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**PREDICTIVE EQUATION APPLICABLE
TO HIGH STRENGTH CONCRETE FOR DIAGONAL
COMPRESSIVE CAPACITY OF RC BEAMS**

by

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A dissertation submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy

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ABSTRACT

For the past decade, the increase in the rate of global warming has led structural engineers to focus on verifying the ultimate limit resistance of concrete structures as well as their durability and sustainability. High strength concrete (HSC) with the compressive strength (f'_c) more than 100 N/mm² and high strength reinforcing bars with yield strength (f_y) more than 685 N/mm² have recently been developed. The utilization of such advanced materials in construction practice can be one of the approaches used to reduce CO₂ emission due to construction activities and to achieve sustainable communities. HSC results in lower material consumption (smaller cross section and number of spans), less construction activities, and longer service life of structures, while maintaining a remarkable structural performance. To fully utilize the advantage of high strength properties, the combination of thin web (T- or I-shaped cross section) HSC beams and high strength stirrups is introduced. However, problems have been arisen as a result of excessive shear reinforcement. This combination will lead to diagonal compression failure, that is, an uncommon type of shear failure caused by the crushing of web concrete prior to stirrup yielding. This type of failure is hazardous, not only because of its brittle behavior at failure, but also the reason that diagonal compressive capacity of reinforced concrete (RC) beams using high strength materials could be less than the typical shear capacity, which is assumed to be the summation of the contributions of concrete and shear reinforcements, leading to an unsafe design of RC structures.

The research on the mechanism of diagonal compression failure is insufficient since it is usually avoided in design. There are some concerns regarding the applicability of the existing predictive equation for diagonal compressive capacity to high strength concrete since most of them were based on experimental results of beam specimens with f'_c less than 40 N/mm². The Standard Specifications for Concrete Structures of Japan Society of Civil Engineers (JSCE) for the diagonal compressive capacity provides inadequate accuracy, only considers f'_c as an influential factor and limits the applicability to concrete with f'_c up to 50 N/mm², resulting in extravagant and inefficient designs of RC beams. Therefore, it is necessary to study and develop a new precise equation applicable to high strength concrete for evaluating the diagonal compressive capacity of RC beams.

This research aims to investigate the effect of various parameters on the diagonal compressive capacity of HSC beams and develop an accurate and simple predictive equation applicable to high strength concrete for the diagonal compressive capacity of RC beams. This research will enable the wider use of high strength materials for the sustainability and the

development of rational designs for the diagonal compressive capacity. An experimental study on the diagonal compressive capacity of HSC beams was conducted. Verification on the effect of compressive strength, stirrup (ratio, diameter, spacing and configuration), flange width to web width ratio, and effective depth was performed.

As a result, the diagonal compressive capacity linearly decreased with wider stirrup spacing regardless of its diameters and configuration when stirrup spacing was narrower than 140 mm; however, the effect of stirrup spacing became negligible when stirrup spacing was wider than 140 mm. With insufficient confinement effects provided by stirrups, the localization of compressive stress in struts occurred. Thus, stirrup spacing is prominent, not stirrup ratio, for evaluating the diagonal compressive capacity of RC beams. However, even when wide stirrup spacing was used, diagonal compressive stress did not concentrate in local area of the specimens and a sufficient confinement effect was achieved by providing two-legged stirrups, and the localization of compressive stress in struts was prevented. Furthermore, the effect of stirrup spacing became more significant with higher compressive strength of concrete. Thus, the effect of compressive strength and stirrup spacing was interrelated. On the other hand, there were slight effects of flange width to web width and effective depth on the diagonal compressive capacity RC beams. Furthermore, there is no clear relationship between the average diagonal crack angle in B-region and compressive strength, stirrup ratio, stirrup spacing, and shear span to effective depth ratio. The angle of principal strain in B-region had no tendency with stirrup ratio, spacing, diameters and configuration.

Subsequently, a predictive equation considering the compressive strength and stirrup spacing for evaluating the diagonal compressive capacity of RC beams was developed on the basis of the experimental findings. The proposed equation covered the application to RC beams with compressive strength up to 165 N/mm^2 and practical stirrup spacing ($45 \text{ mm} \leq s \leq 370 \text{ mm}$). The accuracy of the existing equations for diagonal compressive capacity by Placas et al., JSCE Standard Specifications, ACI Building Code, Eurocode 2 and *fib* Model Code 2010 were evaluated. The existing equations should be improved in both accuracy and appropriateness on the prediction of the diagonal compressive capacity of RC beams. The proposed equation had adequate simplicity and offers precise accuracy ($Avg. = 0.99$) with low variation ($C.V. = 10.5\%$) on the prediction of the diagonal compressive capacity of RC beams. The proposed equation can be implemented to beams with the compressive strength: 19-165 N/mm^2 , stirrup ratio: 0.6-4%, stirrup spacing: 45-370 mm, flange width to web width ratio: 3.75-12.5, shear-span to effective depth ratio: 2.5-4.5, effective depth: 220-563 mm, web width: 40-150 mm and one-legged, two-legged and closed-typed stirrups.

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Chapter 1

Introduction

1.1 General background

For the past decade, the increase in the rate of global warming has led structural engineers to focus on verifying the ultimate limit resistance of concrete structures as well as their durability and sustainability. High strength concrete (HSC) with the compressive strength (f'_c) more than 100 N/mm² and high strength reinforcing bars with yield strength (f_y) more than 685 N/mm² have recently been developed [1]. The utilization of such advanced materials in construction practice can be one of the approaches used to reduce CO₂ emission due to construction activities and to achieve sustainable communities. HSC results in lower material consumption (smaller cross section and number of spans), less construction activities, and longer service life of structures, while maintaining a remarkable structural performance. To fully utilize the advantage of high strength properties, the combination of thin web (T- or I-shaped cross section) HSC beams and high strength stirrups is introduced. However, problems have been arisen as a result of excessive shear reinforcement. This combination will lead to diagonal compression failure, that is, an uncommon type of shear failure caused by the crushing of web concrete prior to stirrup yielding. This type of failure is hazardous, not only because of its brittle behavior at failure, but also the reason that diagonal compressive capacity of reinforced concrete (RC) beams using high strength materials could be less than the typical shear capacity, which is assumed to be the summation of the contributions of concrete and shear reinforcements, leading to an unsafe design of RC structures.

The research on the mechanism of diagonal compression failure is insufficient since it is usually avoided in design. There are some concerns regarding the applicability of the existing predictive equation for diagonal compressive capacity to high strength concrete since most of them were based on experimental results of beam specimens with f'_c less than 40 N/mm². The Standard Specifications for Concrete Structures of Japan Society of Civil Engineers (JSCE) [2] for the diagonal compressive capacity provides inadequate accuracy,

only considers f'_c as an influential factor and limits the applicability to concrete with f'_c up to 50 N/mm², resulting in extravagant and inefficient designs of RC beams. Therefore, it is necessary to study and develop a new precise equation applicable to high strength concrete for evaluating the diagonal compressive capacity of RC beams.

At present, only a few studies have been performed on the diagonal compressive capacity of high strength RC beams with the exception of the research by Kobayashi [3]. Experiments of RC I-beams were carried out to investigate the effect of various parameters on the diagonal compressive capacity. They reported that the factors affecting the diagonal compressive capacity are f'_c , stirrup ratio (r_w), flange width to web width ratio (b_f/b_w) and stirrup spacing (s). The effect of shear span to effective depth ratio (a/d) was found to have less influence. The ranges of their study are f'_c : 35-165 N/mm², r_w : 2-4%, s : 45-160 mm, a/d : 3.0-4.5, b_f/b_w : 6.25, and d : 220 mm. In regard to some factors, e.g. f'_c , r_w and s , there is a lack of experimental data especially in the range $f'_c > 100$ N/mm², $r_w < 2\%$ and $s > 160$ mm. Nevertheless, the conclusion, which b_f/b_w affected the diagonal compressive capacity, was mathematically derived by excluding the effect of f'_c , r_w , s and a/d from the total diagonal compressive capacity without experimental confirmation. As well, the study by Kobayashi [3] neglected the effect of effective depth (d) or size effect, which is a well known negative effect on shear capacity of RC beams. From these reasons, further research on the diagonal compressive capacity of RC beams is required.

1.2 Objectives and scopes of this study

Based on the reasons mentioned in the general background (section 1.1), the main objectives of this research are to investigate the effect of various parameters on the diagonal compressive capacity of HSC beams and develop an accurate and simple predictive equation applicable to high strength concrete for the diagonal compressive capacity of RC beams. This research will enable the wider use of high strength materials for the sustainability and the development of rational designs for the diagonal compressive capacity. An experimental study on the diagonal compressive capacity of HSC beams is conducted. Verification on the effect of compressive strength, stirrup (ratio, diameter, spacing and configuration), flange width to web width ratio, and effective depth is performed. Subsequently, a predictive equation is proposed on the basis of experimental findings. Finally, the accuracy of existing equations reviewed in Chapter 2 on the prediction of the diagonal compressive capacity is verified and compared with the proposed equation.

1.3 Outline of the dissertation

The dissertation consists of six chapters and the contents of each chapter will be explained as the following.

Chapter 1 introduces the general background of diagonal compressive capacity of RC beams, along with the objectives and scope of this study. The brief explanation of the outline of the dissertation is also provided in this chapter.

In Chapter 2, the mechanical model of diagonal compressive capacity is explained. The existing equations for evaluating diagonal compressive capacity of RC beams by various researchers and organizations are reviewed. The brief summary of the study by Kobayashi [3] is also included.

The outline of experiments at the Structural Concrete Laboratory of the Department of Civil Engineering at Tokyo Institute of Technology is presented in Chapter 3. Specimen details, list of experimental cases, material properties, fabrication of specimens, testing procedures and measurement items are described in detail.

In Chapter 4, the experimental results obtained by the methodologies described in Chapter 3 are presented and discussed. The effect of stirrup, compressive strength of concrete, flange width to web width ratio and effective depth on diagonal compressive capacity is investigated.

A new predictive equation on the basis of the experimental findings for evaluating diagonal compressive capacity of HSC beams is proposed in Chapter 5. Subsequently, the accuracy of the existing equations reviewed in Chapter 2 for predicting the diagonal compressive capacity is verified and compared with the proposed equation.

Finally, the general conclusions and recommendations for further study are given in Chapter 6.

2.1 Diagonal compression failure of RC beams

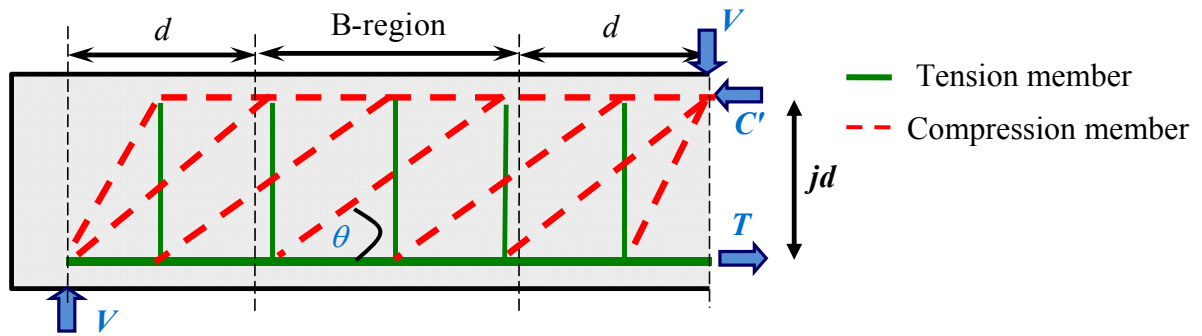


Fig. 2-1 Truss analogy

Shear resisting mechanism of RC slender beams ($a/d > 2.5$) can be represented by the truss analogy as presented in **Fig. 2-1**. The red dash lines are members in compression, while green lines are members in tension. The symbols in the figure are as follows:

V : Applied shear force

T : Tensile force of tensile bars

C' : Compressive force of concrete fiber

jd : Moment lever arm

θ : Angle of inclination of compressive struts to the longitudinal reinforcement

The concept of D-region and B-region was introduced by Schlaich and Weischede [4], [5] in 1982, where D stands for discontinuity or disturbed and B are beam or Bernoulli. In simply-supported beams subjected to one point loading as shown in **Fig. 2-1**, D-regions exist in the distance d away from the support and loading point and the region between D-regions are defined as B-region. In B-region or compression field, the uniform and parallel diagonal

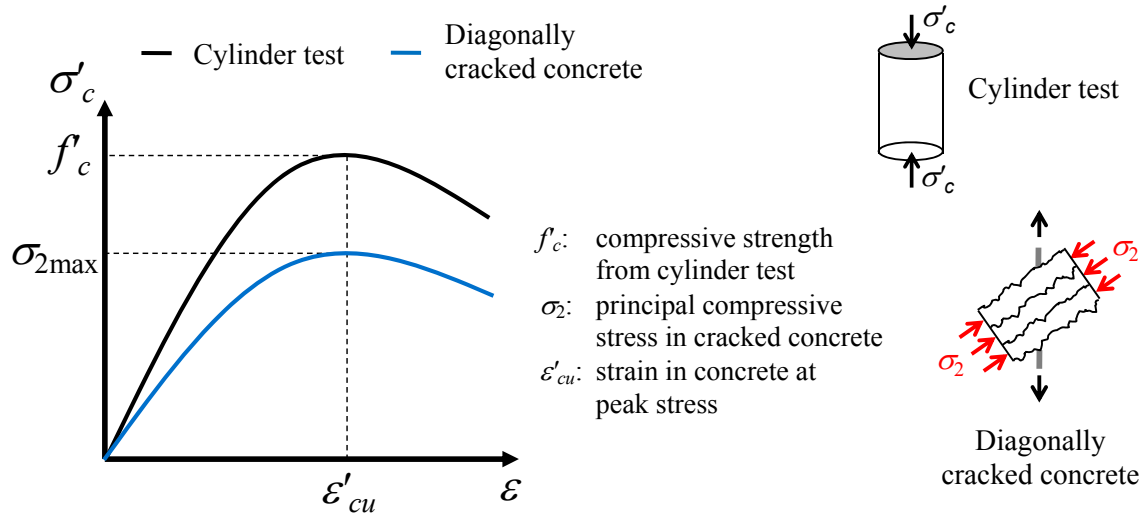


Fig. 2-2 Stress-strain relationship for cracked concrete in compression

compressive stress field takes place. This region is relatively weaker than D-region, and crushing of diagonal compression member usually occurs here in case of thin web RC beams having excessive stirrups. The shear failure of members due to the crushing of diagonal compression member in B-region prior to the yielding of stirrups is defined as the diagonal compression failure. In order to propose a simple predictive equation for the diagonal compressive capacity, it is essential to evaluate parameters affecting the strength of the diagonal compression members and their angle of inclination.

Vecchio and Collins [6] derived the stress-strain relationship for cracked concrete in compression by testing RC panels subjected to biaxial stress and pure shear. The relationship is schematically illustrated in **Fig. 2-2**. The strength of the diagonally cracked web concrete in compression tends to be less than the compressive strength of the same concrete obtained in standard cylinder test. It is because cracked concrete subjected to tensile strains in the direction perpendicular to the compression [6].

At present, the equations for the diagonal compressive capacity were provided by some researchers and design procedures throughout the world. The well-recognized design codes are published by Japan Society of Civil Engineers (JSCE), American Concrete Institute (ACI), European Committee for Standardization (CEN) and International Federation for Structural Concrete (*fib*). In 1971, Placas and Regan [7] proposed an empirical equation based on experimental study of RC beams with normal strength concrete. Only a few studies have been performed on the diagonal compressive capacity of high strength RC beams with the exception of the research by Kobayashi [3].

In the following sections, the features of equations by Placas et al. [7], JSCE Standard [2], ACI Building Code [8], Eurocode 2 [9], *fib* Model Code 2010, Final draft [10], as well as the study of Kobayashi [3] are reviewed.

2.2 Equation by Placas and Regan

Placas and Regan [7] studied the diagonal compressive capacity of RC beams with normal strength concrete and I-shaped cross section. They proposed an empirical equation for evaluating diagonal compressive capacity of RC beams as the following:

$$V_{Placas} = (1.04 + 0.21r_w)\sqrt{f'_c} b_w d \quad (2-1)$$

where; V_{Placas} is the diagonal compressive capacity predicted by the equation by Placas et al. [7].

f'_c : compressive strength of concrete (N/mm²)

b_w : web width (mm)

d : effective depth (mm)

r_w : stirrup ratio ($= 100 \cdot A_w / (b_w s)$) (%)

A_w : cross sectional area of stirrups

s : stirrup spacing

This equation considers the equilibrium conditions of a web crossed by shear cracks. Factors involving the diagonal compressive capacity in this equation are compressive strength of concrete (f'_c) and the stirrup ratio r_w (%). Although there is no upper limit of f'_c stated in this equation, the experimental evidences used to generate this equation are approximate 35 N/mm².

2.3 JSCE Standard Specifications

To avoid compression failure of web concrete without stirrup yielding, the design shear capacity in JSCE Standard Specifications [2] is limited to the diagonal compressive capacity. Only the effect of compressive strength f'_c is considered as the influential parameter of diagonal compressive capacity. Therefore, this equation is quite conservative and only valid

for concrete with f'_c not exceeding 50 N/mm².

$$V_{JSCE} = 1.25\sqrt{f'_c} b_w d \quad (2-2)$$

where; V_{JSCE} is the diagonal compressive capacity calculated by the design equation in JSCE Standard.

f'_c : compressive strength of concrete (N/mm²)

b_w : web width (mm)

d : effective depth (mm)

2.4 ACI Building Code 318M-08

American Concrete Institute (ACI) provides the design recommendation for shear in ACI building code (ACI 318M-08) [8]. The contribution of shear reinforcement (V_s) is limited to be $0.66f'_c{}^{1/2}b_w d$ for preventing from the crushing of concrete in web. Therefore, the maximum shear strength (which is limited by the web crushing) can be expressed as,

$$V_{ACI} = V_c + V_s \leq V_c + 0.66\sqrt{f'_c} b_w d \quad (2-3)$$

where; V_{ACI} is the diagonal compressive capacity calculated by the design equation in ACI 318M-08.

V_c is the contribution of concrete and is equal to $0.17f'_c{}^{1/2}b_w d$.

b_w : web width (mm)

d : effective depth (mm)

The validity of the ACI Code is limited to concrete having compressive strength not greater than 70 N/mm² because of a lack of test data and practical experience. The ACI code also considers only the effect of f'_c .

2.5 Eurocode 2 EN 1992-1-1:2004

European Standards are published by European Committee for Standardization (CEN). Eurocode 2 [9] or EC 2 are the abbreviations for Eurocode 2: Design for concrete structures.

In Eurocode 2, the variable strut inclination based on a truss model was used to design members with shear reinforcement. Designers can select the values of angel of concrete strut (θ) freely from from 21.8 to 45 degrees. In this dissertation, to study the most extreme cases, θ was assumed to be 45 degrees. Hence, the maximum shear force which can be sustained by the member, limited by the crushing of the compression struts, becomes the following:

$$V_{EC2} = 0.45v f'_c b_w d \quad (2-4)$$

where; V_{EC2} is the diagonal compressive capacity calculated by the design equation in Eurocode 2.

b_w : web width (mm)

d : effective depth (mm)

v is a coefficient that takes into account the increase of fragility and the reduction of shear transfer by aggregate interlock with the increase of f'_c . It may be taken to be as following:

$$v = \begin{cases} 0.6 & (f'_c \leq 60 \text{ N/mm}^2) \\ \max(0.9 - \frac{f'_c}{200}, 0.5) & (f'_c > 60 \text{ N/mm}^2) \end{cases} \quad (2-5)$$

2.6 *fib* Model Code 2010

The *fib* model code for Concrete Structures (MC 2010) published in April 2012 introduced the new concept of Levels of approximation [10]. Various levels of approximation are possible for the design and assessment of concrete structures depending on the accuracy and time requirements. The level I methods generally represent the simplest and most straight-forward approach but may results in the lowest accuracy. On the other hands, higher levels require more time and efforts but may lead to higher accuracy and more economical design.

In the design of members with shear reinforcement, four levels of approximation can be chosen as follows:

- I. A variable angle truss model approach

- II. A generalized stress field approach
- III. A general form of sectional shear equations based on the simplified modified compression field theory
- IV. Numerical analysis satisfying the applicable equilibrium conditions and compatibility of strains and using appropriate stress-strain models for the steel and for diagonally cracked concrete.

The design shear resistance of a web is also limited by the crushing of the compression strut in all Level of Approximation. In the case of diagonal compressive capacity, Level III approximation is determined as in the Level II. The basic equation for the diagonal compressive capacity is as follows:

$$V_{MC2010} = 0.9k_c f'_c b_w d \cos\theta \sin\theta \quad (f'_c: \text{N/mm}^2) \quad (2-6)$$

where; V_{MC2010} is the diagonal compressive capacity calculated by the design equation in Model Code 2010.

b_w : web width (mm)

d : effective depth (mm)

θ : the inclination of the compressive stresses field

k_c : the strength reduction factor, which consist of two parts:

$$k_c = k_\epsilon \eta_{fc} \quad (2-7)$$

where; η_{fc} : the effect of more brittle failure behavior of concrete of strength greater than 30 N/mm², which can be taken as

$$\eta_{fc} = (30/f'_c)^{1/3} \leq 1 \quad (f'_c \text{ in N/mm}^2). \quad (2-8)$$

k_ϵ : the state of strain in the webs of beams depending on the Level of Approximation

This study focuses on the accuracy of the existing equations on predicting the diagonal compressive capacity and proposes a new simple predictive equation; hence, Level IV is not in the scope of this study. Level II Approximation requires the iteration of strain state in the web until convergence is achieved. In this study, the simplest approximation (Level I) will be

evaluated.

In MC 2010, a free choice for the inclination of the compressive stresses (θ) between 30 to 45 degrees is allowed. In Level I Approximation, θ shall be taken as 30°. The following expression is obtained.

$$V_{MC2010} = 0.39k_c f'_c b_w d \quad (f'_c: \text{N/mm}^2) \quad (2-9)$$

$$k_c = 0.55(30/f'_c)^{1/3} \leq 0.55 \quad (f'_c: \text{N/mm}^2) \quad (2-10)$$

where; V_{MC2010} is the diagonal compressive capacity in Level I Approximation of Model Code 2010.

k_c : the strength reduction factor, which consist of two parts ($k_c = k_\epsilon \eta_{fc}$)

k_ϵ : the state of strain in the webs of beams = 0.55

2.7 Study by Kobayashi

Kobayashi [3] conducted an experimental study on the diagonal compression failure of RC beams with various compressive strength. He studied the effect of f'_c , stirrup ratio (r_w), stirrup spacing (s) and shear-span to effective depth ratio (a/d) on the diagonal compressive capacity.

Kobayashi also proposed an empirical equation for predicting diagonal compressive capacity as the following.

$$V_{Kob} = \frac{1}{2} \beta_a \beta_b \beta_v f'_{cs} b_w j d \sin 2\theta \quad (2-11)$$

where; V_{kob} is the diagonal compressive capacity predicted by the equation by Kobayashi.

β_a : effect of a/d

$$\beta_a = 3.4 - \frac{0.8a}{d} \geq 1 \quad (1 \leq \beta_a \leq 1.4) \quad (2-12)$$

β_b : effect of b_f/b_w

$$\beta_b = 0.25 \left(\frac{b_f}{b_w} \right)^{3/4} \quad (0.89 \leq \beta_b \leq 1.36) \quad (2-13)$$

β_v : reduction factor with the increase in f'_c

$$\beta_v = 0.65 \left(1 - \frac{f'_c}{550} \right) \quad (\beta_v \geq 0.46) \quad (2-14)$$

f'_{cs} : relationship between f'_c and s

$$f'_{cs} = (f'_c - 30) \left(1 - \frac{0.96s}{d} \right) + 30 \quad (30 < f'_c \leq 165 \text{ N/mm}^2) \quad (2-15)$$

$$f'_{cs} = f'_c \quad (f'_c \leq 30 \text{ N/mm}^2) \quad (2-16)$$

jd : moment lever arm = $(7/8)d$

θ : inclination angle of concrete struts to the longitudinal reinforcement, (degree) = 30°

b_w : web width (mm)

d : effective depth (mm)

This equation considered the effect of f'_c , s , a/d and b_f/b_w . It gives quite high accuracy

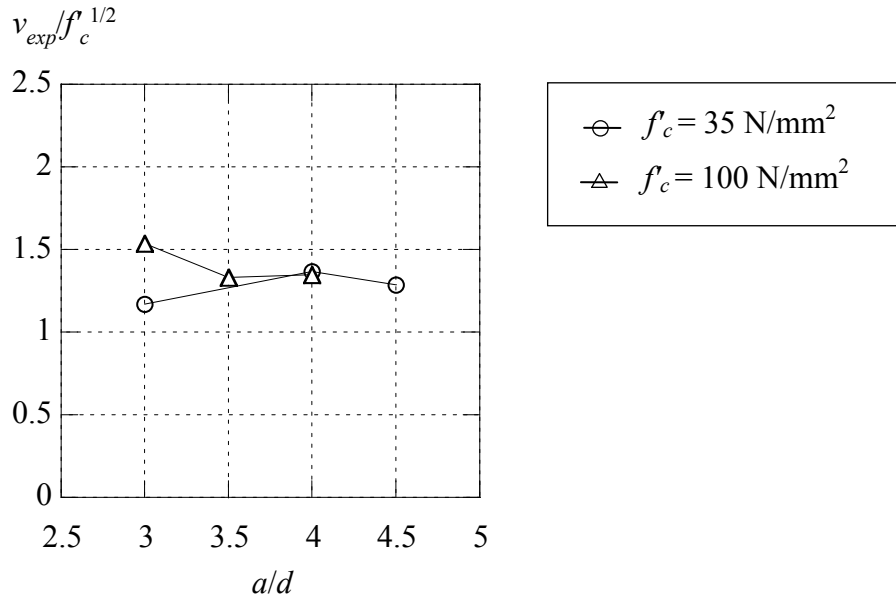


Fig. 2-3 Effect of a/d (Kobayashi [3])

for predicting diagonal compressive capacity and can be used for concrete strength up to 165 N/mm². The ranges of their study are f'_c : 35-165 N/mm², r_w : 2-4%, s : 45-160 mm, a/d : 3.0-4.5, b_f/b_w : 6.25, and d : 220 mm.

In regard to some factors, e.g. f'_c , r_w and s , there is a lack of experimental data especially in the range $f'_c > 100$ N/mm², $r_w < 2\%$ and $s > 160$ mm. Nevertheless, the conclusion, which b_f/b_w affected the diagonal compressive capacity, was mathematically derived by excluding the effect of f'_c , r_w , s and a/d from the total diagonal compressive capacity without experimental confirmation. Hence, his equation does not reflect the actual behavior of the specimen. As well, the study of Kobayashi [3] neglected the effect of effective depth (d) or size effect, which is a well known negative effect on shear capacity of RC beams.

