論文 / 著書情報
Article / Book Information

Title	Analytical Model of Carbon Dioxide Emission with Energy Payback Effect
Authors	motoaki utamura
Citation	Energy, Vol. 30, No. 11, pp. 2073-2088
Pub. date	2004, 9
DOI	http://dx.doi.org/10.1016/j.energy.2004.08.013
Note	このファイルは著者（最終）版です。 This file is author (final) version.
Creative Commons	See next page.

License

ANALYTICAL MODEL OF CARBON DIOXIDE EMISSION WITH ENERGY PAYBACK EFFECT

M. UTAMURA
Hitachi Engineering Co. Ltd.
3-2-1 Saiwai-cho, Hitachi-shi, 317-0073, Japan
E-mail: utasan@ijsnet.ne.jp

Abstract—An analytical model is proposed to account for carbon emission behaviour during replacement of power source from fossil fuel to renewable energy in which sustainability of energy supply is stressed. Logistic function of time is assumed for producing renewable power sources. Analyses show that energy payback time(EPT) should be much shorter than the doubling time of manufacturing cycle to secure adequate available energy during, as well as after, the replacement. A nuclear plant, small hydropower plant, wind power plant and photovoltaic cell are taken as representative candidates and investigated as options to replace fossil power until toward the end of this century. Nuclear or small hydropower plants are promising candidates but the photovoltaic cell needs further development efforts to reduce EPT and avoid energy expense after the replacement.

Carbon dioxide   Power source   Energy payback time   Available energy   Renewable energy   Photovoltaic cell

NOMENCLATURE

A = carbon sent to the atmosphere per unit electric energy supplied (kg/kWh)
a = normalized value of carbon emission ($\equiv A_m/A_0$) (-)
b = energy required to manufacture a new electric power source (kWh)
c = electric energy produced per year by new electric power sources (kWh/y)
$f(t)$ = the cumulative number of new power sources installed until time t (-)
$f'(t)$ = derivative of $f(t)$ with respect to t (1/y)
$G(\phi)$ = normalized energy production by new power sources per year (-)
$J(\phi)$ = normalized new energy introduction rate (-)
K = Fraction of the primary electric energy supply per year to be replaced by new power sources (kWh/y)
N = the number of new power source units equivalent to K ($\equiv K/cX(0)$) (-)
t = time (year) (y)
X = the number of active new power sources ($\equiv f(t) - f(t-\tau_1)$) (-)
ΔX = increment of the number of active new power sources after reference time ($t=0$) ($\equiv X - X(0)$) (-)

Greek letters

α = coefficient of availability (COA) ($\equiv 1 - \beta$) (-)
β = dimensionless energy payback time (DEPT) ($\equiv \varepsilon / \tau_1$) (-)
γ = dimensionless doubling time ($\equiv \tau_2 / \tau_1 \ln 2$) (-)
δ = dimensionless form of τ_f ($\equiv \tau_f / \tau_1$) (-)
ε = energy payback time (EPT) ($\equiv$b/c) (y)
η = non electric energy consumption ratio (-)
σ = energy production ratio (-)
τ_1 = life time of a new power source (y)
τ_2 = doubling time in production of new power sources (y)
ϕ = dimensionless time t ($\equiv t/\tau_1$) (-)

Suffices

o = at reference time ($t=0$)
m = mixed state of power sources
f = state when conventional power source is fully replaced by new ones

INTRODUCTION

The global warming rate is rising more rapidly than ever observed in the past. According to the climate model simulation by IPPC [1] this warming cannot be accounted for without the increase of the concentration of carbon dioxide (CO_2) in the atmosphere. In order to stabilize the CO_2 concentration in the air within an allowable limit at the beginning of the next century, a drastically low carbon future is envisaged. For example, the DNE21 model [2] warned annual carbon emissions should start decreasing in 2040 and eventual CO_2 emission cuts of more than half of today's level should be reached. To achieve the goal, efforts should be focused on the exploitation of carbon free renewable energy. Its energy density, however, is generally low and it thus consumes energy as well as requiring a bigger investment per unit of energy produced than that of fossil fuel.

It is common to evaluate initial cost and running cost to judge whether a new power generation system merits introduction. Energy balance considerations, however, are much more fundamental to determine if a certain system will be an effective measure to reduce CO_2 emissions. In fact, the introduction of renewable energy is not carbon free when it is undergoing substitution for fossil fuel because its manufacturing consumes fossil fuel. Not much work has been done so far on a systematic approach to macroscopic energy systems based on energy balance. Both energy payback time and life cycle time of a certain power generation system are key concepts to determine if introduction of the system is proper.

A simple analytical model was proposed here to predict carbon emissions sent into the atmosphere as a function of time in the course of the introduction of renewable energy. Also, based on calculation results, restrictions regarding energy payback time and life cycle time were discussed for photovoltaic cell.

SUSTAINABILITY OF ENERGY SUPPLY

Steady State Conditions

Rebirth scenarios in general illustrate a future in which global primary energy supply becomes saturated while end demand increases depending on population growth.

Some fundamental inequalities are discussed below for sustainability of the electric power supply.

(1) Effectiveness of power sources

It is apparent that in any meaningful electric power source, energy required to manufacture itself must not exceed total energy produced by itself in its life. This condition leads to $b < c\,\tau_1$ which is reduced to energy payback time ε

$$\varepsilon / \tau_1 < 1. \tag{1}$$

(2) Retention of a given energy supply K

Suppose a group of electric power sources that continue to supply an annual constant energy K where successive dieback and reproduction occur. Then energy requirement similar to the above holds for the group

$Nb / \tau_1 < K$ reduces again to Eq. (1). Thus, Eq. (1) is a necessary condition to sustain the energy system well.

(3) Availability consideration

Left hand side of Eq. (1) is dimensionless energy payback time (DEPT) and is denoted by β . Here, the coefficient of availability(COA) α is defined as the ratio of net available energy produced in life time (total energy produced minus production energy) to the total energy produced. Then, from the definition α is related to β as follows,

$$\alpha = 1 - \beta . \tag{2}$$

Dynamic Conditions

In order to be able to replace a conventional power source by a new one, the amount of electricity generated by an active new power source per year must exceed the energy required to manufacture itself per year. Considering death as well as birth of power source at any time there is

$$cX \geqq bf'(t) \tag{3}$$

where

$$X = f(t) - f(t - \tau_1) \tag{4}$$

in which $t = 0$ denotes reference time(year).

