s_j, s_{-j})$ for any s_j in $S^{j\sigma}_j$.*

Definition 72 (Aggressive Rules for Strings) *Consider decision maker k in $N^{j\sigma}$. For any s_{-j} in $S^{j\sigma}_{-j}$, if $j\sigma$'s perception $r'^{j\sigma}_j$ of rule of decision maker j satisfies that $(r'^{j\sigma}_j(s_{-j}), s_{-j})\ P^{j\sigma}_k\ (s^{*j\sigma}_j, s_{-j})$, then $r'^{j\sigma}_j$ is a decision maker j's* aggressive rule *to decision maker k for σ at s_{-j}, where $s^{*j\sigma}_j$ satisfies that $(s^{*j\sigma}_j, s_{-j})\ \not{P}^{j\sigma}_j\ (s_j, s_{-j})$ for any s_j in $S^{j\sigma}_j$.*

Considering the case that $\sigma = i$, we have the definitions of rational, helping, and aggressive rules for decision maker i given in a preceding chapter in this paper. As the definition of credible information for a decision maker in the chapter, we can also define credible information for a string of decision makers. For any i and j in N, information $\hat{r}_j$ about the rule of decision maker j is said to be *completely credible* for decision maker i, if $\hat{r}_j$ is credible for any string σ in Σ_i. Consider decision maker i and j in N and information $\hat{r}_j$ about the rule of decision maker j.

Definition 73 (Completely Credible Information) *For any σ in Σ_i and any s_{-j} in $S^{j\sigma}_{-j}$, information $\hat{r}_j$ is* credible *for σ at s_{-j}, if we have either*

1. *$\hat{r}_j$ is decision maker j's rational rule for σ at s_{-j},*

2. *there exists decision maker k in $N^{j\sigma}$ such that $e^{j\sigma}_j|_k = +$ and $\hat{r}_j$ is decision maker j's helping rule to decision maker k for σ at s_{-j},*
   or

3. *there exists decision maker k in $N^{j\sigma}$ such that $e^{j\sigma}_j|_k = -$ and $\hat{r}_j$ is decision maker j's aggressive rule to decision maker k for σ at s_{-j}.*

If for any σ in Σ_i and any s_{-j} in $S^{j\sigma}_{-j}$, $\hat{r}_j$ is credible for σ at s_{-j}, then $\hat{r}_j$ is said to be completely credible *for decision maker i.*

Considering the case that $\sigma = i$, we have the definition of credible information for decision maker i. Then we examine relations among credibility, complete credibility, and interperception in terms of emotions. Assume that for any i in N, decision maker i's schemes, $\mathbf{C}_i$, $\mathbf{e}_i$, $\mathbf{P}_i$, and $\mathbf{r}_i$, of base competitions, emotions, preferences, and rules is in inside common knowledge, respectively. Then, we can provide a proposition about one of the relations.

Proposition 21 (Credibility and Complete Credibility) *For any i and j in N and information $\hat{r}_j$ about the rule of decision maker j, $\hat{r}_j$ is completely credible for decision maker i if and only if it is credible for decision maker i.*

(proof) From the definition of completely credible information, it is apparent that if $\hat{r}_j$ is completely credible for decision maker i, then it is credible for decision maker i. Oppositely, assume that $\hat{r}_j$ is credible for decision maker i. Then, for any s_{-j} in S^{ji}_{-j}, we have either

1. $(\hat{r}_j(s_{-j}), s_{-j}) \not{F}^{ji}_j (s_j, s_{-j})$ for any s_j in S^{ji}_j,

2. there exists decision maker k in N^{ji} such that $e^{ji}_j|_k = +$ and $(s^{*ji}_j, s_{-j}) P^{ji}_k (\hat{r}_j(s_{-j}), s_{-j})$,
   or

3. there exists decision maker k in N^{ji} such that $e^{ji}_j|_k = -$ and $(\hat{r}_j(s_{-j}), s_{-j}) P^{ji}_k (s^{*ji}_j, s_{-j})$,

where s^{*ji}_j satisfies that $(s^{*ji}_j, s_{-j}) \not{F}^{ji}_j (s_j, s_{-j})$ for any s_j in S^{ji}_j. Because decision maker i's schemes, $\mathbf{C}_i$, $\mathbf{e}_i$, $\mathbf{P}_i$, and $\mathbf{r}_i$, of base competitions, emotions, preferences, and rules is in inside common knowledge, respectively, it is satisfied that for any σ in Σ_i and any k in $N^{j\sigma}$, $e^{j\sigma}_j|_k = e^{ji}_j|_k$, $F^{j\sigma}_j = F^{ji}_j$, and $P^{j\sigma}_k = P^{ji}_k$. Thus, for any σ in Σ_i and any s_{-j} in $S^{j\sigma}_{-j} = S^{ji}_{-j}$, we have either

1. $\hat{r}_j$ is decision maker j's rational rule for σ at s_{-j},

2. there exists decision maker k in $N^{j\sigma}$ such that $e^{j\sigma}_j|_k = +$ and $\hat{r}_j$ is decision maker j's helping rule to decision maker k for σ at s_{-j},
   or

3. there exists decision maker k in $N^{j\sigma}$ such that $e^{j\sigma}_j|_k = -$ and $\hat{r}_j$ is decision maker j's aggressive rule to decision maker k for σ at s_{-j}.