Figure 2-3 presents the effect of a/d on the diagonal compressive capacity studied by Kobayashi [3]. As a longer span is more preferable in the practical use of high strength concrete girders, this research focused on the range of $a/d > 3.0$. The diagonal compressive capacity normalized by $f'_c{}^{1/2}$ shows slight variation; hence, the effect of a/d can be neglected.

Chapter 3

Experimental Program

This experimental program was carried out at the Structural Concrete Laboratory of the Department of Civil Engineering at the Tokyo Institute of Technology. The methodologies to obtain the test results, as well as specimen details, list of experimental cases, material properties, fabrication of specimens and measurement item are precisely described in this chapter.

3.1 Specimen details and experimental series

In this experimental program, 12 thin web RC beams with shear reinforcement were prepared. All beams were tested by three-point bending. Eight of them were carried out on a 2000 kN capacity hydraulic testing machine, while the test of the other four beams, which have relatively larger size, were conducted by a 3000 kN capacity loading apparatus. The summary of test variables and details of specimens are provided in **Fig. 3-1** and **Table 3-1**. **Figure 3-1(a)** shows the details of all specimens with $d = 220$ mm, while that of C-s100L is shown in **Fig. 3-1(b)**. The re-bars layout and cross section of all specimens having $d = 500$ mm is illustrated in **Fig. 3-1(c)**. The specimens were designed based on the study by Kobayashi [3] in order to discuss and compare with their results. The list of the experimental cases of Kobayashi [3] is given in **Table 3-2**. The effects of compressive strength of concrete (f'_c), stirrup ratio (r_w) with different diameter (ϕ_d) and spacing (s), flange width to web width ratio (b_f/b_w) and effective depth (d) were investigated. The experimental cases can be classified into four series as the following:

- I. The effect of stirrup (ratio, diameter, spacing and configuration) in high strength concrete ($f'_c \sim 100$ N/mm²) beams.
- II. The effect of compressive strength f'_c with different stirrup.
- III. The effect of flange width to web width ratio b_f/b_w .
- IV. The effect of effective depth d or size effect.

Table 3-1 List of the experimental cases

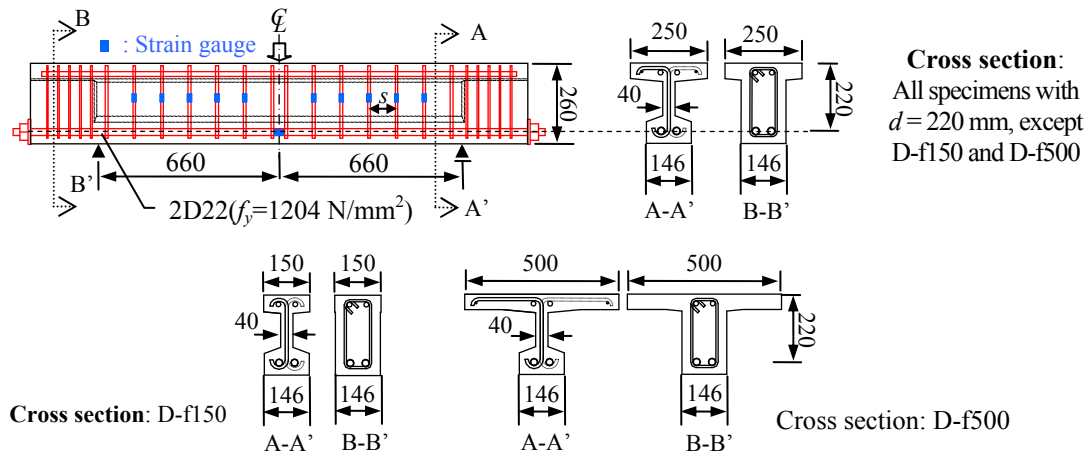
Specimen	f_c' [N/mm ²]	b_f [mm]	b_w [mm]	d [mm]	a [mm]	a/d	Tensile bars properties			Stirrup properties				
							p_w^{*2} [%]	D^{*3} [mm]	f_y [N/mm ²]	ϕ_d^{*4} [mm]	f_{wy}^{*5} [N/mm ²]	No. of legs	r_w^{*6} [%]	s^{*7} [mm]
C-s100-I	100	250	40	220	660	3.0	8.8	22.2	1204	9.53	953	One	1.8	100
C-s120-I													1.5	120
C-s150-I													1.2	150
C-s230-I		310	80	500	1500		8.9	22.2 (top) 25.4 (bottom)	1171	15.9	926	Two	1.08	230
C-s300-II										13.1	1368		1.06	300
C-s370-II(13)										15.9	916		0.88	370
C-s370-II(16)													1.34	
D-s160	130	250	40	220	660	8.8	22.2	930	12.7	955	One	2.0	160	
D-s90								3.5	90					
D-f150		150						1204	9.53	967		3.0	60	
D-f500		500							12.7	931		2.2	100	
C-s100L	100	343	58	319	957	8.6	31.8							

^{*1} upper flange width, ^{*2} longitudinal reinforcement ratio ($=100A_s/(b_w d)$), ^{*3} nominal diameter of tensile bars, ^{*4} nominal diameter of stirrups, ^{*5} yield strength of stirrups, ^{*6} stirrup ratio ($=100A_w/(b_w s)$), ^{*7} spacing of stirrups

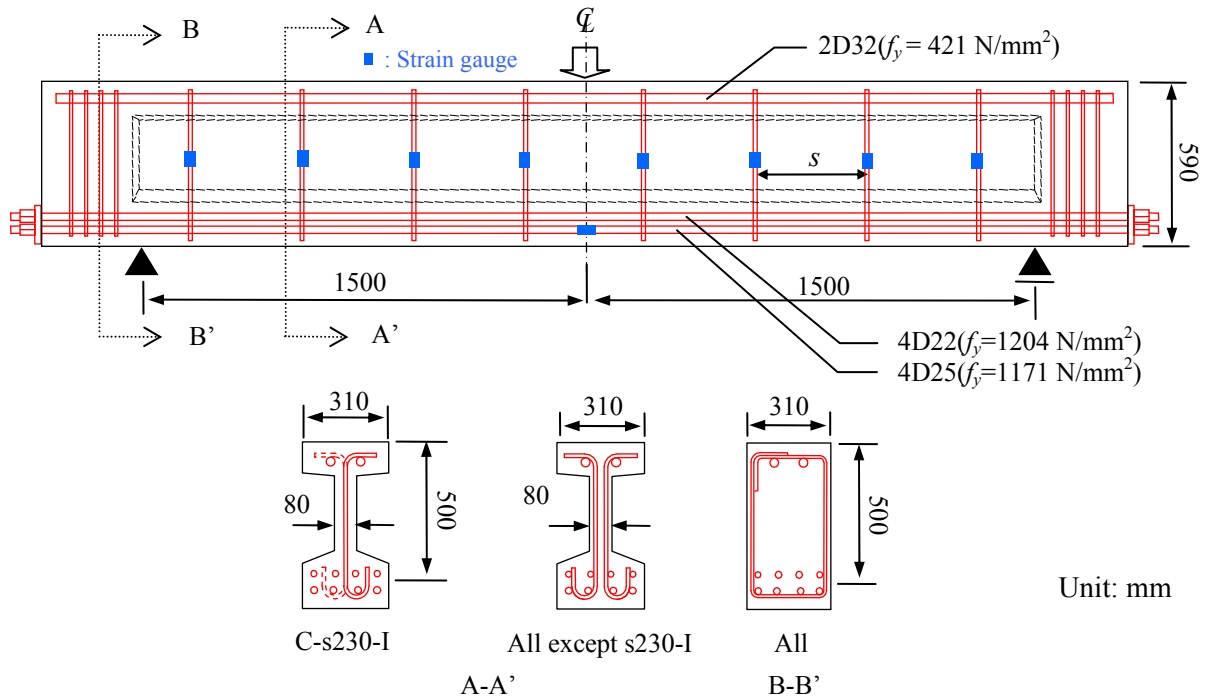
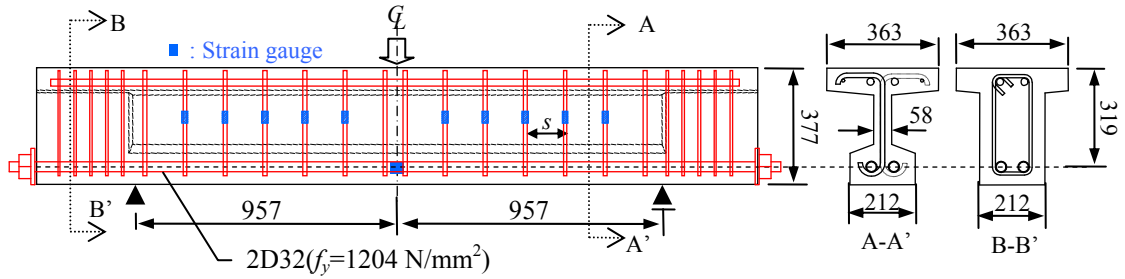
Table 3-2 List of the experimental cases of Kobayashi [3]

Specimen	f'_c [N/mm ²]	b_f [mm]	b_w [mm]	d [mm]	a [mm]	a/d	Tensile bars properties			Stirrup properties				
							p_w^{*2} [%]	D^{*3} [mm]	f_y [N/mm ²]	ϕ_d^{*4} [mm]	f_{wy}^{*5} [N/mm ²]	No. of legs	r_w^{*6} [%]	s^{*7} [mm]
A-s160(7)	35	250	40	220	660	3.0	8.8	22.2	1214	7.1	1425	One	0.63	160
A-s160(9)									930	9.0	1275		1.0	
A-s80													2.0	80
A-s55						2.9			55					
A-ad40						4.0			1198		1350		2.0	80
A-ad45						4.5							2.0	
B-s80	65					930			1275		2.0			
B-s55											2.9		55	
C2-s90-I	100					1198			9.53	954	2.0		90	
C2-s50-I									7.1	1397	3.0		50	
C2-s160-I									1214	12.7			955	160
C-s60-I						1198			9.53	954	4.0		60	
C-s45-I										953	4.0		45	
C-ad35										2.0	90			
C-ad40									4.0	954	3.0		60	
D-s60						130								
E-s60	165					3.0			1214	953				

^{*1} upper flange width, ^{*2} longitudinal reinforcement ratio ($=100A_s/(b_w d)$), ^{*3} nominal diameter of tensile bars, ^{*4} nominal diameter of stirrups, ^{*5} yield strength of stirrups, ^{*6} stirrup ratio ($=100A_w/(b_w s)$), ^{*7} spacing of stirrups



(a) All except C-s100L, C-s230-I, C-s300-II, C-s370-II(13), C-s370(16)



(c) C-s230-I, C-s300-II, C-s370-II(13), C-s370(16)

Fig. 3-1 Dimensions and re-bar layout of beam specimens

The constant variables in all series were longitudinal reinforcement ratio (p_w) of 8.6-8.9% and a/d of 3.0. In series I, due to the limitation of stirrup spacing with effective depth, the effective depth of four specimens (see **Fig. 3-1(c)**), which have $s > 160$ mm, was greatly enlarged than the others in order to provide wider stirrup spacing.

All specimens were designed to be symmetric and be able to resist against the flexure failure and the diagonal tension failure by using high-strength reinforcing bars (f_y and $f_{wy} > 916$ N/mm²) as both of tensile and shear reinforcements. In addition, the combination of thin web cross section with dense reinforcement will cause specimens to exhibit the diagonal compression failure. In order to avoid the local failure, the web width outside support was increased to that of the bottom flange. Anchor plates and nuts were provided as the anchorage, instead of typical bended hook type, to ensure the sufficient anchorage of the tensile bars and prevent the anchorage failure. The photo of anchor plates and nuts are shown in **Photo 3-1**.



Photo 3-1 Anchor nuts and plate

3.2 Materials

3.2.1 Mix proportion of concrete

In this study, the special type of concrete was desired. It was not only needed to have high strength but also high workability since the cross section of the specimens was I-shaped with very thin web (40-80 mm). Self-compacting concrete (SCC) can fulfill this requirement.

Table 3-3 Mix proportion of concrete

Mix	Target f'_c [N/mm ²]	$G_{\max}^{*1}$ [mm]	W/B^{*2} [%]	s/a^{*3} [%]	Unit Quantity [kg/m ³]					
					W^{*4}	C^{*5}	SF^{*6}	S^{*7}	G^{*8}	SP^{*9}
C	100	15	20	44.6	150	675	75	672	861	22.5
D	130	15	16	38.2	150	844	94	526	851	45.7

^{*1} maximum aggregate size, ^{*2} water to binder ratio, ^{*3} sand-aggregate ratio, ^{*4} water,

^{*5} cement (C: high early strength Portland cement (density = 3.14 g/cm³),

D: low heat Portland cement (density = 3.16 g/cm³))

^{*6} silica fume (density = 2.20 g/cm³), ^{*7} fine aggregate, ^{*8} coarse aggregate

^{*9} superplasticizer (type: SP8HU, high-performance water-reducing agent)

By the utilization of SCC, the complete compaction of the specimens, especially in the condense area at web portion, can be assured. Okamura and Ouchi [11] reported that the methods for achieving self-compactability are limiting aggregate content, low water to binder (W/B) ratio and use of superplasticizer. The low W/B ratio characteristic of SCC can also satisfy the need of low W/B ratio in order to acquire high compressive strength. Segregation can be prevented, while maintaining flowability, by using the proper amount of high-performance water-reducing agent as the chemical admixture. Besides, silica fume was used as a replacement of cement content to improve workability and reduce the cement content.

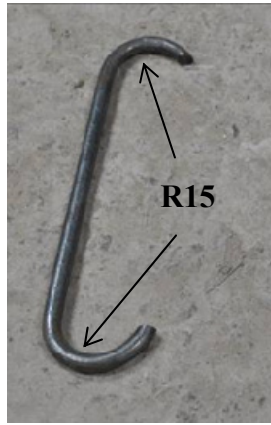
The mix proportion of SCC used in this study is presented in **Table 3-3**. It was designed based on JIS A 1150:2001 standard. Two mix proportions were used in order to obtain the target $f'_c = 100$ and 130 N/mm², respectively. Maximum aggregates size ($G_{\max}$) of 15 mm was used. The W/B ratios were 0.20 and 0.16, respectively. The major difference between these two mix proportions is that high early strength Portland cement was used in C series, while D series used low heat Portland cement. In U series, the target compressive strength can be developed within 7 days, resulting from the properties of high early strength Portland cement. However, even W/B was reduced to 16%, the target compressive strength of 130 N/mm² cannot be acquired within 7 days. For SU, low heat Portland cement was selected because of its long-term strength development and low risk of shrinkage caused by heat generated by hydration reaction. Superplasticizer used in both mix proportions were high-performance water-reducing types (SP8HU: Polycarboxylate ether).

3.2.2 Reinforcing steels

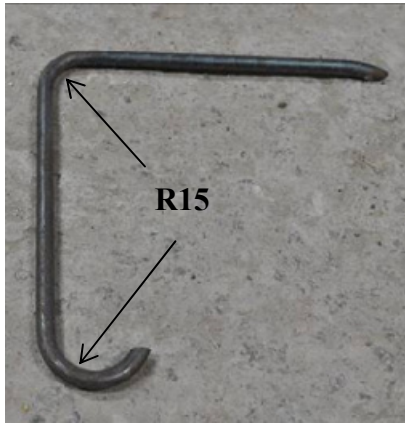
High strength reinforcing bars were selected as tension and shear reinforcements in this study to prevent both of flexure failure and diagonal tension failure and ensure that no yielding occurs during the loading test and specimens would fail in diagonal compression failure. The properties of reinforcing bars are provided in **Table 3-4**. All specimens with $d = 220$ mm used two D22 as longitudinal reinforcements. To keep p_w constant when designing the specimens with larger effective depth, the longitudinal reinforcements were changed to two D32 bars for the case of C-s100L, and two layers of which top layer (Top) was four D22 while four D25 bottom as layer (Bottom) for the case of the specimens with $d = 500$ mm. As for transverse reinforcements (stirrups), one-legged or two-legged stirrups were provided throughout the span. The grade SBPD 930/1080 and SBPD 1080/1230 of the longitudinal reinforcements complies with JIS G 3109, while SBPD 1275/1420 of transverse reinforcement complies with JIS G 3137. The shape of stirrups used in this study was not so common because of the limitation in the specimen size and was changed depending on the difference of flange width and the position of longitudinal reinforcements. **Photo 3-2** shows stirrups used in this research. Stirrups were anchored around tensile bars by the semi-circular hook type. The hook radius was 15 mm (**Photo 3-2(a) and (b)**) and 32.5 mm (**Photo 3-2(c)**)

Table 3-4 Properties of reinforcing steel

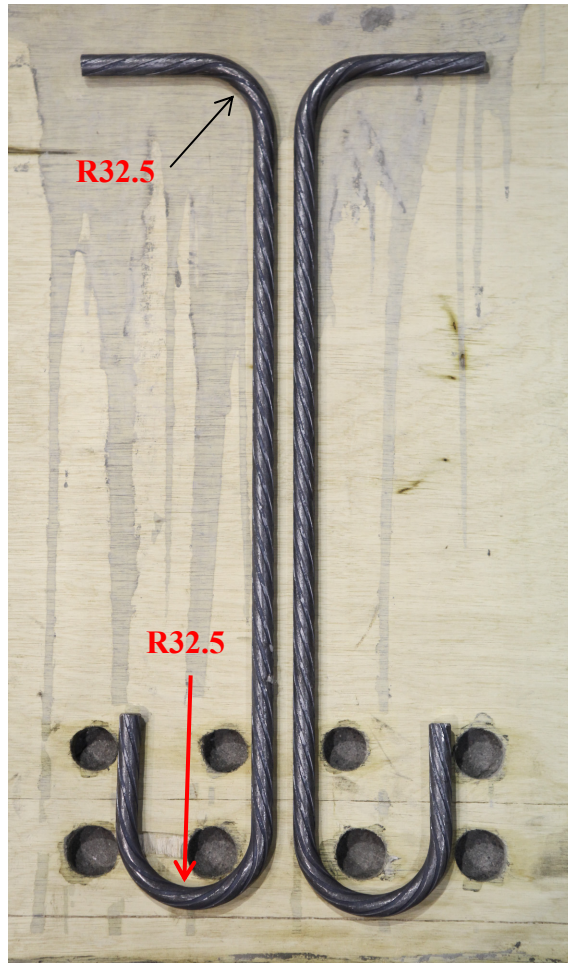
Name	Member	Grade	Nominal diameter [mm]	Cross sectional area [mm ²]	Yield strength or 0.2% offset yield strength [N/mm ²]
D22	Longitudinal	SBPD 930/1080 SBPD 1080/1230	22.2	380.1	≥ 930 ≥ 1080
D25	Longitudinal	SBPD 1080/1230	25.4	490.9	≥ 1080
D32	Longitudinal	SBPD 1080/1230	31.8	804.2	≥ 1080
U13	Transverse	SBPD1275/1420	13.1	125.0	≥ 1386
UD10	Transverse	UB785	9.53	71.33	≥ 785
UD13	Transverse	UB785	12.7	126.7	≥ 785
UD16	Transverse	UB785	15.9	198.6	≥ 785



(a) D-f150



(b) D-f500



(c) C-s300-II, C-s370-II(13)

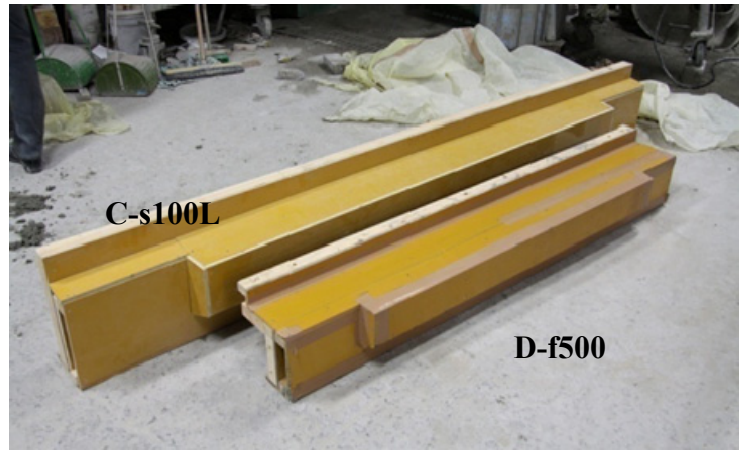
Photo 3-2 Example of stirrups

for the specimens with $d = 220\text{-}319$ mm and $d = 500$ mm, respectively.

3.3 Specimens preparation

3.3.1 Formwork and steel bars arrangement

Side formworks used in this study were made of 12 mm thickness plywood, which is designed for concrete casting, strengthened by woods. The bottom formworks were made of steel. The formworks were removed and reused after casting. The side formworks for the specimen D-f500, C-s100L, and the specimen with $d = 500$ mm were newly made in this study. **Photo 3-3** shows one side of the side formworks for D-f500 and C-s100L, and the



(a) One side of the side formworks for C-s100L and D-f500



(b) Formworks for the specimens with $d = 500$ mm

Photo 3-3 Formworks newly made in this study

formworks for the specimens with $d = 500$ mm.

Reinforcing bars were arranged by hand. Strain gauges (FLA-2-11-5L) were attached to the tensile bar at the mid span while at $d/2$ from the top fiber for stirrups. The locations of these gauges are demonstrated in **Fig. 3-1**. After that, formwork and reinforcing bars were assembled together. **Photo 3-5** displays the specimen C-s370-II(16) ready to be casted. In addition, woods with the same size as the web width were inserted into the web to ensure that



Photo 3-4 Steel bar cage for D-f500



Photo 3-5 C-s370-II(16) ready to be casted

the web width approximated to the design value before the casting. The tops of the formworks were restrained by plywood or steel cramps during casting to prevent lateral pressure of fresh concrete.

3.3.2 Casting of concrete

In this study, 55 liters and 100 liters capacity pan mixer were used to mix concrete depending on the total required volume of the specimen. Mixing of SCC is more difficult and time consuming compared to normal concrete since large amount of powder was used in the mix proportion. **Photo 3-6** shows the mixing of SCC. To achieve a good quality of concrete, SCC was precisely mixed by the following procedures:

- 1) Half of sand was filled into the mixer, following by cement, silica fume and the rest of sand. Start mixing in air for 30 seconds.
- 2) After mixing for 30 seconds, water mixed with superplasticizer was gradually poured into the mixer within 60 seconds.



(1) Filling materials



(2) Pouring water mixed with SP



(3) Stirring cement paste by shovel



(4) Adding coarse aggregate



(5) Mixing mortar with coarse aggregate



(6) Ready to be casted

Photo 3-6 Mixing self-compacting concrete

- 3) Stop mixing and stirring cement paste by shovel.
- 4) After stirring, mix for 1 minute and re-stir with shovel.
- 5) After 1 minute of mixing, add all coarse aggregate into the mixer.
- 6) After continue mixing for 2 minutes, concrete is ready to be casted.

The production of SCC was achieved by the mix proportion and the procedures described above. As SCC has self-compacting ability, it can be poured easily into the prepared formwork without any vibration requirement as shown in **Photo 3-7**. In each specimen, at least 3 of Ø100x200mm cylinder specimen for compression test and Ø150x250mm cylinder specimen for splitting tensile test were prepared. For mix D series (see **Table 3-3**), up to 10 of compression cylinder specimens were provided in order to assure that the desired compressive strength can be obtained on the testing day. Wet clothes were placed



Photo 3-7 Casting self-compacting concrete



Photo 3-8 A specimen after finish casting covered with vinyl sheets

on steel mesh on the top surface of the specimen after finish casting to prevent evaporation and loss of moisture. Finally, the whole specimen was covered with vinyl sheet as presented in **Photo 3-8**.

3.3.3 Curing

All specimens were removed from the formwork one day after the casting as shown in **Photo 3-9**. The full compaction can be confirmed without any vibration requirement even in the lower flange portion. Then, the specimens were cured with wet clothes for 7 days for mix C series and 6 to 8 weeks for mix D series. The difference in curing periods came from the different type of cement used in the mix. Low heat cement used in mix D required more curing periods than high early strength cement used in mix C, but it can develop higher compressive strength in long term. All specimens were covered again by wet clothes and



Photo 3-9 Formwork removal



Photo 3-10 Spraying water

vinyl sheet to prevent evaporation and loss of moisture. In addition, all specimens were sprayed by water every day until the time for the tests. **Photo 3-10** presents that water was directly sprayed to the specimen before it was covered by wet clothes and vinyl sheet.