It is worth noting that if $f(t)$ is a linear function of t, no active new power sources increase in number in $t > 0$. With the assumption that $f(t)$ is an exponential type function being $2^{t/\tau_2}$, an inequality in dimensionless form is obtained

$$\gamma(1 - e^{-1/\gamma}) \geqq \beta \tag{5}$$

where

$$\gamma = \tau_2/\tau_1 \ln 2. \tag{6}$$

It is seen dimensionless doubling time γ has some minimum value as function of dimensionless EPT β to generate available energy.

This restriction guarantees the condition that replacement of the power source, say from fossil fuel to renewable energy should take place. If $\tau_1 >> \tau_2$, the bracketed term on the left hand side of Eq. (5) decreases to unity. Equations (2) and (5) are useful to select an appropriate candidate among various carbon free renewable energies and establish their production schedule.

Figure 1. shows DEPT of various renewable power sources and installation cost normalized by that of coal fired energy derived from the literature [3] in comparison with fossil fired current power source which covers 95% of the primary energy supply of the world at present. It is interesting to note that energy payback time is relatively larger than cost in renewable energies while conventional power sources have much the same except for LNG fired. This implies energy payback consideration cannot be done using cost which has been taken for granted to be valid so far. For example, DEPT of photovoltaic cell is 0.37 from Figure 1. and then Eq. (2) yields $\alpha = 0.63$.

This means maximum achievable COA is 0.63 which is much smaller than that of fossil power source.

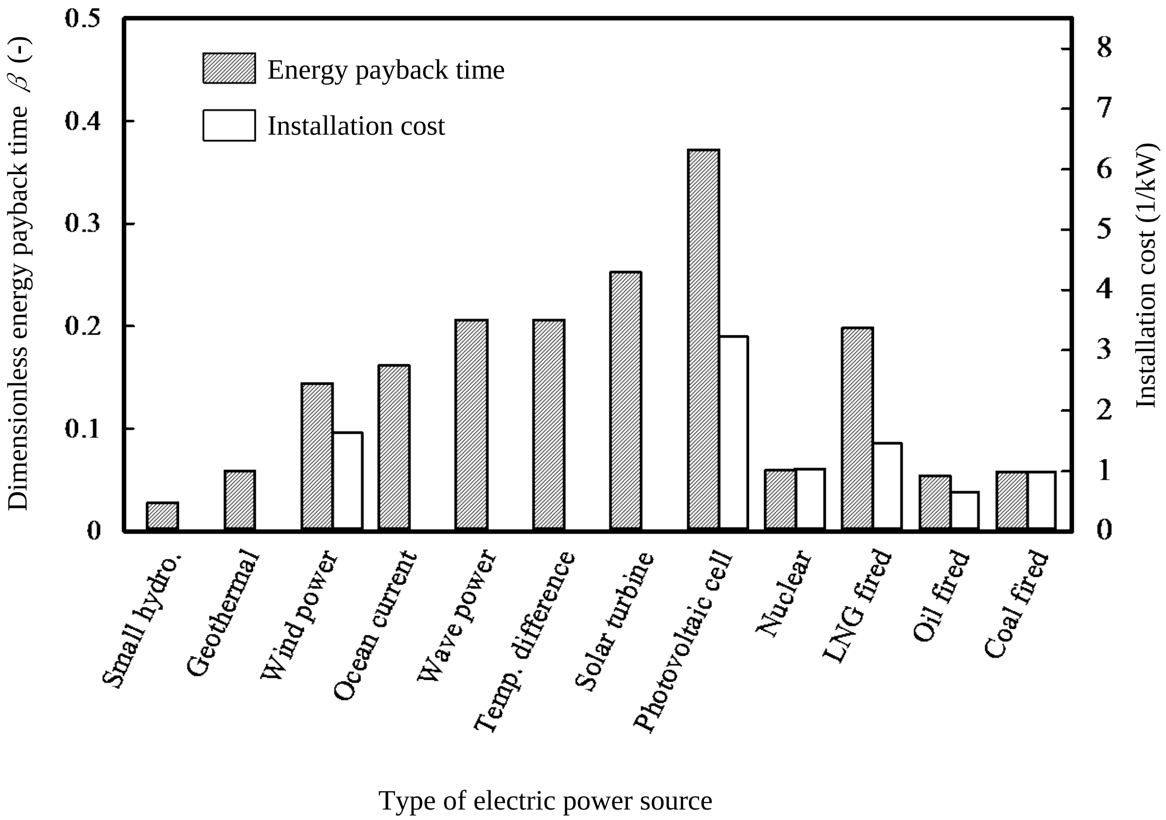

Fig.1. Energy payback time and specific installation cost of various power sources

Figure 2. shows Eq. (5) for doubling time vs. dimensionless energy payback time. Upward convex curve comes from the effect of limited life time of power source. In order to create available energy during replacement from fossil fuel to renewable energy, a small energy payback time is required.

