A Synthesis of Digital Filters with Minimum Pole Sensitivity

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SUMMARY This paper presents an approach to the realization of low-sensitivity second-order digital filters. In contrast to existing methods, which use amplitude sensitivity as a criterion, the proposed method uses pole sensitivity. Pole sensitivity is quite useful for evaluating and comparing the performance of various types of filters because it is not a function of frequency. The relation between pole sensitivity and amplitude sensitivity is clarified, and it is shown that the minimization of amplitude sensitivity is equivalent to that of absolute value of pole sensitivity.

For the minimization of pole sensitivity, the denominator polynomial of a second-order transfer function is expressed in a general form including two multiplier coefficients and some arbitrary integer parameters. Under certain constraints on the range of the parameters, which limit the filter complexity, the values of the parameters are determined through an exclusive search such that the pole sensitivity is minimized. The optimum set of the parameters are fully dependent on the given specifications, i.e. locations of the poles. Therefore the z-plane can be partitioned into some regions, in each of which a single set of the parameters is chosen. The resultant map enables one to realize low-sensitivity digital filters having any given characteristics. Finally, two illustrative examples are included.

1. Introduction

In realizing digital filters, one of the most serious problems is the deviation of the filter response due to finite wordlength multipliers. In general the relation between coefficient errors and variations of the response of the filters is dealt as the problem of the coefficient sensitivity. Since low-sensitivity digital filters can be realized using short wordlength multipliers, one can expect the reduction of the total hardware size with the use of low-sensitivity digital filters.

One method to obtain low-sensitivity digital filters is to simulate the behavior of doubly terminated *LC* filters. Though these types have outstanding sensitivity characteristics, their structures are so complex that they are scarcely used in practical design.

Due to simplicity, the cascade connection of first- or second-order sections is preferred, and the direct form is usually used for the realization of each section. Obviously the substitution of a low-sensitivity building block for each direct form section improves the total sensitivity performance. Accordingly an approach to the realization of low-sensitivity first- and second-order digital filters is considered here.

Coefficient sensitivity is usually proportional to the derivative of a transfer function, and considered to have little direct relation to the realized circuit structure. Therefore, when we treat coefficient sensitivity, it is natural to consider, irrelevantly to the filter structure, how multiplier coefficients appear in the transfer function. Actual circuit structure can be realized from the given transfer function by means of the method in Ref. (1), which enables us to obtain all types of equivalent circuits with a minimal number of multipliers.

Among various types of coefficient sensitivity, amplitude sensitivity has generally been considered up to now. The second-order digital filters having minimal amplitude sensitivity have been proposed in Ref. (2), in which amplitude sensitivities with respect to each of two multiplier coefficients are minimized one after another. However, the criterion is not always applicable for the comparison of various types of filters, because the superiority of one filter to the others may be dependent on frequency and sometimes not both of the two amplitude sensitivities of one filter can be simultaneously minimized.

Since pole sensitivity is not a function of frequency, it is a good index to evaluate and compare the sensitivity performance of various types of filters. Mitra and Sherwood developed an analytical technique for the evaluation of pole displacements with the use of pole sensitivity[(3)].

In this paper the relation between pole sensitivity and amplitude sensitivity is clarified, and a method for minimizing the sum of squared values of pole sensitivities with respect to each of all multiplier coefficients is proposed. Finally examples are given for the comparison of the sensitivity of the obtained filter and other types.

2. Amplitude Sensitivity and Pole Sensitivity

2.1 Quantities Characterizing a Filter

In this chapter, a second-order digital filter is considered. The transfer function of a second-order digital filter is given in the following general form;

$$H(z^{-1}) = \frac{N(z^{-1})}{1+g_1 z^{-1}+g_2 z^{-2}}, \tag{1}$$

where $N(z^{-1})$ is the numerator polynomial and it is determined depending on the filter types, i.e. lowpass, highpass or bandpass type. g_1 and g_2 are constants depending on the multiplier coefficients.

