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Magnetohydrodynamics control of capillary Z-pinch discharge by using a triangular current pulse for lasing a H-like N recombination soft x-ray laser

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In expansion cooling phase of pinched nitrogen plasma generated by fast capillary discharge, it might be possible to realize lasing a Blamer α recombination SXRL, which requires a rapid cooling of nonequilibrium plasma. It is effective to decrease the discharge current rapidly in reducing the additional heating caused by the joule heating and the magnetic compression of plasma as quickly as possible. The shaping of discharge current waveform was demonstrated with a transmission line and its effect on expanding plasma dynamics were investigated through magnetohydrodynamics (MHD) calculation, and validity of the MHD calculation in the expansion phase was shown using the discharge photographs taken by using a high speed camera. As a result, strong radiation from the H-like N ion at the maximum pinch, which is in the current decay phase of the triangular current with peak amplitude of over 70 kA and pulse width of 60 ns, has been confirmed in x-ray photodiode signals at wavelength of less than 2.5 nm, to clarify the existence of the Lyman series and continuum of the H-like N ion. Without additional heating by the discharge current after the generation of the fully stripped nitrogen ions, it might be possible to generate the population inversion between the principal quantum number n=2 and 3. © 2010 American Institute of Physics.

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I. INTRODUCTION

Due to the high photon energy, the soft x-ray laser (SXRL) is capable of strongly interacting with a molecular and an atomic level in matter. It becomes a tool to elucidate the problem of solid state physics, nanotechnology, and life science. However, it is difficult to realize lasing a SXRL because the pumping power required to amplify a collisional excitation SXRL is inversely proportional to its wavelength and thus it requires a large system. There has been a great advance owing to the recent progress in the pulsed power technology. In 1985, using an ultrashort pulsed laser for collisional or recombination excitation, a SXRL at a wavelength of around 20 nm has been demonstrated in Lawrence Livermore Laboratory and Princeton Plasma Physics Laboratory, respectively.1,2

In 1994, utilizing a fast capillary Z-pinch discharge to generate a plasma column efficiently, Ne-like Ar SXRL at a wavelength of 46.9 nm was demonstrated by Rocca et al.3 In addition, suppressing the growth of magnetohydrodynamics (MHD) instability by use of a predischarge, Ne-like Ar laser was reproducibly observed.4,5 Moreover, the amplification of Ne-like Ar laser with higher coherence was realized using a 45-cm-long capillary6 and after then the demonstration of realizing a capillary discharge for the SXRL was successfully reported by several groups worldwide, which leads to the improvement of technical bases for the capillary discharge scheme.7–9 Concerning the generation of a shorter wavelength SXRL pumped by capillary discharge scheme, much effort has been made resulting in an amplification of a 13.2 nm Ni-like Cd line and a 18.2 nm H-like C VI recombination line.10–15 However, there is no demonstration of lasing SXRLs of these wavelengths up to saturation because it is still difficult to generate a highly uniform long plasma column with a several or a several tens cm length.

In this study, we investigate a possibility of lasing the H-like N Balmer α recombination SXRL at a wavelength of 13.4 nm which is relevant to the shorter wavelength laser pumped by the capillary discharge scheme. The wavelength of 13.4 nm is in an extreme ultraviolet range so that a Mo/Si multilayer mirror can be widely applied to the capillary discharge for SXRLs. The scheme of the possible H-like N recombination SXRL is shown in Fig. 1, by the population inversion between principal quantum number n=2 and 3, which can be generated by three body recombination in rapidly cooled plasma after the generation of fully stripped nitrogen ion in the maximum pinch phase. For the fast capillary discharge scheme, a hot and dense plasma column is generated by the implosion of current sheet initiated at the

N7+ + e− → N6+ + 3body recombination

Cascade decay
n=4

Continuum
n=3

Hα: 13.4 nm
n=2

Fast radiative decay,
Lyα: 2.5 nm
n=1

FIG. 1. Balmer α (Hα) line of H-like N.