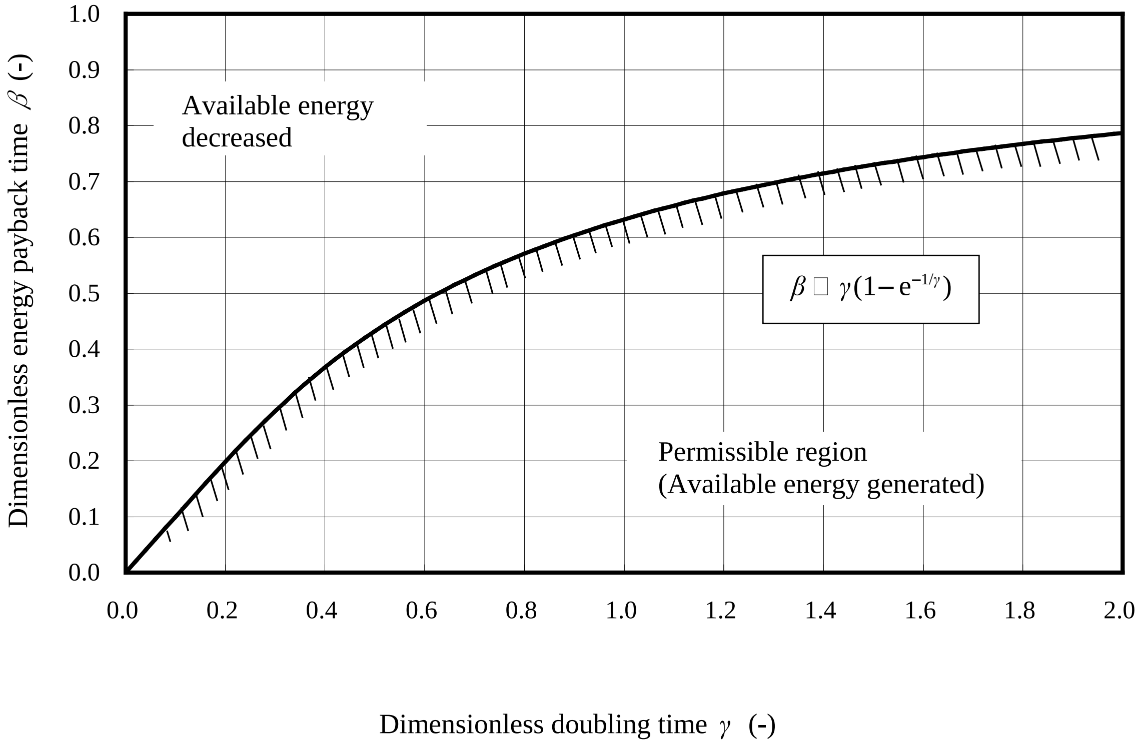

Fig.2. Permissible range of energy payback time

ANALYSIS OF CARBON EMISSIONS FROM MIXED POWER SOURCES IN REPLACEMENT

Model Considerations

CO_2 emissions from power plants result from three ways, i.e. ① manufacturing and installation of the facility, ② operation and maintenance (O & M) and ③ fuel combustion. Fossil power meeting 95% of the world's electricity demand at the moment emits most CO_2 due to fuel combustion. On the other hand, natural energy as an alterative to fossil energy for CO_2 abatement is not necessarily free from CO_2 emission because fossil energy is consumed during its manufacturing and installation before replacement is completed. Its portion is reported as almost 100% for a small hydropower plant and more than 90% for both photovoltaic cell and wind power [3]. From above, following assumptions are made in the analysis for simplicity.

(1) Power sources can be divided into two categories, conventional (fossil) and new (renewable) ones. Suppose the former is replaced by the latter with time.

(2) Conventional power source is represented by a coal fired plant and it emits the most CO_2 in the process of fuel combustion and has a constant energy payback time $(1-\alpha_0)$.

(3) New power source emits CO_2 during manufacturing and installation only and no CO_2 during operation.

Figure 3 shows the energy flow considered in the present model. Total carbon emitted from power sources is the sum of contributions from both the conventional and the renewable.

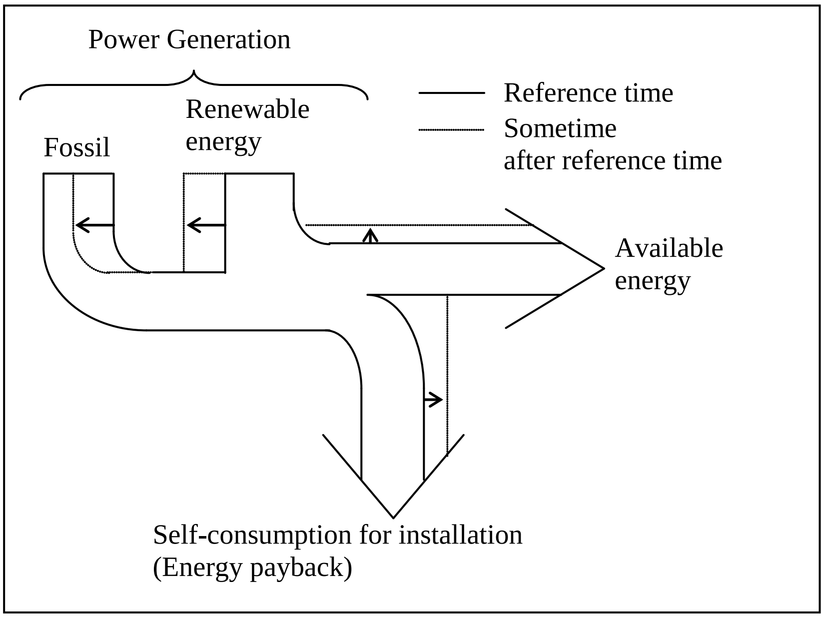

Fig.3. Electric energy balance

Increased Available Energy Supply Scheme

Here the energy supply is assumed to be increased by those required for manufacturing new renewable power sources. Then noting that the energy supplies due to conventional and new power sources become $K-(c\Delta X-bf'(t))$ and $c\Delta X$, respectively, the carbon balance of the mixed power system yields

$$A_m\ (K+bf'(t)) = A_0\ (K-(c\Delta X-bf'(t))) + A_m bf'(t)\eta \tag{7}$$

under the condition that $cX \geqq bf'(t)$ which is Eq. (3).
Here, the second term of the right-hand side of Eq. (7) denotes carbon enissions from energy consumption by those relevant to manufacturing, transportation, construction, maintenance and so on of new power sources. The energy consumption by new power sources consist of electric energy and non electric one, in which the carbon emissions due to electric energy is included in the first term of the right-hand side of Eq. (7). Thus, the second term should represent carbon emissions due to non electric energy consumption i.e. direct fossil fuel burning. To do this, parameter η is introduced that expresses non electric to total energy consumption ratio and ranges between zero and unity. $\eta=1$ denotes an extremity where energy is compossed of solely non electric one and yields maximum possible carbon emissions. To the contrary, $\eta=0$ means all energy is consumed by electric energy only, envisaging mimum possible carbon emissions.