Thus, $\hat{r}_j$ is completely credible for decision maker i. ■

This proposition implies that if decision maker i's schemes, $\mathbf{C}_i$, $\mathbf{e}_i$, $\mathbf{P}_i$, and $\mathbf{r}_i$, of base competitions, emotions, preferences, and rules is in inside common knowledge, respectively, then a piece of information about the rule of decision makers is credible for decision maker i if and only if it is completely credible for decision maker i. Thus under the condition that a decision maker has schemes in inside common knowledge, the concept of credibility is equivalent to that of complete credibility.

4.3 A Solution Concept Involving Emotions

Many solution concepts, such as Nash equilibrium [23] in game theory, metagame equilibrium [41] in metagame theory, sequential stability [100] in hypergame theory, and so on, have been proposed, but they are defined not referring to emotional aspects of decision makers. When we analyze a competitive situation in which emotional aspects of decision making is engaged, we need solution concepts that involve the emotional aspect in the definition. Considering a competitive situation as in Table 11, in which economic rationality of decision makers is common knowledge, we can have the case that information in terms of the way of selection is incomplete. In fact the situation has two Nash equilibria, (s_1, s_2) and (t_1, t_2), and we cannot specify which outcome will be selected as the final outcome, because economic rationality of the actions of one decision maker depends on the other's selection. The final outcome of the situation can be even (s_1, t_2) or (t_1, s_2) if each of the decision makers incorrectly infers the other's selection. In that case each decision maker changes his/her perceptions of the others' way of selection, because he/she can find that he/she has made an incorrect inference. On the other hand, if one of the Nash equilibria is achieved, decision makers do not change their perceptions, because each of them can see that he/she correctly infers the others' actions. Thus we can regard Nash equilibria as outcomes that do not induce changes of decision makers' perceptions. Applying this view of Nash equilibria to our frameworks, we can define a solution concept called *emotional equilibrium*. In the definition we see the final outcome of a situation as a piece of information about the rule of decision makers. An *emotional equilibrium* is defined as an outcome that does not induce any changes of decision makers' schemes. Moreover, we provide a sufficient condition for an outcome to be an emotional equilibrium.

		2: s_2	2: t_2
1	s_1	(4, 4)	(2, 1)
1	t_1	(1, 2)	(3, 3)

Table 11. A competitive situation with two Nash Equilibria.

4.3.1 Emotional Equilibria

Consider the set N of all decision makers in a situation and decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers. Suppose, moreover, decision makers' schemes, $\mathbf{C}$, $\mathbf{e}$, $\mathbf{P}$, $\mathbf{f}$, $\mathbf{r}$, and $\mathbf{d}$ of base competitions, emotions, preferences, rule functions, rules, and decision functions, respectively. For any i in N, decision maker i chooses just one action s_i^i in S_i^i employing his/her decision function d_i^i and his/her scheme $\mathbf{r}_i$ of rules, that is, $d_i^i(\mathbf{r}_i) = s_i^i$. He/she, at the same time, makes inferences about the others' selections. For any j in N^i, if it is satisfied that $d_j^{ji}(\mathbf{r}_{ji}) = s_j^{ji}$, decision maker i thinks that decision maker j will select s_j^{ji} in S_j^{ji}. Moreover, decision maker i makes inferences about the other decision makers' inferences about decision makers' selections. Generally, for any $\sigma = i_1 i_2 \cdots i_p$ in Σ_i and any j in N^σ, if $d_j^{j\sigma}(\mathbf{r}_{j\sigma}) = s_j^{j\sigma}$, then decision maker $i = i_p$ thinks that decision maker i_{p-1} thinks that $\ldots$ decision maker i_1 thinks that decision maker j will choose $s_j^{j\sigma}$. Thus gathering $s_j^{j\sigma}$ for any σ in Σ_i and any j in N^σ, we can form a scheme of actions, that is, $(s^\sigma)_{\sigma \in \Sigma_i}$. We call it decision maker i's *scheme of final selections*, denoted by $\mathbf{s}_i$. Moreover, a list $(\mathbf{s}_i)_{i \in N}$ of decision maker i's schemes $\mathbf{s}_i$ of final outcomes for each i in N is called decision makers' *scheme of final outcomes*, denoted by $\mathbf{s}$.

After each of the decision makers selects an action, the final outcome $s = (s_i^i)_{i \in N}$ is determined. Then each decision maker may modify his/her schemes so as to be consistent with the final outcome. If the final outcome s does not cause any modifications of decision makers' schemes, then the outcome is said to be an *emotional equilibrium*.

Definition 74 (Emotional Equilibria) *Consider for any i in N, decision maker i's schemes, $\mathbf{C}_i$, $\mathbf{e}_i$, $\mathbf{P}_i$, $\mathbf{f}_i$, $\mathbf{r}_i$, and $\mathbf{d}_i$ of base competitions, emotions, preferences, rule functions, rules, and decision functions, respectively. Suppose that the final outcome $s = (d_i^i(\mathbf{r}_i))_{i \in N}$ is given, and modifications of decision maker i's scheme $\mathbf{X}_i$ of component X in terms of s, where X can be C, e, P, f, r, or d, is expressed by decision maker i's scheme function $g_i^i(s)_X$ that indicates new scheme $\mathbf{X}_i'$ corresponding to old scheme $\mathbf{X}_i$, that is, $g_i^i(s)_X(\mathbf{X}_i) = \mathbf{X}_i'$. If we have that for any i in N and any X, $g_i^i(s)_X(\mathbf{X}_i) = \mathbf{X}_i$, then the final outcome s is an* emotional equilibrium.

