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Author	Hirone Komatsu, Gen Endo, Ryuichi Hodoshima, Shigeo Hirose, Edwardo F. Fukushima
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Basic Consideration about Optimal Control of a Quadruped Walking Robot during Slope Walking Motion

Hirone Komatsu, Gen Endo, Ryuichi Hodoshima, Shigeo Hirose, and Edwardo F. Fukushima

***Abstract*— Quadruped walking robots are expected to be utilized on rugged terrain because of its high terrain adaptability compared with wheeled and crawler robots. To utilize quadruped walking robots on such environments, realization of walking motion with high energy efficiency is one of the most important problem. In this paper, we propose new method which directly optimize energy efficiency during walking motion on sloping surface. To verify the validity of proposed method, we make numerical simulations to calculate energy efficiency during walking on the slope.**

I. INTRODUCTION

Quadruped walking robots have some characteristics as follow:

1) it has the minimum numbers of the leg to maintain static stability thus it can be lightweight and simple.
2) it can perform omni-directional locomotion with no slippage at the contact points.
3) it can work on rugged terrain by using the leg as arm with attaching the tool at the tip of the leg.

For these unique characteristics, quadruped walking robots are expected to be utilized for operations on uneven terrain such as construction in mountainous area, agriculture, forestry and so on. It has been studied for a long time. Based on those prospects, the authors have studied mechanism and control theory of a quadruped walking robot [1]. We have developed a quadruped walking robot "TITAN XII" for large obstacle climbing shown in Fig.1 and its basic performance has been verified by basic experiments [2]. Furthermore as the first step to make gait control theory for large obstacle climbing, we focused on the body rising motion which essentially requires energy consumption and basic control method to achieve high energy efficiency was considered by analyzing this motion. To extend our former research, we will consider improvement in energy efficiency of slope walking motion which is the simplest walking motion essentially consumes energy.

Improvement of energy efficiency of walking robots is one of the most important subject, thus it has been studied for a long time. Optimal posture to achieve energy efficient walking motion has been studied by some researchers [3]-[5]. Furthermore, optimal force distributions of multi-legged robots to minimize consumed energy have been studied [6]-[10]. Although consumed energy is calculated from joint torque in most of these previous researches, it is necessary to consider the effect to the consumed energy of moving velocity.

H. Komatsu, G. Endo, S. Hirose, and E. F. Fukushima are with the Department of Mechanical and Aerospace Engineering, Tokyo Institute of Technology, Tokyo, Japan, komatsu@robotics.mes.titech.ac.jp
R. Hodoshima is with the Department of Mechanical Engineering, Saitama University, Saitama, Japan,hodoshima@mech.saitama-u.ac.jp

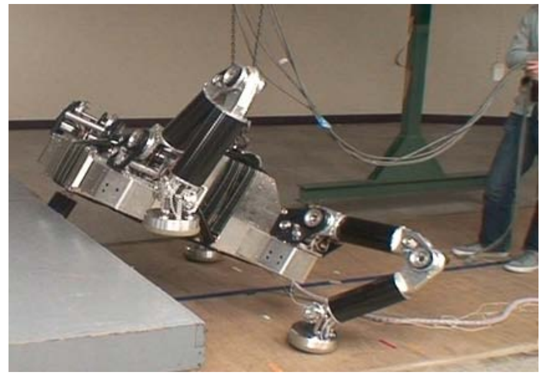


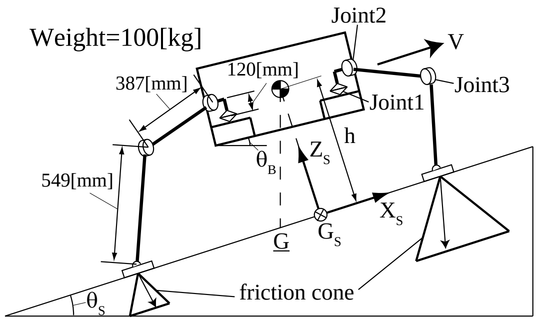
Fig. 1. Prototype of TITAN XII

In this paper, basic whole strategy to optimize energy efficiency of the slope walking motion is discussed in Section 2. we propose new method to optimize specific resistance which is a criterion of energy efficiency directly based on fractional programming and formulate evaluation function, constraints, and optimization algorithm. Finally, effectiveness of proposed method is verified by numerical simulations in Section 4.