3.3.4 Loading test and measuring items

Three-point bending was carried out using the 2000 kN capacity loading machine. The specimens with $d = 500$ mm were too long to be tested on the loading machine; hence, their loading test were conducted on the 3000 kN capacity loading apparatus. **Photo 3-11** illustrates

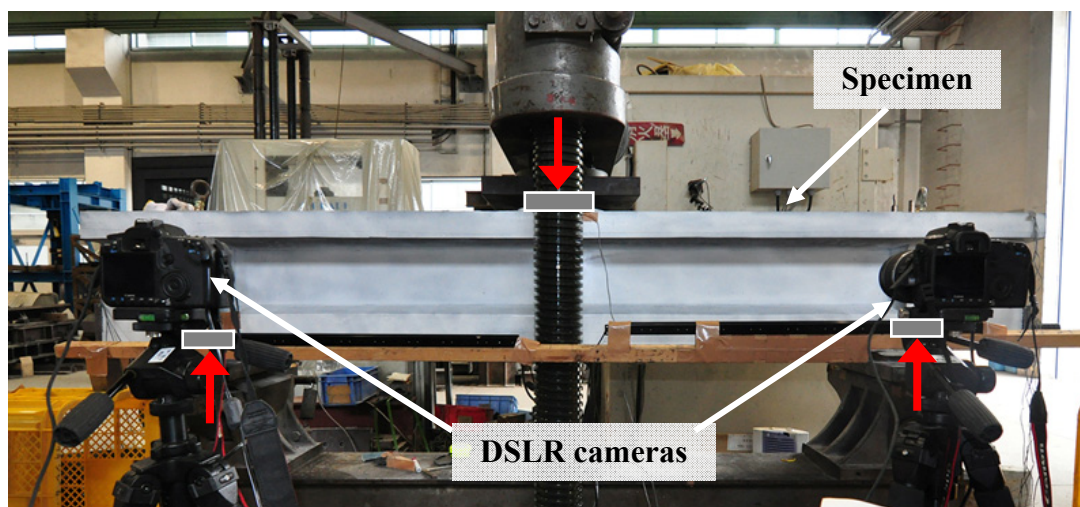


Photo 3-11 Loading test setup on the 2000 kN loading machine

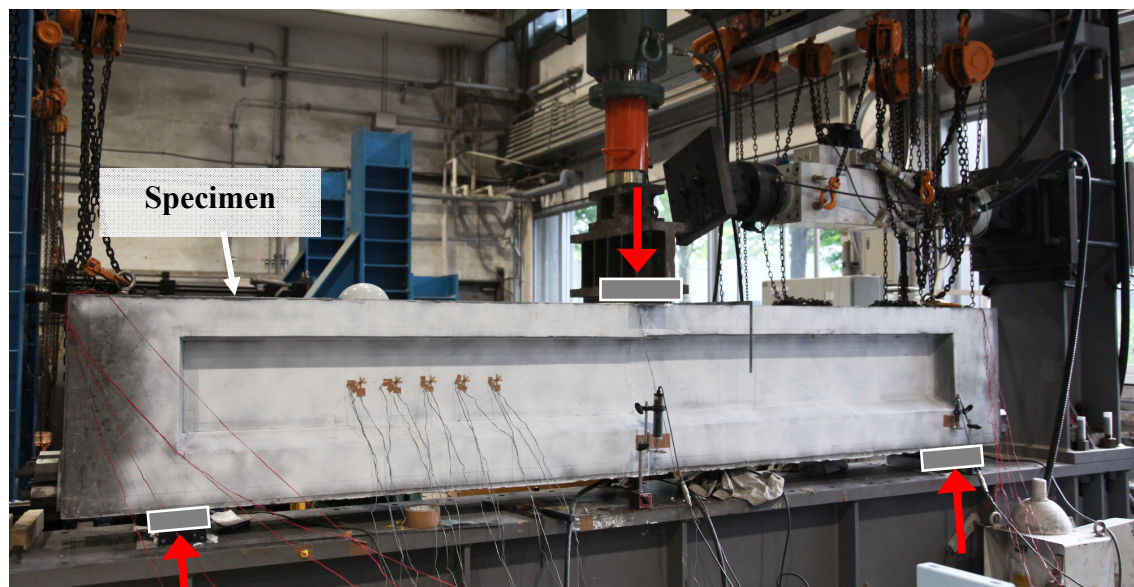
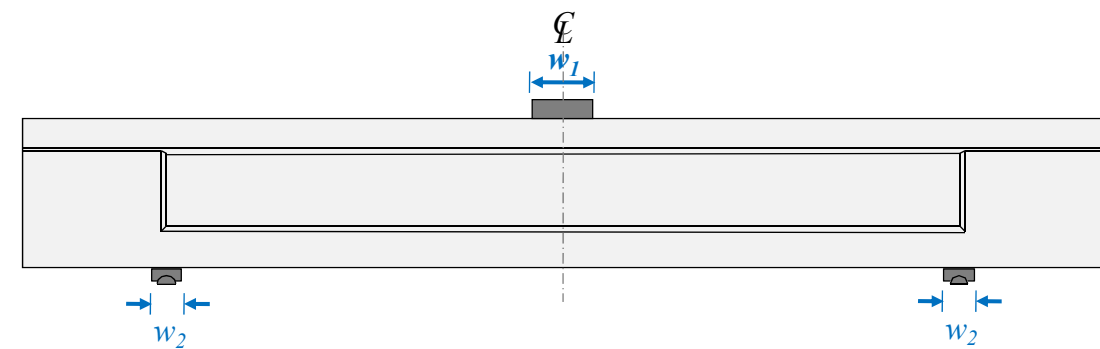


Photo 3-12 Loading test setup at the 3000 kN loading apparatus

the loading test setup for the case using the loading machine, while the case of using the loading apparatus is shown in **Photo 3-12**. For all specimens, the applied load, mid-span deflection and strains of concrete, tensile bars and stirrups were measured and recorded by data logger. Four linear voltage differential transducers (LVDT) were used to monitor the vertical deflection at the mid span. Strain of steel bars was measured by 2 mm length strain gauges (FLA-2-11-5L) attached at the mid span to measure the strain of tensile bars whereas



Specimen	w_1 (mm)	w_2 (mm)	Thickness (mm)
$d = 220$ mm	100	50	30
C-s100L	145	73	30
$d = 500$ mm	150	90	30

Fig. 3-2 Width of loading and support plates

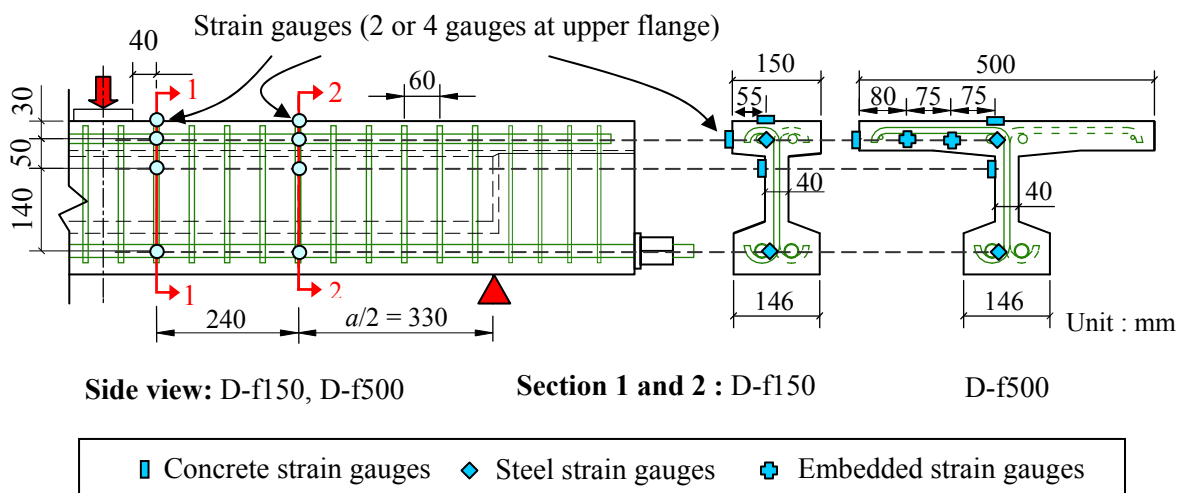


Fig. 3-3 Location of the stain gauges of D-f150 and D-f500.

at the distance of $d/2$ from the compression fiber for stirrups (see **Fig. 3-1**). Strain of concrete at extreme compression fiber was measured by 60 mm length strain gauge (PL-60-11) attached on the surface of the specimen. Width of loading plate and support plate are shown in **Fig. 3-3**. It can be divided into 3 cases depending on the size of specimens. All plates had thickness of 30 mm and covered the full flange width (both upper and lower).

In addition, the additional strain measurements were performed for evaluating the

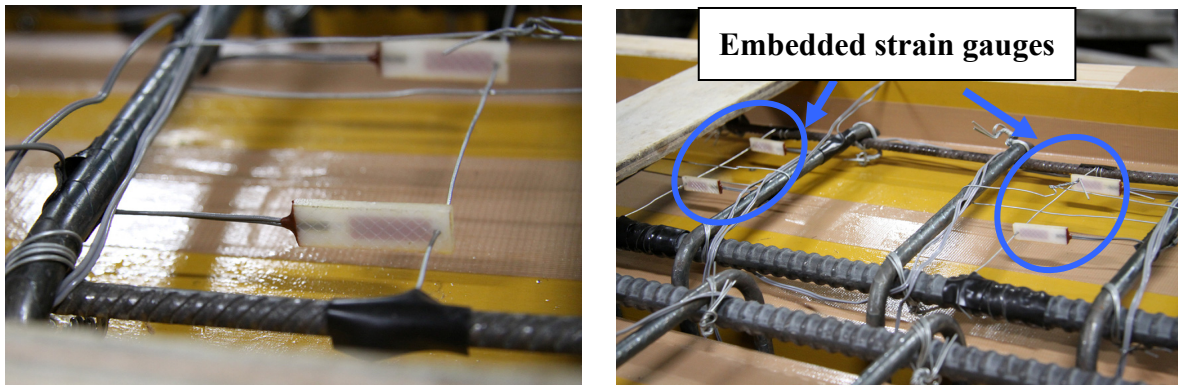
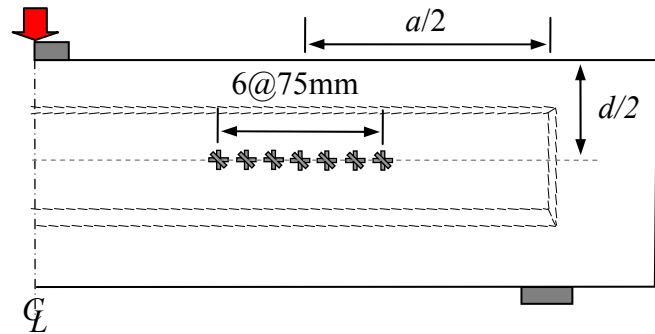
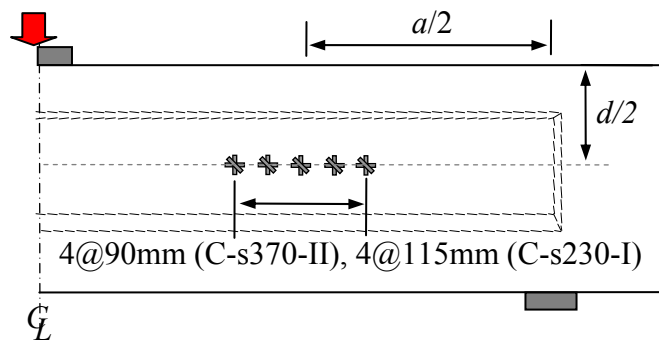


Photo 3-13 Embedded strain gauges in D-f500



(a) For C-s300-II



(b) For C-s370-II(13), C-s370-II(16) and C-s230-I

Fig. 3-4 Location of tri-axis strain gauges

beam and arch action in series III (D-f150 and D-f500) as shown in **Fig. 3-3**. It will be explained in detail later in section 3.4. For D-f500, embedded strain gauges (**Photo 3-13**) were installed inside the concrete in the upper flange portion to measure strain distribution in the flange. The locations of embedded strain gauges are shown in **Fig. 3-3**. As for the specimens with $d = 500$ mm, the angle of principle strain of web concrete was additionally measured by tri-axis strain gauges attached on the specimen's surface (left span in **Photo 3-12**). The locations of these strain gauges are illustrated in **Fig. 3-4**.

Furthermore, both side surfaces of all specimens were painted in white to ease the drawing and observation of crack during the experiments. Pictures were taken every 10 kN by two digital single-lens reflex (DSLR) cameras in both shear spans until a flat inclination of load-deflection curve was noticed. Then, pictures were taken more frequently to capture the state at the peak load.

3.4 Method for evaluating beam and arch actions by strain distribution measurement

In RC beams, the total shear force (V) is equal to the derivative of the moment as shown in Eq. 3-1.

$$V = \frac{dM}{dx} \quad \text{and} \quad M = T \cdot jd \quad (3-1)$$

where, T and jd are tensile force in the section and the moment lever arm, respectively.

The products from this differentiation will consist of two components:

- (1) Variable tensile force acting over a constant moment lever arm, $jd \frac{dT}{dx}$
- (2) Constant tensile force acting over a variable moment lever arm, $T \frac{d(jd)}{dx}$.

Thus Eq. 3-1 becomes the following.

$$V = \frac{d(T \cdot jd)}{dx} = jd \cdot \frac{dT}{dx} + T \cdot \frac{d(jd)}{dx} \quad (3-2)$$

The former term is called 'beam action' while the latter component represents 'arch

action'. These actions are combined together to resist the total shear force. Although the beam action normally controls the shear strength of a beam when the shear span to effective depth ratio a/d is comparatively large, the situation in case of RC T-shaped or I-beam can be changed because of the presence of top flange [12].

In series III (effect of b_f/b_w), the changing of governing mechanism was used to explain the effect of b_f/b_w on the diagonal compressive capacity. A method for evaluating the beam and arch actions in the experiment was adapted from the study of Pansuk and Sato [12]. To evaluate the value of these two actions between the two known sections of the beam, Eq. 3-2 can be rewritten as,

$$V = jd \cdot \frac{\Delta T}{\Delta x} + T \cdot \frac{\Delta(jd)}{\Delta x} \quad (3-3)$$

By measuring strain distribution in a section with an assumption that the stress-strain relationship of concrete is elastic in compression and the stress of concrete in tension is negligible, the value of T and jd of that section can be obtained at each load level. To calculate beam and arch actions between two sections, jd and T are taken to be the average of two sections while ΔT and Δjd equal to $T_1 - T_2$ and $jd_1 - jd_2$, respectively. Then, Eq. 3-3 becomes the following:

$$V = \frac{(jd_1 + jd_2)}{2} \cdot \frac{(T_1 - T_2)}{\Delta x} + \frac{(T_1 + T_2)}{2} \cdot \frac{(jd_1 - jd_2)}{\Delta x} \quad (3-4)$$

where; jd_1 and jd_2 are the moment lever arm at section 1 and 2, respectively.

T_1 and T_2 are the tensile force of tensile bars at section 1 and 2, respectively.

Δx is the distance between the two sections.

The locations of strain gauges which were used for evaluating beam and arch actions for D-f150 and D-f500 were revealed in **Fig. 3-3**.

3.5 Experimental data analyzing method

3.5.1 Load at first flexure crack (P_{cr})

The applied load when the first flexure crack occurs can be determined by the

relationship between the applied load and the strain of longitudinal bars and visual observation during the loading.

3.5.2 Average crack spacing ($s_{c,avg}$) and crack angle (β_{avg}) at the peak load

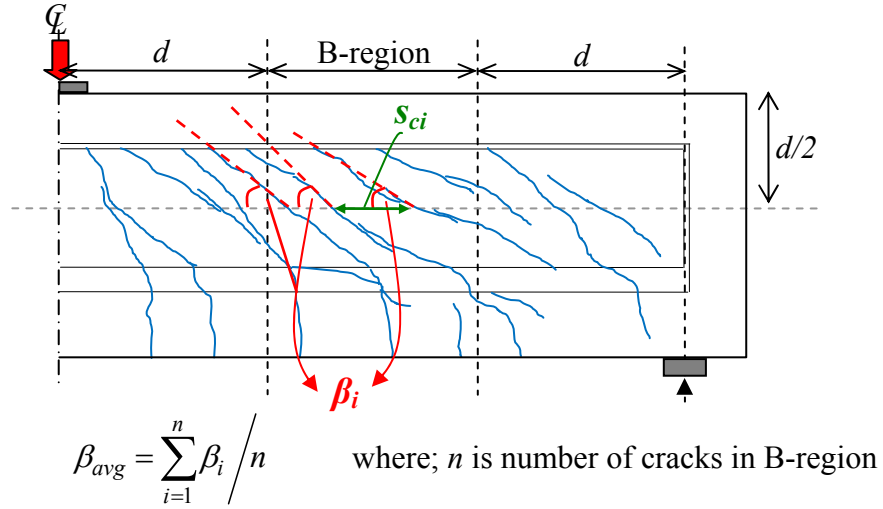


Fig. 3-5 Example of crack spacing and angle

From pictures taken at the peak load, the horizontal cracks spacing at the peak load (s_{ci}) and the crack angle (β_i) were measured at the middle height of the web. The example of s_{ci} and β_i measurement of a crack is demonstrated in **Fig. 3-5**. The average of s_{ci} of cracks in the shear span and the average of β_i of cracks in B-region (β_{avg}) will be used in the later discussion since it was observed that the crushing area, which is corresponding to the failure region, was mostly found in B-region (the portion outside the distance approximately d away from the loading point and supports).

Chapter 4

Experimental results

In order to clarify the effect of each parameter on diagonal compressive capacity, 12 RC I-beams with different compressive strength (f'_c), stirrup ratio (r_w), stirrup spacing (s), stirrup configuration, flange width to web width ratio (b_f/b_w) and effective depth (d) were tested by three-point bending. In this chapter, the experimental results and all measured data are presented and discussed.

4.1 General behavior of diagonal compression failure

Load-deflection relationships of specimens in series I and II with $d = 220$ mm and $d = 500$ mm are illustrated in **Fig. 4-1(a) and (b)**, respectively. Similar behavior is found in all

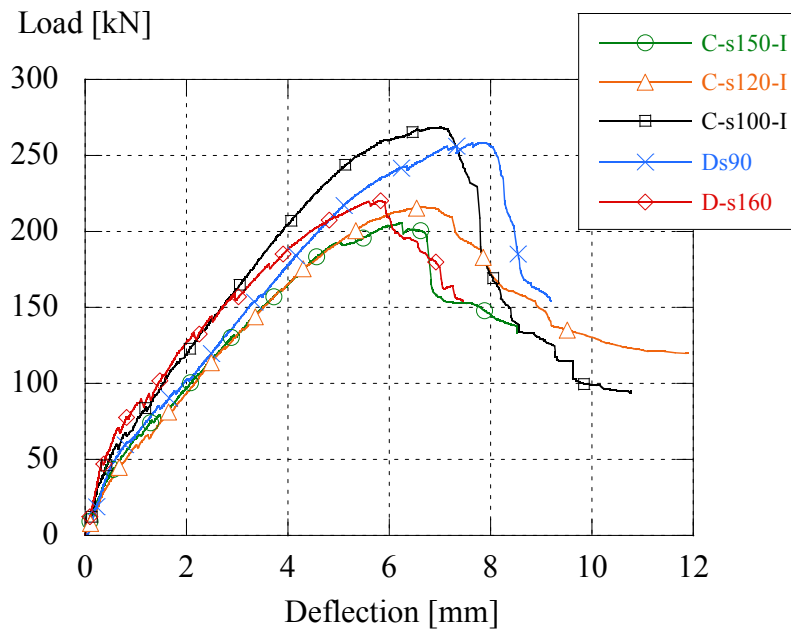


Fig. 4-1(a) Load-deflection relationships of specimens with $d = 220$ mm.

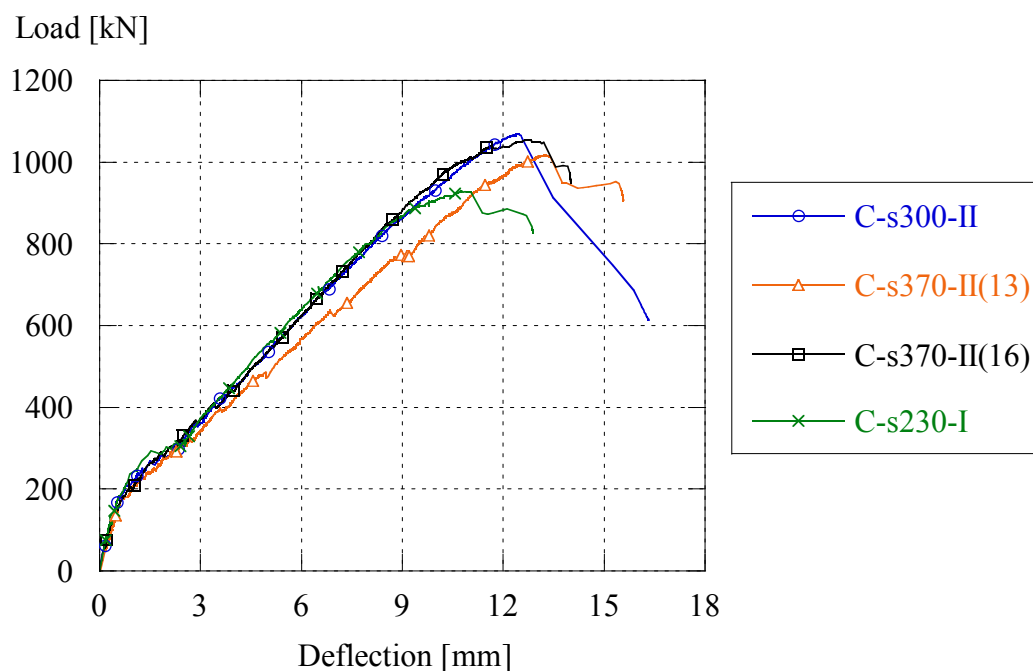


Fig. 4-1(b) Load-deflection relationships of specimens with $d = 500$ mm.

specimens and can be considered as the general behavior of diagonal compression failure. Firstly, specimens behaved in elastic manner until the first flexural crack occurred in the bottom flange near the mid span, which is reflected in the graph as a rate of inclination decreases. After the first flexural crack, the load-deflection curve remained to advance linearly with the continuous initiation of diagonal cracks at the web concrete. Near the peak load, the deflection increased with a relatively small increase in applied load as the web concrete began to crush. Afterwards, the applied load reached to the peak. After the peak load, applied load rapidly decreased. The experimental results including Kobayashi [3] are summarized in **Table 4-1**. Data of the stresses of longitudinal bars and stirrups revealed no yielding at the peak load and the values were much less than their yield strengths. It implies that the failure mode was neither the flexure failure nor the diagonal tension failure. The web concrete crushed at the peak load and splitting cracks along the member axis near tensile bars did not initiate at that time; hence, the cause of failure was not by anchorage failure of both tensile bars and stirrups. It can be concluded by considering these observations that the failure mode of all specimens was designated as the diagonal compression failure. The diagonal compression failure in which the web concrete crushed before the yielding of stirrup exhibited the brittle mode and should be prevented in the design of RC beams.

Table 4-1 Experimental results (This study)

Specimen	f_c' [N/mm ²]	d [mm]	a/d	b_f/b_w	r_w [%]	s [mm]	ϕ_d [mm]	No. of stirrup legs	f_y [N/mm ²]	$\sigma_{s, \max}^{*1}$ [N/mm ²]	f_{wy} [N/mm ²]	$\sigma_{w, \max}^{*2}$ [N/mm ²]	$s_{c, avg}^{*3}$ [mm]	β_{avg}^{*4} [degree]	V_{cr}^{*5} [kN]	V_{exp}^{*6} [kN]
C-s100-I	107	220	3.0	6.25	1.8	100	9.53	One	1204	613.2	953	549.6	92.5	36.7	19	134.3
C-s120-I	101				1.5	120				454.2		403.4	99.0	40.5	13	108.0
C-s150-I	105				1.2	150				401.4		550.8	91.0	38.8	13	102.7
C-s230-I	123	500		3.88	1.1	230	15.9	Two	1171	515.2	916	809.0	155.9	36.8	88	465.0
C-s300-II	104				1.1	300	13.1			400.8	1368	883	121.6	36.0	90.5	535.0
C-s370-II(13)	105				0.9	370				724.4		904.8	144.9	33.7	83.5	508.5
C-s370-II(16)	102				1.3		15.9			297.6	926	731.8	164.5	33.3	75	527.0
D-s160	128	220		6.25	2.0	160	12.7	One	1198	402.0	955	377.8	111.3	28.7	25	110.1
D-s90	131				3.5	90			930	594.4		363.4	73.2	40.7	18	129.3
D-f150	130			3.75	3.0	60	9.53		1204	665.8	967	359.4	53.0	39.8	23.5	159.9
D-f500	135			12.5						583.2		508.2	49.0	47.0	20	161.4
C-s100L	108	319		6.25	2.2	100	12.7			1187	641.6	931	627.2	128.4	33.7	20

^{*1} maximum stress in longitudinal bars, ^{*2} maximum stress in stirrups, ^{*3} average crack spacing in horizontal direction at peak load, ^{*4} average crack angle in B-region at peak load, ^{*5} shear force at first flexure crack, ^{*6} diagonal compressive capacity.

Table 4-1 (cont.) Experimental results (Kobayashi [3])

Specimen	f'_c [N/mm ²]	d [mm]	a/d	b_f/b_w	r_w [%]	s [mm]	ϕ_d [mm]	No. of stirrup legs	f_y [N/mm ²]	$\sigma_{s, \max}^{*1}$ [N/mm ²]	f_{wy} [N/mm ²]	$\sigma_{w, \max}^{*2}$ [N/mm ²]	$s_{c, avg}^{*3}$ [mm]	β_{avg}^{*4} [degree]	V_{cr}^{*5} [kN]	V_{exp}^{*6} [kN]	
A-s160(7)	32	220	3.0	6.25	0.63	160	7.1	One	1214	246.0	1425	434.8	142.5	32	14	59.4	
A-s160(9)	33.4				1.0		9.0		930	280.2	1275	372.2	85.5	40.0	12.5	58.6	
A-s80	35.8				2.0					80		224.8	295.6	114.8	35.5	11	61.6
A-s55	35.0				2.9					55		244.0	186.0	87.7	33	10	61.7
A-ad40	32.0		4.0		2.0	80	9.0		1198	411.4	1350	282.6	74.9	37.4	15.5	68.1	
A-ad45	34.9		4.5		2.0					426.2		275.8	103.9	35	20	67.0	
B-s80	73.8		3.0		2.0				930	519.5	1275	390.0	103.7	27	15.5	114.5	
B-s55	61.5				2.9					55		518.4	374.4	-	-	14	107.8
C2-s90-I	102				2.0	90	9.53		1198	601.0	954	561.2	69.9	31	7.5	137.0	
C2-s50-I	110					50	7.1			765.0	1397	721.0	60.0	36.5	18	168.5	
C2-s160-I	115					160	12.7		1214	420.6	955	300.8	86.0	35	17	96.6	
C-s60-I	98.2		3.5		3.0	60	9.53		1198	700.0	954	538.6	-	-	16	144.9	
C-s45-I	99.1				4.0	45				724.6		372.8	56.5	33	15	167.6	
C-ad35	110				2.0	90				641.8	953	544.6	77.2	39.5	16	122.7	
C-ad40	104		691.6							587.8		83.3	45	12.5	121.1		
D-s60	133		3.0		3.0	60				722.2	954	525.6	60.5	41.7	15	162.0	
E-s60	165								1214	834.6	953	496.4	59.9	34	23.5	192.1	

*¹ maximum stress in longitudinal bars, *² maximum stress in stirrups, *³ average crack spacing in horizontal direction at peak load, *⁴ average crack angle in B-region at peak load, *⁵ shear force at first flexure crack, *⁶ diagonal compressive capacity.