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surface of the capillary wall toward the capillary axis. After the maximum pinch, a plasma column expands and an electron temperature is rapidly cooled down in 10–20 ns. Utilizing these pinch dynamics, it might be possible to obtain a small signal gain of over 1 cm⁻¹ according to MHD calculation results and experimental investigation has been done.¹⁶,¹⁷ Fast decay of discharge current after the maximum pinch instant causes the rapid decrease in the magnetic pressure on the pinched plasma and the rapid decrease in the additional joule heating that possibly leads to the increase in the laser gain.

In this paper, the rapid cooling of the pinched plasma is attempted by the pulse waveform shaping. At first, the effects of current waveform on the expansion dynamics were investigated through an MHD calculation. Then, the rapid radial plasma expansion was observed experimentally by using a high speed camera in order to validate the expectation with MHD model. As a result, strong radiation from the H-like N ions at the maximum pinch, which is in the current decay phase of the triangular current with peak amplitude of over 70 kA and pulse width of 60 ns, was confirmed in x-ray photodiode (XRD) signals at wavelength of less than 2.5 nm to clarify the existence of the Lyman (Ly) series and the continuum of the H-like N ion. Without additional heating by the discharge current after the generation of the fully stripped nitrogen ions, it might be possible to generate the population inversion between the principal quantum number \( n=2 \) and 3.

II. THEORETICAL ASSUMPTION OF THE RAPID CURRENT DECAY EFFECT ON GENERATION OF THE POPULATION INVERSION

In order to ionize a nitrogen ion up to NVIII in only several ns, it is required to generate a hot and dense plasma column with electron temperature of about 150 eV and electron number density of about \( n_e=1 \times 10^{20} \) cm⁻³. Assuming that MHD equilibrium is established at the maximum pinch instant with column radius \( r_p=200–300 \) μm, the current amplitude of over 50 kA is necessary to generate such a hot and dense pinched plasma by using the following Bennet’s relation ¹⁸

\[
I = \sqrt{8 \pi^2 r_p^2 n_e T_e k / \mu_0},
\]

(1)

Here, \( T_e \) is electron temperature, \( n_e \) is an electron number density, \( \mu_0 \) is the permeability of vacuum, \( I \) is a discharge current at the maximum pinch, and \( k \) is the Boltzmann constant.

In the recombining phase, in order to enhance the collisional ladderlike cascade decay rate to the quantum principal number \( n=3 \) from the higher excited levels, Byron’s boundary \( P_{\text{Byron}} \) (Refs. 19 and 20)

\[
P_{\text{Byron}} \approx \left( \frac{z^2 a_0}{3 k T_e} \right)^{1/2}
\]

(2)

is preferable to be higher than \( n=2 \) where \( z \) is a nuclear charge and \( a_0 \) is the Bohr radius. Such a condition could be obtained with an electron temperature of less than 50 eV, as shown in Fig. 2. In addition, in order to enhance the capture radiative cascade especially for the laser lower state \( n=2 \), the Griem’s boundary \( P_{\text{Griem}} \) (Ref. 21)

\[
P_{\text{Griem}} = \left[ (7 \times 10^{18}) \frac{z^7}{n_e} \frac{k T_e}{e^2 E_H} \right]^{2/17}
\]

(3)

is preferable to be 3–4, as also shown in Fig. 2. Where, \( E_H \) is the ionization energy of the hydrogen.

Assuming that after the maximum pinch, free expansion cooling progresses isentropically, and the average charge state of the nitrogen ions is nearly constant, electron temperature and electron number density are related with each other according to adiabatic relation as follows:

\[
n_{e, \text{expansion}} \approx n_{e, \text{pinch}} \left( \frac{T_{e, \text{expansion}}}{T_{e, \text{pinch}}} \right)^{1/(\gamma-1)},
\]

(4)

where specific heat ratio \( \gamma=5/3 \). With this relation, when the pinched H-like N plasma column having electron temperature of 150 eV and electron number density of \( 1 \times 10^{20} \) cm⁻³ adiabatically cools down to electron temperature of below 50 eV, electron number density of about \( 1 \times 10^{19} \) cm⁻³ is supposed to be obtained, which possibly realizes the generation of population inversion between \( n=2 \) and 3.