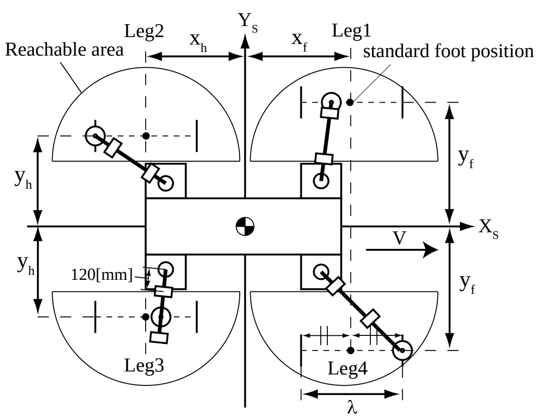
II. BASIC STRATEGY OF OPTIMIZATION

We consider that parameters to optimize energy efficiency of a quadruped walking robot is classified into two types. One type is defined as trajectory parameters which are kept constant during one cycle regular gait, and the other is instantaneous parameters which are independently determined at each moment of one cycle regular gait. Fig.2 shows parameters to define gait of a quadruped walking on the slope, weight and link parameters of a quadruped walking robot TITAN XII. We consider trajectory parameters to define crawl gait as follows:as x_f, y_f are standard foot positions of fore legs, x_h, y_h are standard foot positions of hind legs, λ is the stroke of supporting legs, h is body height from the slope, θ_B is body angle of inclination, and β is duty factor of crawl gait.

The sequence to decide parameters of the slope walking motion is considered as below. First, body height h and body angle of inclination θ_B is determined so that reachable area of each leg on the contact plane is decided. Second, standard foot positions are determined with satisfying the constraint which stability margin becomes positive. Then trajectory of



(a) Side view



(b) Top view

Fig. 2. Trajectory parameters of the slope walking motion

the robot is decided. Finally we have to decide instantaneous parameters, body velocity and foot forces at each moment on the trajectory. Instantaneous parameters are decided at each moment during one cycle regular gait and current instantaneous parameters don't influence on the determination of next instantaneous parameters. Therefore, the energy efficiency of current one cycle regular gait is optimized by optimizing instantaneous parameters at each moment. To optimize trajectory parameters, optimization result of instantaneous parameters on current trajectory parameters is required. For this reason, optimization method of the energy efficiency by instantaneous parameters is required first when we consider entire optimization of the slope walking motion by a quadruped walking robot.

In next section, we will formulate the method to optimize energy efficiency of a quadruped walking robot during slope walking motion by optimizing instantaneous parameters.

III. Formulation

A. Assumption

In this paper, we assume model of a quadruped walking robot as follows. The robot has massless legs and the mass of the robot concentrates on the center of the body. Each leg of the robot requires energy consumption during only supporting phase. One leg of the robot is composed of three active joints and the ankle does not consume power though TITAN XII is equipped with active ankles. We assume quasi-static walking and consider only static forces and moments. Note that i means the number of supporting leg and j means the number of joint.

B. Equilibrium Equations

Equilibrium of forces affect to the robot is given below, where the gravitational acceleration g, the mass of the robot m, slope angle of inclination θ_s, number of supported leg N (3 or 4):

$$\sum_{i=1}^{N} f_{xi} = mg\sin\theta_s \tag{1}$$

$$\sum_{i=1}^{N} f_{yi} = 0 \tag{2}$$

$$\sum_{i=1}^{N} f_{zi} = mg\cos\theta_s \tag{3}$$

where, f_{xi}, f_{yi}, f_{zi} is each component of foot force of supporting leg i.

In the same way, equilibrium of moments affect to the robot is given as follows:

$$\sum_{i=1}^{N} (y_i f_{zi} + z_i f_{yi}) \quad = 0 \tag{4}$$

$$\sum_{i=1}^{N} (z_i f_{xi} + x_i f_{zi}) \quad = 0 \tag{5}$$

$$\sum_{i=1}^{N} (x_i f_{yi} + y_i f_{xi}) \quad = 0 \tag{6}$$

where, x_i, y_i, z_i is foot position of supporting leg i.