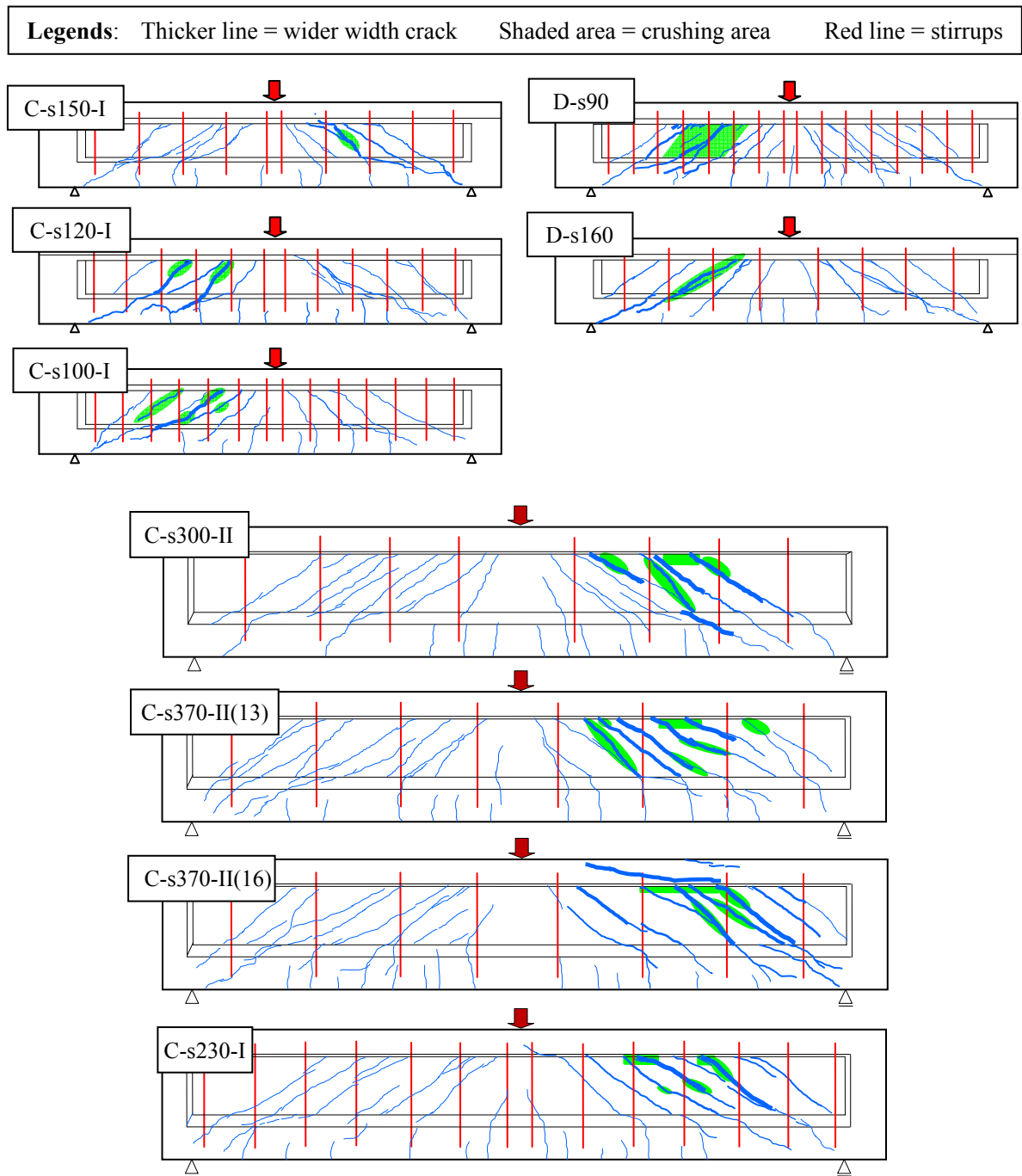


Fig. 4-2 Crack patterns just before the peak load

Crack patterns just before the peak load of all specimens are demonstrated in **Fig. 4-2**. The thicker lines, the shaded areas and red lines represent the wider crack, the crushing areas, and the location of stirrups, respectively. The actual photos at peak load of failure span and at the end of loading can be found in Appendix A. At the end of loading, web concrete crushed more extensively.

4.2 Series I - Effect of stirrup

Kobayashi [3] adapted a method to eliminate the effect of f'_c by normalizing the obtained shear strength ($v_{exp} = V_{exp}/(b_w d)$) by $f'_c{}^{1/2}$ which is used in the design equation of JSCE [2] and the predictive equation by Placas et al. [7]. This method is also applied in this study since the experimental results in this study support the expression of diagonal compressive capacity as a function of $f'_c{}^{1/2}$. The relationships between r_w and $v_{exp}/f'_c{}^{1/2}$ including the specimens in the study by Kobayashi [3], Rangan [13] and Leonhardt et al. [14] are demonstrated in **Fig. 4-3**, where v_{exp} is the shear strength ($=V_{exp}/b_w d$). The details of specimens in the study by Rangan [13] and Leonhardt et al. [14] are provided in **Table 4-2**.

With the increase in r_w , the diagonal compressive capacity tends to increase. However, considering the case of constant value of $r_w = 2\%$ with different ϕ_d and s (C-s50-I, C-s90-I, C-s160-I), $v_{exp}/f'_c{}^{1/2}$ differs significantly. The specimen with the widest s (C-s160-I) had the lowest diagonal compressive capacity, while the specimens with closer s (C-s90-I, C2-s50-I) resulted in 49.5% and 77.7% higher diagonal compressive capacity than C2-s160, respectively. The same tendency can be observed in the case of the specimens with the same $r_w = 1.1\%$ (C-s230-I and C-s300-II) as well. The smaller difference in diagonal compressive capacity in the latter case was found. The specimens with the same s but different ϕ_d (C-s370-II(13) and C-s370-II(16)) also have slight difference in the diagonal compressive capacity, confirming that the diameter of stirrup does not affect the diagonal compressive capacity. From **Table 4-1**, the average crack spacing at the peak load ($s_{c,avg}$) among three specimens

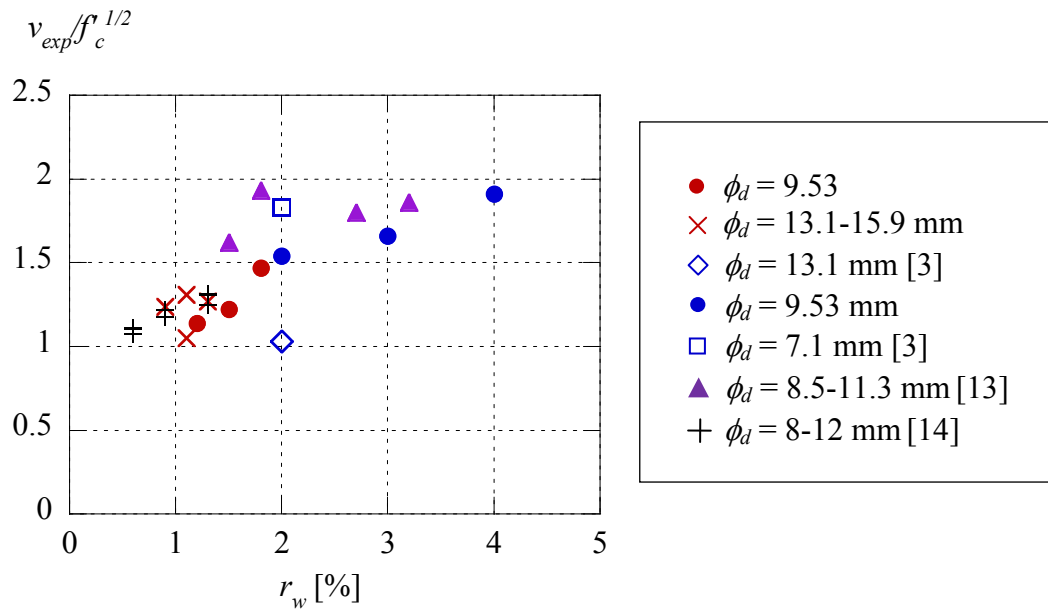


Fig. 4-3 Effect of r_w

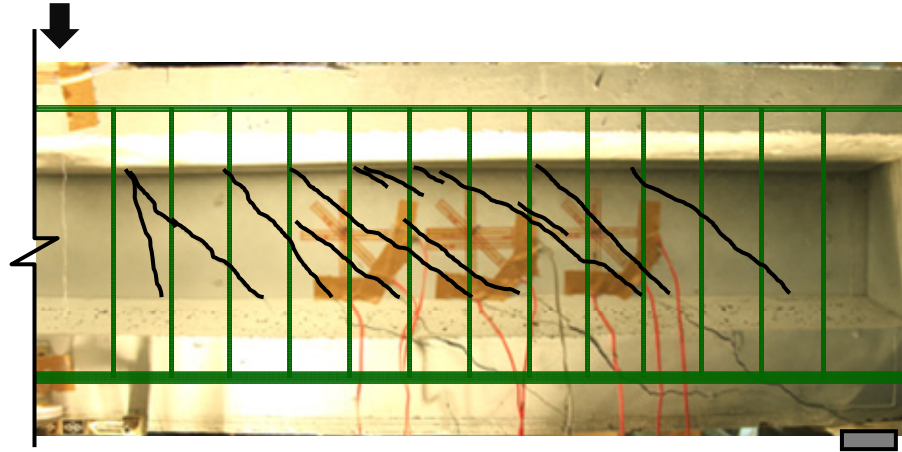
with $r_w = 2\%$ (C2-s50-I, C2-s90-I, C2-s160-I) is also different. The measuring method of $s_{c,avg}$ was explained in section 3.5.2. The comparison of crack patterns at the peak load between C2-s50-I and C2-s160-I is presented in **Fig. 4-4**. Apparently, the specimens with closer stirrup spacing (C2-s50-I) had larger number of cracks, finer crack distribution and closer $s_{c,avg}$ than the specimen with wider stirrup spacing (C2-s160-I), even the identical r_w was provided in both specimens. Diagonal cracks were localized at the middle of shear span in the case of C2-s160. Moreover, from **Table 4-1**, $s_{c,avg}$ also increases with the increase in stirrup spacing. In conclusion, stirrup ratio is not an appropriate indicator for the diagonal compressive capacity, but stirrup spacing. The diagonal compressive capacity will be discussed by s in the following section.

The experimental results of the same set of data are plotted against s despite of r_w in **Fig. 4-5**. The specimens of Kobayashi [3] and the author in the range of $s = 45$ -230 mm used one-legged stirrup, while two-legged stirrup was provided in the beams with $s = 300$ and 370 mm. Considering the range of $s = 45$ -160 mm, the diagonal compressive capacity linearly decreases with wider s regardless of its diameter and configuration. On the contrary, the

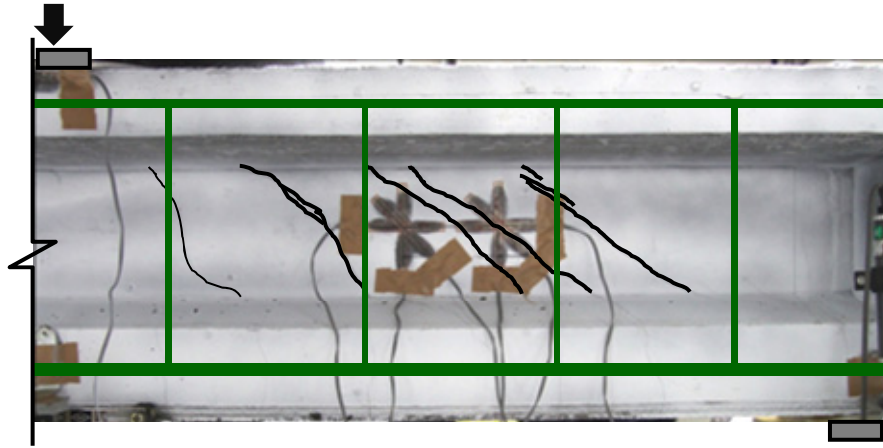
Table 4-2 List of detail of the specimens and results of the other authors

Specimen	f'_c [N/mm ²]	b_w [mm]	d [mm]	a/d	p_w [%]	r_w [%]	s [mm]	ϕ_d [mm]	Stirrup type	f_{wy} [N/mm ²]	V_{exp} [kN]
I1 ¹⁾	36.5	74	563	2.5	8.4	2.7	50	11.3	Two- legged	485	453
I2 ¹⁾	30.2					1.5		8.5			371
I3 ¹⁾	31.2	63			9.8	3.2		11.3			369
I4 ¹⁾	35.7	64			9.7	1.8		8.5			416
TA1 ²⁾	20.6	150	375	3.3	4.7	1.3	113	12	Closed - typed	426	335
TA2 ²⁾	20.6					0.86		10		425	301
TA3 ²⁾	19.1					0.59		8		417	272
TA13 ²⁾	23.7					1.29		12		426	343
TA14 ²⁾	23.7					0.86		10		425	333
TA15 ²⁾	23.2					0.59		8		417	292

¹⁾ Rangan [13], ²⁾ Leonhardt et al. [14]



C2-s45-I ($s = 45 \text{ mm}$, $\phi_d = 7.1 \text{ mm}$, $s_{c,avg} = 60 \text{ mm}$)



C2-s160-I ($s = 45 \text{ mm}$, $\phi_d = 7.1 \text{ mm}$, $s_{c,avg} = 86 \text{ mm}$)

Fig. 4-4 Cracks patterns comparison between C2-s50-I and C2-s160-I

different tendency of the diagonal compressive capacity in the case of relatively wider s is observed. **Figure 4-5** demonstrates that the change in $v_{exp}/f_c^{1/2}$ becomes negligible when s is wider than approximately 140-160 mm. The diagonal compressive capacity of s230-I was lower than those of the other three specimens with wider s (C-s300-II, C-s370-II(13), C-s370-II(16)) even s is narrower. This is caused by difference in the number of stirrup legs among these specimens. It will be explained in the following parts.

The reduction in the diagonal compressive capacity with the wider stirrup spacing is resulted from two mechanisms caused by the confinement effect by stirrups. One is smaller diagonal crack width (w) caused by providing closer shear reinforcements; therefore, the critical average stress in web concrete (σ_{2max}) would be greater because w affects the diagonal compressive capacity as also reported by Schäfer et al. [5] and Reineck[15]. The other is the

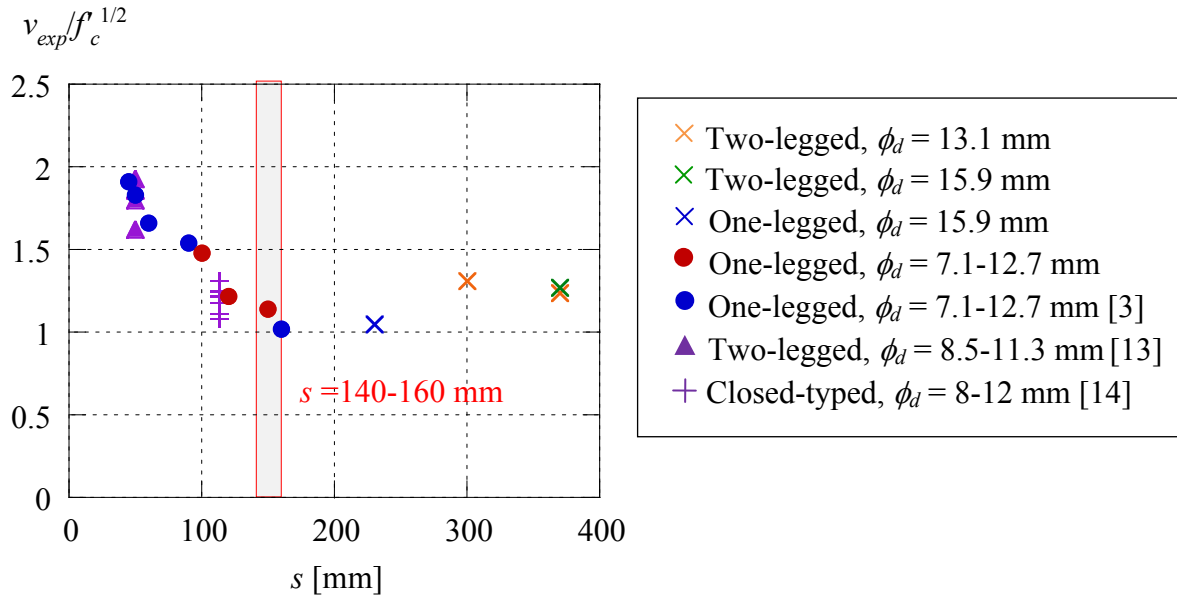


Fig. 4-5 Effect of s

localization of compressive strut, which resulted from an insufficient confinement effect provided by stirrups. When the confinement effect by stirrups is insufficient, stirrups cannot prevent the excessive crack opening so that the stress cannot distribute along the beam axis. This stress concentration causes the early crushing of web concrete and the reduction of diagonal compressive capacity. In addition, it should be noted that the confinement effect provided by stirrups in thin web beams is different from one provided by tie reinforcements in columns, which increases the strength of core concrete inside the tie reinforcements. Such confinement effect scarcely exists in thin web RC beams (large web height to web width ratio), in which diagonal compression failure generally occurs. The diagonal compression failure is not likely to occur in RC beams with adequately low ratio of web height to web width to cause considerable confinement effect.

The failure localization relates to the distribution of diagonal cracks and crushing area. In the case of one-legged stirrup with closer-spacing beams (C-s100-I, D-s90), cracks more finely distribute and the crushing area at web is more widespread than that of wider-spacing cases (C-s150-I, D-s160-I, C-s230-I), and the average crack spacing at the peak load ($s_{c,avg}$) shows a decreasing tendency as can be observed from the crack patterns just before the peak load (**Fig. 4-2**) and **Table 4-1**, respectively. It implies that the wider stirrup spacing will lead to the inadequate confinement effect and failure localization. In addition, Zakaria et al. [16] reported that larger beams caused greater average crack spacing at the peak load. Thus, $s_{c,avg}$ should be related to the increase in s and d .

However, the sufficient confinement effect in beams with wider stirrup spacing can also be achieved by providing two-legged stirrups. From the experimental results, the fine cracks distribution, widespread of crushing area and comparatively narrow $s_{c,avg}$, not only be observed in the case of closer-spacing specimens, but also in the case of using two-legged stirrups with wider-spacing stirrup (C-s300-II, C-s370-II(13) and C-s370-II(16)), even s and d of these specimens are comparatively larger. The effect of stirrup configuration on the failure localization can be confirmed by comparing specimens having the same r_w but different s , ϕ_d and number of legs (C-s230-I and C-s300-II) as shown in **Fig. 4-6**. Finer cracks distribution and smaller $s_{c,avg}$ can be observed for C-s300-II even C-s230-I had narrower s and larger ϕ_d . It is because the use of two-legged stirrups resulted in a larger effective area in which the confinement effect (due to the stirrup) can control excessive crack opening, while the use of one-legged stirrup (for which the effective area was comparatively smaller), induced the localization of compressive strut. The difference in the effective area between one-legged and two-legged stirrups is demonstrated in **Fig. 4-7**. From these reasons, the diagonal compressive

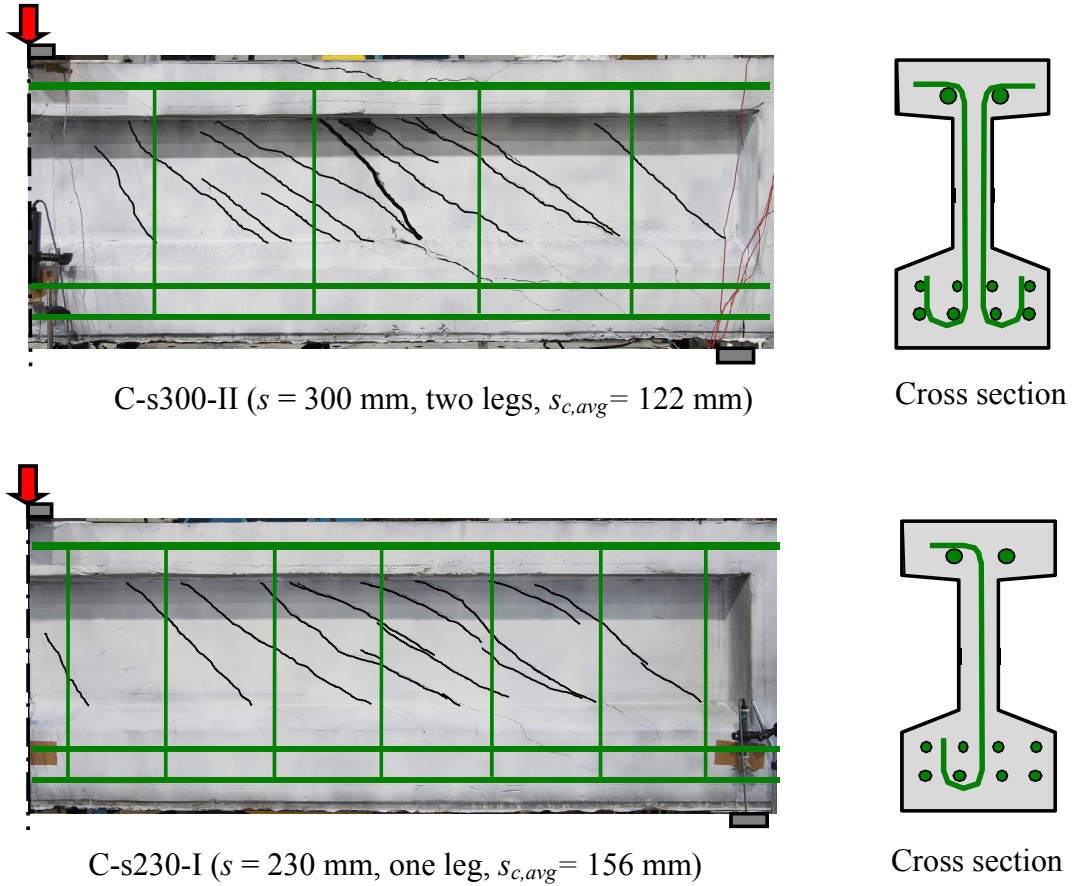


Fig. 4-6 Cracks patterns comparison between C-s230-I and C-s300-II

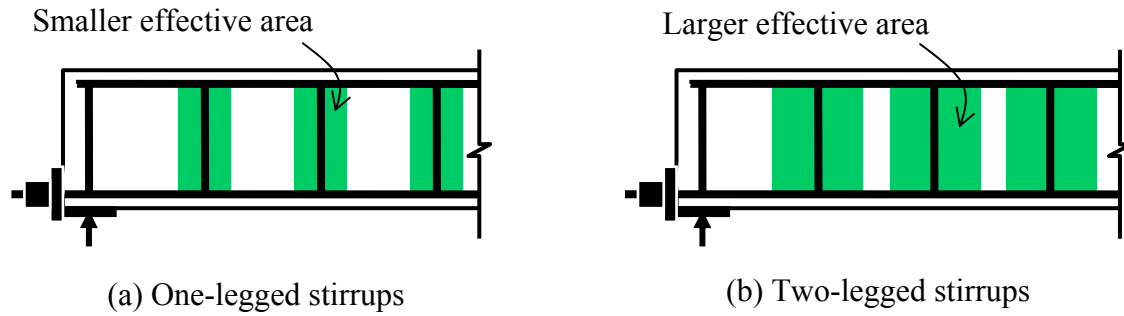


Fig. 4-7 Difference in effective area between one-legged and two-legged stirrups

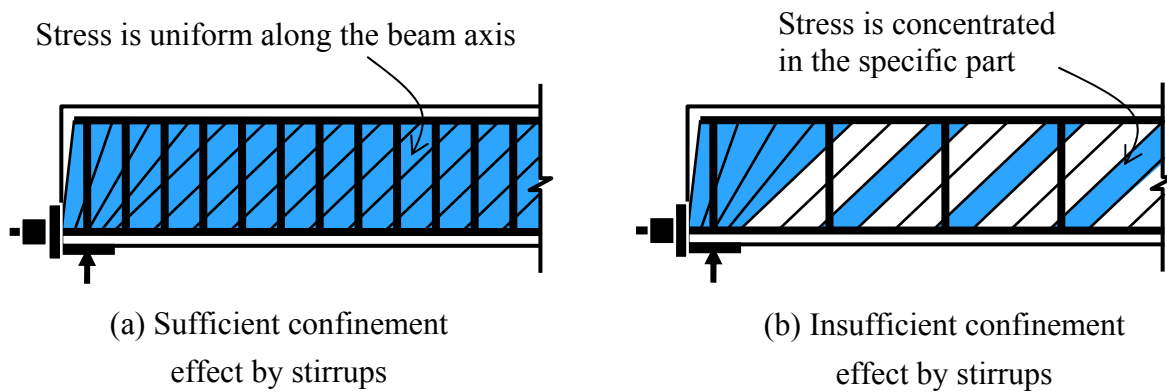


Fig. 4-8 Formation of compressive struts

capacity did not decrease when using two-legged stirrups (C-s300-II, C-s370-II(13) and C-s370-II(16)) compared to C-s230-I.

Figure 4-8 explains the model of compressive strut formation. **Figure 4-8(a)** demonstrates the case of sufficient confinement effect by providing close-spacing stirrups or two-legged stirrup. Diagonal stress generates uniformly along the beam axis. In contrary, as shown in **Fig. 4-8(b)**, diagonal stress is concentrated in a local portion of the beam with insufficient confinement effect by stirrups (one-legged wide-spacing stirrups).

4.3 Series II - Effect of f'_c with different stirrup spacing

In Series II, the additional 10 specimens including the specimens of Kobayashi [3] (A-s160(7), A-s160(9), A-s80, A-s55, B-s80, B-s55, D-s60, D-s90, D-s160 and E-s60) were combined with Series I in order to discuss the effect of f'_c with different stirrup spacing. All specimens had f'_c from 35 N/mm² to 165 N/mm². **Figure 4-9** displays $v_{exp}/f'_c{}^{1/2}$ as a function of s for specimens with different f'_c . It is confirmed that the effect of s depends on f'_c . There is a certain variation in $v_{exp}/f'_c{}^{1/2}$ by the change in s when using normal strength concrete while the greater effect of s can be observed in high strength concrete beams. As for beams with $f'_c > 65$ N/mm², the diagonal compressive capacity reduces as s increases in a linear relationship. With higher f'_c , the inclination of trend line increases with a limitation when f'_c approximates 100 N/mm². It can be explained by considering that the diagonal compressive capacity depends on not only f'_c but also the width of compressive strut. If the width of compressive strut becomes smaller with higher f'_c , the area loss of the struts reduces the diagonal compressive capacity. Thus the diagonal compressive capacity is limited.

Figure 4-10 plots the shear strength v_{exp} ($=V_{exp}/b_wd$) as a function of f'_c . Since there was slight effect of s when using $f'_c = 35$ N/mm², the specimen A-s55 [3] is used as a representative of specimen with $f'_c = 35$ N/mm² and $s = 60$ mm. It can be observed that v_{exp} increases with the increase in f'_c . As discussed in the previous paragraph, the f'_c and s are interrelated. The incremental rate of v_{exp} is different when using different s .