$f(t)$ is the cumulative number of the renewable power sources manufactured until time t which is assumed as follows,

(1) at the early stage of the introduction of new power source, the number of installations increases exponentially, which yields

$$\lim_{t\to-\infty} f(t) = f(0)\,2^{\frac{t}{t_2}}$$

(2) no active new power source increases in number at the time of complete replacement by new power source, which leads to

$$f''(\tau_f)=0$$

A trial function to meet above condition may be selected from so called logistic function as shown in Figure.4. having the following form

$$f(t) = \frac{\nu}{2^{-\frac{t}{t_2}} + 1} \tag{8}$$

In dimensionless from this reduces to

$$f(\phi) = \frac{e^{\frac{\delta}{\gamma}} + 1}{e^{-\frac{\phi-\delta}{\gamma}} + 1} \cdot \frac{e^{\frac{1}{\gamma}} + e^{-\frac{\delta}{\gamma}}}{e^{\frac{1}{\gamma}} - 1} \tag{9}$$

$$(0 \leqq \phi \leqq \delta)$$

Then, Eq. (7) leads to

$$a \equiv \frac{A_m}{A_0} = \frac{1 - G(\phi) + J(\phi)}{1 + (1-\eta)\ J(\phi)} \tag{10}$$

where

$$\left.\begin{aligned} G(\phi) &= (G(\phi)\ - 1)\ /\ N \\ J(\phi) &= \beta h(\phi)\ /\ \gamma N \\ G(\phi) &= \frac{e^{-\phi/\gamma}\ (e^{\delta/\gamma} + 1)\ (e^{(\delta+1)/\gamma} + 1)}{(e^{-(\phi-\delta)/\gamma} + 1)(e^{-(\phi-\delta-1)/\gamma} + 1)} \\ H(\phi) &= \frac{e^{-\phi/\gamma}\ (e^{\delta/\gamma} + 1)\ (e^{(\delta+1)/\gamma} + 1)}{(e^{1/\gamma} - 1)(e^{-(\phi-\delta)/\gamma} + 1)^2} \end{aligned}\right\} \tag{11}$$

The total production energy is $K + bf'(t)$ and then expressed in dimensionless form σ

$$\sigma = 1 + J(\phi). \tag{12}$$

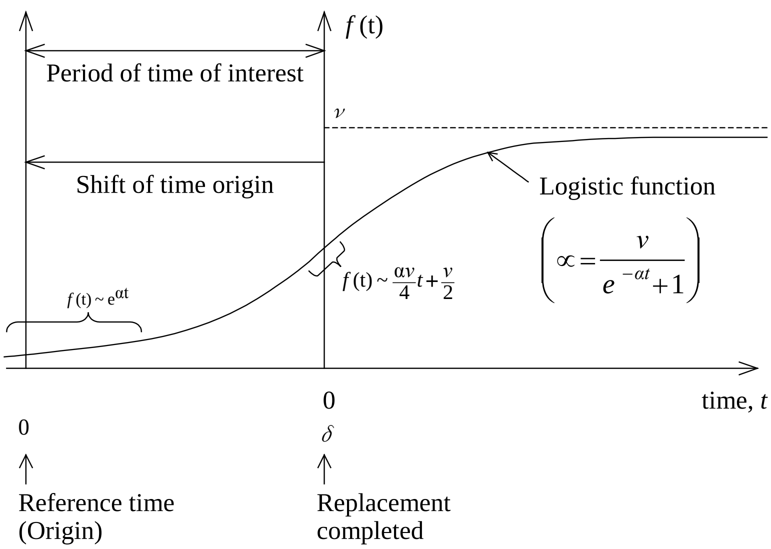

Fig.4. Trial function of cumulative installation number of new power plants

Then, the amount of carbon emissions from the mixed power sources may be expressed by the product $a\sigma$. Full replacement occurs at the time $\tau_{\rm f}$ when

$$K - (c\varDelta X_{\rm f} - bf'(t)) = 0 \tag{13}$$

holds. Solving Eq. (13) the dimensionless form δ for $\tau_{\rm f}$ is obtained see (Appendix)

Available energy supply is $K\alpha_0 + (1 - \alpha_0)(c\varDelta X - bf'(t))$ and COA $\alpha_{\rm m}$ can be shown to become

$$\alpha_{\rm m}(\phi) = \alpha_0 + (1-\alpha_0)(G(\phi) - J(\phi)). \tag{14}$$

When replacement is completed, $\alpha_{\rm m}(\delta)$ becomes unity as is expected from the assumption.

Gross Carbon Emissions

The amount of carbon release in the air during the time course of replacement is evaluated by the following integral using Eqs. (10) and (12).

$$I = \int_0^{\delta} a\sigma d\varphi \tag{15}$$

RESULTS AND DISCUSSIONS

Prediction of Power Conversion Time and Carbon Emissions for Representative Carbon Free Energy Sources

Among candidates for renewable power sources, nuclear plant, small hydropower plant, wind power and photovoltaic cell were chosen and analysed. They are representative because nuclear power is already well established, small hydropower plant has a large amount of resources to be exploited with smallest energy payback time (EPT), wind power is widely available and photovoltaic cell is easy to install anywhere but it has the biggest EPT at present as shown in Figure 1.

Adopting the model in the preceding section, where increased available energy is guaranteed, time of complete replacement and carbon release were analysed.

Calculation procedure was as follows.

(1) Specify renewable energy and obtain β from Figure 1.
(2) Input K and $cX(0)$, and calculate $N(= K/cX(0))$.
(3) Guess permissible γ referring to Figure 2.
(4) Calculate δ using Eq. (A-8) (A-9).
(5) Calculate $\tau_{\rm f}$ and compare with target value.
(6) Calculate $\sigma_{\rm f}$, the total power at the time of complete replacement using Eq. (12).

Results are given in Table 1 and Figure 5. under the condition that $\eta = 1$.

K value of 600TWh/y was electric energy produced by burning fossil fuel in Japan in 1996 which is to be replaced by carbon free energy. Photovoltaic cell power capacity reached about 200MW in 1999 in Japan, which is the highest level worldwide [4]. The value of $cX(0)$ was determined assuming 90% availability for nuclear plant and 60% for a small hydropower plant, 35% for wind power and 15% for photovoltaic cell [4].