Because an emotional equilibrium in a situation is the outcome that is chosen by decision makers as the final outcome and does not cause any modification of decision makers' schemes, we have that if the decision makers repeatedly participate in the same situation, and once they achieve an emotional equilibrium, then they will choose the equilibrium in every time after that.

4.3.2 Emotional Equilibrium and Correctness of Inferences

In order to give a sufficient condition for an outcome to be an emotional equilibrium, we provide some definitions of states of a situation. Consider outcome $s = (s_i)_{i \in N}$ is chosen by decision makers in a situation as the final outcome, that is, for any i in N, $d_i^i(\mathbf{r}_i) = s_i$. For any i in N, modifications of decision maker i's scheme $\mathbf{X}_i$ of X is expressed by decision maker i's schemes, $\mathbf{g}_i(s)_X$, $\mathbf{h}_i(s)_X$, and $\mathbf{l}_i(s)_X$, of scheme functions, perception functions, and view functions, in terms of s.

Definition 75 (Correct Inferences) *For any i in N, decision maker i has* correct inferences *if for any j in $N^i \backslash \{i\}$, $d_j^{ji}(\mathbf{r}_{ji}) = s_j$.*

If a decision maker has correct inferences, then the decision maker exactly infers the others' selections.

Definition 76 (Perception Stability from Coincidence) *For any i in N, decision maker i has* perception stability from coincidence *if we have that for any σ in Σ_i and any k in N^σ, if for any j in $N^{k\sigma}\backslash\{k\}$, $s_j = d_j^{jk\sigma}(\mathbf{r}_{jk\sigma})$, then for any X, $h_k^{k\sigma}(s)_X(\mathbf{X}_{k\sigma}) = X^{k\sigma}$.*

If decision maker i has perception stability from coincidence, then considering the case that $\sigma = i$ and $k = i$, we have particularly that if for any j in $N^i\backslash\{i\}$, $s_j = d_j^{ji}(\mathbf{r}_{ji})$, then decision maker i's perceptions of component X does not modified for any X.

Definition 77 (Scheme Stability from Coincidence) *For any i in N, decision maker i has* scheme stability from coincidence *if we have that for any σ in Σ_i, any k in N^σ, and any j in $N^{k\sigma}\backslash\{k\}$, if $s_j = d_j^{jk\sigma}(\mathbf{r}_{jk\sigma})$, then for any X, $\pi_j \circ l_k^{k\sigma}(s)_X(\mathbf{X}_{k\sigma}) = \mathbf{X}_{jk\sigma}$.*

If decision maker i has scheme stability from coincidence, then considering the case that $\sigma = i$ and $k = i$, we have particularly that for any j in $N^i\backslash\{i\}$, if $s_j = d_j^{ji}(\mathbf{r}_{ji})$, then decision maker i's perception of decision maker j's scheme of X is not modified for any X.

Employing these definitions, we provide a sufficient condition for the final outcome s of a situation to be an emotional equilibrium. Assume that decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers is in common knowledge, and is not modified through scheme functions.

Proposition 22 (A Condition for Emotional Equilibria) *If it is satisfied for any i in N that decision makers' scheme $\mathbf{s}$ of final selections is in inside common knowledge for decision maker i, and decision maker i has correct inferences, perception stability from coincidence, and scheme stability from coincidence, then the final outcome s of the situation is an emotional equilibrium.*

(proof) It is suffice to show that for any i in N, any component X, decision maker i's scheme $\mathbf{X}_i$ of X is not modified by the scheme function $g_i^i(s)_X$, that is, if $\mathbf{X}'_i$ denotes $g_i^i(s)_X(\mathbf{X}_i) =$, then it is enough to prove that for any σ in Σ_i, $X^\sigma = X'^\sigma$. Because decision makers' scheme $\mathbf{s}$ of final selections is in inside common knowledge for decision maker i, we have that for any σ in Σ_i, and any k in N^σ, and any j in $N^{k\sigma}\backslash\{k\}$, $s_j^{jk\sigma} = s_j^{ji}$. Decision maker i has correct inferences, thus we have that $d_j^{ji}(\mathbf{r}_{ji}) = s_j$. Since $s_j^{ji} = d_j^{ji}(\mathbf{r}_{ji})$, it is satisfied that $s_j^{jk\sigma} = s_j$. From the perception stability from coincidence, the scheme stability from coincidence, and the relations among $\mathbf{g}_i(s)_X$, $\mathbf{h}_i(s)_X$, and $\mathbf{l}_i(s)_X$ such that for any σ in Σ_i and any j in N^σ,

$$g_j^\sigma(s)_X = (h_j^\sigma(s)_X, (g_k^{j\sigma}(s)_X)_{k\in N^{j\sigma}\backslash\{j\}} \circ l_j^\sigma(s)_X),$$

then we have that for any σ in Σ_i, $X^\sigma = X'^\sigma$. In fact, because it is satisfied that $h_i^i(s)_X(\mathbf{X}_i) = X^i$ from the perception stability, we have that $X^i = X'^i$. Moreover, from the scheme stability, we have that $\pi_j \circ l_i^i(s)_X(\mathbf{X}_i) = \mathbf{X}_{ji}$, and $X'^{ji} = h_j^{ji}(s)_X \circ \pi_j \circ l_i^i(s)_X(\mathbf{X}_i) = h_j^{ji}(s)_X(\mathbf{X}_{ji})$. Because the perception stability implies that $h_j^{ji}(s)_X(\mathbf{X}_{ji}) = X^{ji}$. Thus we have that $X'^{ji} = X^{ji}$. Considering inductively, we have the result. ■

Intuitively, we think that if each decision maker in a situation correctly infers the others' selections, then decision makers' perceptions of the situation is not modified. This proposition shows, however, that in order that no modification of decision makers' perceptions is implied by the correct inferences, we need such conditions as inside commonality, perception stability, and scheme stability. Under the conditions, moreover, we have that if the final outcome is not an emotional equilibrium, then there exists a decision maker whose inferences of the others' selections are incorrect.