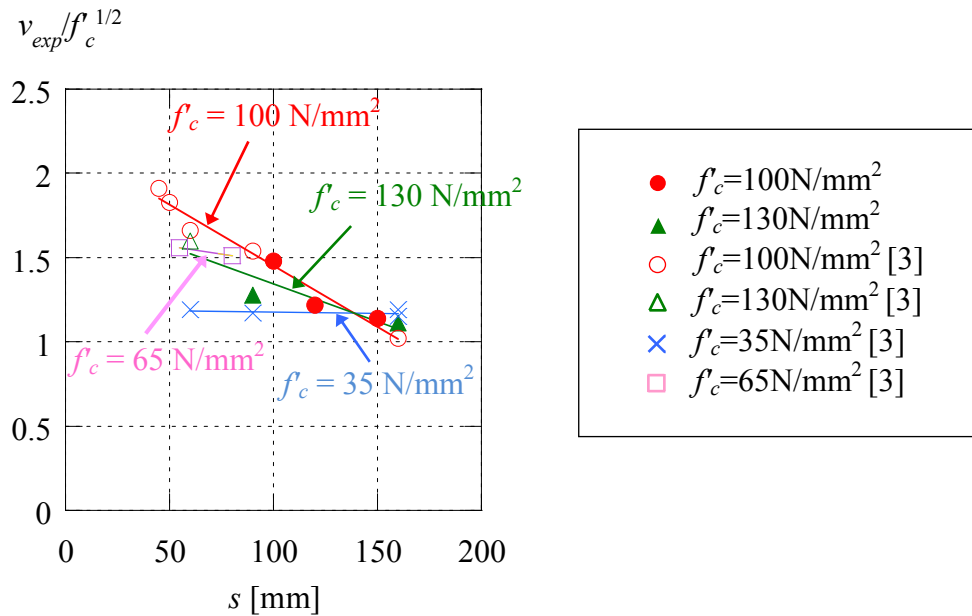


Fig. 4-9 Effect of s with different f'_c

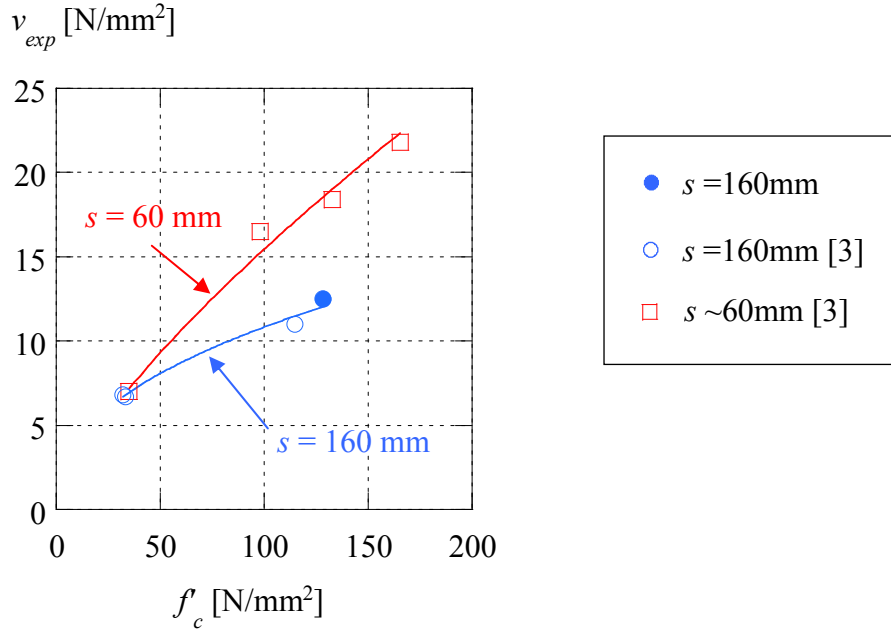


Fig. 4-10 Effect of f'_c (v_{exp} - f'_c)

4.4 Series III - Effect of b_f/b_w

Series III (b_f/b_w series) comprises of two specimens (D-f150 and D-f500) originally tested by the author combined with one specimen of Kobayashi [3] (D-s60). In this series, web width was designed to be constant and b_f/b_w was varied by increasing the upper flange width. All specimens had f'_c approximates 130 N/mm^2 with stirrup spacing equals to 60 mm (one-legged). Load-deflection relationships of this series are illustrated in **Fig. 4-11**. The diagonal compression failure occurred in all cases. The maximum loads are almost the same level in all specimens and no significant difference can be observed except that the deflection at peak slightly increases with the increase in the upper flange width. After the peak load, the applied load drastically drops and the failure was sudden and explosive. The normalized shear capacity against b_f/b_w is drawn in **Fig. 4-12** again to represent the effect of b_f/b_w . It can be seen that the normalized shear strength shows nearly constant value with the increase of b_f/b_w ; hence no effect of b_f/b_w on the diagonal compressive capacity of RC I-beams within this range. It can be explained by two reasons.

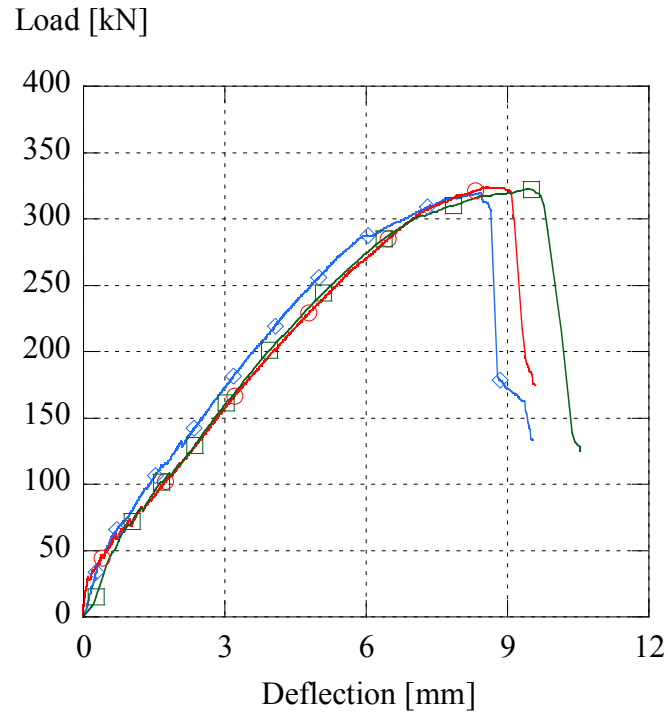
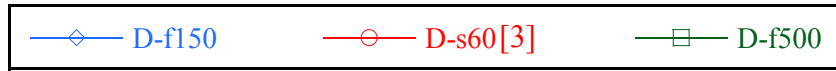


Fig. 4-11 Load-deflection relationships

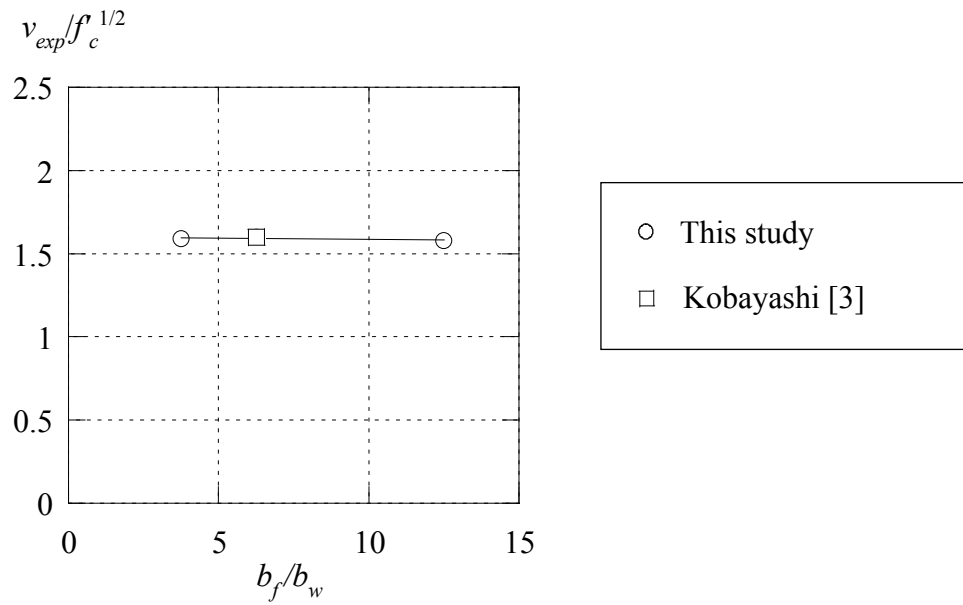


Fig. 4-12 Effect of b_f/b_w

Firstly, the strain distribution along the top flange (in the perpendicular direction to beam axis) of D-f150 and D-f500 is presented in **Fig. 4-13**. The positive value indicates

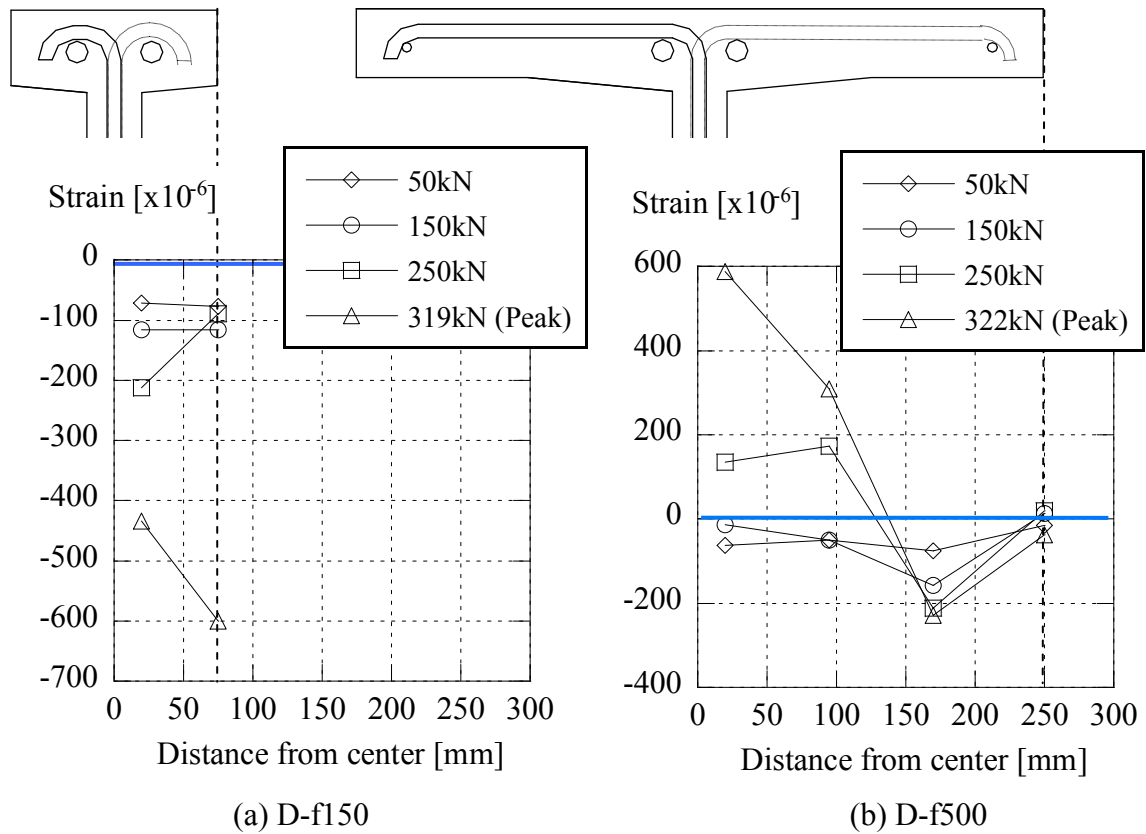


Fig. 4-13 Strain distribution in upper flange measured at the middle of shear span

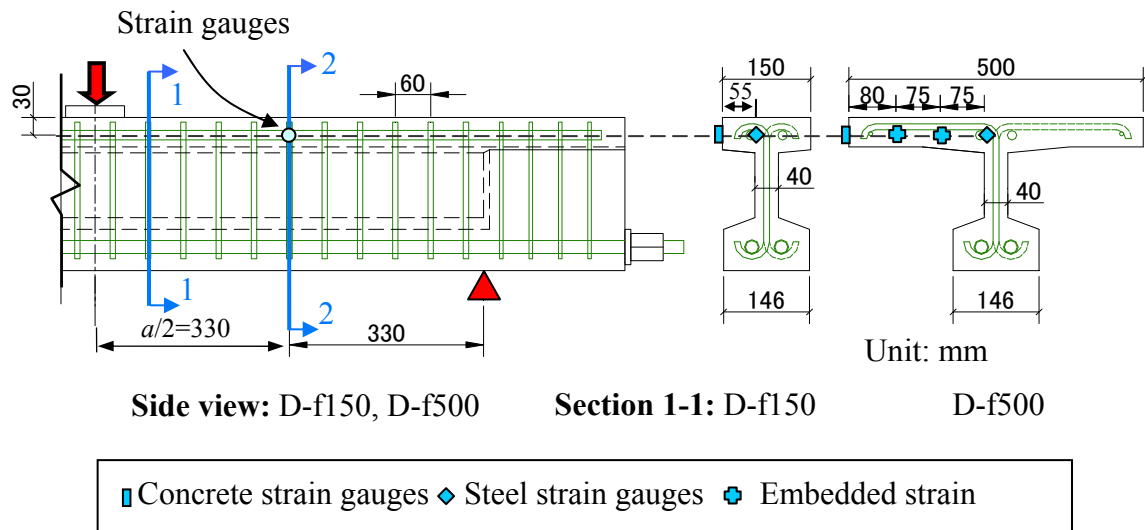
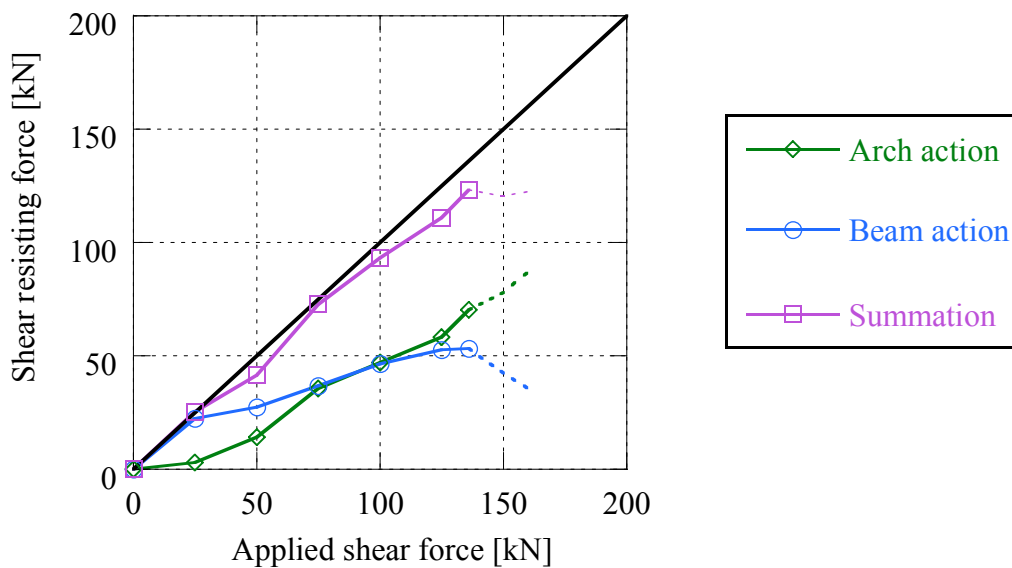


Fig. 4-14 Location of strain gauges of D-f150 and D-f500

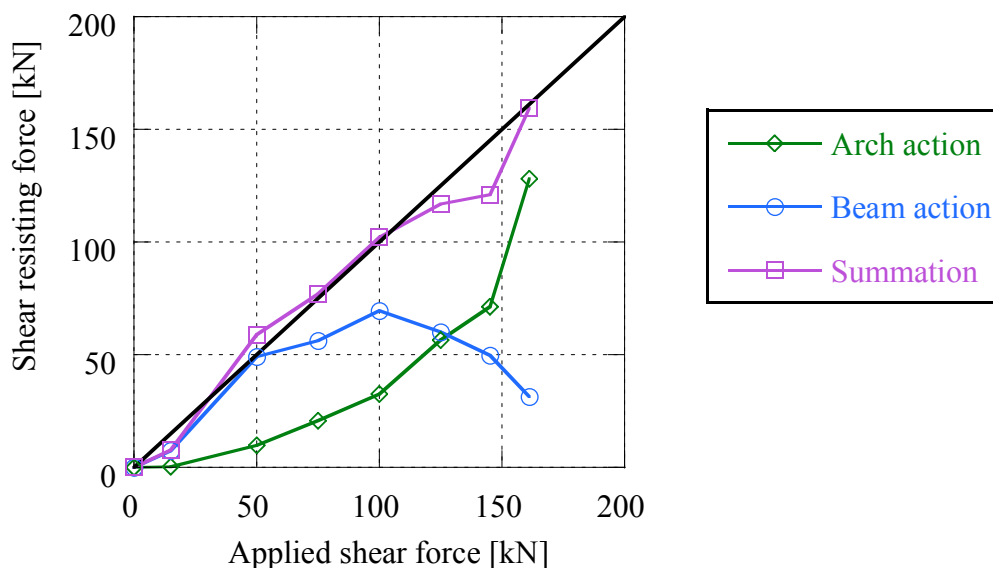
tensile strain. The strain was measured at the middle of shear span. The locations of these strain gauges are shown in **Fig. 4-14**. As for the case of D-f500, the strain concentration can be observed at the zone near the center of the beam. This tensile strain is resulted from the

development of high diagonal compressive stress in the web. Furthermore, as the distance from the beam center increases, the strain decreases and becomes comparatively small at some distance. Thus the outside flange portion resists less stress and can be considered as ineffective area. Since the increase in flange width becomes ineffective, diagonal compressive capacity is constant.

Secondly, the results of beam and arch actions between section 1 and 2 shown in **Fig.**



(a) D-f150



(b) D-f500

Fig. 4-15 Shear resisting components

4-14 calculated from the experimental results of D-f150 and D-f500 using Eq. 3-4 are demonstrated in **Fig 4-15**. The calculation will be accurate in case that the summation of both actions approximates the applied shear force (thick diagonal line in the graph). For both specimens, the beam action controls the shear resistance at the early stage. After that, the arch action gradually increases with the increase of the total shear resistance. Finally, the arch action becomes dominant in both specimens. Although the ratio between beam and arch actions are different in the early stage because of the difference in the upper flange width, it becomes similar just before the peak. This result reveals that the mechanism at failure does not change with wider upper flange width. As a result, the diagonal compressive capacity becomes constant whether b_f/b_w is changed. In addition, it is noted that the measured region included both B- and D-region. Based on the experimental observation that the failure occurred in the B-region, the result of this measurement will not be considered in the equation formulation.

In the equation by Kobayashi [3] (Eq. 2-10), the diagonal compressive capacity from experimental results was divided by the effect of the other parameters ($=V_{exp}/0.379\beta_a\beta_vf''_{cs}$) and the quotient was taken from the effect of b_f/b_w . In contrast, the result discussed in this section revealed no effect of b_f/b_w . Therefore, the equation by Kobayashi [3] does not reflect the actual behavior and is not evaluated in chapter 5.

4.5 Series IV - Effect of d or size effect

This series consists of two specimens, C-s100-I and C-s100L, respectively. Size effect was examined by comparing between C-s100-I and C-s100L. Size of C-s100L was magnified by a 1.45 scale factors in three dimensions, width, height and length. The f'_c , p_w , a/d , b_f/b_w as well as stirrup spacing in this series were designed to be constant. As a result of series I, stirrup spacing was chosen to be constant, rather than stirrup ratio. Load-deflection response of this series is displayed in **Fig. 4-16**. In larger size specimen, failure was more brittle and explosive as representing in a sharp drop of the applied load after the peak load. **Figure 4-17** plots the shear strength v_{exp} as a function of d . It exhibits slight variation of diagonal compressive capacity with the change of d . Therefore, the effect of d from 220 to 319 mm cannot be observed and can be considered as insignificant in the prediction of diagonal compressive capacity.

From **Table 4-1**, $s_{c,avg}$ is wider in larger specimen and approximate 1.45 times larger. The specimens, which have the smallest effective depth in this and Zakaria's study [16], were

selected as the reference beams. The relationship between the average crack spacing at the peak load normalized by the average crack spacing of the reference beam and the effective depth is illustrated in **Fig. 4-18**. It can be observed that the normalized crack spacing increases proportionally with the increase in effective depth of specimens.

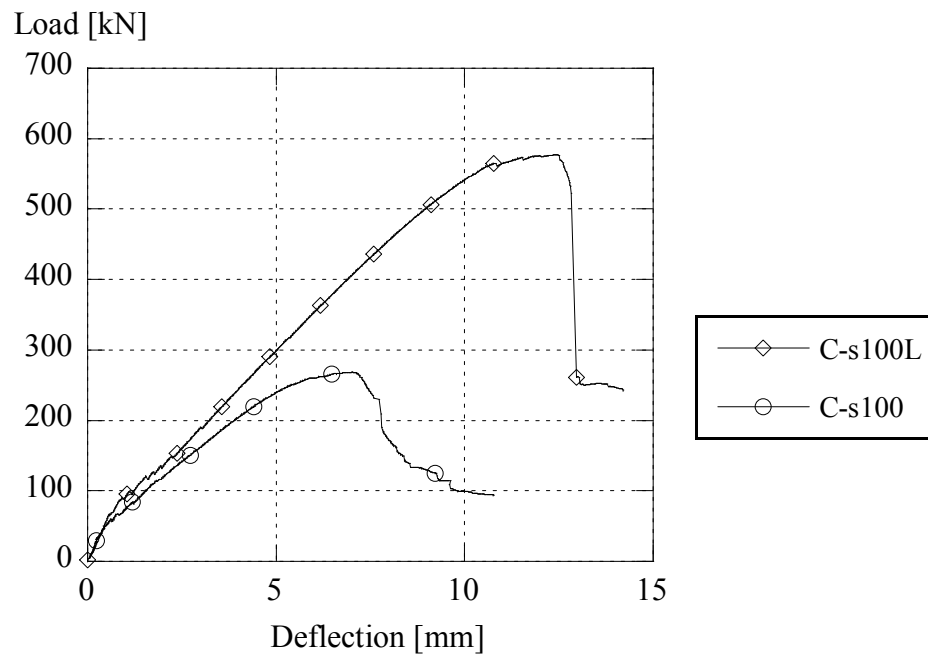


Fig. 4-16 Load-deflection relationships

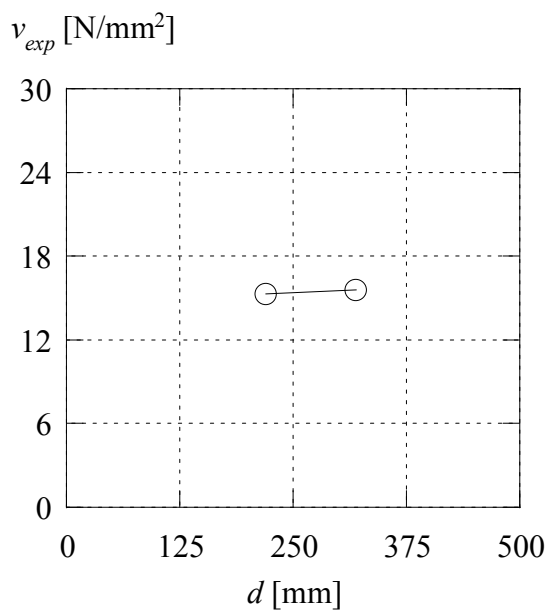


Fig. 4-17 Effect of d

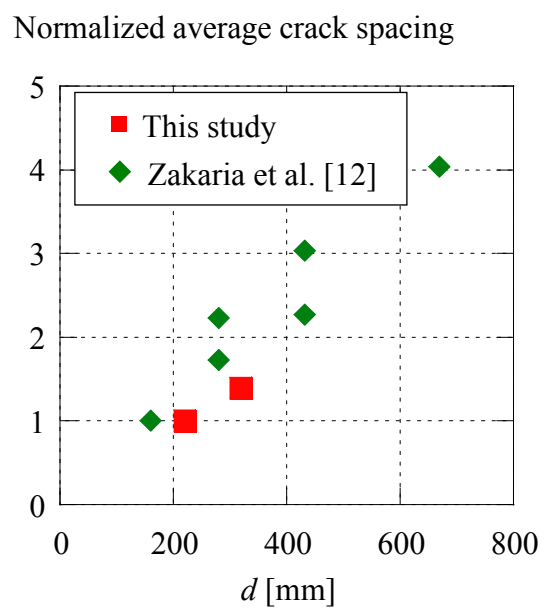


Fig. 4-18 Effect of d on diagonal crack spacing

4.6 Angle of diagonal cracks and principal strain

From the photos taken at the peak load, the diagonal crack angle (β_i) of each crack was measured at the middle height of the web. The total of 27 data (12 specimens from this study

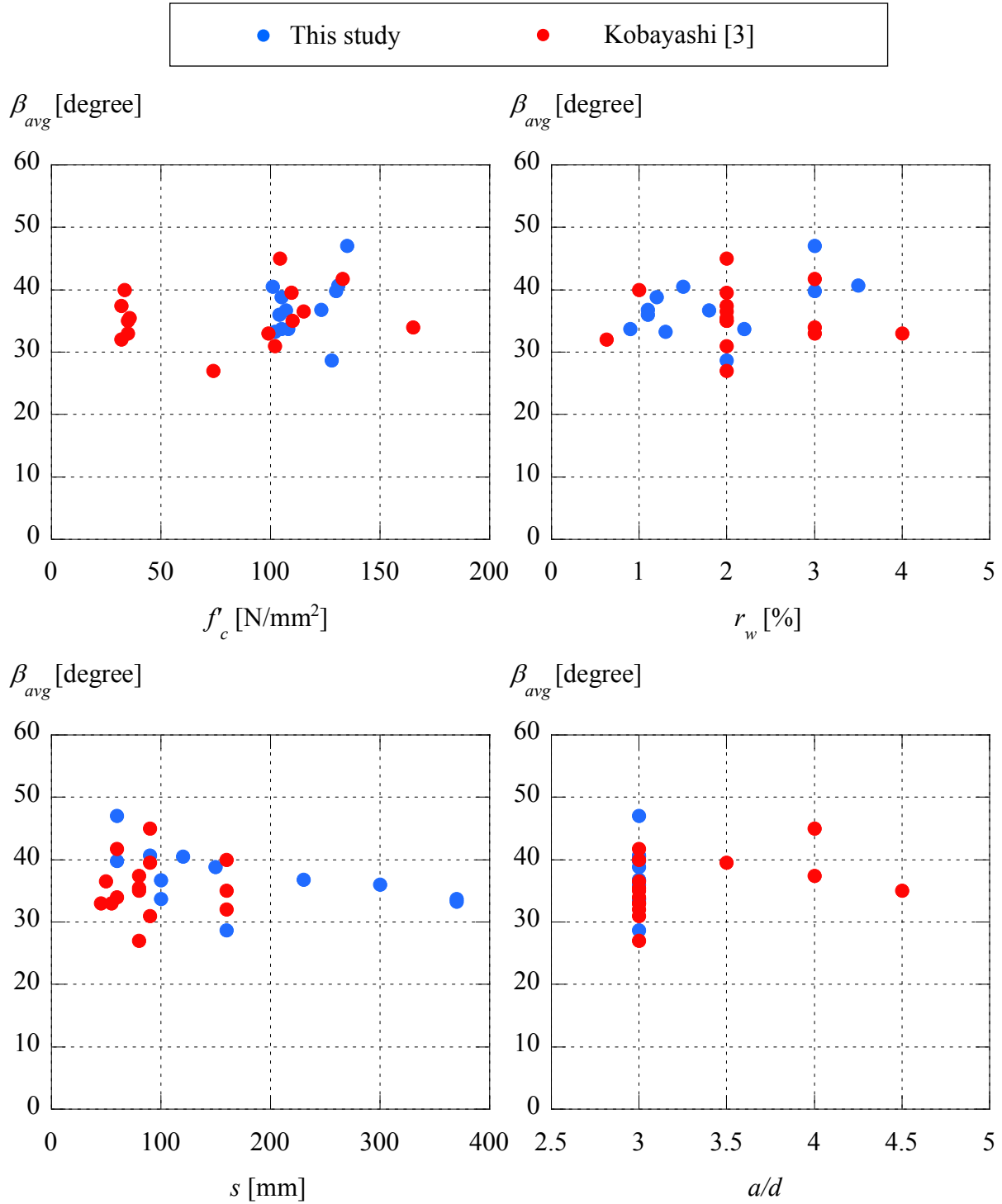


Fig. 4-19 Relationship of the average diagonal crack angle in B-region

plus 15 specimens of Kobayashi [3]) was used in this evaluation. The example of β_i measurement of a crack is demonstrated in **Fig. 3-4**. The measurement of β_i was focused in B-region since it was observed that the crushing areas, which are corresponding to the failure regions, were mostly found in that region. The relationship between the average diagonal crack angles in B-region at the peak load (β_{avg}) and f'_c , r_w , s and a/d are illustrated in **Fig. 4-19**. It can be seen that the values of β_{avg} are scattering in the range between 27 to 47 degrees. A clear tendency of β_{avg} and each possible factor influencing the crack angle cannot be found. The average value of β_{avg} for all specimens was 36.3 degrees with the coefficient of variation 12.6%.