In actual plasma, radiative cooling of the plasma may decrease the electron temperature less than that of adiabatic free expansion. In this case, assuming that the radiative cooling through the radiative recombination from NVIII to NVII is considered to dominate and plasma is cooled down to 50 eV in about 10 ns, decrease of electron temperature is estimated as

\[
\Delta T_{e, \text{rad}} = \int_{t_{\text{pinch}}}^{t} \frac{N_{T_7}(t) n_e(t) R_{\text{rad}}(T_e(t), T_r(t)) dT_e(t) dt}{N_{T_7}^2(t) + n_e(t)}
\]

\[
\approx \frac{N_{T_7}(t_{100} \text{ [eV]}) n_e(t_{100} \text{ [eV]}) R_{\text{rad}}(100 \text{ [eV]}) (100 \text{ [eV]})}{N_{T_7}(t_{50} \text{ [eV]}) + n_e(t_{50} \text{ [eV]})}
\]

\[
\times 10 \text{ [ns]} \approx -10 \text{ to } -20 \text{ [eV]},
\]

(5)

where \( t \) is time, \( t_{\text{pinch}} \) is the time at the maximum pinch, \( t_{100} \text{ eV} \) and \( t_{50} \text{ eV} \) is the time when the plasma adiabatically cools down to 100 and 50 eV, duration time \( \Delta t \) from \( t_{\text{pinch}} \) to \( t_{50} \text{ eV} \) is set to 10 ns, \( N_{T_7}^{2+} \) is the number density of the fully stripped nitrogen, and \( R_{\text{rad}} \) is the Seaton’s radiative recombination rate coefficient given as ²²
The discharge system mainly consists of a water capacitor charging section and a capillary discharge section. The water capacitor charging system consists of a two-stage LC inversion generator, a 2:54 pulsed transformer and a 3 nF coaxial water capacitor. The discharge section consists of a relatively low inductance gap switch, a transmission line and a capillary load. Figure 3 shows a schematic diagram of the discharge section with the gap switch and the capillary. The gap switch is pressurized with SF\textsubscript{6} to decrease a gap distance to about 3 cm. Furthermore, several insulating barriers are used to prevent the surface discharge in the gap switch and the capillary section. Resulting inductances of the gap switch and the capillary section are about 75 and 50 nH, respectively. A water transmission line is inserted between the gap switch and the capillary as shown in Fig. 4. Shaping of current waveform was done by optimizing the length of transmission line. Thus, it is possible to control a delay time between a forward and a reflected voltage wave. It enables to superpose or separate two kinds of voltage waves for shaping the current waveform. The measured current waveforms were predicted by the wave model of distributed constant circuit. Waveforms shown in Fig. 5 are the calculation results for a transmission line with a length of 25–80 cm. The longer the transmission line is, the longer the pulse width of a current waveform generated is. For the transmission line of 25–40 cm, the triangular current waveform with higher peak current is generated because a forward wave and a reflected wave are superposed in phase at the center of the pulse. In this case, when the charging voltage of water capacitor is about 300 kV, the triangular current wave with amplitude of over 70 kA and pulse width of about 60–80 ns can be generated. In this paper, a transmission line with a line length of 25 or 40 cm and a surge impedance of 3 Ω was used.

In the experiment, an alumina capillary with a diameter of 3 mm and a length of 75 mm was used. Capillary plasma was preionized to generate axially uniform plasma in pinch phase by utilizing a RC discharge with initial current amplitude of 20 A and a decay time constant of 3 μs. The expanding plasma after the maximum pinch was observed from the axial end of the capillary by using a high density Balmer α laser.