4.4 Deadlock in a Meeting

A meeting is often held to make a group decision. Since a meeting is often time-consuming, it is harmful for persons to attend the meeting. Delay of the decision affects the efficiency of the organization for which the meeting is held, because the organization cannot make actions before the decision is made. Moreover, the decision made not in time is often not optimal for the organization. Thus, the meeting should be carried out smoothly. We can often see several stages in a meeting. At the first stage, decision maker in the meeting have to learn the components of the meeting; which decision makers are participating in the meeting, which alternatives are available, decision makers' favors for the alternatives, the rule of final voting, and so on. At the next stage, the decision makers interact each other. Each of the decision makers tries to persuade others to agree on the alternative that is moat preferable for the decision maker. Some decision makers may compromise and others not. The final stage is devoted to the actual voting, and an alternative is chosen by using a given voting rule. Because one of the most time-consuming stages is the interaction stage, we should examine the stage in detail. In spite of the existence of formal models for the last stage [80], however, there are only verbal models for the interaction stage [85]. We propose a framework for formal treatment of interaction among decision makers in a meeting, particularly, persuasion and compromise by the decision makers.

The most time-consuming event in the interaction stage of a meeting is that the meeting reaches a state in which a disagreement cannot be settled. We call the state a *deadlock*, and formally describe it as a state that there is no alternative that seems to be chosen, but no interaction such as persuasion and compromise is occurred. Because we regard compromise lead from persuasion as helping behavior, the concept of emotions plays an important role in the formal definition of a deadlock. We also define the concept of a *completely deadlock*, applying the idea of deadlock to higher degrees of perceptions. The concept of complete deadlock implies that of deadlock. Thus we first give a sufficient condition for a meeting not to reach a complete deadlock, and then we provide a sufficient condition for a meeting not to reach a deadlock. We also show an example that implies the importance of stability of decision makers' schemes of emotions for smooth progression of a meeting.

4.4.1 Deadlock and Complete Deadlock

Consider the set N of all decision makers in a situation and decision makers' pair $(\mathbf{N}, \Sigma)$ of a scheme and a set of strings of decision makers. Suppose decision makers' schemes, $\mathbf{M}$, $\mathbf{e}$, $\mathbf{P}$, of base meetings, emotions, and preferences, respectively, where $\mathbf{M} = (\mathbf{M}_i)_{i \in N}$ and $\mathbf{M}_i = (\mathbf{N}_i, \mathbf{W}_i, \mathbf{A}_i, \mathbf{F}_i)$ for any i in N. Assume that $\mathbf{M}$ is in common knowledge.

Regarding functions of emotions, we have assumed that positive and negative emotions from a decision maker to another cause his/her helping and aggressive behaviors to another, respectively. Applying the assumptions to relations among persuasion, compromise, and emotions, we can assume that if decision maker i has positive emotions to decision maker j, then decision maker j's persuasion to decision maker i will succeed, and that if decision maker i has negative emotions to decision maker j, then decision maker j's persuasion to decision maker i will end in failure (Figure 8).

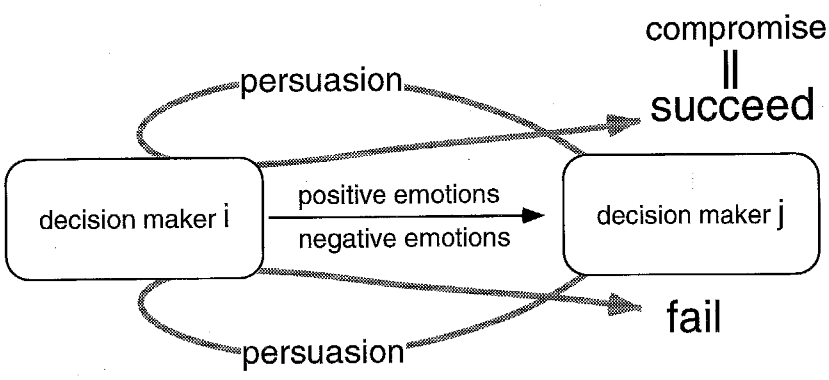

Figure 8. Relations among persuasion, compromise, and emotions.

If a decision maker thinks that persuasion to another will succeed, then the decision maker will make the persuasion, because we assume that if decision maker j's persuasion to decision maker i has succeeded, then decision maker i comes to prefer most the alternative that decision maker j prefers most. The changes of preferences of decision maker i is a special type of helping behaviors of decision maker i to decision maker j. In spite of the existence of other types, we adopt the type in this case, because we focus on meetings with majority rule, and other types of changes of preferences do not affect the outcome of the final voting.

We assume, moreover, that the relations among persuasion, compromise, and emotions are common knowledge among decision makers. Thus if decision maker i thinks that decision maker j has negative emotions to decision maker i, then decision maker i will not persuade decision maker j, since decision maker i thinks that decision maker j will not compromise. Because of the relations, a meeting sometimes reaches a situation in which a disagreement cannon be settled. We call the situation a *deadlock*. A deadlock is described as a situation in which each decision maker thinks that there is no alternative that seems to be selected in the voting stage under the current states of decision makers' preferences, and that there is no opponent who seems to cooperate with the decision maker under the current states of decision makers' emotions. Our formal definition of a deadlock requires some additional concepts such as *agreement* and *winning decision makers*. For any set A and any linear ordering P on A, $maxP$ denotes the highest alternative in terms of the ordering P.