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When the filter of Eq. (1) has complex conjugate poles $p=re^{\pm j\theta}$, the transfer function can also be rewritten in the following form;

$$H(z^{-1}) = \frac{N(z^{-1})}{(1-re^{j\theta}z^{-1})(1-re^{-j\theta}z^{-1})}. \tag{2}$$

Equating Eqs. (1) and (2) yields the following relations;

$$g_1 = -2r\cos\theta, \tag{3a}$$

$$g_2 = r^2. \tag{3b}$$

In case of analog filters, the center angular frequency and Q are usually used to specify the denominator polynomial of second-order transfer function. These quantities can be defined from either the pole position or the frequency response. Usually the center angular frequency can be normalized to unity without loss of generality, thus, the frequency characteristics can be expressed using only one parameter Q.

In the digital counterpart the normalized center angular frequency ω_0 and the 3 dB bandwidth ω_b are defined as shown in Fig. 1. It can be shown that they are expressed in terms of g_1 and g_2 or r and θ as follows;

$$\cos\omega_0 = \frac{-g_1}{1+g_2} = \frac{2r\cos\theta}{1+r^2}, \tag{4a}$$

$$\tan\frac{\omega_b}{2} = \frac{1-g_2}{1+g_2} = \frac{1-r^2}{1+r^2}. \tag{4b}$$

Even if Q of a digital filter is defined by the ratio of ω_0 and ω_b, the relation between Q and g_1, g_2 cannot be represented in a simple form. Furthermore, since ω_0 is already normalized by the sampling frequency, it is impossible to normalize ω_0 to unity. Therefore it is suitable to use ω_0 and ω_b to represent the denominator of a second-order digital filter transfer function.

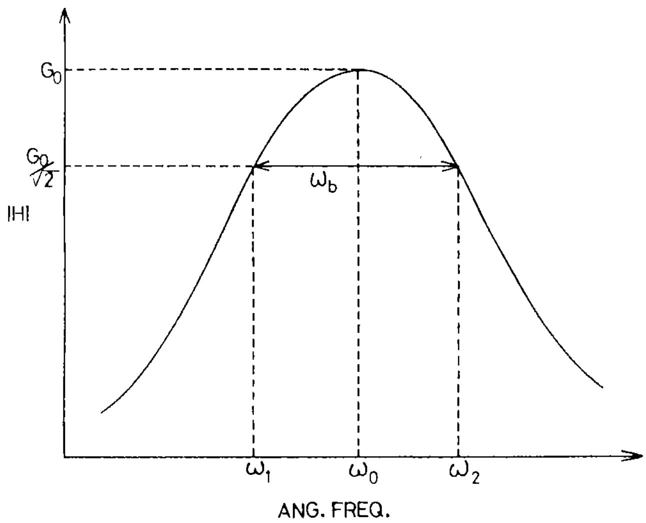

Fig. 1 Amplitude characteristics of the second-order bandpass filter with the numerator $N(z^{-1})=g_0(1-z^{-2})$.

Since sensitivity problems become significant especially for narrow band filters, the poles of filters in consideration can be considered to be very near the unit circle. Applying the approximation of $r\cong 1$ to Eq. (4), the expressions of ω_0 and ω_b are simplified as follows;

$$\omega_0 \cong \theta, \tag{5a}$$

$$\frac{\omega_b}{2} \cong \frac{1-r}{r}. \tag{5b}$$

For narrow band filters, ω_0 is approximately located at the center of ω_1 and ω_2 in Fig. 1, hence ω_1 and ω_2 are expressed by

$$\omega_1,\ \omega_2 \cong \theta \mp \frac{1-r}{r}, \tag{6}$$

where minus sign and plus sign correspond to the cases when $\omega=\omega_1$, and $\omega=\omega_2$ respectively.

2.2 Sensitivity

Various types of sensitivity can be defined depending on the criterion of network performance. Among them, the most popular one is amplitude sensitivity defined by

$$S_x^{|H|} = \frac{x}{|H|}\frac{\partial |H|}{\partial x}, \tag{7}$$

where x is the multiplier coefficient. $S_x^{|H|}$ is a function of frequency since $|H|$ is a function of frequency.