Relation between output velocity $\boldsymbol{v_i}$ and joint angular velocity $\boldsymbol{\omega_i}$ of supporting leg i is given from differential relation between joint and leg's end point.

$$\boldsymbol{\omega_i} = \boldsymbol{J_i}^{\boldsymbol{-1}}\boldsymbol{v_i} \tag{7}$$

where, $\boldsymbol{J_i}^{\boldsymbol{-1}}$ is inverse of jacobian matrix of supporting leg i. Note that $\boldsymbol{v_i} = -\boldsymbol{V}$ during slope walking motion, where $\boldsymbol{V}$ is body velocity vector.

Relation between output force of the leg $\boldsymbol{f_i}$ and joint torque $\boldsymbol{\tau_i}$ of supporting leg i is given from the principle of virtual work.

$$\boldsymbol{\tau_i} = \boldsymbol{J_i}^{\boldsymbol{T}}\boldsymbol{f_i} \tag{8}$$

where, $\boldsymbol{J_i}^{\boldsymbol{T}}$ is transpose of jacobian matrix of supporting leg i.

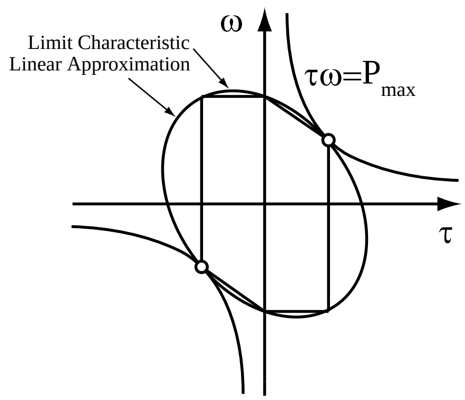


Fig. 3. Limit characteristic of DC motor and its linear approximation

C. Consideration of Friction Constraints

To discuss internal forces loaded at the foot, it is necessary to consider constraints determined by static friction forces. This limitation range is generally called friction cone. Within this cone, foot forces can be generated with no slippage at the foot. Here, we approximate friction cone as quadrangular pyramid for later optimization algorithm.

$$|f_{xi}| \leq \frac{\mu}{\sqrt{2}}|f_{zi}| \quad (9)$$

$$|f_{yi}| \leq \frac{\mu}{\sqrt{2}}|f_{zi}| \quad (10)$$

where, μ is the coefficient of static friction.

Since walking robots cannot generate adhesion forces at the foot, vertical foot forces f_{zi} of supporting leg i must satisfy below limitation.

$$f_{zi} < 0 \quad (11)$$

D. Actuator Model

Next, we will consider mathematical model of installed actuators. In this paper, we consider actuator as DC motor which is commonly used for robots. DC motor cannot be driven continuously over the heat threshold because of structural limits. By using equivalent circuit model of DC motor, this continuous driving range is represented as inside of a ellipse written as follows [3][11]:

$$\frac{R_a}{K^2}\frac{1}{\xi^2}{\tau_{ij}}^2 + \frac{2R_a}{R_h}\tau_{ij}\omega_{ij} + \frac{K^2}{R_h}\left(\frac{R_a}{R_h}+1\right)\xi^2{\omega_{ij}}^2 \leq \frac{2P_{\max}}{\sqrt{1+R_h/R_a}-1} \quad (12)$$

Here, joint torque τ_{ij}, joint angular velocity ω_{ij}, representing resistance of copper loss R_a, representing resistance of iron and windage loss R_h, torque constant K, maximum output power $P_{\max}$, and reduction ratio of each joint ξ. In order to simplify the equation of heat limitation of DC motor, we approximate heat limitation ellipse as a hexagon [12]. By this approximation, the heat limitation is written as three linear inequality constraints below:

$$\left|\frac{\tau_{ij}}{\xi}\right| \leq \tau_{P_{\max}} \quad (13)$$

$$|\xi\omega_{ij}| \leq k\omega_{P_{\max}} \quad (14)$$

$$\left|(1-k)\frac{\omega_{P_{\max}}}{\tau_{P_{\max}}}\frac{\tau_{ij}}{\xi} + \xi\omega_{ij}\right| \leq k\omega_{P_{\max}} \quad (15)$$

where $\tau_{P_{\max}}$ is maximum torque, $\omega_{P_{\max}}$ is maximum angular velocity, and k is a parameter.