Definition 78 (Agreement) *For any i in N, any σ in Σ_i, and any j and k in N^σ, decision maker j* agrees *with decision maker k for σ, if $maxP^\sigma_j = maxP^\sigma_k$.*

If decision maker i agrees with decision maker j, then they prefer the same alternative most. A decision maker whose the most preferable alternative seems to be selected at the final voting stage is called a *winning* decision maker.

Definition 79 (Winning Decision Makers) *For any i in N, any σ in Σ_i, and any j in N^σ, decision maker j is* winning *for σ, if there exists a winning coalition S in $W^{j\sigma}$ such that for any k in S, decision maker k agrees with decision maker j for $j\sigma$.*

In order to express a meeting at a deadlock formally, we should consider when a decision maker persuades another. Firstly, we can obviously say that if the decision maker is a winning decision maker, then it is not necessary for the decision maker to persuade others to change their preferences, and it is useless for others to persuade the decision maker, because it will end in failure. Secondly, it is also obvious that if two decision makers agree on an alternative, then persuasion will not occur between them. Thus when a decision maker tries to persuade another, there must be a disagreement between them. Thirdly, because of the assumptions about the function of emotions, we should have that if decision maker i persuades decision maker j, then decision maker i thinks that decision maker j has positive emotions to decision maker i. Using this consideration about persuasion and the definition of winning decision makers, we define a meeting at a deadlock.

Definition 80 (Meetings at a Deadlock) *A meeting is said to be at a* deadlock*, if for any i in N, it is satisfied that for any j in N^i, decision maker j is not winning for i, and decision maker j agrees with decision maker i for i and/or $e^i_j|_i = -$.*

Applying the idea of deadlock to higher degrees of perceptions, we can define the concept of *complete deadlock.* If a meeting is at a complete deadlock, then each decision maker in the meeting thinks that that the meeting is at a deadlock is common knowledge among decision makers.

Definition 81 (Meetings at a Complete Deadlock) *A meeting is said to be at a* complete deadlock*, if for any i in N, any σ in Σ_i, and any k in N^σ, it is satisfied that for any j in $N^{k\sigma}$, decision maker j is not winning for $k\sigma$, and decision maker j agrees with decision maker k for $k\sigma$ and/or $e^{k\sigma}_j|_k = -$.*

We should notice that a meeting at a complete deadlock is also at a deadlock. Thus to be at a complete deadlock is a stronger condition for a meeting than to be at a deadlock.

4.4.2 Impossibility to Reach a Deadlock or a Complete Deadlock

We need some concepts to provide propositions about a deadlock and a complete deadlock in a meeting. First, we define meetings with *majority rule.*

Definition 82 (Majority Rule) *A meeting is said to be with* majority rule*, if it is satisfied that for any i in N and any σ in Σ_i, S is included in W^σ if and only if $|S|$ is larger than $|N^\sigma|/2$.*

We treat only meetings with *odd number of decision makers* for simplicity.

Definition 83 (Odd Number of Decision Makers) *A meeting is said to be with* odd number of decision makers *if for any i in N and any σ in Σ_i, there exists a non-negative integer n such that $|N^\sigma| = 2n + 1$.*

For any i in N, any σ in Σ_i, and any j in N^σ, let D^σ_j be the set $\{l \in N^{j\sigma} \mid maxP^{j\sigma}_l \neq maxP^{j\sigma}_j$ and $e^{j\sigma}_l|_j = -\}$, called decision maker j's *depressing set* for σ. Considering the case that $\sigma = i$, if decision maker l is an element of decision maker j's depressing set for i, then decision maker i thinks that decision maker j believes that decision maker l does not agree with decision maker j, and decision maker i thinks that decision maker j believes that decision maker l has negative emotions toward decision maker j. That is, decision maker i thinks that decision maker j will not persuade decision maker l. In terms of depressing set, we have the following proposition.

Proposition 23 (Intersection Theorem under a Complete Deadlock) *If a meeting at a complete deadlock, then for any i in N, any σ in Σ_i, any j in N^σ, and any S in $W^{j\sigma}$, S intersects decision maker j's depressing set D^σ_j for σ.*

(proof) For any i in N, any σ in Σ_i, any j in N^{σ}, consider decision maker j's depressing set D_j^{σ} for σ, that is, $\{l \in N^{j\sigma} \mid maxP_l^{j\sigma} \neq maxP_j^{j\sigma}$ and $e_l^{j\sigma}|_j = -\}$. A meeting is at a complete deadlock if and only if it is satisfied that for any i in N, any σ in Σ_i, any k in N^{σ}, and any j in $N^{k\sigma}$, we have that for any S in $W^{jk\sigma}$, there exists decision maker l in S such that $maxP_l^{jk\sigma} \neq maxP_j^{jk\sigma}$, and that $maxP_j^{k\sigma} = maxP_k^{k\sigma}$ and/or $e_j^{k\sigma}|_k = -$. Supposing the case that $k = j$, we have that for any i in N, any σ in Σ_i, any j in N^{σ}, and any S in $W^{j\sigma}$, there exists decision maker l in S such that $maxP_l^{j\sigma} \neq maxP_j^{j\sigma}$. Because S in a subset of $N^{j\sigma}$, we have that l is an element of $N^{j\sigma}$. Thus from the assumption that the meeting is at a complete deadlock, we also have that for any i in N, any σ in Σ_i, any j in N^{σ}, $maxP_l^{j\sigma} = maxP_j^{j\sigma}$ and/or $e_l^{j\sigma}|_j = -$. Therefore, we have that for any i in N, any σ in Σ_i, any j in N^{σ}, and any S in $W^{j\sigma}$, there exists decision maker l in S such that $maxP_l^{j\sigma} \neq maxP_j^{j\sigma}$ and $e_l^{j\sigma}|_j = -$. Because decision maker l in S satisfies the condition to be an element of decision maker j's depressing set D_j^{σ} for σ, we have that S and D_j^{σ} are mutually intersect. ■