III. EXPERIMENTAL SETUP

The discharge system mainly consists of a water capacitor charging section and a capillary discharge section. The water capacitor charging system consists of a two-stage LC inversion generator, a 2:54 pulsed transformer and a 3 nF coaxial water capacitor. The discharge section consists of a relatively low inductance gap switch, a transmission line and a capillary load. Figure 3 shows a schematic diagram of the discharge section with the gap switch and the capillary. The

\[ R_{rad} = 1 \times 10^{-14} \eta_j \frac{\Delta E}{T_e} \sqrt{T_e} \text{[cm}^3\text{s]} \].

(6)

In this assumption, ionization potential \( \Delta E = 667 \text{ eV} \) of NVII to NVIII and average electron temperature of 100 eV is used.

However, in a fast Z-pinch scheme, additional joule heating by the discharge current is considered to be significant. Increase in electron temperature is estimated as follows by assuming that discharge current of 50 kA continues during the expansion phase and using the average plasma radius of about 500 μm.

\[ \Delta E_{joule} = \frac{1}{2} \eta_j j^2 \Delta t \approx 10 \text{ to } 20 \text{ [eV]}. \]

(7)

where \( \eta_j \) is the conductivity of the Lorentz gas and \( j \) is the current density. Therefore, continuous discharge current after the maximum pinch may cause critical increase in the electron temperature, being higher than that of adiabatic free expansion, which prevents the generation of the population inversion. Furthermore, strong magnetic field of continuous discharge may also enhance the magnetic compression which has possibility to cause the double pinch, resulting in overheating of the plasma. In addition, rapid expansion in Z-pinch discharge leads to larger Doppler broadening, which may decrease the opacity of the plasma, resulting in the reduction in the radiation trapping of the laser lower level.

As a consequence, if the discharge current rapidly decreased to less than half value of peak amplitude after the maximum pinch, increase in electron temperature could be suppressed to several eV because the joule heating is proportional to current density squared. It possibly leads to the rapid cooling of the plasma and the electron temperature becomes lower than that obtained assuming the adiabatic expansion, resulting in the possible generation of population inversion for Balmer α laser.
speed camera, PI-MAX System of Princeton Instruments, with a gate time of 2 ns. Flaming photographs of expanding plasma were taken at visible wavelength and no filter was used.

Time evolution of the soft x-ray radiation from the nitrogen plasma was measured by using a XRD to confirm the maximum pinch time and to demonstrate the generation of the H-like N and the fully stripped nitrogen ions in the decay phase of the triangular current. The XRD with a gold photocathode, placed at a half meter from the end of the capillary, was biased to $-1 \text{kV}$ to avoid the space charge current limitation. To observe the radiation at wavelengths of less than 2.5 nm, a 2-$\mu$m-thick Al filter was used to measure the radiation of Ly $\alpha$ series and the continuum radiation from H-like N ion.