As mentioned earlier, carbon release in the air should start decreasing until mid-century in order to stabilise the concentration of carbon dioxide at the beginning of the next century so as to reach the value two times larger than that of the pre-industrial era. Considering this situation, results show that nuclear, small hydropower, and wind power plants may be good candidates in terms of CO_2 abatement. Here, resource care has to be taken that nuclear energy is not a renewable but an exhaustible energy resource, and not reputable among natural resources. Also, hydropower may affect eco-system around rivers during and after exploitation.

The photovoltaic cell, however, requires 92 years from 1999 for complete replacement and also total carbon release in the air even increases for true decades before complete replacement according to the present performance characteristics, which is pessimistic to realize the rebirth scenario. Besides, $\sigma_{\rm f}$ is very large two times larger than present one, which implies half of the energy is consumed to make photovoltaic cells themselves. This means the present specifications i.e. 20 year average life and 7.4 year energy payback time (EPT) need to be improved. Then, these were modified to 3 years for EPT and 30 years for life, respectively, which is similar to those of wind power in terms of β and γ and calculated again. Results were much improved in terms of σ_f but not so much in view of replacement time. On the other hand, in the cases of nuclear and small hydropower plants, excess power is estimated to be about 10% and 3%, respectively, both of which are very small.

In these new power sources, longer replacement completion tactics look tolerable even at the expense of allowance of a bit more release of carbon. It is important to note that once complete replacement is reached, power generation scale cannot be reduced without loss of available energy Figure 6. shows comparison of carbon emissions $a\sigma$ among new power sources. Carbon saving is unexpectedly dependent on the type of power source during power conversion. Small hydro looks most effective in view of effective CO_2 abatement.

Table 1 Assessment of renewable energy candidates

	Parameter \ Power source	Nuclear plant	Small Hydropower plant	Wind power	Photovoltaic cell: Present spec.	Photovoltaic cell: Target spec.
Input	K (TWh /y)	600	600	600	600	600
	$cX(0)$ (TWh /y)	330 (41GW)	82 (15GW)	0.25 (80MW)	0.26 (200MW)	0.26 (200MW)
	β	0.06	0.02	0.1	0.37	0.1
	τ_1 (y)	30	30	30	20	30
	N	1.8	7.3	2400	2300	2300
Output	γ	1.5	0.5	0.2	0.5	0.19
	δ	3.01	1.45	1.76	4.61	2.9
	τ_2 (y)	31	10	4	7	4
	τ_f (y)	90	44	53	92	87
	σ_f	1.10	1.03	1.34	1.95	1.36

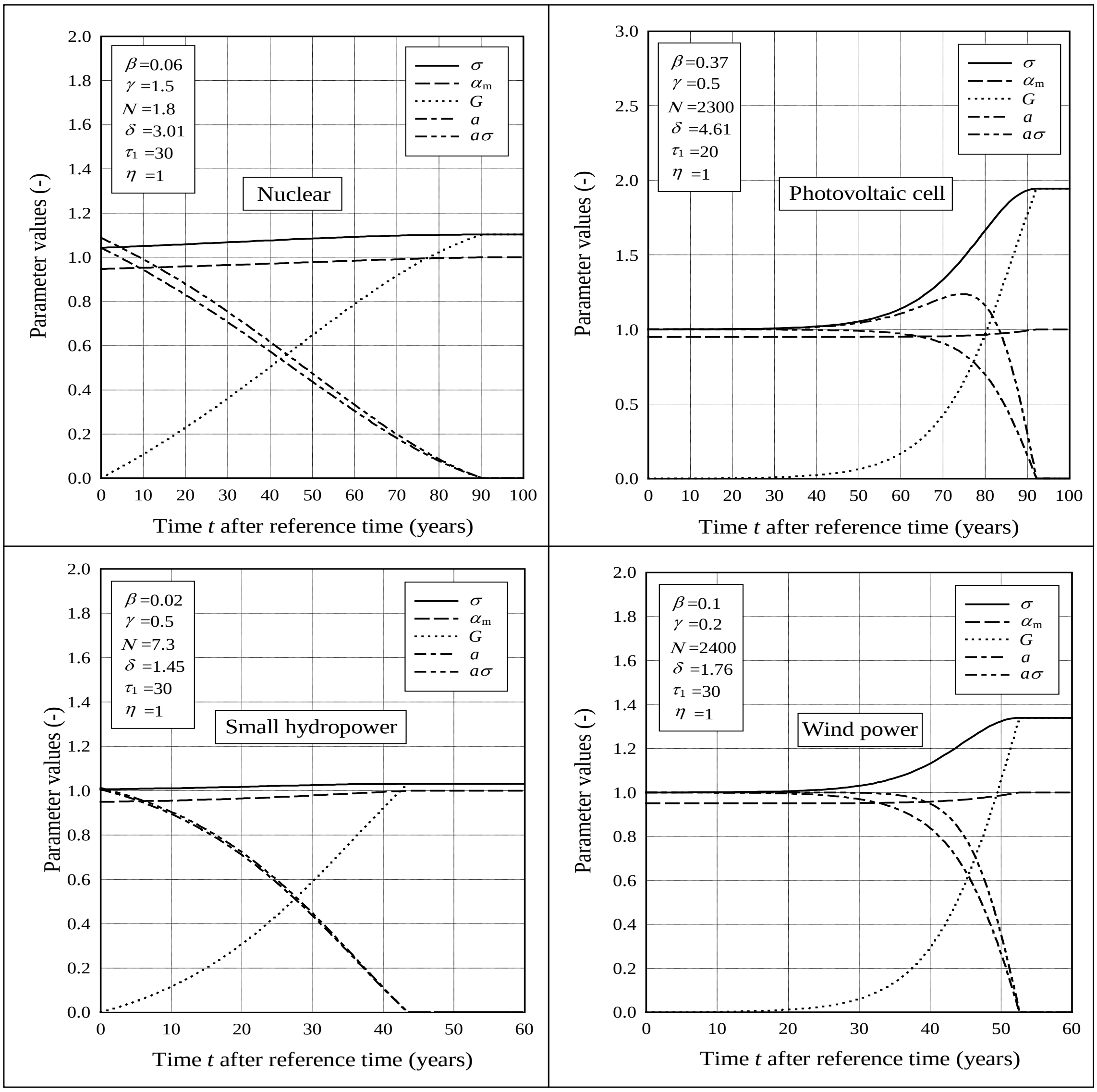

Fig.5. Energy production and carbon emission during replacement of coal fired power source by various non-fossil power sources