Now, we can provide a proposition that gives a sufficient condition for a meeting not to reach a complete deadlock.

Proposition 24 (Impossibility to Reach a Complete Deadlock) *If a meeting has majority rule, odd number of decision makers, and completely stable scheme of emotions, then the meeting is not at a complete deadlock.*

(proof) Assume that a meeting has majority rule, odd number of decision makers, and completely stable scheme of emotions. Since we are considering a meeting with decision makers' $\mathbf{N}$ scheme of decision makers in common knowledge, it is satisfied that for any i in N and any σ in $\Sigma_i = \Sigma_i^*$, $N^{\sigma} = N$, and from the assumption that the meeting has odd number of decision makers, we have that there exists a non negative integer n such that for any i in N and any σ in $\Sigma_i = \Sigma_i^*$, $|N^{\sigma}| = 2n + 1$. From the separation theorem for completely stable schemes of emotions, $N^{\sigma} = N$ can be partitioned into two subsets, N_1 and N_2 such that for any j and k in N^{σ}, $e_j^{\sigma}|_k = +$ if j and k are belong to the same subset, and $e_j^{\sigma}|_k = -$ if j and k are belong to different subsets. We may assume that $|N_1|$ is larger than n. Then, we have that N_1 is an element of W^{σ} for any i in N and any σ in $\Sigma_i = \Sigma_i^*$, because the meeting has decision makers' scheme $\mathbf{W}$ of winning coalitions in common knowledge and majority rule. For any l in N_1, $D_l^{\sigma} \cap N_1 = \phi$, because $D_l^{\sigma} \subseteq N_2$. Thus, the meeting is not at a complete deadlock from the intersection theorem under a complete deadlock. ■

Example 19 (Possibility to Reach a Complete Deadlock) *Consider a meeting with the set $N = \{1, 2, 3\}$ of all decision makers and the set $A = \{a, b, c\}$ of all alternatives. Assume that decision makers' schemes, $\mathbf{S}$, $\mathbf{A}$, $\mathbf{W}$, and $\mathbf{F}$, of decision makers, alternatives, winning coalitions, and favors, are in common knowledge, respectively. Assume, moreover, that for any i in N and any σ in Σ_i, it is satisfied that $maxP_1^{\sigma} = a$, $maxP_2^{\sigma} = b$, and $maxP_3^{\sigma} = c$, and that $e_j^{\sigma}|_k = -$ for any j and k in $N^{\sigma} = N$ such that $j \neq k$. If the meeting has majority rule, then the meeting is at a complete deadlock.*

Since $e_1^{\sigma}|_2 \times e_2^{1\sigma}|_3 \neq e_1^{\sigma}|_3$, the meeting does not have completely stable scheme of emotions. The proposition and example implies that complete stability of emotions is essential for a meeting not to reach a complete deadlock under the condition of majority rule and odd number of decision makers.

In order to verify a proposition that implies impossibility to reach a deadlock, we need an intersection theorem under a deadlock.

Proposition 25 (Intersection Theorem under a Deadlock) *If a meeting is at a deadlock, then for any i in N, and any S in W^i, S intersects decision maker i's depressing set D^i_i for i.*

(proof) For any i in N, consider decision maker j's depressing set D^i_i for i, that is, $\{l \in N^i \mid maxP^i_l \neq maxP^i_i$ and $e^i_l|_i = -\}$. Since the meeting is at a deadlock, it is satisfied that for any i in N, any j in N^i, and any S in W^{ji}, there exists k in S such that $maxP^{ji}_k \neq maxP^{ji}_j$. Particularly, considering the case that $j = i$, we have that for any i and any S in W^i, there exists l in S such that $maxP^i_l \neq maxP^i_i$. On the other hand, from the assumption that the meeting is at a deadlock, we have that $maxP^i_l = maxP^i_i$ and/or $e^i_l|_i = -$. Therefore, for any i in N and any S in W^i, there exists k in S such that $maxP^i_l \neq maxP^i_i$ and $e^i_l|_i = -$. Because k satisfies the condition to be an element of D^i_i, we have the result. ■

Now, we can provide a proposition that gives a sufficient condition for a meeting not to reach a deadlock.