Fig.3 shows limit characteristic of DC motor and its linear approximation.

E. Evaluation Function

We use specific resistance to evaluate the efficiency of locomotion. Specific resistance ϵ [13] is dimensionless quantity which expresses the energy required for moving unit distance and used widely in robotics research.

$$\epsilon = \frac{E}{mg{\cdot}L} \quad (16)$$

Here, E is the required energy for locomotion, m is the mass of the robot, g is gravitational acceleration, and L is a traveling distance. It is possible to rewrite the above equation with a power consumption P and velocity V by differentiating both numerator and denominator as follows.

$$\epsilon = \frac{dE/dt}{mg{\cdot}dL/dt} = \frac{P}{mg{\cdot}V} \quad (17)$$

The smaller specific resistance indicates the higher energy efficiency. The supplied power from power source to installed DC motors P_{in} is described by the summation of mechanical consumed power P_{mech} and dissipated heat P_{heat}.

$$P_{in} = P_{mech} + P_{heat} \quad (18)$$

Note that power consumption P in Eq.(17) is equivalent to total supplied power to whole installed DC motors P_{in} in Eq.(18).

It is difficult to install regeneration systems in walking robots generally because it is heavy and complicated, thus negative power consumption is not regenerated and dissipated as heat. For this reason, total mechanical power consumption at joint P_{mech} is given as below:

$$P_{mech} = \sum_{i=1}^{N}\sum_{j=1}^{3} a_{ij}\tau_{ij}\omega_{ij} \quad (19)$$

$$a_{ij} = \begin{cases} 1, & \tau_{ij}\omega_{ij} \geq 0 \\ 0, & \tau_{ij}\omega_{ij} < 0 \end{cases}$$

Then DC motor heat loss is described in left side of Eq.12. Therefore total dissipated heat P_{heat} is,

$$P_{heat} = \sum_{i=1}^{N}\sum_{j=1}^{3}\left[\frac{R_a}{K^2}\frac{{\tau_{ij}}^2}{\xi^2} + \frac{2R_a}{R_h}\tau_{ij}\omega_{ij} + \frac{K^2}{R_h}\left(\frac{R_a}{R_h}+1\right)\xi^2{\omega_{ij}}^2\right] \quad (20)$$

By using output power of the robot P_{out}, mechanical consumed power P_{mech} is rewritten as follows [14]:

$$P_{mech} = \frac{P_{out} + \sum_{i=1}^{N}\sum_{j=1}^{3} |\tau_{ij}\omega_{ij}|}{2} \quad (21)$$

Here, P_{out} is equal to the increase of potential energy per unit time.

$$P_{out} = mgV \sin\theta_S \quad (22)$$

where, V is body velocity.

F. Optimization Variables

We consider variables as joint torque τ_{ij} and body velocity V. Joint angular velocity ω_{ij} is written by body velocity V and $\hat{\omega_{ij}}$ which is joint angular velocity to generate unit velocity vector at the foot as below.

$$\omega_{ij} = V\hat{\omega}_{ij} \quad (23)$$

$\hat{\omega}_{ij}$ can be considered as an invariable because direction of output velocity of the leg and jacobian matrix $\boldsymbol{J}$ is decided.

Evaluation function includes absolute value of mechanical joint power so that we introduce new non-negative parameter p_{ij}.

$$p_{ij} = |\tau_{ij}\hat{\omega}_{ij}| \quad (24)$$

Since it is difficult to deal with this equality constraint directly in optimization algorithm, this equality constraint is transformed into two equivalent inequality constraints.

$$\tau_{ij}\hat{\omega}_{ij} \leq p_{ij} \quad (25)$$

$$-p_{ij} \leq \tau_{ij}\hat{\omega}_{ij} \quad (26)$$

When we try to minimize p_{ij} subject to Eq.(25) and (26), optimized parameter satisfies Eq.(24).