We use pole sensitivity as a measure of filter performance, instead of amplitude sensitivity. Pole sensitivity with respect to a multiplier coefficient is defined by

$$S_x^P = x\frac{\partial P}{\partial x}. \tag{8}$$

This is physically interpreted as a variation of the pole location caused by small deviation of the multiplier coefficient from its nominal value. Since pole sensitivity is a complex quantity, it can be decomposed into r and θ directional components as follows;

$$S_x^r = x\frac{\partial r}{\partial x}, \tag{9a}$$

$$S_x^\theta = x\frac{\partial \theta}{\partial x}. \tag{9b}$$

Then we obtain

$$S_x^P = P\left(\frac{1}{r}S_x^r + j\,S_x^\theta\right). \tag{10}$$

Since pole variation is not directly measurable, we define ω_0 sensitivity and ω_b sensitivity as follows;

$$S_x^{\omega_0} = x\frac{\partial \omega_0}{\partial x}, \tag{11a}$$

$$S_x^{\omega_b} = x\frac{\partial \omega_b}{\partial x}. \tag{11b}$$

Then they are closely related with pole sensitivity as follows;

$$S_x^{\omega_0} = S_x^{\theta} - (1-r)\cot\theta\ S_x^{r}, \tag{12a}$$

$$S_x^{\omega_b} = -2\,S_x^{r}. \tag{12b}$$

These expressions are quite meaningful since center frequency and bandwidth variation can be easily measured in the laboratory.

In contrast with amplitude sensitivity, pole sensitivity is constant with respect to frequency. Therefore it is a useful index to evaluate and compare the performance of various types of filters. In what follows, the relation between amplitude sensitivity and pole sensitivity is investigated.

Amplitude sensitivity can be expressed in terms of pole sensitivity as;

$$S_x^{|H|} = S_x^{|N|} - S_r^{|D|} S_x^{r} - S_\theta^{|D|} S_x^{\theta}, \tag{13}$$

where N and D denote the numerator and the denominator polynomials of the transfer function. $S_r^{|D|}$ and $S_\theta^{|D|}$ are defined by

$$S_r^{|D|} = \frac{1}{|D|}\frac{\partial\,|D|}{\partial\,r}, \tag{14a}$$

$$S_\theta^{|D|} = \frac{1}{|D|}\frac{\partial\,|D|}{\partial\,\theta}. \tag{14b}$$

Note that $S_r^{|D|}$ and $S_\theta^{|D|}$ are independent of the filter structure. In general $S_x^{|N|} \leqq 1$ in the passband.

It can be shown that $S_r^{|D|}$ takes a maximum value at $\omega=\omega_0$ and $S_\theta^{|D|}$ reaches its maximum value at $\omega \cong \omega_1, \omega_2$. Evaluation of Eq. (13) at $\omega=\omega_0$ and $\omega=\omega_1$, ω_2 yields the following equations.

At $\omega=\omega_0$

$$S_x^{|H|} = S_x^{|N|} + \frac{1}{1-r} S_x^{r} - \frac{1}{2}\cot\theta\ S_x^{\theta}. \tag{15a}$$

And at $\omega=\omega_1$, ω_2

$$S_x^{|H|} = S_x^{|N|} - \frac{1}{1-r}\frac{\sin\theta \mp (1-r)\cos\theta}{2\sin\theta \mp (1-r)\cos\theta} S_x^{r} \pm \frac{1}{2(1-r)}\frac{2\sin\theta \pm (1-r)\cos\theta}{2\sin\theta \mp (1-r)\cos\theta} S_x^{\theta}, \tag{15b}$$

where the upper and lower signs correspond to the cases when $\omega=\omega_1$ and $\omega=\omega_2$ respectively.

It should be noted that Eq. (15a) corresponds to the Moschytz' euqation[4] in analog filters. And the physical interpretation of Eq. (15) is that the closer to the unit circle the poles of filter are located, the larger the coefficients of S_x^{r} and S_x^{θ} become, which leads to increase in amplitude sensitivity.

In Eq. (15b), the signs before S_x^{r} and S_x^{θ} corresponding to the cases when $\omega=\omega_1$ and $\omega=\omega_2$ are different. Therefore it is impossible to reduce amplitude sensitivity by cancellation of the terms. In order to obtain a low -sensitivity digital filters, both S_x^{r} and S_x^{θ} must be reduced. In other words, it is necessary to minimize the absolute value of pole sensitivity. In this sense, the minimization of amplitude sensitivity is equivalent to the minimization of the absolute value of pole sensitivity. In the next chapter a method for the synthesis of the minimum pole sensitivity digital filters is considered.