Vecchio and Collins [6] reported that the angle of principal stress differed from the angle of principal strain within ± 10 degrees. For a reasonable simplification, the angle of principal stress can be assumed to coincide with the angle of principal strain. The angle of principal strain was measured in the four specimens with $d = 500$ mm (C-s300-II, C-s370-II(13), C-s370-II(16) and C-s230-I). The location of the measurement is shown in **Fig. 3-3**. The results measured at the peak load are exhibited in **Fig. 4-20**. To avoid excessive measured tensile strain, some gauges, which were propagated through by diagonal cracks, were defined as broken and were excluded from the calculation. The angle of principal strain varies from 21.3 to 36.9 degrees. The average angle

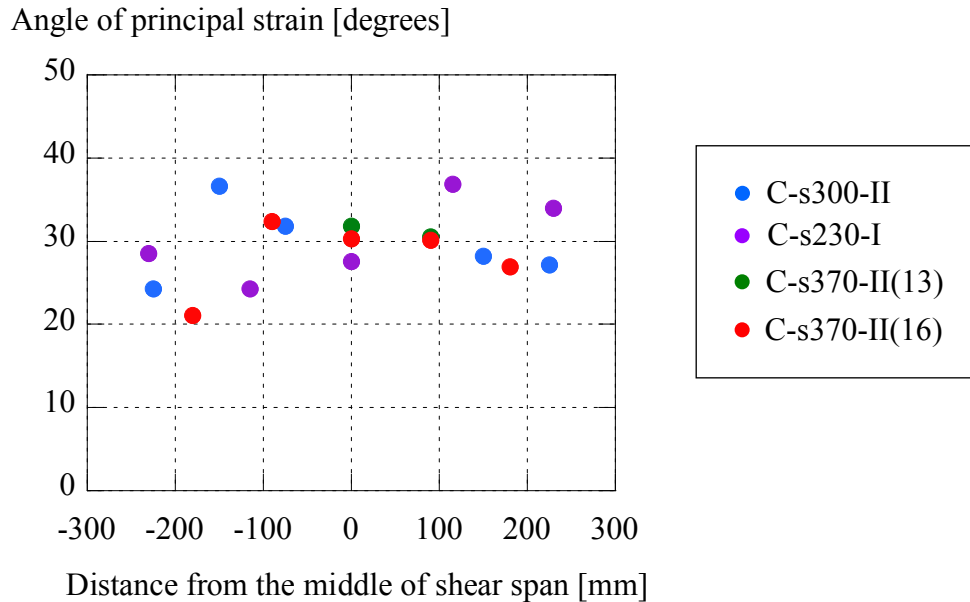


Fig. 4-20 Distribution of angle of principal strain

of principal strain of C-s300-II, C-s370-II(13), C-s370-II(16) and C-s230-I are 29.6, 31.2, 28.2 and 30.3 degrees, respectively. As well, the angle of principal strain has no tendency with stirrup ratio, spacing, diameters and configuration.

The average of diagonal cracks angle is slightly steeper than the average of principal strain angle (6 degrees). Thus, it can be assumed that the angle of diagonal cracks parallel to the angle of principal strain, as well as the principal stress.

4.7 Conclusions in chapter 4

In order to investigate the effect of each parameter on diagonal compressive capacity of RC beams, the loading test of 12 RC I-beams with different compressive strength (f'_c), stirrup ratio (r_w), stirrup spacing (s), flange width to web width ratio (b_f/b_w) and effective depth (d) were carried out. As a result, the below conclusions can be drawn.

- 1) By considering the crushing of concrete at the web combined with crack patterns and no yielding of stirrups, it indicated that the diagonal compression failure occurs even stirrup ratio is less than 2%.
- 2) With insufficient confinement effects provided by wider spacing of stirrups, the localization of compressive stress in struts occurred. Thus, stirrup spacing is prominent, not stirrup ratio, for evaluating diagonal compressive capacity of RC beams.
- 3) The diagonal compressive capacity linearly decreases with wider stirrup spacing regardless of its diameters and configuration when stirrup spacing is narrower than 140 mm; however, the effect of spacing becomes negligible when stirrup spacing is wider than 140 mm.
- 4) Even though wide stirrup spacing is provided, the sufficient confinement effect is achieved by providing two-legged stirrups and prevents the localization of compressive stress in struts.
- 5) The greater effect of the stirrup spacing on the diagonal compressive capacity of RC beams was observed in higher strength concrete. Thus, the effect of compressive strength of concrete and stirrup spacing is interrelated.
- 6) The ratios of flange width to web width from 3.75 to 12.5 and effective depth from 220 mm to 319 mm have almost no influence on the diagonal compressive capacity of

RC beams.

- 7) There is no clear relationship between the average diagonal crack angle in B-region and compressive strength, stirrup ratio, stirrup spacing, and shear span to effective depth ratio.
- 8) The angle of principal strain in B-region had no tendency with stirrup ratio, spacing, diameters and configuration.

Chapter 5

Proposal of predictive equation

At present, the equations for diagonal compressive capacity were provided by some researchers and associations throughout the world. These equations were reviewed in Chapter 2. As stated before, the applicability of these equations to high strength concrete were suspected. In this chapter, an accurate and simple equation, applicable to high strength concrete, for predicting diagonal compressive capacity of RC beams is proposed on the basis of the experimental results including Kobayashi [3]. Subsequently, the accuracy of diagonal compressive capacity calculated by the proposed equation is compared with those evaluated by JSCE Standard specifications [2], ACI Building Code [8], Eurocode 2 [9], *fib* Model Code 2010 [10] and the equation by Placas and Regan [7]. However, the equation by Kobayashi [3] does not reflect actual behavior as discussed in section 4.5 and is not evaluated in this chapter.

5.1 Development of the equation

A free body diagram cut in a vertical direction at the critical section is illustrated in **Fig. 5-1**. The forces acting on cutting surface are:

- 1) Compressive force of concrete fiber, C'
- 2) Diagonal compressive force of strut in web, D'
- 3) Tensile force of longitudinal reinforcement, T

Considering the equilibrium in a vertical direction, the shear force (V) must be resisted by the vertical component of D' . Since it is difficult to indicate the exact width of compressive struts from experiments, this research assumed a uniform distribution of diagonal compressive stress in web. Subsequently, the average diagonal compressive stress in web, σ_2 is equal to $D'/(b_w j d \cdot \cos\theta)$. With the assumptions that the diagonal compressive force causes the failure of the member without yielding of stirrups, the following expression can be obtained.

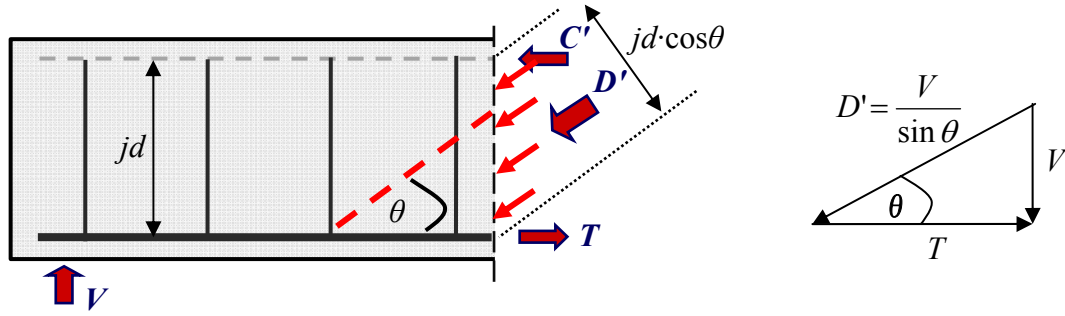


Fig. 5-1 Free-body diagram cut at the critical section

$$V_{\max} = D'_{\max} \sin \theta = \sigma_{2\max} b_w jd \cos \theta \sin \theta \quad (5-1)$$

where; $V_{\max}$: shear force carried by web at web crushing stage (N).

θ : angle of inclination of concrete struts to the longitudinal reinforcement (degree).

$\sigma_{2\max}$: critical average diagonal compressive stress in the web concrete (N/mm²).

b_w : web width (mm).

jd : moment lever arm (mm).

With the use of trigonometry, Eq. 5-2 is proposed based on Eq. 5-1.

$$V_{\max} = \frac{1}{2} \sigma_{2\max} b_w jd \sin 2\theta \quad (5-2)$$

The $\sigma_{2\max}$ of the diagonally cracked concrete tends to less than compressive strength of the same concrete obtained in standard cylinder test. It is because diagonally cracked concrete was subjected to tensile strains in the direction perpendicular to the compression [6]. An appropriate formula for $\sigma_{2\max}$ was empirically generated by using the trends of the experimental results presented in the last chapter. The data indicated that compressive strength (f'_c) and stirrup spacing (s) had a major effect on the diagonal compressive capacity of RC beams, whereas the influence of flange width to web width ratio (b_f/b_w) from 3.75 to 12.5 and effective depth (d) from 220 mm to 319 mm was insignificant. As well, the review of the study by Kobayashi [3] in Chapter 2 revealed that the effect of shear-span to effective depth ratio (a/d) was very slight for the cases of $a/d > 3.0$. Hence, only the effect of f'_c and s

was taken into account in the development of predictive equation.

As mentioned in section 4.2, the diagonal compressive capacity linearly decreased with wider s regardless of its diameters and configuration when s is narrower than 140 mm. The effect of s became negligible when s is approximately wider than 140 mm. Section 4.3 indicated that the greater effect of s was observed in higher strength concrete and the diagonal compressive capacity increased with the increase in f'_c with the different incremental ratio depending on the value of s ; hence, the effect of f'_c and s was interrelated. Since the effect of d is negligible, the proposed equation was the function of s , rather than s/d .

As for the stirrup configuration, even though one-legged stirrups with wide spacing resulted in lower diagonal compressive capacity and the closed-typed stirrups are more preferable for the reinforcement detailing in most design standards, but, as a result, the proposed equation becomes rather safer side and there is no problem in practical use of one-legged stirrups. Therefore, the effect of stirrup configuration in wide spacing was averaged in the modification.

The 3-D expression of v_{exp} - f'_c - s relationship was used to formulate a function of σ_{2max} having the minimal variation and corresponding to the relationship observed from the experimental results. This function of σ_{2max} was considered to have adequate simplicity and was developed through a surface-fitting technique as the following equation:

$$\sigma_{2max} = k_1 \sqrt{f'_c} \quad (f'_c: \text{N/mm}^2) \quad (5-3)$$

$$\text{where } k_1 = (4.57 - \frac{s}{79.1}) > 2.79 \quad (s: \text{mm}) \quad (5-4)$$

The residual plot of v_{exp} - f'_c - s demonstrated in **Fig. 5-2** manifests the accuracy of the fitting surface. The coefficient of determination (R^2) of this fitting is 0.9135, which indicates that the data are well fit the curve. Even the experimental tendency in **Fig. 4-10** shows f'_c to the power of variable s , for simplicity, Eq. 5-3 is simplified by using f'_c to the constant power of 0.5, which is also simplified from 0.53.

As for the angle of compressive strut (θ), this study assumed it to be the same as that of the average crack angle in B-region (β_{avg}). Even though it is widely accepted that the angle of compressive strut is variable; however, it was chosen to be constant in this research since an obvious tendency of β_{avg} cannot be discovered as explained in section 4.7. Various researchers have proposed indeterminate truss systems representing coincident behavior of

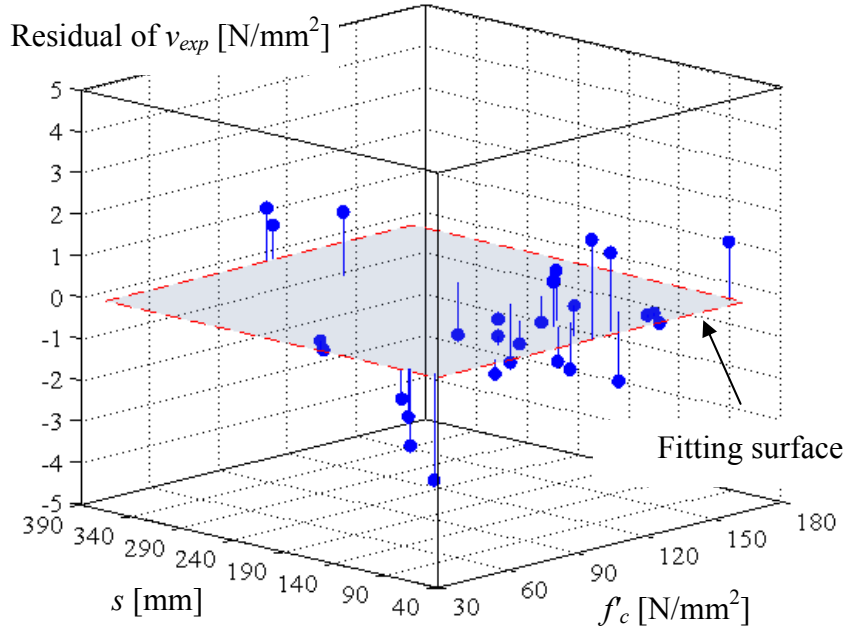


Fig. 5-2 The residual plot of v_{exp} - f'_c - s

tied arch and beam [17–19]. Considering these models, when $a/d \geq 3.0$, the arch action have insignificant effect on θ which was also observed from the experiment. Hence, the simplified of average value of β_{avg} (36 degrees) was selected as the constant value of θ . By substituting Eq. 5-3 into Eq. 5-2, $jd = (7/8)d$ as recommended in JSCE Standard [2] and $\theta = 36$ degrees, the diagonal compressive capacity is given by the following equation.

$$V_{cal} = k_s \sqrt{f'_c} b_w d \quad (f'_c: \text{N/mm}^2) \quad (5-5)$$

$$\text{where; } k_s = 1.9 - \frac{s}{190} \geq 1.16 \quad (s: \text{mm}) \quad (5-6)$$

V_{cal} : the diagonal compressive capacity predicted by the proposed equation.

b_w : web width (mm)

d : effective depth (mm)

The range of application of Eq. 5-5 are $45 \text{ mm} \leq s \leq 370 \text{ mm}$, $0.6\% \leq r_w \leq 4\%$, $19 \text{ N/mm}^2 \leq f'_c \leq 165 \text{ N/mm}^2$, $3.75 \leq b_f/b_w \leq 12.5$ and $2.5 \leq a/d \leq 4.5$.

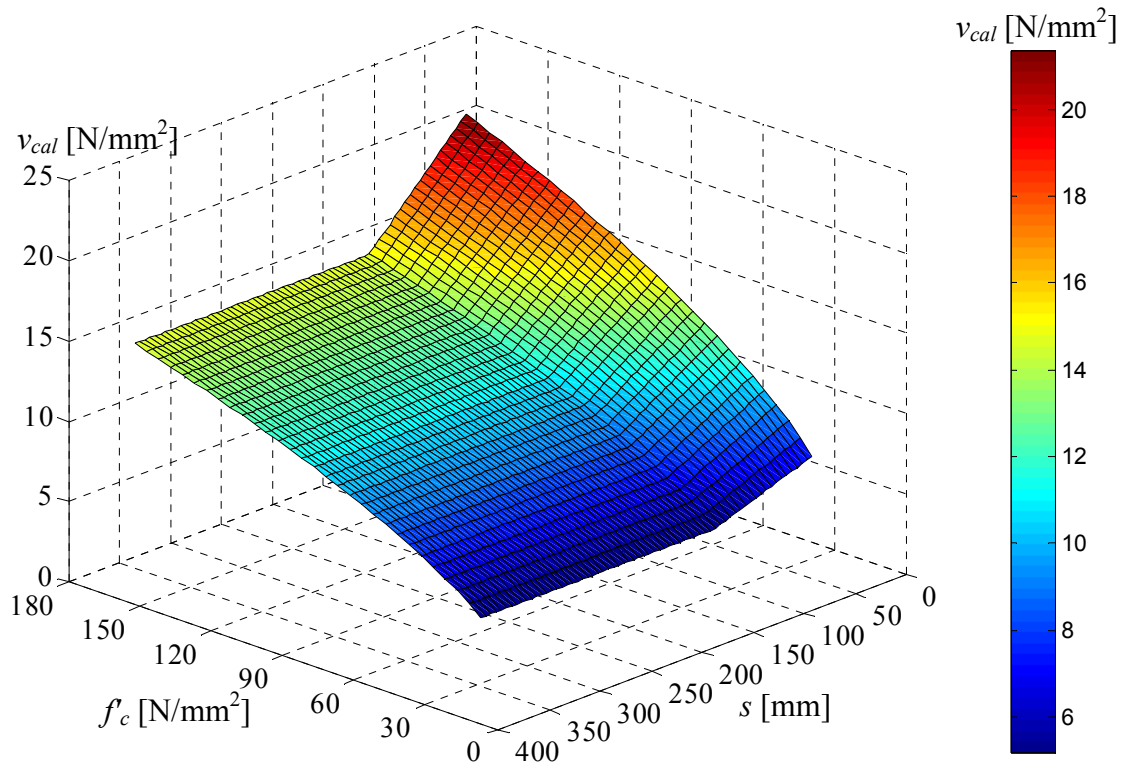


Fig. 5-3 Shape of the proposed equation (Eq. 5-5)

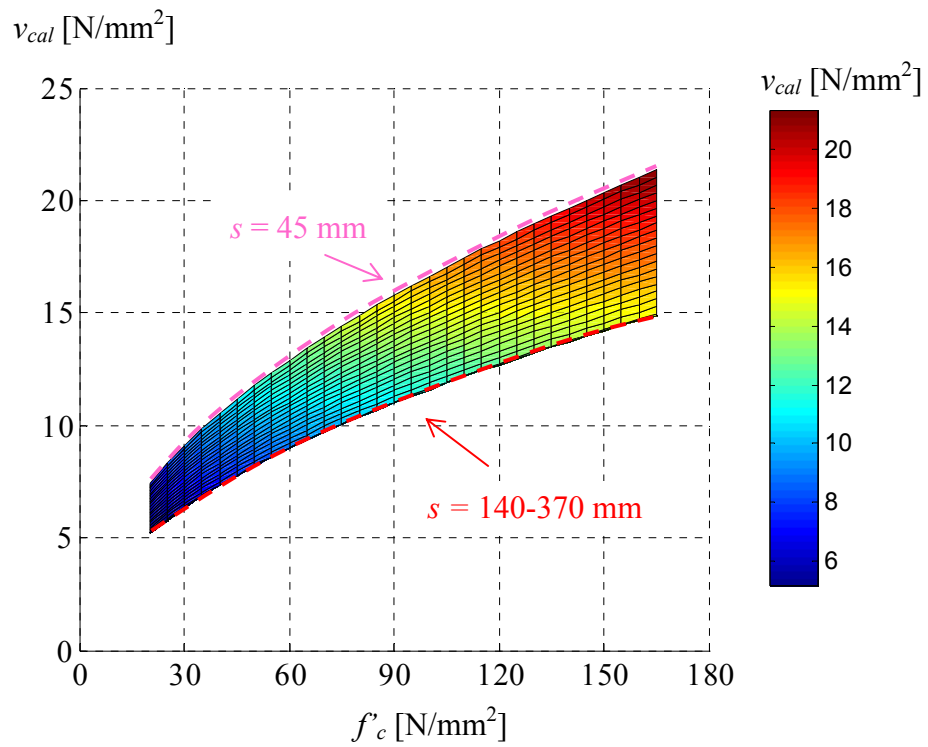


Fig. 5-4 Shape of the proposed equation (v - f'_c view)

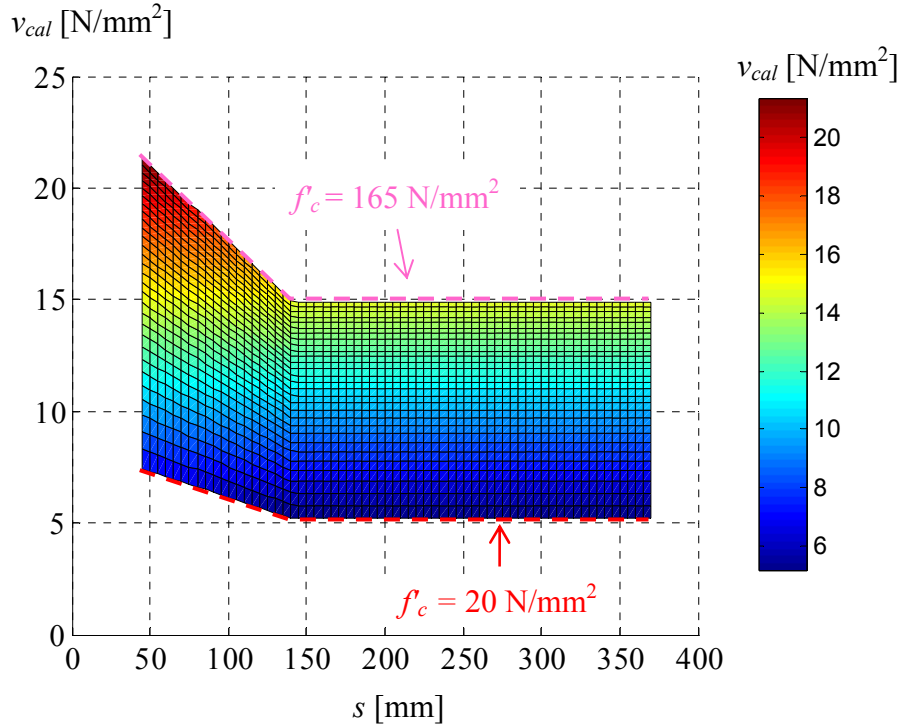


Fig. 5-5 Shape of the proposed equation (v - s view)

The shape of Eq. 5-5 in 3D relationship of v_{cal} - f'_c - s is illustrated in **Fig. 5-3** ($v_{cal} = V_{cal}/b_w d$). For better understanding, the shapes of Eq.5-4 in v - f'_c and v - s view are presented in **Fig. 5-4** and **5-5**, respectively. **Figure 5-4** shows that the increment of v_{cal} with the increase in f'_c depends on s . The greater f'_c results in the greater effect of s . However, **Fig. 5-4** may not be consistent with the actual tendency shown in **Fig. 4-10**. It is because **Fig. 5-4** is the result of simplification by using f'_c to the constant power of $1/2$. The relationship between v_{cal} and s is represented by a bilinear curve as shown in **Fig. 5-5**. The predicted diagonal compressive capacity by the proposed equation linearly decreases with the increase of s and become constant regardless of s when s is wider than 140 mm.

The strength reduction factor of Eurocode 2 [9] (Eq. 2-5), MC2010 [10] (Eq. 2-7) and the proposed equation ($= \sigma_{2max}/f'_c^{1/2}$) against f'_c is illustrated in **Fig. 5-6**. The factor in the proposed equation and MC2010 non-linearly decreases with the increase in f'_c . In case of the proposed equation, when f'_c increases from 20 to 165 N/mm², the factor decreases from 0.89 to 0.31 when using $s = 45$ mm and from 0.62 to 0.22 when using $s > 140$ mm, while MC2010 decreases from 0.55 to 0.31. The factor of Model Code 2010 is close to that of the proposed equation with $s = 140$ mm at f'_c around 25 N/mm². As f'_c increases, the Model Code intersects the proposed equation with closer s and converge with $s = 45$ mm at $f'_c = 150$ N/mm². As for

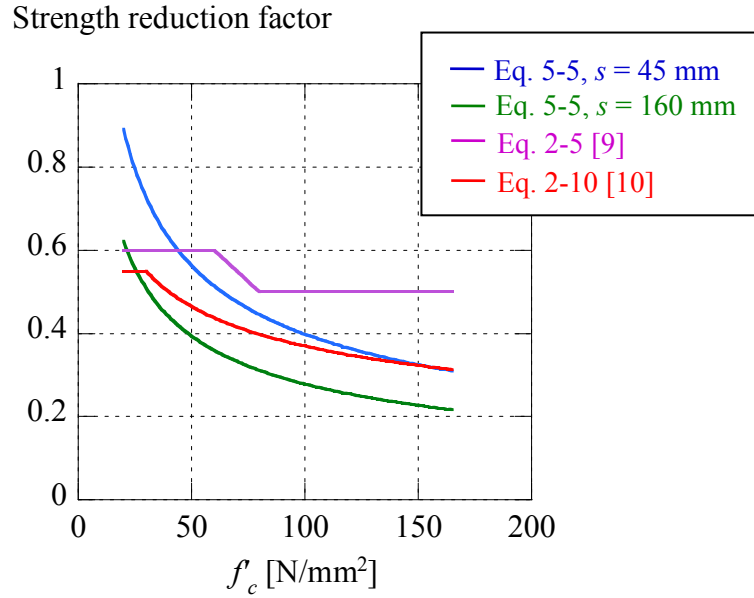


Fig. 5-6 Comparison of strength reduction factors

Eurocode 2, the factor starts to decrease linearly at $f'_c > 60$ N/mm² and becomes constant when $f'_c > 80$ N/mm². Eurocode 2 has higher strength reduction factor than MC2010 in the whole range. The similar relationship between Eurocode 2 (Eq. 5-5) and Eq. 2-5 when $s = 45$ mm can be observed in the range of $f'_c = 60$ -80 N/mm². It is worth mentioning that the Model Code 2010 and Eurocode 2 are based on the variable angle truss model.