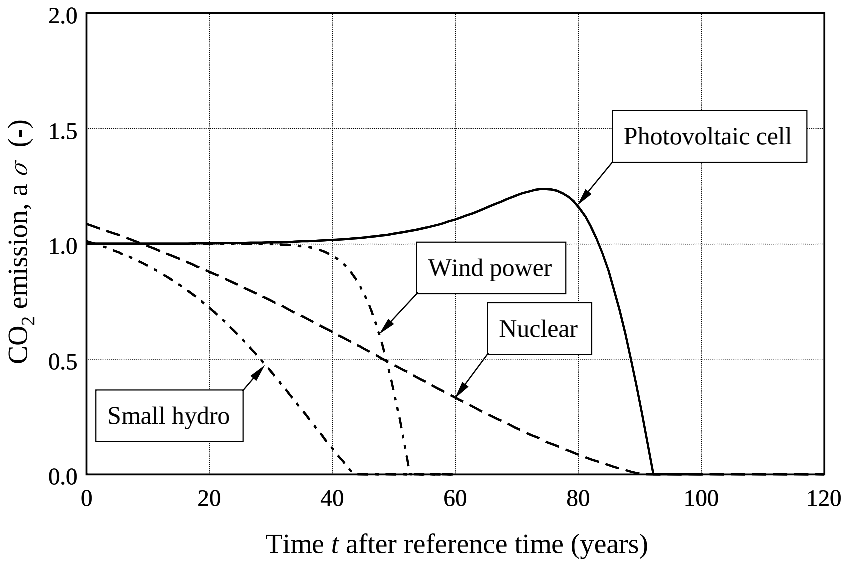

Fig.6. Comparison of CO_2 emission behavior among non-fossil power sources

Restrictions on the Energy Payback, Average Life and Doubling Time in Manufacturing Renewable Power Sources

Preferable design and manufacturing conditions are discussed in this section, which serves to determine the goal of renewable energy resource development. In view of energy saving, an energy strategy to maximise the available energy as well as to minimise the number of installations of new power sources is desired. Its dominant parameters are sought.

(1) The number of installations of new power sources

Using the relation $\sigma_f = 1 + J(\delta) = G(\delta)$, the number of active power sources at the end of replacement σ_f is expressed by

$$\sigma_f = \frac{Z - 1 + \frac{\xi}{Z}(Z+1)}{Z - 1 - \xi(Z+1)} \tag{16}$$

Where

$$Z = e^{1/\gamma}, \ \xi = \frac{\beta}{2\gamma}$$

With $N \gg 1$, this expression can be approximated by

$$1/\sigma_f = 1 - \xi\ (Z+1)/(Z-1) \tag{17}$$

which strongly depends on the value β / γ. Under the condition $\gamma \ll 1$,equation (17) may be further simplified to $1/\sigma_f = 1 - \beta / 2\gamma$. It is important to note that 1-β / 2γ is independent of life time τ_1. The inequality

$$\varepsilon \ll \tau_2 \tag{18}$$

is desirable from the standpoint of energy saving. This implies that power source manufacturing rate(1/γ) of the next generation power source should have an allowable upper limit as a function of energy payback time β. Parametric effect on total energy production σ are shown in Figure 7.

Dimensionless energy payback time β strongly affects the value of σ_f at the complete replacement. To the contrary, the effect of N, the number of new power source units equipment to K is small on σ_f but is significant on τ_f, the time of complete replacement.