Finally, vector of optimization variables $\boldsymbol{x}$ are described below.

$$\boldsymbol{x} = \begin{bmatrix}\tau_{11} & \ldots & \tau_{N3} & p_{11} & \ldots & p_{N3} & V\end{bmatrix}^T \quad (27)$$

G. Optimization Algorithm

Specific resistance ϵ is a fractional function, and optimization problem whose evaluation function $f(\boldsymbol{x})$ is expressed as fractional function is classified into fractional programming.

$$f(x) = \frac{N(\boldsymbol{x})}{D(\boldsymbol{x})} \quad (28)$$

Here, $N(\boldsymbol{x})$ is numerator function and $D(\boldsymbol{x})$ is denominator function. We solve this optimization problem of specific resistance by dinkelbach's method [15]. First, convergence judgement function $g(q_k)$ is defined by using a new parameter q_k as below.

$$g(q_k) = N(\boldsymbol{x}) - q_k D(\boldsymbol{x}) \quad (29)$$

Optimization algorithm of fractional programming is described as follows:

1) Set $q_1 = 0, k = 1$ and go to step2.
2) Set q_k in convergence judgement function $g(q_k)$, and optimize $g(q_k)$ by parameters $\boldsymbol{x}$. After the optimization, optimal parameters $\boldsymbol{x_k}$ are obtained.
3) If $g(q_k) = 0$, set $\boldsymbol{x_0} = \boldsymbol{x_k}$, $q_0 = q_k$ and finish.
4) Else if $g(q_k) > 0$, set $q_{k+1} = N(\boldsymbol{x_k})/D(\boldsymbol{x_k})$, $k = k+1$ and return to step2.

Note that $g(q_k)$ always becomes non-negative in this algorithm [15].

q_0 is minimum value of original evaluation function $f(\boldsymbol{x})$ and $\boldsymbol{x_0}$ is optimal parameters. In this algorithm, we have to optimize $g(q_k)$ by some optimization method in step2. The evaluation function in step2 is described as below:

$$g(q_k) = \frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{c^T}\boldsymbol{x} \quad (30)$$

Since $g(q_k)$ is expressed as quadratic function of parameters $\boldsymbol{x}$, step2 corresponds to solving the quadratic programming(QP).

The detail of matrix $\boldsymbol{Q}$ and vector $\boldsymbol{c}$ are written in Appendix. As matrix $\boldsymbol{Q}$ is positive semidefinite, convergence judgement function $g(q_k)$ is convex and global optimal parameters can be obtained. We consider equality constraints as Eq.(1)∼(6) and inequality constraints as Eq.(9)∼(11), Eq.(13)∼(15), and Eq.(25), (26) for the QP.

TABLE I

DC MOTOR PARAMETERS

Physical quantity	Symbol	Value
Representing resistance of copper loss	R_a	1.16[Ω]
Representing resistance of iron and windage loss	R_h	667[Ω]
Torque constant	K	0.0603[Nm/A]
Maximum output power	$P_{\max}$	150[w]
Reduction ratio	ξ	600:1
Maximum torque	$\tau_{P_{\max}}$	0.3[Nm]
Maximum angular velocity	$\omega_{P_{\max}}$	773[rad/s]
Heat limitation parameter	k	1.5

IV. Numerical Simulation

Here, we calculate specific resistance when the robot walks on the slope by crawl gait during one cycle. In this simulation, it is assumed that the simulation model has the same weight and size as TITAN XII. Table.I shows the parameters of installed motors in the simulation. Joint 1 and 3 are driven by one DC motor and Joint 2 is by two DC motors. Thus total consumed power during slope walking motion is calculated from consumed power of 16 motors in total.