5.2 Accuracy of the existing equations for diagonal compressive capacity

The experimental results of the total of 39 beams including Kobayashi [3], Rangan [13], and Leonhardt et al. [14] were gathered. All specimens had compressive strength from 19 to 165 N/mm², $a/d = 2.5$ -4.5 (slender beams) and were simply supported and reported to fail in the diagonal compression failure without yielding of stirrups. Some test results were excluded, even it was reported to fail in the diagonal compression, since stirrups had already yielded at the failure (Reineck [15]) and the specimens had a/d lower than 2.5 (Takagi et al. [20], Nishiura et al. [21], Matsuzaki et al. [22]). The range of $a/d = 2.5$ -4.5 was selected because when $a/d > 4.5$, the beam tends to fail in flexure as summarized by Macgregor and Wight [23]. In order to clarify the applicability of the existing equations presented in chapter 2 to high strength concrete, the equations were applied beyond their limitation of compressive strength. The comparison of the shear capacity from experiments and the results calculated by the equations reviewed in chapter 2 are listed in **Table 5-1**. Safety factors were not considered in this calculation of diagonal compressive capacity in **Table 5-1** by the design codes (JSCE

Table 5-1 Comparison of the test results with various equations

Specimen	f'_c [N/mm ²]	s [mm]	a/d	d [mm]	V_{exp} [kN]	$V_{exp}/$ V_{Placas}	$V_{exp}/$ V_{JSCE}	$V_{exp}/$ V_{ACI}	$V_{exp}/$ V_{EC2}	$V_{exp}/$ V_{MC2010}	$V_{exp}/$ V_{cal}
C-s150-I	105	150	3.0	220	102.7	0.84	0.91	1.37	0.49	0.78	0.98
C-s120-I	101	120			108.0	0.86	0.98	1.47	0.54	0.85	0.96
C-s100-I	107	100			134.3	0.99	1.18	1.78	0.63	1.01	1.07
C-s230-I	123	270		500	465.0	0.83	0.84	1.26	0.42	0.90	0.90
C-s300-II	104	300			535.0	1.04	1.05	1.58	0.57	0.85	1.07
C-s370-II(13)	105	370			508.5	1.01	0.99	1.50	0.54	0.89	1.11
C-s370-II(16)	102	370			527.0	0.97	1.03	1.55	0.57	0.70	1.13
D-s90	131	90		220	129.3	0.69	1.03	1.55	0.50	0.85	0.90
D-s160	128	160			110.1	0.72	0.88	1.33	0.43	0.73	0.95
D-f150	130	60			159.9	0.91	1.27	1.92	0.62	1.06	1.01
D-f500	135	60			161.4	0.90	1.26	1.90	0.60	1.04	1.00
C-s100L	108	100		319	288.7	1.38	1.20	1.81	0.64	1.03	1.09

Table 5-1 (cont.) Comparison of the test results with various equations

Specimen	f'_c [N/mm ²]	s [mm]	a/d	d [mm]	V_{exp} [kN]	$V_{exp}/$ V_{Placas}	$V_{exp}/$ V_{JSCE}	$V_{exp}/$ V_{ACI}	$V_{exp}/$ V_{EC2}	$V_{exp}/$ V_{MC2010}	$V_{exp}/$ V_{cal}
C-s45 ¹⁾	99.1	45	3.0	220	167.6	0.97	1.53	2.31	0.85	1.33	1.15
C-s60 ¹⁾	98.2	60			144.9	0.95	1.33	2.00	0.75	1.16	1.05
C-s90 ¹⁾	102	90			137.0	1.00	1.23	1.85	0.68	1.06	1.08
C-s50 ¹⁾	110	50			168.5	1.19	1.46	2.20	0.78	1.25	1.12
C-s160 ¹⁾	115	160			96.6	0.67	0.82	1.23	0.43	0.69	0.88
D-s60 ¹⁾	133	60			162.0	0.96	1.28	1.93	0.62	1.06	1.01
E-s60 ¹⁾	165				59.4	0.97	1.36	2.05	0.59	1.08	1.07
A-s160(7) ¹⁾	32	160			58.6	0.97	0.95	1.44	0.78	1.00	1.03
A-s160(9) ¹⁾	33.4				61.6	0.88	0.92	1.39	0.74	0.96	0.99
A-s80 ¹⁾	35.8	80			61.7	0.76	0.94	1.41	0.72	0.97	0.79
A-s55 ¹⁾	35.0	55			114.5	0.68	0.95	1.43	0.74	0.98	0.74
B-s80 ¹⁾	73.8	80			107.8	0.99	1.21	1.82	0.74	1.11	1.02
B-s55 ¹⁾	61.5	55			192.1	0.89	1.25	1.88	0.75	1.18	0.97
A-ad4 ¹⁾	32	80	4.0	563	68.1	0.89	1.09	1.65	0.90	1.15	0.92
A-ad4.5 ¹⁾	34.9	80	4.5		67.0	0.84	1.03	1.55	0.81	1.07	0.87
C-ad3.5 ¹⁾	110	90	3.5		122.7	0.87	1.07	1.60	0.57	0.91	0.93
C-ad4.0 ¹⁾	104	90	4.0		121.1	0.88	1.08	1.62	0.59	0.93	0.94
I1 ²⁾	36.5	50	2.5	563	453.1	1.12	1.44	2.17	1.10	1.48	1.1
I2 ²⁾	30.2				371.0	1.20	1.30	1.95	1.09	1.38	0.99
I3 ²⁾	31.2				369.1	1.09	1.49	2.24	1.24	1.58	1.14
I4 ²⁾	35.7				416.0	1.36	1.55	2.33	1.20	1.60	1.18
TA1 ³⁾	20.6	113	3.3	375	335.2	1.00	1.05	1.58	1.07	1.35	1.01
TA2 ³⁾	20.6				300.9	0.97	0.94	1.42	0.96	1.21	0.90
TA3 ³⁾	19.1				272.0	0.95	0.88	1.33	0.94	1.18	0.85
TA13 ³⁾	23.7				343.0	0.96	1.00	1.51	0.95	1.20	0.96
TA14 ³⁾	23.7				333.2	1.00	0.97	1.47	0.93	1.17	0.93
TA15 ³⁾	23.2				292.0	0.93	0.86	1.30	0.83	1.04	0.83

¹⁾ Kobayashi [3], ²⁾ Rangan [13], ³⁾ Leonhardt et al. [14]

Standard [2], ACI Building Code [8], Eurocodes 2 [9] and *fib* Model Code 2010 [10]). The relationship between the prediction by the equations and compressive strength along with the average of these ratios (*avg.*) with a coefficient of variation (*C.V.*) are presented in **Figs. 5-7, 5-8 and 5-9**. The value 1.00 indicates that the evaluation from the equation corresponded with the test results. Hence the value above 1.00 (blue line) represents the underestimation of the capacity, or conservative. The lower *C.V.* represents the smaller scattering.

Figure 5-7(a) compares the diagonal compressive capacity from test results with those obtained by the equation by Placas et al. [7] (Eq. 2-1). The average of V_{exp}/V_{Placas} equals to 0.94 with a *C.V.* of 14.9 %. It implies that Placas's equation can evaluate an average value of the diagonal compressive capacity even f'_c approximates 100 N/mm². On the contrary, this equation exhibits unacceptable variation as *C.V.* = 14.9%. However, it gives the lowest variation among these five equations since it takes into account the effect of stirrup. In addition, Eq. 2-1 overestimates the diagonal compressive capacity in most of the cases except

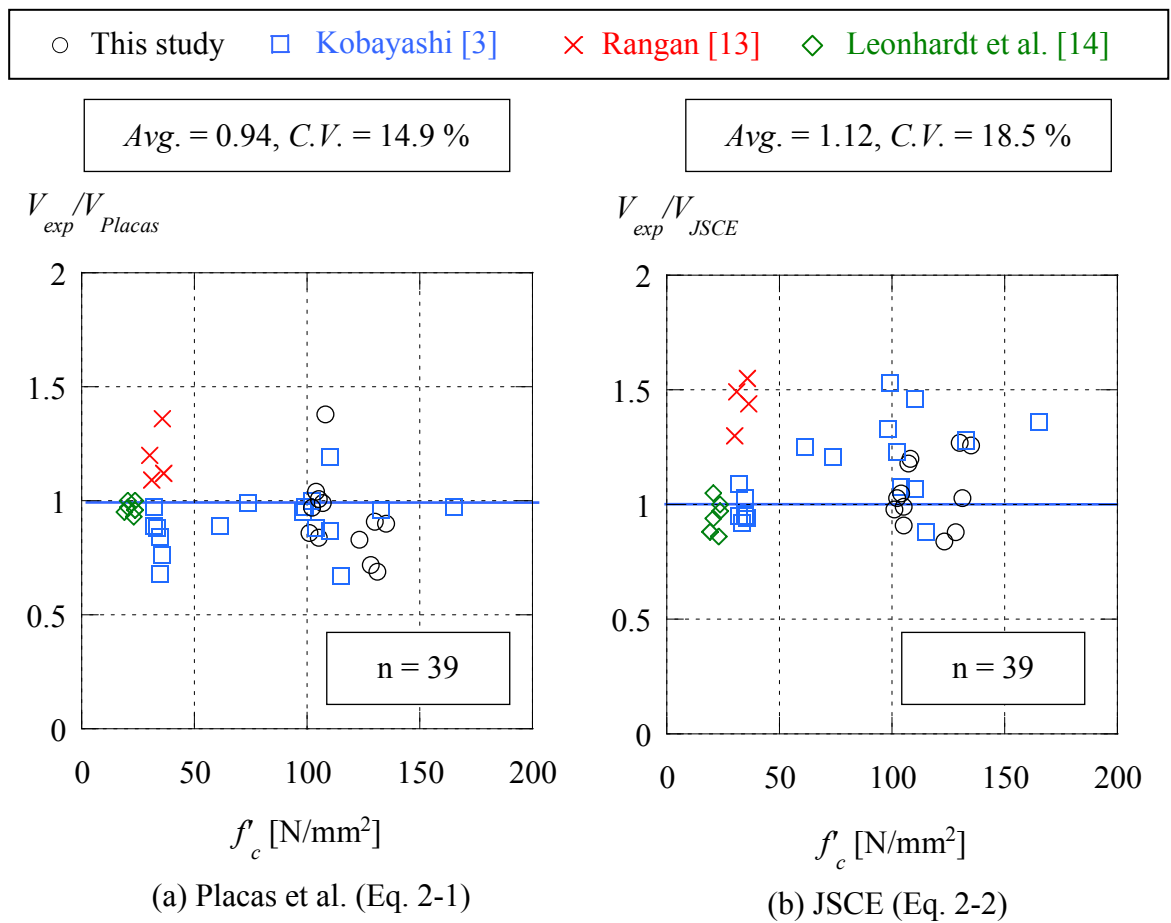


Fig. 5-7 Accuracy of Placas and JSCE equations

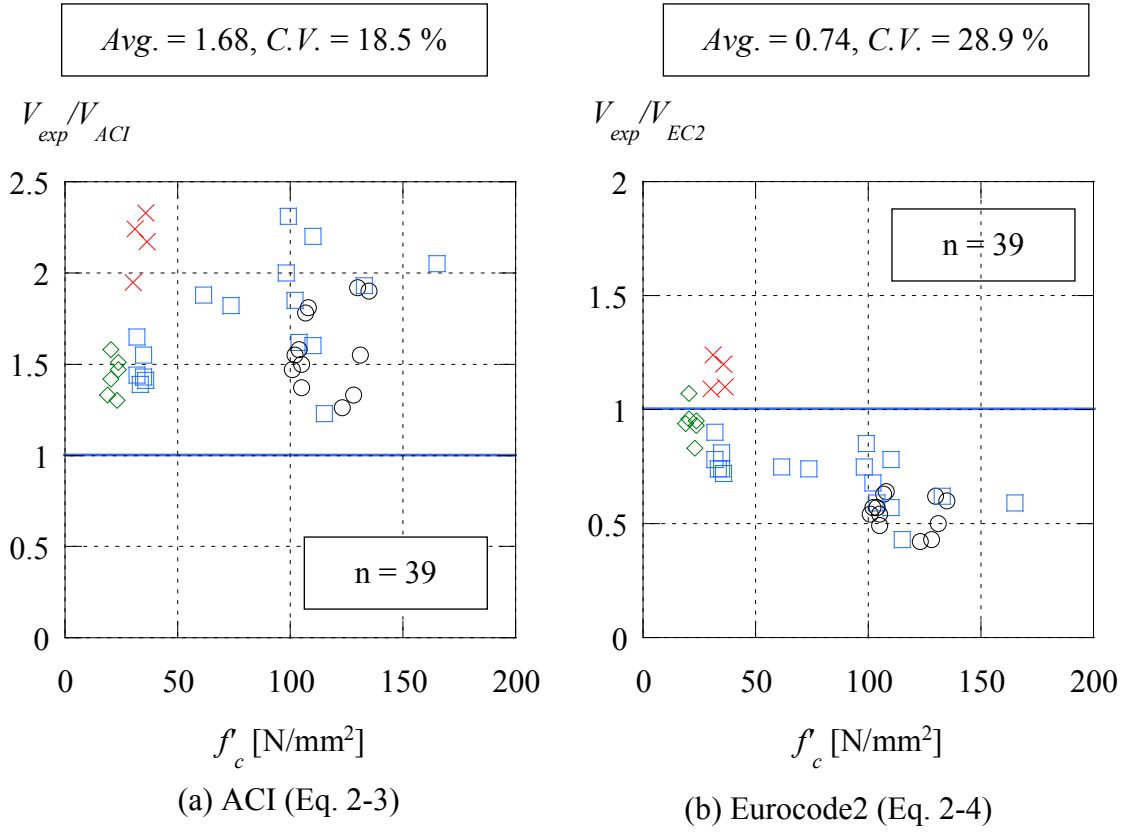


Fig. 5-8 Accuracy of ACI and EC2 equations

for the data of Rangan [13]. Despite the application of Placas's equation within its applicable compressive strength, the equation does not provide comparatively higher accuracy as the average is 0.98 with *C.V.* of 16.9% (n=16).

JSCE Standard Specifications (Eq. 2-2) demonstrate the average of $V_{exp}/V_{JSCE} = 1.12$. **Figure 5-7(b)** indicates that JSCE equation underestimates the capacity in almost all specimens. This is obvious since it is design equation which is normally conservative because of safety reason. In the same way as Placas's equation, the results calculated by Eq. 2-2 shows large variation as *C.V.* = 18.3%. Evaluating the accuracy of JSCE equation within its limitation of concrete strength, the average of $V_{exp}/V_{JSCE} = 1.09$ with *C.V.* of 21.0% (n=16) is obtained. It also gives insubstantial difference of accuracy.

The accuracy of ACI Building Code (Eq. 2-3) is represented in **Fig. 5-8(a)**. It can be seen that the ACI 318-08 does not correlate with the test data as average = 1.68 and *C.V.* = 18.5%. Equation 2-3 greatly underestimates the diagonal compressive capacity throughout the

range of data and is the most conservative among these five equations. As well, the acquired average = 1.66 and $C.V. = 19.8\%$ ($n=16$) when verifying the accuracy of ACI code inside its regulation for compressive strength reveal similar accurateness.

Similarly, the evaluation of the Eurocode 2 (Eq. 2-4) is shown in **Fig. 5-8(b)**. The *avg.* of V_{exp}/V_{EC2} is 0.74 with a $C.V.$ of 28.9 %. The selection of 45-degree θ results the prediction by Eurocode 2 in severe overestimation, especially in the case of high strength concrete, even it is based on the lower bound theory of plasticity. This result comes from that V_{EC2} is proportional to f'_c . Equation 2-4 gives huge scattering and the largest among these equations. Afterwards, the prediction of Eurocode 2 within its cover range of f'_c is assessed ($f'_c < 90$ N/mm²). The average is 0.92 with $C.V.$ of 18.2 % ($n=18$). Equation 2-4 performs much better in its range of application; however, its accuracy is unsatisfactory.

The accuracy of Level I Approximation of *fib* Model Code 2010 (Eq. 2-9) is presented in **Fig. 5-9**. The average is 1.07 with $C.V.$ of 20.4%. This equation can predict the average value with large variation as it also neglects the influence of stirrup. In normal strength concrete, MC2010 provides a safe estimation on the diagonal compressive capacity. However, it overestimates in most of the data in this study. The ratio of V_{exp}/V_{MC2010} decreases as f'_c

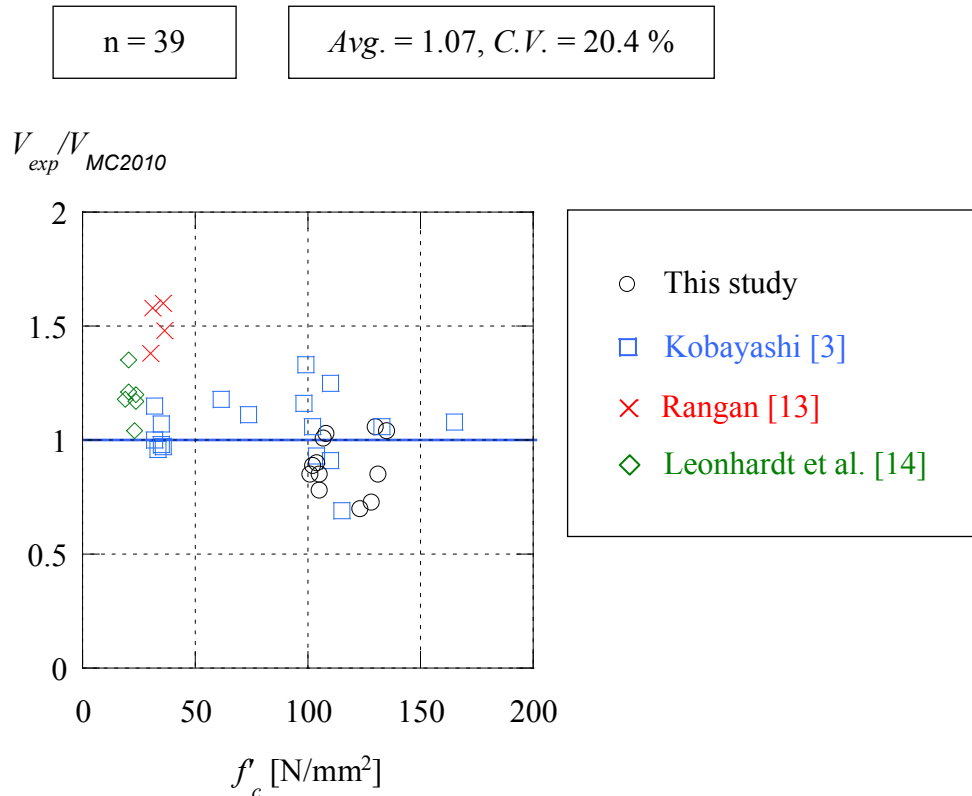


Fig. 5-9 Accuracy of MC2010 (Eq. 2-9)

increase.

From the reasons mentioned above, the existing equations should be improved in both accuracy and appropriateness. Equation 2-1 considers r_w as a representative of the effect of stirrup which contrasts to the reported insignificant effect of stirrup diameter in this study. The other equations provide large variation as it does not consider the influence of stirrup on the diagonal compressive capacity.

5.3 Accuracy of the proposed equation

The validity of the developed equation (Eq. 5-5) was established by the comparison between the predictions from the proposed equation with the experimental results. In **Fig. 5-10**, the accuracy of the proposed equation was evaluated with the same database used in section 5.2. The average of V_{exp}/V_{cal} is 0.99 with a coefficient of variation of 10.5%. From **Fig. 5-10**, the proposed equation may overestimate in low strength concrete ($f'_c \sim 30 \text{ N/mm}^2$) which comes from the attempt to simplify the equation by using f'_c to the constant power of $1/2$; thus, an appropriate safety factor should be selected by designers to ensure the safety.

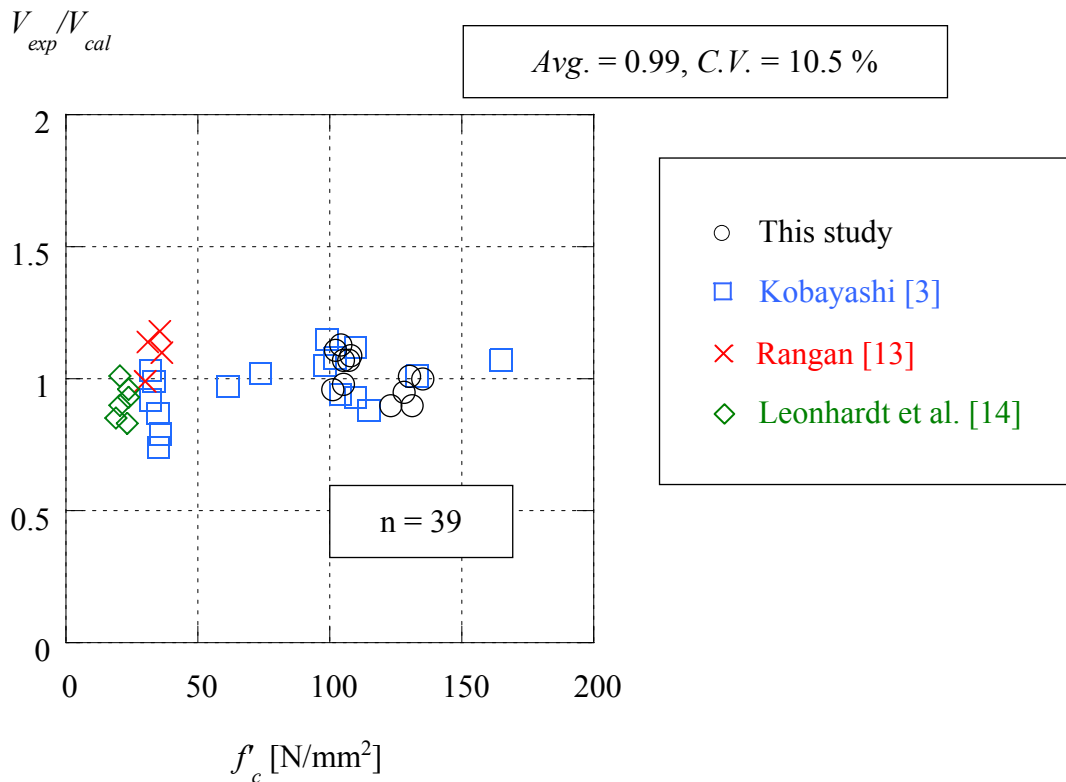


Fig. 5-10 Accuracy of the proposed equation (Eq. 5-5)

Table 5-2 Summary of the accuracy of the equations

Cases	No.	Average (<i>Avg.</i>)						Coefficient of variation (%)					
		Placas	JSCE	ACI	EC2	MC 2010	Proposed Equation	Placas	JSCE	ACI	EC2	MC 2010	Proposed Equation
All	39	0.94	1.12	1.68	0.74	1.07	0.99	14.9	18.5	18.5	28.9	20.4	10.5
$f'_c \geq 70$ N/mm ²	22	0.92	1.14	1.71	0.60	0.97	1.02	13.1	17.3	17.4	19.0	17.5	7.9
$f'_c < 70$ N/mm ²	17	0.97	1.09	1.65	0.93	1.21	0.95	16.5	20.4	20.2	17.9	17.0	12.6
$s = 45\text{-}140$ mm	30	0.95	1.16	1.75	0.79	1.13	0.98	15.5	17.9	17.9	26.7	18.3	11.1
$s = 140\text{-}370$ mm	9	0.88	0.93	1.41	0.55	0.84	1.01	14.7	8.6	8.7	23.9	13.6	8.7
$r_w \geq 2\%$	22	0.90	1.19	1.79	0.71	1.08	0.99	15.4	16.9	17.0	27.3	19.4	11.2
$r_w < 2\%$	17	0.99	1.02	1.54	0.78	1.06	0.99	13.1	17.3	17.1	30.6	22.3	9.8

The accuracy and *C.V.* of all equations are summarized in **Table 5-2**. It is categorized into seven cases: all cases, $f'_c \geq 70$ N/mm², $f'_c < 70$ N/mm², $s = 45$ -140 mm, $s = 140$ -370 mm ($k_s = 1.16$), $r_w \geq 2\%$, $r_w < 2\%$. The comparison for the cases: all cases, $f'_c \geq 70$ N/mm², $f'_c < 70$ N/mm², $s = 45$ -140 mm ($k_s > 1.16$), $s = 140$ -370 mm ($k_s = 1.16$) are illustrated in **Fig. 5-11**. The proposed equation offers a better correlation with this set of data and can predict diagonal compressive capacity more accurately with lower variation compared to the other existing equations in all cases. In the case $s = 140$ -370 mm ($k_s = 1.16$), the proposed equation always

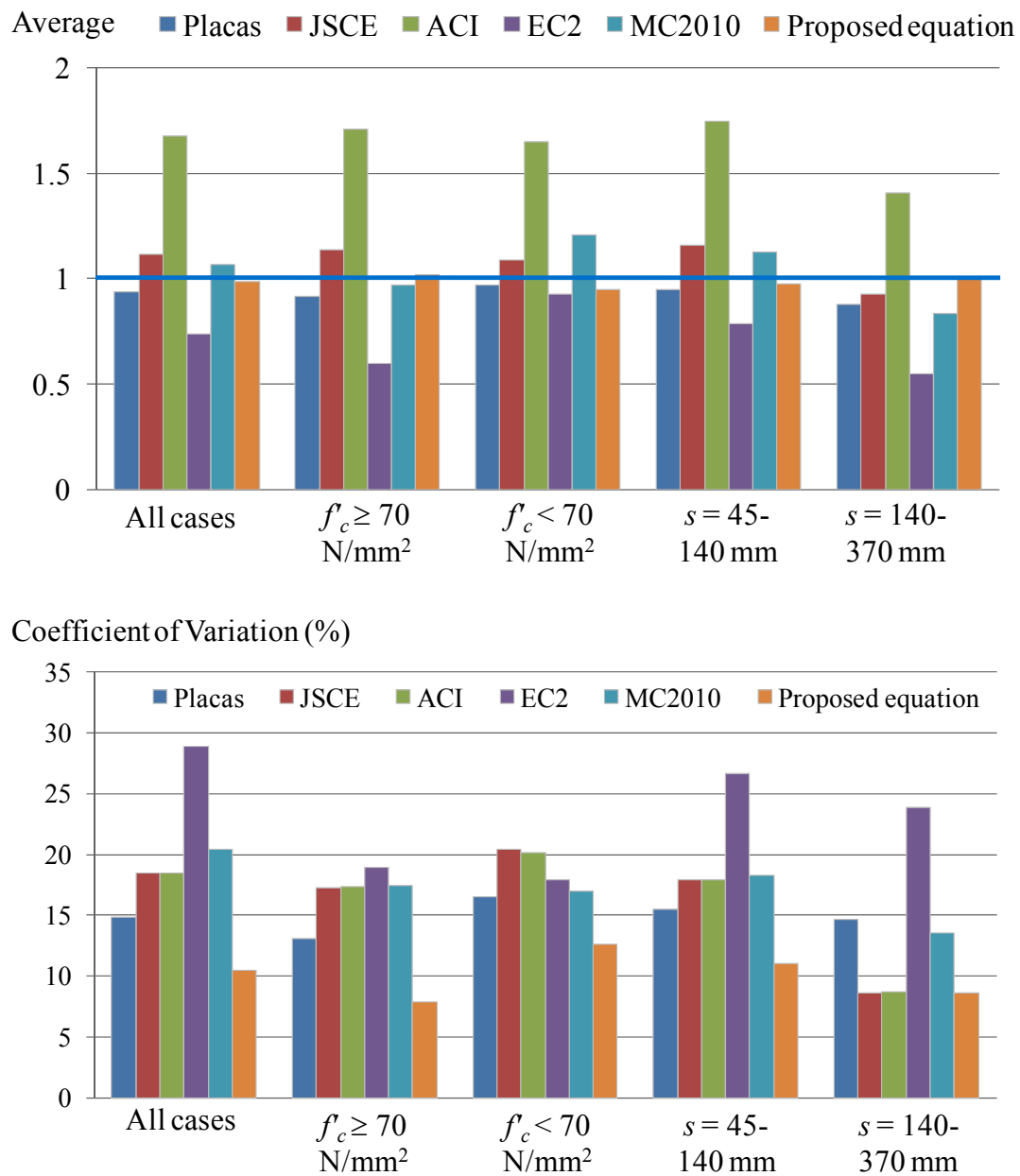


Fig. 5-11 Accuracy of the equations categorized by cases

gives smaller diagonal compressive capacity than JSCE Standard; however, **Table 5-2** and **Fig. 5-11** reveals that, in this case, the proposed equation shows better agreement ($Avg. = 1.01$) than JSCE Standard ($Avg. = 0.93$). ACI code is the most conservative among these equations in all cases. Even though this would cause an extravagant design but it can ensure safety of structures without the need of safety factors in all cases. Eurocode 2 severely overestimates in the case of concrete with $f'_c \geq 70 \text{ N/mm}^2$ since it is proportional to f'_c ; however, the better prediction can be achieved when $f'_c < 70 \text{ N/mm}^2$. It also has the largest variation among these equations considering all cases except for $f'_c < 70 \text{ N/mm}^2$. When it is categorized by f'_c , the smaller variation can be obtained.