IV. EFFECTS OF CURRENT WAVEFORM SHAPING ON MHD DYNAMICS IN CALCULATION

To investigate the effects of current waveform shaping on rapid expansion cooling of the pinched plasma, one-dimensional (1D) MHD calculation has been done. Calculated current density, magnetic flux, axial velocity in the Lagrangian cells, electron temperature, and electron density for the triangular current pulse with a pulse width of about 50 ns and for a sinusoidal current with a half cycle time of about 100 ns are shown in Fig. 6, respectively. The triangular and the sinusoidal pulse waveforms were obtained by computing a telegraph equation with a 25-cm-long transmission line and without a transmission line but a circuit stray inductance of 250 nH, respectively. In the calculation, it is assumed that the initial electron temperature is 4 eV and nitrogen molecules are fully ionized to NII with number density of $3 \times 10^{17}$ cm$^{-3}$. In Fig. 6, the start time of a current flow is set to 0. The maximum pinch occurs at 40 ns in both cases, where current densities or magnetic flux densities are nearly the same in both cases. The magnetic flux density at the maximum pinch instant, which confines the plasma, is about 40 T. In addition, the current density profiles before the maximum pinch instant have similar trends in both cases. Consequently, the energies dissipated in the plasma before the maximum pinch are nearly the same in both cases, so that the maximum electron temperature and the maximum electron number density reach about 150 eV and $1 \times 10^{20}$ cm$^{-3}$, respectively, in both cases, which are required values in order to ionize the nitrogen ions up to NVIII. After the maximum pinch in the case of using a sinusoidal current, the peak current amplitude is maintained for about 10 ns. On the other hand, in the case of using a triangular current, the current and the current density decrease rapidly after the maximum pinch. As a result, there is a discrepancy of expansion velocity between two cases. In the case of sinusoidal one, maximum expansion velocity is $1.0 \times 10^5$ m/s and plasma tends to implode. However, in the case of triangular one, maximum expansion velocity is increased to $1.5 \times 10^5$ m/s. Consequently, in the off-axis region, at 10 and 15 ns after the maximum pinch, electron temperature cooled down to 40–60 eV and $<40$ eV in the case of sinusoidal one and triangular one with electron density of the order of $10^{18}$–$10^{19}$ cm$^{-3}$, respectively, as shown in Fig. 7. This is ascribed to the fast reduction in magnetic pressure used for plasma confinement which is caused by the rapid decay of a discharge current. Moreover, the rapid current decay reduces additional joule heating of plasma after the maximum pinch. These effects lead to the rapid cooling of the plasma column, which possibly leads the generation of the population inversion between $n=2–3$.

V. EXPERIMENTAL RESULTS

The discharge current waveforms experimentally obtained are shown in Fig. 8. Dashed line and gray line indicate the current waveform obtained by using a conventional discharge section with inductance of 250 nH [5] and the discharge section with inductance of 125 nH, as shown in Fig. 3, respectively. Discharge current obtained by inserting a transmission line of 40 cm long with a surge impedance of 3 $\Omega$ between the gap switch and the capillary load. As a result, a sinusoidal current waveform is shaped to a triangular one with a pulse width of about 60 ns and the amplitude is increased 1.5 times higher than that of current obtained using a load of 125 nH, which coincides with the calculation results.

To verify the effect of MHD control by current waveform shaping on plasma dynamics, the images of expansion plasma were taken by a high speed camera at lower current as shown in Fig. 9. Figure 10 shows the MHD calculation results of the electron temperature, the electron number density and the Lagrangian cell position, which were based on experimental conditions such as the discharge current waveform and the initial gas pressure. The discharge current used in the experiment was triangular one with an amplitude of 30 kA and a pulse width of about 80 ns which is generated by using a transmission line of 40 cm. In this experiment with peak current of 30 kA which is relatively low, a 40-cm-long transmission line was selected to synchronize the maximum pinch time with current decaying phase, which was considered to be optimum. The inner radius of capillary was 1.5 mm and an initial gas pressure of nitrogen molecule was 133 Pa. The reason of the asymmetry in 10 ns image on Fig. 9 is due to the experimental misalignment between the capillary axis and the high speed camera and relatively good reproducibility was confirmed in shot by shot. In Figs. 9 and 10, the origin of time is set to the instant when plasma starts to expand. In the experiment, a plasma column with 1.5 mm in radius is compressed to the maximum pinched plasma column with a radius of about 200–300 $\mu$m. After that, plasma starts to expand and reaches the capillary inner wall in about 15 ns. At $t=20$ ns, the ring shaped radiation image is observed due to the strong emission from the ablated capillary inner wall material, which indicates the end of the expansion phase. During the plasma expansion, it was confirmed that the plasma keeps its axial symmetry and rapidly expands toward a capillary inner wall rather uniformly without any significant MHD instability growth and double pinch. These experimental results have a similar trend to the 1D MHD calculation results which enables to estimate the dynamics of expansion plasma after the maximum pinch. In addition, the expanding plasma is considered to have a radially uniform
density profile that is similar to the MHD calculation result. In such a uniform plasma column, although it is difficult to anticipate the effect of wave guiding for efficient amplification of the x-ray without divergence, it may still be possible to amplify the soft x-ray radiation in the plasma column with radially uniform electron density profile. However, discrepancy of the pinch time by about 10–20 ns between the calculation and experimental results was observed. It is considered that the ablation from the capillary inner wall occurs and the ideal snow plow model cannot be adopted for the actual implosion plasma. In order to realize lasing at shorter wavelength, it is necessary to increase the discharge current over 50 kA. It may lead to the increase in the amount of ablated plasma from a capillary inner wall, resulting in a larger discrepancy between the experimental and the calculation results. Actually, with required peak current of over 50 kA, larger discrepancy between the experimental and the MHD calculation results occurs and significant decrease in reproducibility prevents the reliable observation of the pinch plasma. Therefore, ablation effect must be included in the MHD calculation in order to investigate the detailed pinch dynamics for higher discharge current. Increase in the amount of ablated material entered into plasma causes a decrease in an electron temperature at the maximum pinch instant so that the current amplitude required to the ionization of the nitrogen ions up to NVIII will be higher than that.