Proposition 26 (Impossibility to Reach a Deadlock) *If a meeting has majority rule, odd number of decision makers, and stable scheme of emotions, then the meeting is not at a deadlock.*

(proof) Assume that a meeting has majority rule, odd number of decision makers, and stable scheme of emotions. Since we are considering a meeting with decision makers' $\mathbf{N}$ scheme of decision makers in common knowledge, it is satisfied that for any i in N and any σ in $\Sigma_i = \Sigma^*_i$, $N^\sigma = N$, and from the assumption that the meeting has odd number of decision makers, we have that there exists a non negative integer n such that for any i in N and any σ in $\Sigma_i = \Sigma^*_i$, $|N^\sigma| = 2n + 1$. From the separation theorem for stable schemes of emotions, $N^i = N$ can be partitioned into two subsets, N_1 and N_2 such that for any j and k in N^i, $e^i_j|_k = +$ if j and k are belong to the same subset, and $e^i_j|_k = -$ if j and k are belong to different subsets. We may assume that $|N_1|$ is larger than n. Then, we have that N_1 is an element of W^i for any i in N, because the meeting has decision makers' scheme $\mathbf{W}$ of winning coalitions in common knowledge and majority rule. For any l in N_1, $D^l_l \cap N_1 = \phi$, because $D^l_l \subseteq N_2$. Thus, the meeting is not at a deadlock from the intersection theorem under a deadlock. ■

Because *not* to be at a deadlock implies *not* to be at a complete deadlock, the proposition about impossibility to reach a deadlock refines the proposition about impossibility to reach a complete deadlock. That is, we have that if a meeting has majority rule odd number of decision makers, and stable scheme of emotions, then the meeting is not at a complete deadlock. Moreover, the example about possibility to reach a complete deadlock implies that a meeting that has majority rule, odd numbers of decision makers, and *not* stable scheme of emotions can reach a deadlock. These propositions and the example, therefore, indicate the importance of stability of emotions for smooth progression of a meeting.

Chapter 5

Conclusions

5.1 Summary

In this paper we dealt with situations of decision making with *interperception* by decision makers. In spite of the existence of the aspect of interperception by decision makers in the real world, the aspect has been treated only in limited extent in existing theories for decision making such as game theory [23, 73, 80], metagame theory [41], hypergame theory [5, 100], and 'soft' game theory [44]. Game theory and metagame theory have the assumptions of complete information and economic rationality, the assumptions that restrict treatment of the aspect of interperception. Although the assumption of complete information in terms of components of a situation is eliminated in hypergame theory, we still have the assumption of economic rationality in the theory. Because 'soft' game theory focuses on economically irrational behaviors of decision makers, we do not have the assumption of economic rationality in the theory, but completeness of information in terms of components of a situation, especially, in terms of *emotions*, is assumed. In this paper we eliminated both of the assumptions, and analyzed situations of decision making with interperception.

In order to make analyses of situations of decision making with interperception, we need frameworks for treating incompleteness of information and economically irrational behaviors, respectively. Even in hypergame theory and 'soft' game theory, however, only unsuitable frameworks have been provided. The framework for investigating the incompleteness in hypergame theory [5, 100] deal with decision makers' perceptions of the set of all decision makers inappropriately. 'Soft' game theory [44] does not have a formal framework for coping with the economical irrationality, thus arguments in the theory often have some extent of vagueness. In this paper we provided formal and strict frameworks for describing decision makers' misperception and emotional behaviors, respectively. We expressed decision makers' perceptions of a component of a situation by introducing the concept of *schemes*. Effects of emotions to decision making were described by the concept of *credibility* of information. Thanks to these frameworks we could give acceptable examinations of incompleteness of information and economic irrationality.

Moreover, for satisfactory analyses of situations of decision making with interperception, we provided a framework for dealing with exchanges of information and changes of perceptions. Existing theories such as hypergame theory [5, 100] and information structure [1, 24] do not supply satisfactory frameworks that meet our purposes in this paper. The former does not focus on changes of perceptions, and the latter requires a special hypothesis. Thus we provided a more general framework for examinations of the aspect of exchanges of information and changes of perceptions, where three types of functions, *scheme functions*, *perception functions*, and *view functions*, express transitions of perceptions. The framework makes it possible to investigate the aspect strictly.

Employing the frameworks above, we defined concepts such as *inside common knowledge*, *integration of perceptions*, and *stability of emotions*, the concepts that enable us to describe decision makers' perceptions of a component of a situation in more detail than existing concepts such as common knowledge [1, 24], composition of games [96], and balancedness [13, 35]. The concept of inside common knowledge focuses individual aspect of common knowledge, and expresses the states that a decision maker believes that a piece of information is common knowledge among all decision makers in a situation. Integration of perceptions provides us a formal method to construct a model of a whole situation that consists of many smaller models and interactions among them. We can apply the method to more general situations that those currently in use [82, 85]. The concept of stability of emotions is a formal and proper expression of the idea of balancedness that has been treated inappropriately. Thanks to the formal expression, we could employ several mathematical properties such as the separation theorem [13, 28] to our analyses. In this paper all these concepts encouraged our analyses of situations of decision making with interperception by decision makers. Actually, the concept of inside common knowledge and its generalizations were used to classify types of structures of decision makers' perceptions. The concept of *generation of schemes* was defined by using integration of perceptions, and we could categorize situations with interperceptions by decision makers by the concept. The concept of stability of emotions bore the concept of *complete stability of emotions*, and these could be applied to classification of structures of decision makers' perceptions of emotions.

In terms of these concepts, in this paper, we verified properties essential for analyses of situations of decision making with interperception by decision makers. For example, we showed that schemes in common knowledge is also in inside common knowledge, and this implies that if a decision maker in a situation does not think that a piece of information is *not* in common knowledge, then the information is *not* in common knowledge actually. Then, regarding integration and generation of schemes, we provided an example that means that situations generated from the same basis can reach different final outcomes. Moreover, in terms of stability and complete stability of emotions, we proved that stable and completely stable schemes of emotions exist, respectively, and that the equivalence of stability and complete stability under the condition of inside common knowledge. By using these properties, we could efficiently examine situations of decision making with interperception by decision makers.