3. Realization of Minimum Pole Sensitivity Digital Filters

Since the coefficient values of the transfer function of a narrow band filter are close to integers, a method to reduce the sensitivity by separating the integers from the coefficient values has been proposed[5]. In that method, the density of possible pole locations is used as a criterion. However, there are many degrees of freedom to extract the integer multipliers, and there are a lot of citcuits having comparable possible pole density. Hence, the density of pole locations provides only a rough estimation of the performance of filters.

The design method proposed here uses the objective function defined by the sum of squared values of pole sensitivity with respect to each of all the multiplier coefficients. In general the coefficients g_i 's of the denominator polynomial are of the form

$$g_i = \sum_j m_{ij} P_j, \tag{16}$$

where P_j is some product of multiplier coefficients and m_{ij} is a constant. The number of multipliers used is restricted to the minimum i.e. the degree of freedom of the filter, so that the resultant filter structure is not excessively complicated. If m_{ij} 's are small integers or 2^n, they may be realized without any extra multipliers and they cause no errors in performing the multiplications. From these considerations the values of m_{ij} 's are restricted to 0, ±1 or ±2 in this paper. By the aid of an exclusive search the values of m_{ij} 's are chosen among the above values such that the objective function is minimized. If one extends the range of m_{ij} 's, the less sensitive digital filters having more complex structure may be obtained.

3.1 First-Order Filter

The transfer function of a first-order digital filter is represented using the pole location $p=r$ by

$$H(z^{-1}) = \frac{N(z^{-1})}{1+g_1 z^{-1}} = \frac{N(z^{-1})}{1-r z^{-1}}. \tag{17}$$

From Eq. (17), we have

$$g_1 = -r. \tag{18}$$

Since this filter has one degree of freedom, it can be realized using one multiplier, say $\boldsymbol{a}$. Then g_1 is expressed by the linear function of $\boldsymbol{a}$ as given below

$$g_1 = m_1 + m_2\,\boldsymbol{a}, \tag{19}$$

where m_i 's ($i=1, 2$) are integers to be determined. From Eqs. (18) and (19), pole sensitivity is calculated as follows;

$$S_{\boldsymbol{a}}^{p} = S_{\boldsymbol{a}}^{r} = m_1 + r. \tag{20}$$

In case of narrow band filters, the value of r is close to unity, and we should choose m_1 to be -1 to minimize the value of S_a^p. The values of m_2 and $\boldsymbol{a}$ can be determined from the following equation.

$$m_2 \boldsymbol{a} = g_1 + 1 \tag{21}$$

The resultant circuit is just the same as that proposed in Ref. (1).

3.2 Second-Order Filter

A general second-order transfer function is already shown in Eq. (1). When it has not a finite transmission zero, the degree of freedom is two. Hence it can be realized using two multipliers, say $\boldsymbol{a}$ and $\boldsymbol{b}$. Then g_1 and g_2 in Eq. (1) are expressed by linear functions with respect to each of $\boldsymbol{a}$ and $\boldsymbol{b}$ as

$$g_1 = m_{11} + m_{12}\boldsymbol{a} + m_{13}\boldsymbol{b} + m_{14}\boldsymbol{ab} \quad , \tag{22a}$$

$$g_2 = m_{21} + m_{22}\boldsymbol{a} + m_{23}\boldsymbol{b} + m_{24}\boldsymbol{ab} \quad . \tag{22b}$$

From Eqs. (3) and (22), the following relations are derived.

$$-2r\cos\theta = m_{11} + m_{12}\boldsymbol{a} + m_{13}\boldsymbol{b} + m_{14}\boldsymbol{ab} \quad , \tag{23a}$$

$$r^2 = m_{21} + m_{22}\boldsymbol{a} + m_{23}\boldsymbol{b} + m_{24}\boldsymbol{ab} \quad . \tag{23b}$$