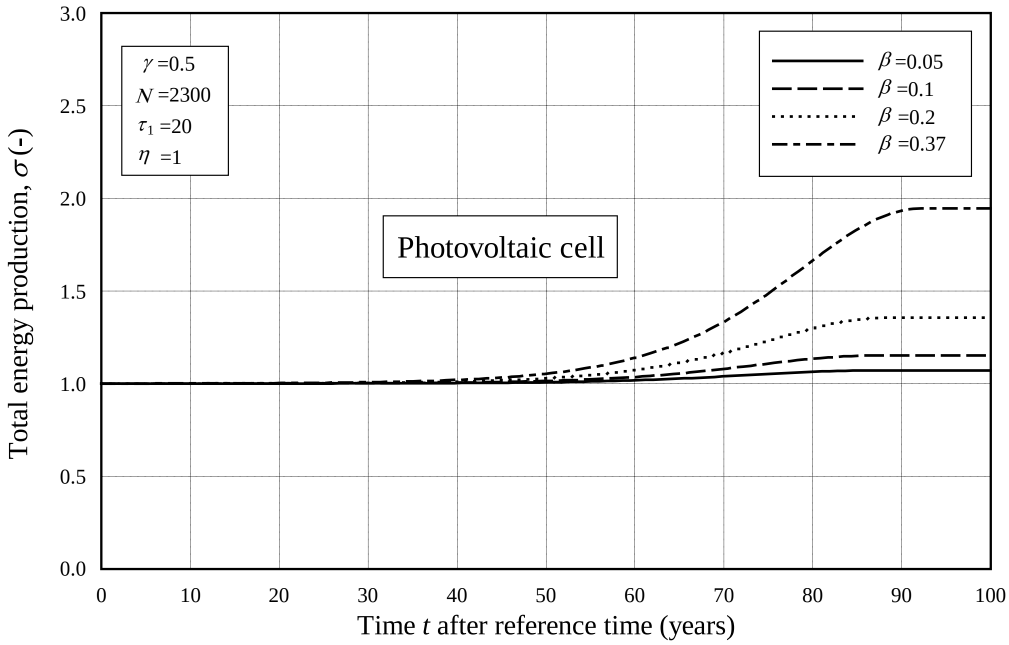

Fig.7-A Total energy production with β variation

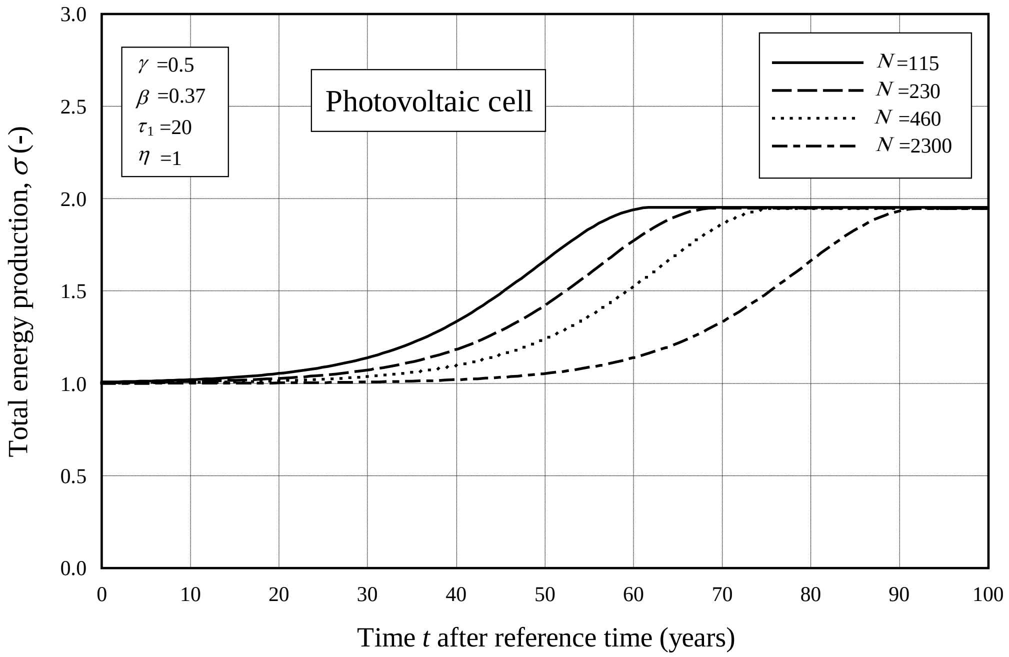

Fig.7-B Total energy production with N variation

Fig.7. Parametric effects on total energy production

(2) Reduction of carbon emissions

Figure 7. and 8 shows parametric effects on total carbon emissions $a\sigma$ with time.

Both parameters η and β affect carbon emission behavior strongly at the later period but not the time of complete replacement. To the contrary, N and γ affect the time of complete replacement greatly. Peak value of aσ is not so much influenced by N as is by the choice of doubliy time γ. Too fast replacement (small γ) results in a large increase in carbon emissions toward the end of complete replacement. Smaller β and large γ i.e. smaller ratio of β / γ is as advisable as in the case of the number of installations of new power sources σ_f.

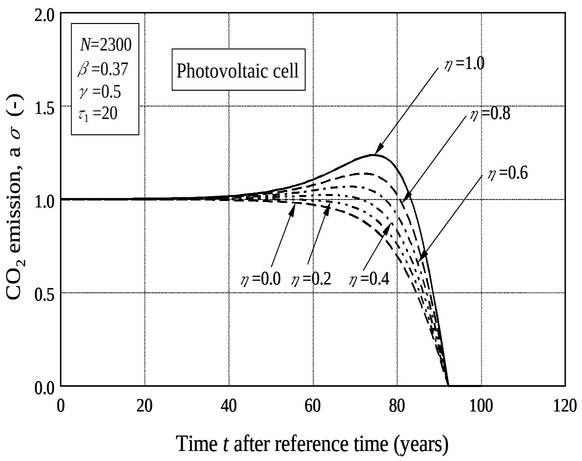

Fig.8-A. Variation of CO_2 emission behavior with η

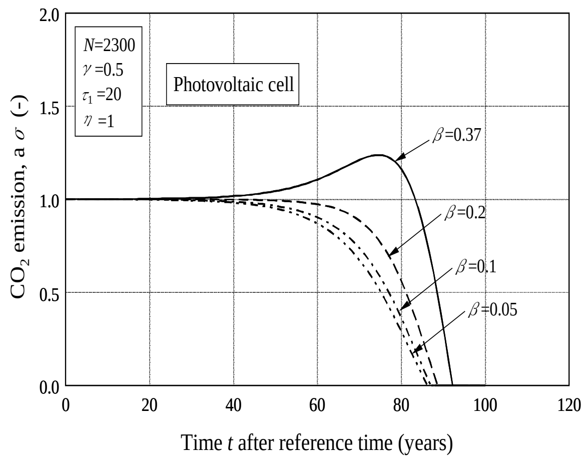

Fig.8-B. Variation of CO_2 emission behavior with β

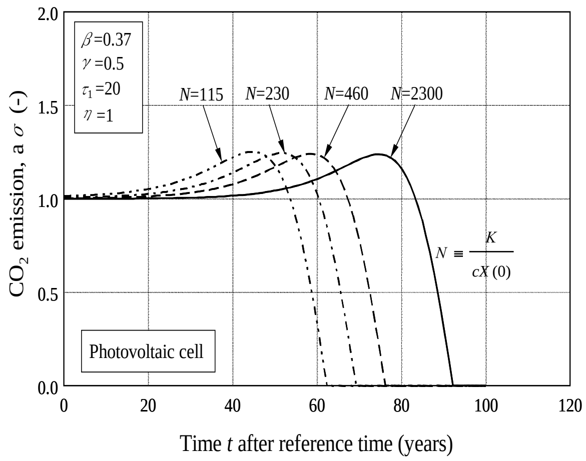

Fig.8-C. Variation of CO_2 emission behavior with N

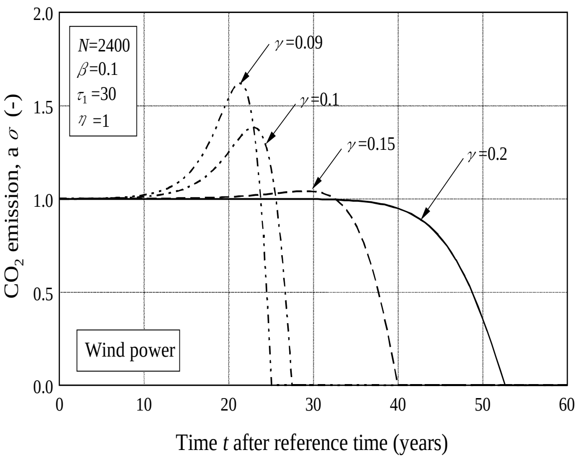

Fig.8-D. Variation of CO_2 emission behavior with γ

Fig.8. Parametric effect on carbon emission

CONCLUSIONS

Carbon dioxide(CO_2) abatement by the introduction of renewable energy sources in place of fossil fuel was modelled and analysed with the overall energy balance taken into account. Limited operation time of the new power sources were considered. Exponential function of time was assumed as a trial function to express the number of new power sources installed during replacement process.