FIG. 6. (Color online) Effect of current waveforms on expansion phase of plasma in 1D MHD calculation. Left: for sinusoidal current with half cycle of about 100 ns, right: for triangular current with pulse duration of about 50 ns, capillary radius: 1.5 mm, initial number density: $3 \times 10^{17}$ cm$^{-3}$, initial electron temperature: 4 eV.
estimated by 1D MHD calculation. However, as far as dynamics of expansion plasma after the maximum pinch is concerned, it is expected that plasma expands axisymmetrically without a radial gradient of electron density near the axis in which amplification of soft x-ray radiation is possible. The abovementioned results show that it might be possible to control the dynamics of the expanding plasma by shaping the current waveform.

Time evolution of the soft x-ray radiation in XRD signals from the nitrogen plasma using a triangular current with peak amplitude of over 70 kA and pulse width of 60 ns are shown in Fig. 11. In Fig. 11, green line, which shows the radiation at wavelength of less than about 20 nm indicating radiation from the lower charge state of the ionizing nitrogen ions, gradually increases with the rise of the triangular current in the implosion phase of the pinch plasma. A few tens ns after the initiation of the radiation at longer wavelength, a sharp XRD signal at wavelength of less than 2.5 nm has been observed, which indicates the existence of the Ly series and continuum of the H-like N ions. Thus, existence of the fully stripped nitrogen ion at the maximum pinch required for recombination laser scheme is anticipated. After the strong peak of the XRD signal at shorter wavelength, it slowly decreases in several tens ns, which suggests the recombining plasma with the rapid current decay. The radiation at shorter wavelength is observed in the decay phase of the triangular

\[ \text{Te} \times 10^{-3} \]
triangular current with the generation of the H-like N and control the dynamics of the expanding plasma by using the abovementioned results show that it might be possible to inversion between principal quantum number expected, which possibly leads to the realization of population pinch where existence of the highly ionized NVIII is ex- suf ciently rapid triangular current decay, it is possible to zero. By utilizing such a rapidly expansion plasma caused by after the maximum pinch, triangular current decays to nearly to a half value of peak amplitude. In following 10–20 ns cooling rate of the plasma. Installing a transmission line be- comes nearly zero. In such the expansion phase after the ionization of the H-like N ions, the triangular current contin-

VI. CONCLUSION

joule heating and the magnetic compression could be maximum pinch with rapid current decrease, the additional comes nearly zero. In such the expansion phase after the values to rapidly decay to less than another half value and be-

References