3.2.1 Restrictions on m_{ij}'s

Since $\boldsymbol{a}$ and $\boldsymbol{b}$ are real and must be determined uniquely from given r and θ, there are some restrictions on the values of m_{ij}'s. First, all m_{1i}'s and m_{2j}'s ($i \neq j$, $i, j = 2, 3, 4$) must not be zero at the same time. Next the following discriminant must be non-negative.

$$D = B^2 - 4AC \quad , \tag{24}$$

where $A = m_{13}m_{24} - m_{14}m_{23}$,

$$B = (r^2 - m_{21})m_{14} + (2r\cos\theta + m_{11})m_{24} + m_{13}m_{22} - m_{12}m_{23} \quad ,$$

$$C = (2r\cos\theta + m_{11})m_{22} + (r^2 - m_{21})m_{12} \quad .$$

3.2.2 Realization of Filters

From Eq. (23), r and θ sensitivity are calculated as follows.

With respect to $\boldsymbol{a}$;

$$S_{\boldsymbol{a}}^{r} = \frac{\boldsymbol{a}}{2r}(m_{22} + m_{24}\boldsymbol{b}), \tag{25a}$$

$$S_{\boldsymbol{a}}^{\theta} = \frac{\boldsymbol{a}}{2r^2\sin\theta}\{r(m_{12} + m_{14}\boldsymbol{b}) + \cos\theta(m_{22} + m_{24}\boldsymbol{b})\}, \tag{25b}$$

and with respect to $\boldsymbol{b}$;

$$S_{\boldsymbol{b}}^{r} = \frac{\boldsymbol{b}}{2r}(m_{23} + m_{24}\boldsymbol{a}), \tag{26a}$$

$$S_{\boldsymbol{b}}^{\theta} = \frac{\boldsymbol{b}}{2r^2\sin\theta}\{r(m_{13} + m_{14}\boldsymbol{a}) + \cos\theta(m_{23} + m_{24}\boldsymbol{a})\}. \tag{26b}$$

As an objective function, the sum of squared values of pole sensitivities with respect to $\boldsymbol{a}$ and $\boldsymbol{b}$ is adopted.

$$S = |S_{\boldsymbol{a}}^{p}|^2 + |S_{\boldsymbol{b}}^{p}|^2 = (S_{\boldsymbol{a}}^{r})^2 + r^2(S_{\boldsymbol{a}}^{\theta})^2 + (S_{\boldsymbol{b}}^{r})^2 + r^2(S_{\boldsymbol{b}}^{\theta})^2 \tag{27}$$

Using this objective function both of the two pole sensitivities can be evaluated at the same time.

The parameters m_{ij}'s are chosen from 0, ±1, or ±2 such that S is minimized through an exclusive search. The resultant values are fully dependent on the given specifications i.e. r and θ. For filters having similar pole locations, the same set of m_{ij}'s are obtained. Therefore z-plane can be divided into several regions depending on the optimum set of m_{ij}'s. The resultant classification map is shown in Fig. 2. Figure 2 shows the first quadrant of z-plane, and other quadrants can be derived from symmetry. The values of m_{ij}'s corresponding to each region in Fig. 2 are listed in Table 1.

Applying the realization method in Ref. (1), it becomes clear that the narrow band filters which have their poles near $z = 1$ in z-plane (types Y, Z, AD, BD, CD and DD) can be realized using similar network configurations. They can be generalized as shown in Fig. 3. In these circuits the following equations hold;