To verify the effectiveness of proposed method, we calculated specific resistance by two methods. First one is the proposed method which was explained in Section 3 in detail. Second one is normal method which is described as follows. Foot forces are calculated in consideration of only equality constraints and internal foot forces are not generated. In case of three-leg-supporting, foot forces are decided uniquely. In case of four-leg-supporting, foot forces are decided to minimize square sum of them. Body velocity is calculated from

TABLE II

PARAMETERS FOR SLOPE WALKING MOTION

Physical quantity	Symbol	Value
Slope angle of inclination	θ_S	10[deg]
Body angle of inclination	θ_B	10[deg]
Duty factor	β	0.85
Stroke of supporting leg	λ	0.2[m]
Static friction coefficient	μ	0.8
Standard position in x coordinate of fore leg	x_f	0.4[m]
Standard position in x coordinate of hind leg	x_h	0.55[m]

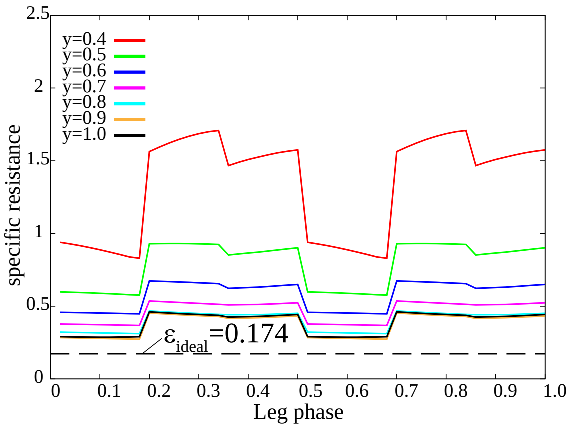


Fig. 4. Specific resistance derived by proposed method

average of optimal velocity of proposed method. Table.II shows trajectory parameters of the slope walking motion in this simulation. Here, standard position in y coordinate of fore and hind leg is the same. Standard position in y coordinate is changed from 0.4∼1.0 to investigate change of optimal parameters by walking posture. Simulations in this section are analyzed by MATLAB and we solved QP by QP solver of Optimization Toolbox of MATLAB.

Simulation results of specific resistance are shown in Fig.4 and Fig.5. Each broken line in Fig.4 and Fig.5 shows ideal specific resistance ϵ_{ideal} for this slope walking motion, and

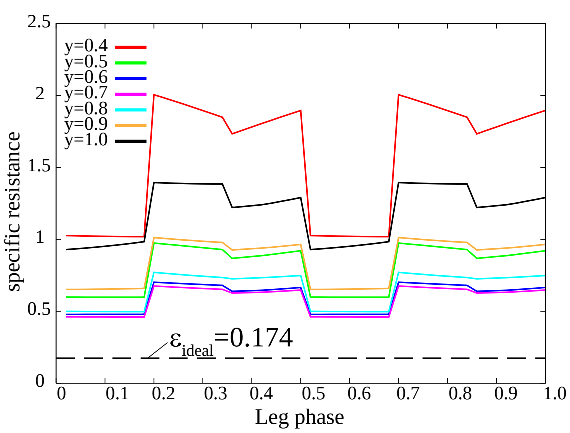


Fig. 5. Specific resistance derived by normal method

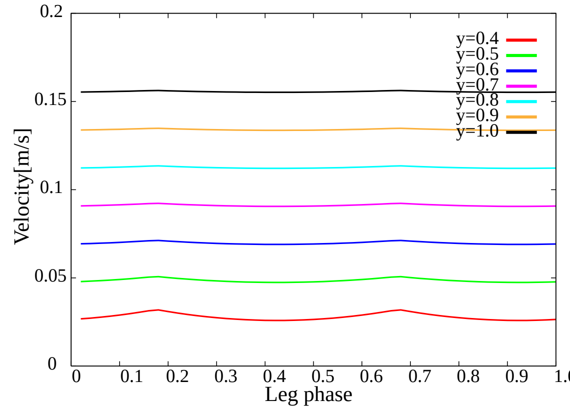
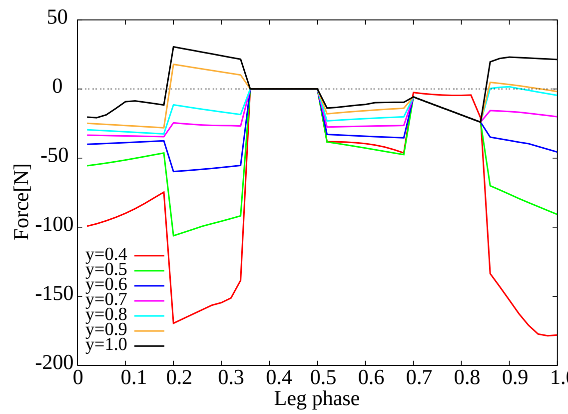
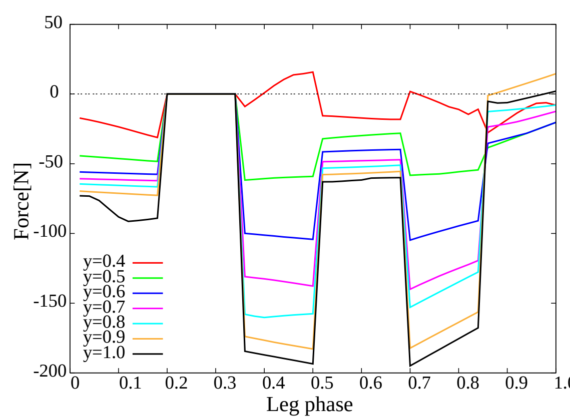