(1) Several useful inequalities and relations among characteristic parameters of energy payback time(EPT), average life(τ_1) and manufacturing doubling time (τ_2) were obtained in non-dimensional forms.

(2) Energy payback time should be kept much shorter than doubling time so that sufficient available energy remains when the replacement is completed.

(3) Nuclear and small hydropower plants were promising candidates but the photovoltaic cell, to avoid a large energy expense after the replacement, would need substantial development effort to reduce EPT.

(4) Energy balance issue should be considered prior to economic issues in determining energy source option or policies such as subsidy, incentives etc. for CO_2 abatement.

Future research efforts are being directed to include non-electric energy flow and cost as well in a model. Also, the present model may be extended for solving an inverse problem to obtain the new energy production scheme and to determine target specification of EPT, τ_1 and τ_2 of renewable energy power sources to a given CO_2 abatement scheme.

ACKNOWLEDGMENTS

The author is indebted to Professor H.Yoshiki, Tokyo University for his kind disclosures of information necessary to conduct this work. He also expresses his thanks to Mr. S. Kirikami, Hitachi Ltd. for his valuable discussions and Mr. K. Watanabe, Hitachi Engineering Co. Ltd. for his cooperation in carrying out calculations.

APPENDIX

Derivation of Eq. (10)

Using dimensionless time ϕ $(\equiv t/\tau_1)$ and doubling time γ $(\equiv \tau_2/\tau_1 \ln 2)$, we can rewrite Eq. (8) as

$$f(\phi) = \frac{e^{\delta/\gamma}+1}{e^{-\frac{(\phi-\delta)}{\gamma}}+1} \cdot f(0) \qquad \text{(A-1)}$$

It is seen Eq. (A-1) meets the assumption (2) i.e. $f''(\tau_f) = 0$

Because

$$f''(\phi) = \frac{\psi\, e^{-\frac{(\phi-\delta)}{\gamma}}}{\gamma^2\, \mathrm{M}^3} \left(e^{-\frac{(\phi-\delta)}{\gamma}} - 1\right) \qquad \text{(A-2)}$$

Where

$$\psi = (e^{\frac{\delta}{\gamma}} + 1)\ f(0)$$

$$M = e^{-\frac{(\phi-\delta)}{\gamma}} + 1$$

and $f''(\delta) = 0$ holds.

Since $X(0) = f(0) - f(-\tau_1)$ from Eq. (4) or in a dimensionless form $X(0) = f(0) - f(-1)$ one can write incorporating Eq. (A-1)

$$f(0) = \frac{e^{(\delta+1)/\gamma} + 1}{e^{\delta/\gamma}(e^{1/\gamma} - 1)} \times (0) \qquad \text{(A-3)}$$

Following relations exist

$$\mathit{\Delta} X(0) = X(\phi) - X(0) \qquad \text{(A-4)}$$

$$X(\phi) = f(\phi) - f(\phi - 1) \qquad \text{(A-5)}$$

Then, combination of Eqs. (A-3) and (A-5) gives

$$X(\phi) = G(\phi) - X(0)$$

and

$$\mathit{\Delta} X(\phi) = (G(\phi) - 1)\, X(0)$$

where

$$G(\phi) = e^{-\phi/\gamma} \cdot \frac{(e^{\delta/\gamma}+1) \cdot (e^{(\delta+1)/\gamma}+1)}{(e^{-(\phi-\delta)/\gamma}+1) \cdot (e^{-(\phi-\delta-1)/\gamma}+1)} \cdot X(0) \qquad \text{(A-6)}$$

Derivation of δ

From the condition $a = 0$ (Eq. (10)) at $\phi = \delta$ where carbon emission ceases,

$$1 - G(\delta) + J(\delta) = 0 \qquad \text{(A-7)}$$

Form Eq. (A-7), quadratic equation is obtained

$$Zy^2 - \frac{(2\mathrm{N}+1)(Z^2-1) + \xi(Z+1)^2}{(Z-1) - \xi(Z+1)}\, y + 1 = 0 \qquad \text{(A-8)}$$

Where

$$Z = e^{1/\gamma}$$

$$\xi = \frac{\beta}{2\gamma}$$

$$Z = e^{\delta/\gamma}$$

Solving Eq. (A-8), one can obtain solution y and δ from

$$\delta = \gamma \ln y \qquad \text{(A-9)}$$

Derivation of formula for σ_f

From the condition $a = 0$ (Rq. (10)) at $\phi = 0$ where carbon emission ceases,

Since $$\sigma_f = 1 + J(\delta) = G(\delta)$$

$$G(\delta) = \left[\frac{(y+1)(Zy+1)}{2y(Z+1)} - 1 \right] / N$$

$$= \frac{Zy^2 + 1 - (y+1)Z}{2y(Z+1)} \cdot \frac{1}{N} \qquad \text{(A-10)}$$

Combining Eq. (A-8) with Eq. (A-10), $Zy^2 + 1$ may be replaced, and we have

$$G(\delta) = \frac{Z - 1 + \dfrac{\xi}{N}(Z+1)}{Z - 1 + \xi(Z+1)} \qquad \text{(A-11)}$$

REFERENCES

[1] Godrej, D. Climate Change. London: Verso. 2001.
[2] Kaya, Y., Carbon Dioxide Abatement Strategy (in Japanese). Tokyo: Nikkan-Kougyo, 2000, PP.179-217.
[3] Uchiyama, Y. and Yamamoto, H., Electric Economy Research 1991, 29, P7
[4] Kando, M., Photovoltaic Power Generation (in Japanese). Journal of Thermal and Nuclear Power 2001, 52 (10)
[5] Hondo, H. et al., CRIEPI Report No.Y99009, 2000
[6] Utamura, M., Proceeding of ASME TURBO EXPO 2002-Land, Sea & Air, GT-2002-30448, 2002