$$m_{11} = -2\,,\ m_{21} = 1 \quad \text{and} \quad m_{13} = -m_{23}, \tag{28}$$

It should be noted again that each m_{ij} in Fig. 3 takes

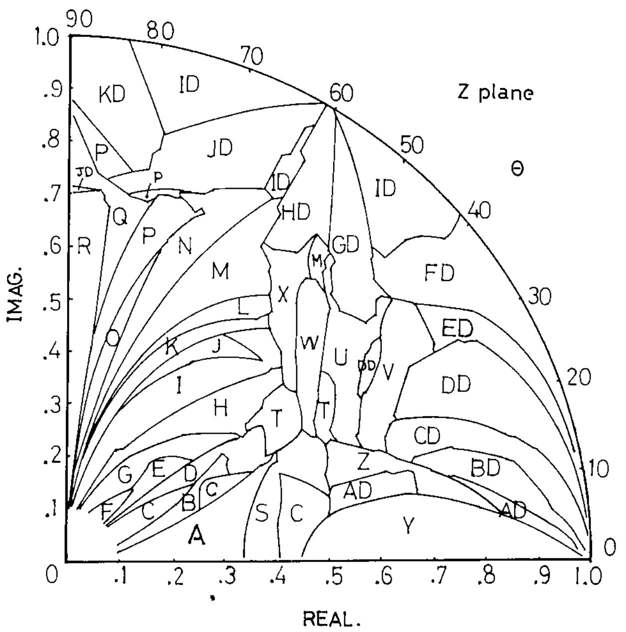

Fig. 2 Classification map.

Table 1 The values of m_{ij} 's corresponding to each region in Fig. 2.

A	0	-1	-2	1
	0	0	0	2
B	0	-1	-1	1
	0	0	0	1
C	0	-1	-1	-2
	0	0	0	2
D	0	-1	-1	-1
	0	0	0	2
E	0	-1	-1	1
	0	0	0	2
F	0	-1	-2	-2
	0	0	1	2
G	0	-1	-2	-2
	0	0	1	1
H	0	-1	-2	1
	0	0	1	0
I	0	-1	-2	2
	0	0	1	0
J	0	-2	-2	-2
	0	1	1	-2
K	0	-2	-1	-2
	0	1	1	-2
L	0	-2	-1	0
	0	1	1	-2
M	0	-2	-1	2
	0	1	1	-2
N	0	-2	-1	2
	0	1	2	-2
O	0	-2	0	0
	0	1	1	-2
P	0	0	-1	0
	0	1	1	-2
Q	0	-1	0	1
	0	1	1	-2
R	0	-1	0	2
	0	1	1	-2
S	-1	-1	1	-1
	0	2	0	-2
T	-1	-1	1	-2
	0	2	0	-1
U	-1	-1	1	-2
	0	2	1	-2
V	-1	-1	0	-2
	0	2	1	-2
W	-1	0	1	-2
	0	1	1	-2
X	-1	0	1	-1
	0	1	1	-2
Y	-2	1	2	2
	1	-1	-2	1
Z	-2	2	1	1
	1	-2	-1	2
AD	-2	1	1	2
	1	-1	-1	0
BD	-2	1	1	1
	1	-1	-1	2
CD	-2	2	2	0
	1	-1	-2	0
DD	-2	2	1	-1
	1	-1	-1	1
ED	-2	1	2	-2
	1	0	-1	-2
FD	-2	2	1	-2
	1	-1	0	0
GD	-1	0	-1	2
	1	-1	-1	2
HD	-1	1	0	2
	1	-2	-1	2
ID	-1	0	-1	2
	1	-2	0	2
JD	-1	2	1	-2
	1	-1	-1	2
KD	0	-1	-1	2
	1	-2	-1	2

m11 m12 m13 m14
m21 m22 m23 m24

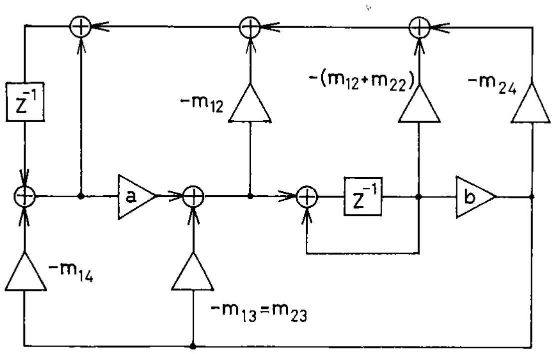

Fig. 3 An example of general configurations for the realization of the filter types Y, Z, AD, BD, CD and DD.

the value among 0, ±1 or ±2, all of which can be performed very easily without any actual multipliers.