Fig. 6. Optimal body velocity V



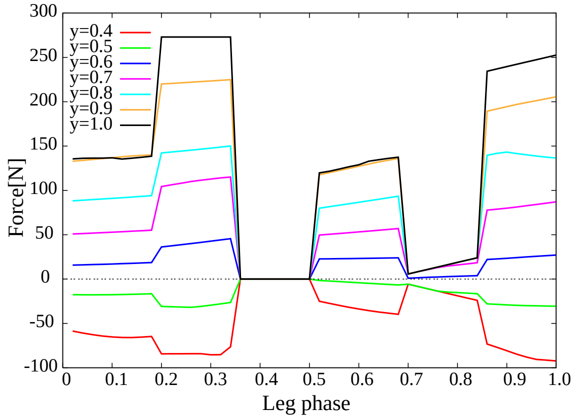
(a) Optimal foot force f_{x1}



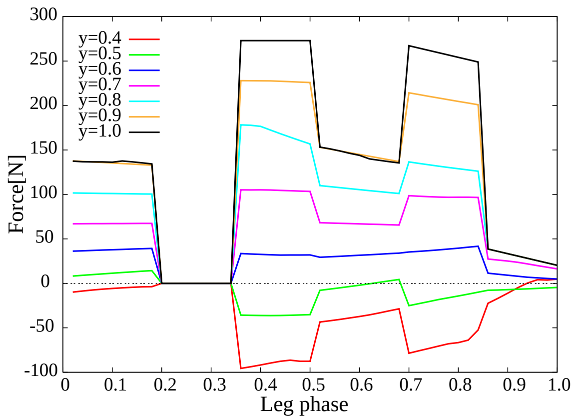
(b) Optimal foot force f_{x2}

Fig. 7. Optimal foot force f_x

it is equal to $\sin\theta_s$ which is derived from Eq.(17) and Eq.(22). As shown in Fig.4, specific resistance ϵ derived by proposed method is large where the leg is close to the body. Then, it becomes small as the leg width y get larger during y=0.4∼0.9. Finally, it becomes large where y = 1.0.