In this paper analyses were done in terms of four topics. First, we examined relations among decision makers' perceptions of situations, decision makers' emotions, and final outcomes of situations. We showed that misperception can be occurred in a situation even if only honest and trustful decision makers are participating in the situation. We intuitively thought that *honest* and *confident* decision makers can correctly perceive each other, but this intuition was denied by the proposition. We proved, moreover, that when all decision makers prefer the same outcome, if each of the decision makers has positive emotions to the others, and believes that he/she is thought to select the action that is required to realize the outcome, then the outcome will be achieved. Thus we could justify Howard's [44] claim that if 'naive' decision makers unanimously intend to get the same outcome, then the outcome will be chosen. This proposition also implies that perceptions in inside common knowledge and positive emotions are needed to conclude the outcome that is hoped by all decision makers to be realized. Secondly, we treated information in situations of decision making. In this paper we focused information about rules of decision makers' behaviors. Then we proposed two types of strategyproofness, *inside strategyproofness* and *outside strategyproofness*, and provided sufficient conditions to satisfy the states, respectively. Because we have that a deceiver can induce more profitable outcome though deception than that concluded by sincere exchanges of information under the condition that decision makers' preferences are mutually independent [9], we could see that the aspect of interperception, especially that of interdependence of preferences, greatly affects the final outcome of a situation. We also dealt with the concept of *credibility* and *complete credibility* of information, and showed a proposition that implies the equivalence of credibility and complete credibility under the condition of inside common knowledge. We can utilize this proposition when we categorize types of information in situations of decision making. Thirdly, we proposed a solution concept, called *emotional equilibrium*, and showed a sufficient condition for an outcome to be an emotional equilibrium. Roughly, we defined an emotional equilibrium in a situation as an outcome that does not induce any changes of decision makers' perceptions of the situation, and the proposition implies importance of inside commonality of perception for an outcome to be an emotional equilibrium. Fourthly, we focused cooperative situations of decision making at a *deadlock*. We examined meetings with majority rule and odd number of decision makers, and proved that it is impossible for a meeting with stable scheme of emotions to reach a deadlock. We also provided an example that shows a meeting with *not* stable scheme of emotions can reach a deadlock, thus stability of emotions plays important roles in smooth progression of a meeting.

To sum up, properties shown in this paper implies great differences between situations under completeness of information and economic rationality and those under incompleteness of information and economic irrationality, and thus importance of analyses of situations of decision making with interperception by decision making. The frameworks and the concepts provided in this paper is useful for formal analyses of the situations.

5.2 Comments for Further Researches

We have further the topics concerning analyses of situations of decision making with interperception by decision makers.

- Other types of relations between profits and preferences. In this paper we employed the concept of *emotions* to describe a type of relations between profits and preferences. We can, however, consider other types of relations, particularly, the relations induced from three or four types of emotions [44, 49]. We should examine the relations generally. What kind of relations can be identified by three or four emotions? What properties should the relations have? Are the properties mutually consistent? We should answer these questions.

- Treatment of a group as a decision maker. We provided a sufficient condition for a group of decision makers in a competitive situation to achieve cooperation. If a group always obtain the outcome that they should get, then the group can be regarded as a decision maker as a whole. We should formally define the groups that can be seen as a decision maker, and examine their behaviors in a situation of decision making. They may misperceive the situation they are participating, and may act economically irrational. Thus we need to determine groups' perceptions, groups' emotions, and so on, in order to treat a group as a decision maker. Moreover, since groups can compose a group as a decision maker, we can think of a group of groups. Analyses of the hierarchy of decision makers are useful for understanding of behaviors of organizations.

- Situations between competition and cooperation. In this paper decision makers' cooperation induced from their emotions is occurred in a competitive situation. Thus the situation is not purely competitive. Similarly, we treated aggressive behaviors in cooperative situations. The situation, therefore, is not purely cooperative. Generally, we should analyze situations between purely competitive ones and purely cooperative ones. Such analyses will help us to understand behaviors of organizations.

- Distance of decision makers' preferences. We proposed a metric on the set of possible patterns of preferences of a decision maker. We should investigate the metric and the other metrics in more detail. Especially, we should examine the validity of the metrics in terms of the definitions of helping and aggressive behaviors of decision makers.

- Information in decision making. We treated in this paper only information about rules of decision makers' behaviors. We can, however, think of various types of information, and thus we should deal with exchanges of other types of information and changes of perceptions induced by the information. In the analyses we should describe relations between types of information and changes of perceptions.

- Convergence of perceptions. If decision makers participate a situation, and repeat exchanges of information, then their perceptions of the situation may change subsequently. We should examine the state that the perceptions will reach, and relations between the state and the emotional equilibrium. The examinations is useful to infer actions of decision makers in a situation.

- Classification of situations. We provided several concepts useful for classification of situation. The concepts, *inside common knowledge*, *integration*, and *generation* can be employed to categorize situations in terms of structures of decision makers' perceptions of the situations. We should proceed the classification for efficient analyses of the situations. Moreover, we should explore relations among the concepts.

- Strategyproofness. The property of strategyproofness is required for a situation to induce sincere exchanges of information. We proposed in this paper two types of strategyproofness. We should investigate other types as well. Moreover, we should provide necessary conditions to satisfy strategyproofness. We treated manipulation of information in only competitive situations, but manipulation of information can be occurred in cooperative situations. Thus, insincere information exchanges and strategyproofness in cooperative situations should be analyzed.

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