4. Examples

4.1 Example 1

Consider the realization of a second-order low-pass Butterworth characteristics whose normalized cut-off angular frequency is $\pi/10$. The transfer function is

$$H(z^{-1}) = \frac{0.020084(1+z^{-1})^2}{1-1.561018z^{-1}+0.641353z^{-2}}, \tag{29}$$

and its amplitude response is shown in Fig. 4. From Eq. (29) the pole locations of this filter are obtained as follows;

$$r = 0.800845,$$

$$\theta = \pm 12.9395°.$$

Therefore, from Fig. 2 the best choice of the filter type is BD. Then from Table 1, the transfer function is expressed by

$$H(z^{-1}) = \frac{\frac{3}{4}\boldsymbol{ab}(1+z^{-1})^2}{1+(-2+\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{ab})z^{-1}+(1-\boldsymbol{a}-\boldsymbol{b}+2\boldsymbol{ab})z^{-2}}, \tag{30}$$

and the multiplier coefficients are calculated to be

$$\boldsymbol{a} = 0.0808249,$$

$$\boldsymbol{b} = 0.3314014.$$

Table 2 shows the value of S in this case together with

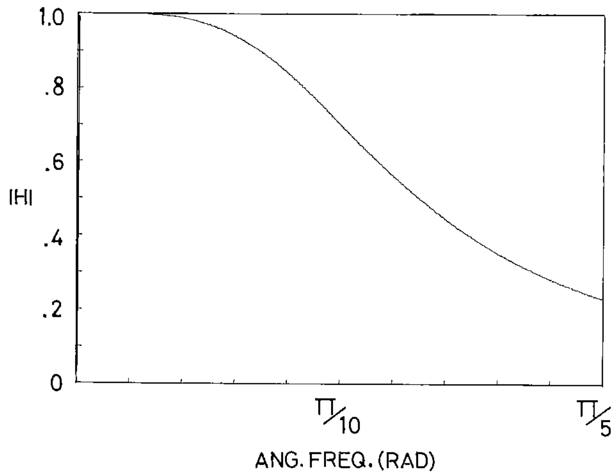

Fig. 4 Amplitude response of the filter in example 1.

Table 2 Comparison of sensitivities for example 1.

filter type	value of S
BD	6.000×10^{-2}
CD	7.062×10^{-2}
DD	7.277×10^{-2}
Biquad[1]	1.125×10^{-1}
Second Agarwal-Burrus[6]	1.705×10^{-1}
Avenhaus type E[5]	1.003×10^{-1}

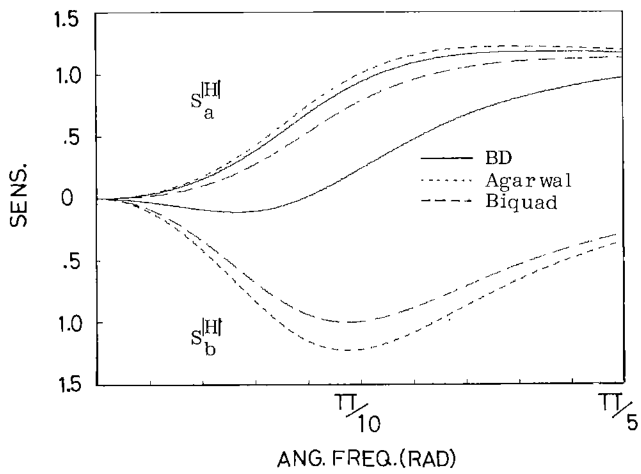

Fig. 5 Comparison of amplitude sensitivities.

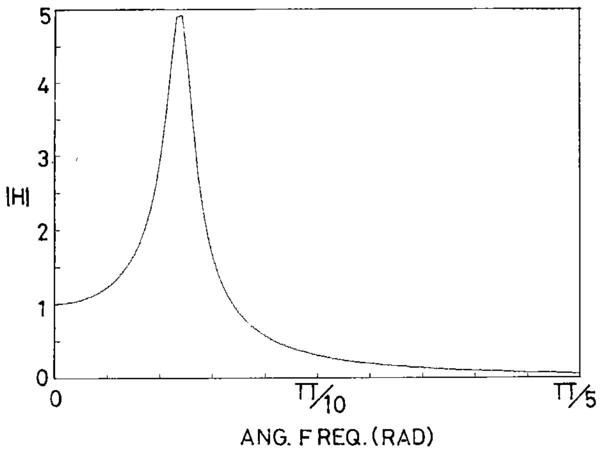

Fig. 6 Amplitude response of the filter in example 2.