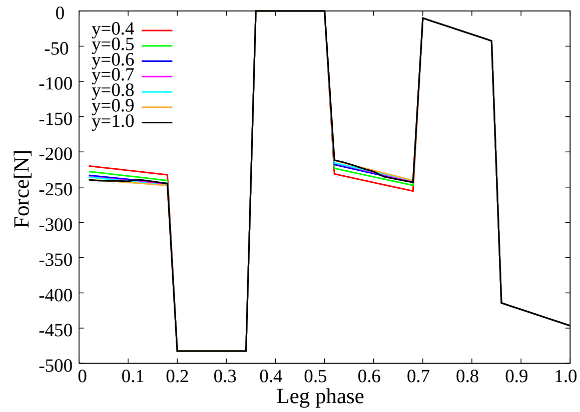


(a) Optimal foot force f_{y1}

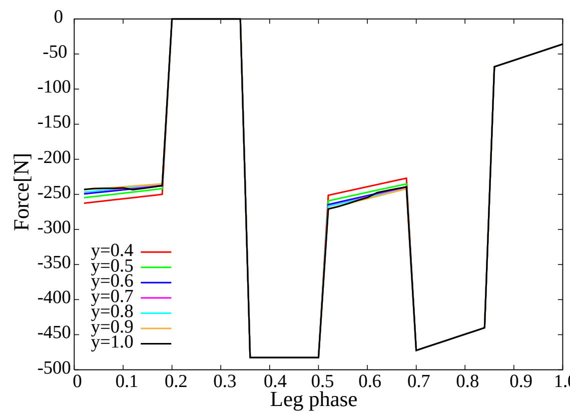


(b) Optimal foot force f_{y2}

Fig. 8. Optimal foot force f_y



(a) Optimal foot force f_{z1}



(b) Optimal foot force f_{z2}

Fig. 9. Optimal foot force f_z

As shown in Fig.5, specific resistance ϵ derived by normal method is large where the leg is close to the body. Then, it becomes small as the leg width y get large during y = 0.4∼0.7. Finally, it becomes large as the leg width y get larger during y=0.8∼1.0. The difference of each result is small where y is close to 0.5∼0.6 and difference becomes large where y is larger than 0.7.

We consider characteristics of optimal body velocity derived by proposed method. Fig.6 shows body velocity V optimized by proposed method. As the leg width y becomes large, body velocity V becomes large in Fig.6. This is because Joint 1 is driven with a maximum rate and distance between Joint 1 and foot become large.

We also consider characteristics of optimal foot forces derived by proposed method. Fig.7∼9 shows change of foot forces and only forces of Leg 1 and Leg 2 are shown because of symmetry on either side of walking per one cycle crawl gait. f_x and f_y is greatly changed by leg width y. However, f_z is almost same in each y. The reason of above results about optimal foot forces are considered as follows. If the leg width y get larger, each leg are more extended. Then joint moment arms and joint torques can be small by generating internal foot forces appropriately. Therefore, decrease in joint torques is considered to be the main causes of decrease in specific resistance ϵ where y is large as shown in Fig.4.

We confirmed that specific resistance was improved 28.9% on average by proposed method compared with normal method, thus effectiveness of proposed method was verified although improvement rate was greatly changed by walking posture.

V. Conclusions

In this paper, basic strategy to optimize the energy efficiency of the slope walking motion by a quadruped walking robot was considered and the problem was classified into two problems. The method to optimize the specific resistance which is the criterion of energy efficiency directly in consideration of characteristics of actuators and formulated as a nonlinear programming. Finally, the effectiveness of the proposed method was verified by the numerical simulation.

For future works, it is necessary to develop optimization method to optimize trajectory parameters of the slope walking motion. Moreover, validity of simulation results have to be verified by the experiment using a quadruped walking robot TITAN XII.

Appendix

Matrix $\boldsymbol{Q}$ and vector $\boldsymbol{c}$ in Eq.(30) are described as follows:

$$\boldsymbol{Q} = \begin{bmatrix} \frac{2R_a}{\xi^2 K^2} & 0 & \cdots & \cdots & \cdots & 0 & \frac{2R_a}{R_h}\xi\hat{\omega}_{11} \\ 0 & \ddots & \ddots & & & \vdots & \vdots \\ \vdots & \ddots & \frac{2R_a}{\xi^2 K^2} & 0 & \cdots & 0 & \frac{2R_a}{R_h}\xi\hat{\omega}_{N3} \\ \vdots & & 0 & 0 & \cdots & 0 & \frac{1}{2} \\ \vdots & & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \frac{1}{2} \\ \frac{2R_a}{R_h}\xi\hat{\omega}_{11} & \cdots & \frac{2R_a}{R_h}\xi\hat{\omega}_{N3} & \frac{1}{2} & \cdots & \frac{1}{2} & \sum_{i=1}^{N}\sum_{j=1}^{3}\frac{K^2}{R_h}\left(\frac{R_a}{R_h}+1\right)\xi^2\hat{\omega}_{ij}^2 \end{bmatrix} \quad (31)$$

$$\boldsymbol{c} = \begin{bmatrix} 0 & \ldots & 0 & mg\left(\frac{1}{2}\sin\theta_S - q_k\right) \end{bmatrix} \quad (32)$$

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