that of other types of filters having the same specifications. In Fig. 5, the amplitude sensitivity characteristics of this filter are compared with those of two other types, realization 2 of Agarwal-Burrus[6] and Digital Biquad[1], which are known to have low sensitivity. It is apparent that the amplitude sensitivity with respect to $\boldsymbol{a}$ of the obtained filter is almost the same as that of other filters, and that with respect to $\boldsymbol{b}$ is much smaller.

4.2 Example 2

As another example, a filter which has more stringent characteristics than that in example 1 is realized. The amplitude response of this filter is shown in Fig. 6, and the transfer function is

$$H(z^{-1}) = \frac{0.006061(1+z^{-1})^2}{1-1.94495z^{-1}+0.969195z^{-2}}. \tag{31}$$

The cut-off angular frequency is $\pi/20$.

From Eq. (31) the poles are located at

$$r = 0.984477,$$

$$\theta = \pm 8.955762°.$$

When we adopt the filter type DD from Fig. 2, the transfer function and the multiplier coefficients are obtained as follows;

$$H(z^{-1}) = \frac{\frac{\boldsymbol{a}}{4}(1+z^{-1})^2}{1+(-2+2\boldsymbol{a}+\boldsymbol{b}-\boldsymbol{a}\boldsymbol{b})z^{-1}+(1-\boldsymbol{a}-\boldsymbol{b}+\boldsymbol{a}\boldsymbol{b})z^{-2}}, \tag{32}$$

$$\boldsymbol{a} = 0.024245,$$

$$\boldsymbol{b} = 0.0067233.$$

The values of S of this filter as well as other types are listed in Table 3. Figure 7 shows the comparison of the amplitude sensitivity of this filter with that of the other low-sensitivity filters. The amplitude sensitivity with respect to $\boldsymbol{b}$ is much smaller than that of others.

From these examples, it has been shown that the

Table 3 Comparison of sensitivities for example 2.

filter type	value of S
BD	unrealizable for the specifications
CD	6.186×10^{-3}
DD	6.184×10^{-3}
Biquad[1]	6.309×10^{-3}
Second Agarwal-Burrus[6]	7.039×10^{-3}
Avenhaus type E[5]	1.251×10^{-2}

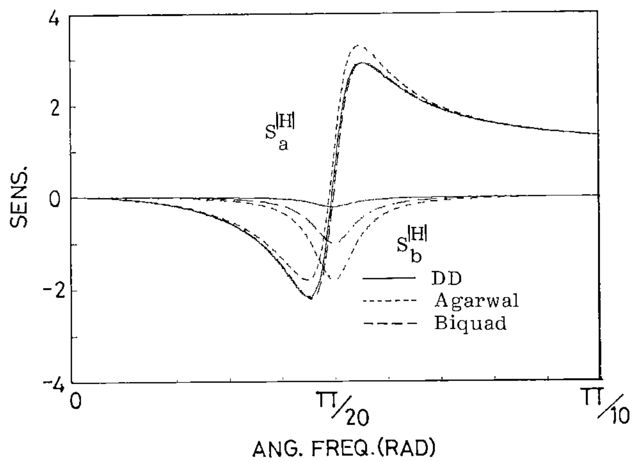

Fig. 7 Comparison of amplitude sensitivities.

obtained filters have outstanding sensitivity performance. And it has also been shown that the pole sensitivity is a good index for the evaluation of various types of filters.

5. Conclusion

The pole sensitivity has been proposed as an index to consider the sensitivity problems. Relation between the conventional amplitude sensitivity and pole sensitivity has been clarified, and it has been shown that pole sensitivity is a good index to compare the performance of various types of filters.

The denominator polynomial of a second-order transfer function is expressed in a general form through the introduction of arbitrary integer parameters. The values of the parameters and the multiplier coefficients are determined through an exclusive search such that the sum of squared values of pole sensitivity becomes minimum. The resultant filters are found to possess low amplitude sensitivity as was expected.

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