It should be noted that, as the proposed equation is an empirical type, it can be applied only in the range of the study. This equation can be implemented to beams with the compressive strength from 19 to 165 N/mm^2 , the stirrup ratio in the range from 0.6 to 4%, the stirrup spacing in the range from 45 to 370 mm, the flange width to web width ratio from 3.75 to 12.5, the shear-span to effective depth ratio from 2.5 to 4.5, effective depth from 220 to 563 mm and web width from 40 to 150 mm. In addition, it is required that yielding of stirrups must not occur. As for stirrup arrangement, the proposed predictive equation covers one-legged (as used in this study), two-legged (as used in this study and Rangan [13]) and closed-type (as used by Leonhardt et al. [14]). However, the practical application of one-legged stirrups is not desirable.

There is a possibility that the size effect in the diagonal compression failure may exist when the effective depth of beams is larger than the upper limit in this study since Walraven and Lehwalter [17] reported significant size effect in shear compression failure of short beams. The proposed equation might be extended to members with several webs, i.e., box section, since in JSCE Standard shear capacity of the members shall be considered as the same as beams having b_w equal to the total widths of web [2]. Further investigation, however, is required to validate the applicability to this case.

In conclusion, the proposed predictive equation covers the application to concrete with compressive strength up to 165 N/mm^2 and practical stirrup spacing. It provides the sufficient simplicity and precise prediction of the diagonal compressive capacity of RC beams. This research will enable the wider use of high strength materials for the sustainability and the development of rational designs for the diagonal compressive capacity.

5.4 Conclusions in chapter 5

- 1) A predictive equation considering the compressive strength and stirrup spacing for evaluating the diagonal compressive capacity of RC beams was developed on the basis of the experimental findings. The proposed equation covers the application to RC beams with compressive strength up to 165 N/mm^2 and practical stirrup spacing ($45 \text{ mm} \leq s \leq 370 \text{ mm}$).
- 2) The accuracy of the existing equations for diagonal compressive capacity by Placas et al., JSCE Standard Specifications, ACI Building Code, Eurocode 2 and *fib* Model Code 2010 were evaluated. As a result, the existing equations should be improved in both accuracy and appropriateness on the prediction of the diagonal compressive capacity of RC beams.
- 3) The proposed equation has adequate simplicity and offers precise accuracy ($\text{Avg.} = 0.99$) with low variation ($C.V. = 10.5\%$) on the prediction of the diagonal compressive capacity of RC beams.
- 4) The proposed equation can be implemented to beams with the compressive strength: $19\text{-}165 \text{ N/mm}^2$, stirrup ratio: $0.6\text{-}4\%$, stirrup spacing: $45\text{-}370 \text{ mm}$, flange width to web width ratio: $3.75\text{-}12.5$, shear-span to effective depth ratio: $2.5\text{-}4.5$, effective depth: $220\text{-}563 \text{ mm}$, web width: $40\text{-}150 \text{ mm}$ and one-legged, two-legged and closed-typed stirrups.

Chapter 6

Conclusions and recommendations

6.1 General conclusions

To promote more widespread use of high strength concrete (HSC) for the efficient design of reinforced concrete (RC) beams, the experimental investigation on the influence of compressive strength (f'_c), stirrup ratio (r_w), stirrup spacing (s), flange width to web width ratio (b_f/b_w) and effective depth (d) on diagonal compressive capacity of RC beams has been performed and the new predictive equation for diagonal compressive capacity of RC beams was proposed on the basis of experimental findings. The loading tests of 12 thin web HSC beams ($f'_c = 100\text{-}135 \text{ N/mm}^2$) were carried out at the Structural Concrete Laboratory of the Department of Civil Engineering at Tokyo Institute of Technology. The experimental cases can be classified into four series: the effect stirrup ratio ($r_w = 0.9\text{-}3.5\%$) and spacing ($s = 90\text{-}370 \text{ mm}$), the effect of compressive strength with different stirrup, the effect of flange width to web width ratio ($b_f/b_w = 3.75, 6.25 \text{ and } 12.5$) and size effect ($d = 220\text{-}319 \text{ mm}$). the accuracy of diagonal compressive capacity calculated by the proposed equation is compared with those evaluated by JSCE Standard specifications [2], ACI Building Code [8], Eurocode 2 [9], *fib* Model Code 2010 [10] and the equation by Placas and Regan [7]. As a result, the following conclusions can be drawn.

- 1) By considering the crushing of concrete at the web combined with crack patterns and no yielding of stirrups, it indicated that the diagonal compression failure occurs even stirrup ratio is less than 2%.
- 2) With insufficient confinement effects provided by stirrups, the localization of compressive stress in struts occurred. Thus, stirrup spacing is prominent, not stirrup ratio, for evaluating diagonal compressive capacity of RC beams.

- 3) The diagonal compressive capacity linearly decreases with wider stirrup spacing regardless of its diameters and configuration when stirrup spacing is narrower than 140 mm; however, the effect of spacing becomes negligible when stirrup spacing is wider than 140 mm.
- 4) Even though wide stirrup spacing is provided, the sufficient confinement effect is achieved by providing two-legged stirrups and prevents the localization of compressive stress in struts.
- 5) The greater effect of the stirrup spacing on the diagonal compressive capacity of RC beams is observed in higher strength concrete. Thus, the effect of compressive strength of concrete and stirrup spacing is interrelated.
- 6) The ratios of flange width to web width from 3.75 to 12.5 and effective depth from 220 mm to 319 mm have almost no influence on the diagonal compressive capacity of RC beams.
- 7) There is no clear relationship between the average diagonal crack angle in B-region and compressive strength, stirrup ratio, stirrup spacing, and shear span to effective depth ratio.
- 8) The angle of principal strain in B-region had no tendency with stirrup ratio, spacing, diameters and configuration.
- 9) A predictive equation considering the compressive strength and stirrup spacing for evaluating the diagonal compressive capacity of RC beams is developed on the basis of the experimental findings. The proposed equation covers the application to RC beams with compressive strength up to 165 N/mm^2 and practical stirrup spacing ($45 \text{ mm} \leq s \leq 370 \text{ mm}$). The proposed equation has adequate simplicity and offers precise accuracy ($Avg. = 0.99$) with low variation ($C.V. = 10.5\%$) on the prediction of the diagonal compressive capacity of RC beams. The proposed equation can be implemented to beams with the compressive strength: 19-165 N/mm^2 , stirrup ratio: 0.6-4%, stirrup spacing: 45-370 mm, flange width to web width ratio: 3.75-12.5, shear-span to effective depth ratio: 2.5-4.5, effective depth: 220-563 mm, web width: 40-150 mm and one-legged, two-legged and closed-typed stirrups.
- 10) The accuracy of the existing equations for diagonal compressive capacity by Placas et al., JSCE Standard Specifications, ACI Building Code, Eurocode 2 and *fib* Model Code 2010 was evaluated. As a result, the existing equations should be improved in both accuracy and appropriateness on the prediction of the diagonal compressive capacity of RC beams.

6.2 Recommendations for further study

The proposed equation offers precise prediction and requires less effort and time to calculate the diagonal compressive capacity of RC beams. It covers the application to RC beams with compressive strength up to 165 N/mm^2 and practical stirrup spacing ($45 \text{ mm} \leq s \leq 370 \text{ mm}$). However, it is an empirical equation which may not be applicable outside of its range of study. Further investigation may be required to validate the applicability of the proposed equation to these cases.

Firstly, there is a possibility that the size effect in the diagonal compression failure can be occurred when the effective depth of beams is larger than the upper limit in this study since Walraven and Lehwalter [17] reported significant size effect in shear compression failure of short beams. Secondly, the proposed equation might be extended to members with several webs, i.e., box section, since in JSCE Standard, shear capacity of the members shall be considered as the same as beams having b_w equal to the total widths of web [2]. Finally, few studies on the diagonal compressive capacity of RC beams with high strength concrete have been performed except for Kobayashi [3]. It is recommended to calibrate the proposed equation by using more independent data using high strength concrete.

Furthermore, from Eq. 5-1, to evaluate the diagonal compressive capacity, it is necessary to determine the value of angle of compressive struts (θ). However, it is difficult to indicate the location of compressive struts exactly and measure the width and angle of compressive strut correctly in experiments. The actual stress flow is still unknown. Hence, further study by some analytical approach such as Finite Element Analysis is also recommended.

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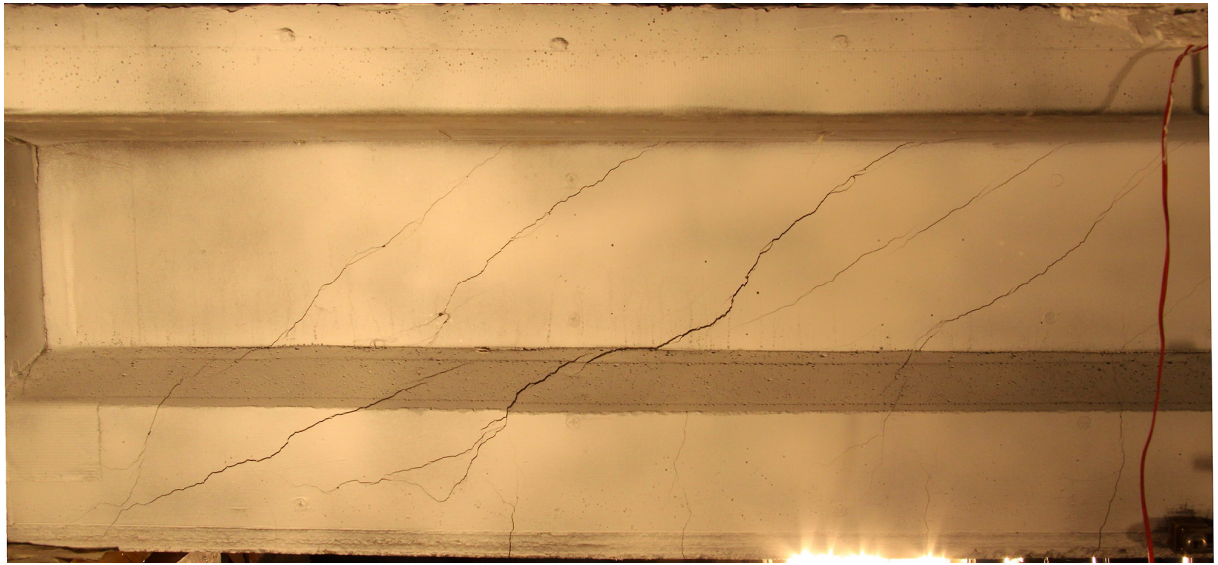
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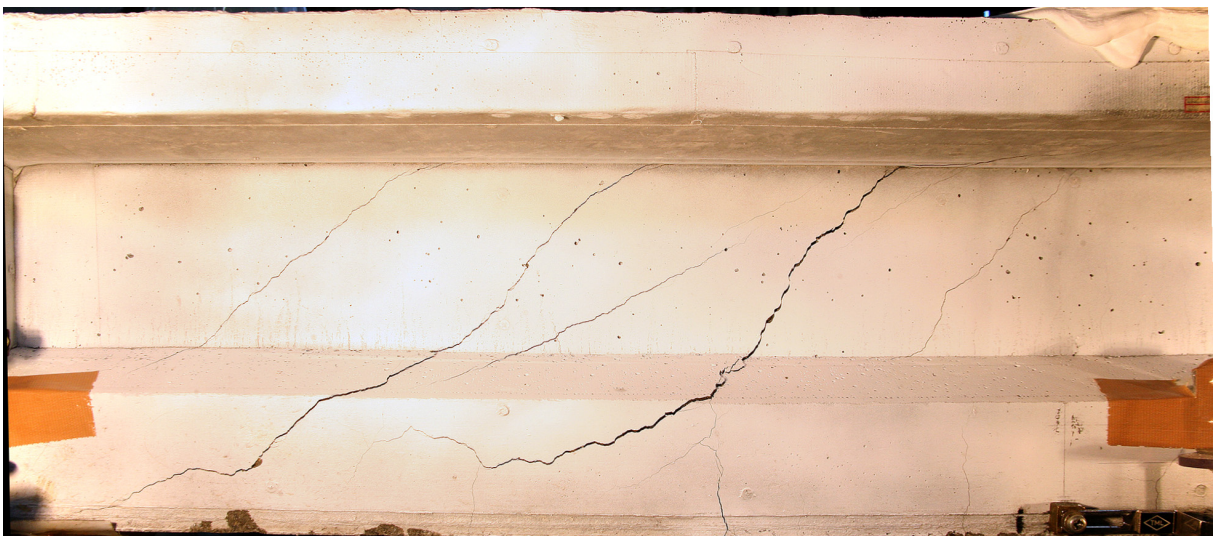
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Appendix A

Photo of specimens at the peak load and after loading



Crack pattern at the peak load: C-s100-I



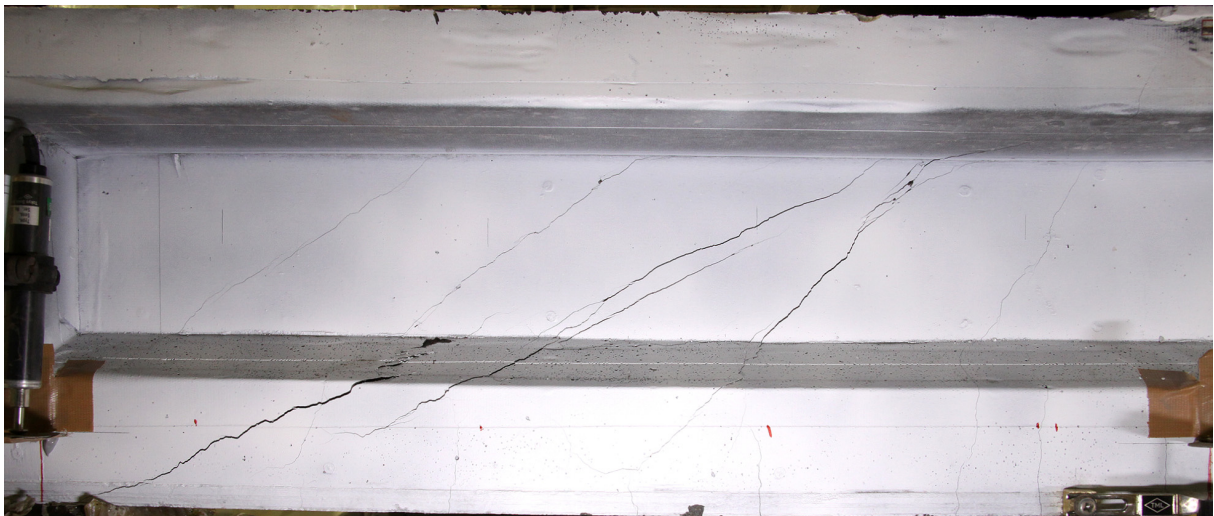
Crack pattern at the peak load: C-s120-I



Crack pattern at the peak load: C-s150-I



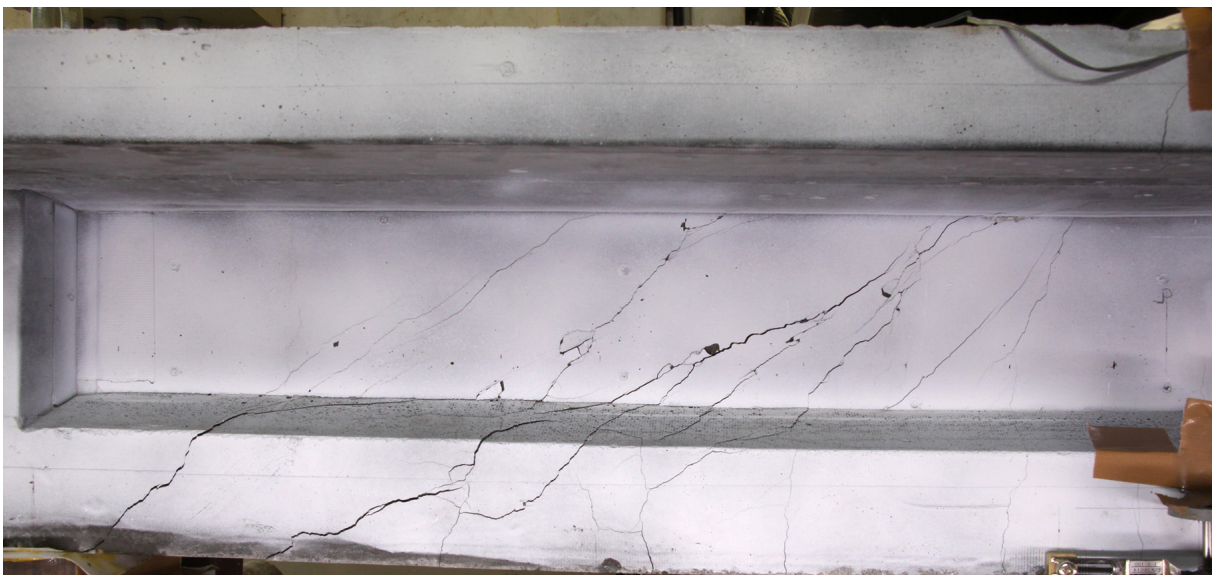
Crack pattern at the peak load: D-s90



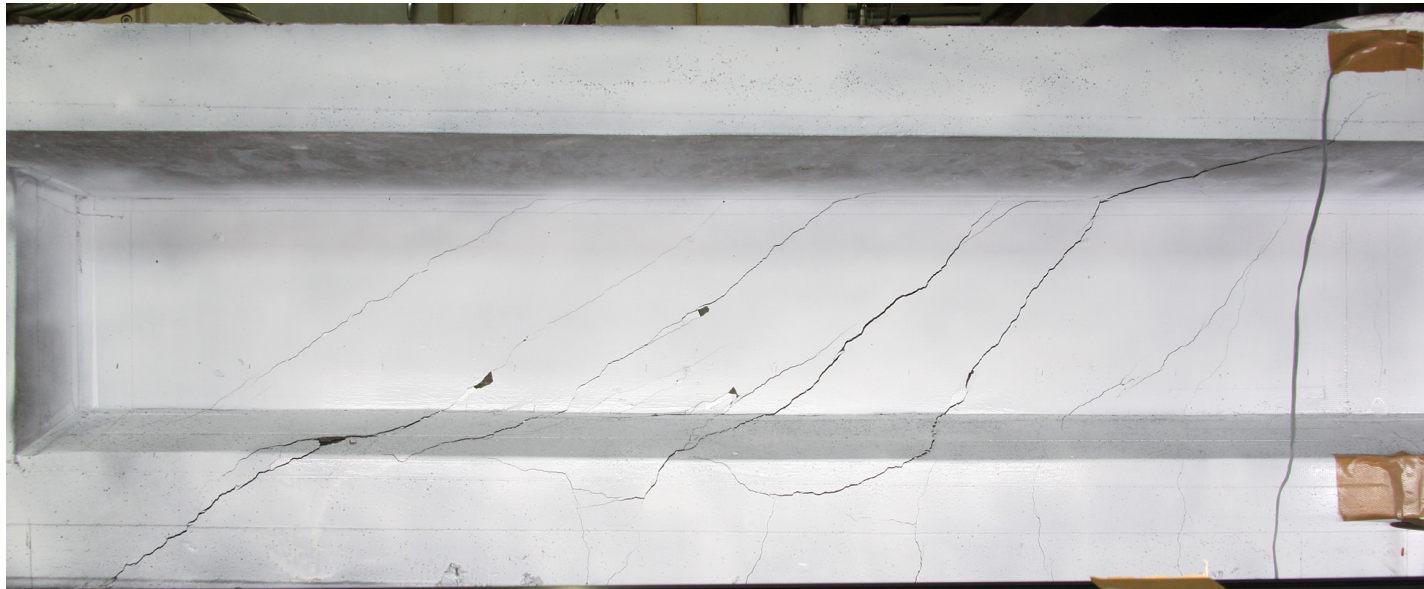
Crack pattern at the peak load: D-s160



Crack pattern at the peak load: D-f150



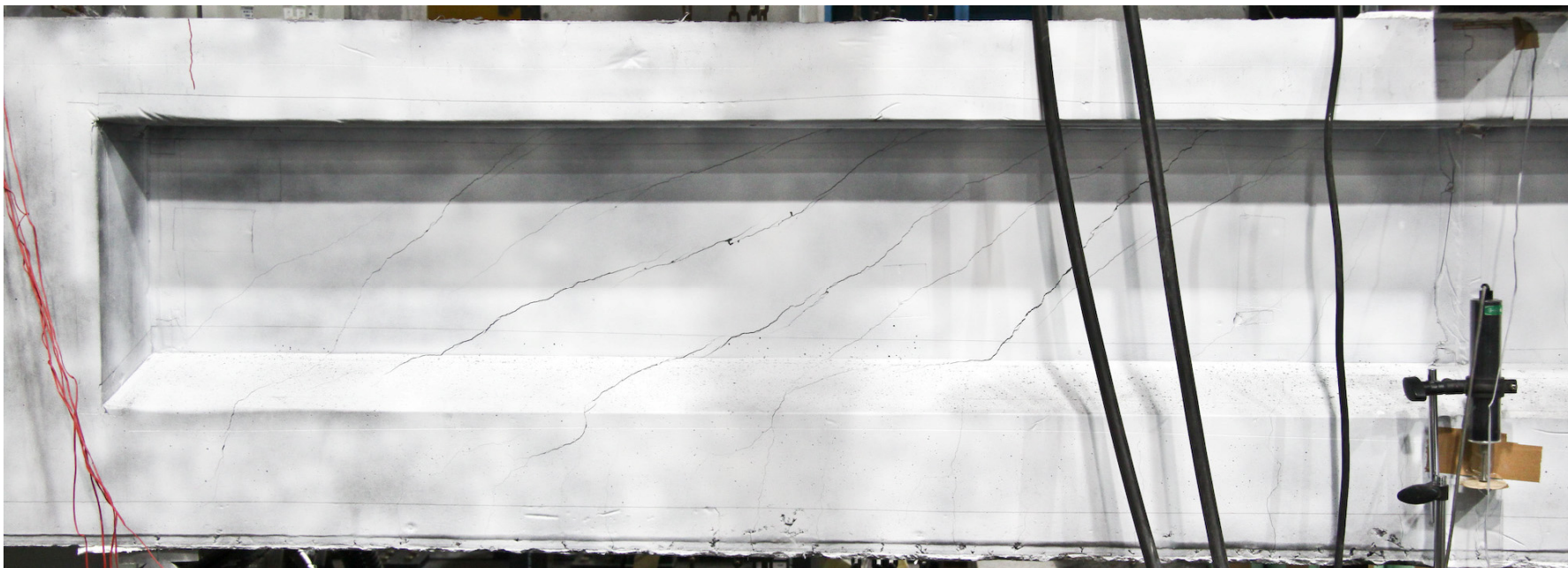
Crack pattern at the peak load: D-f500



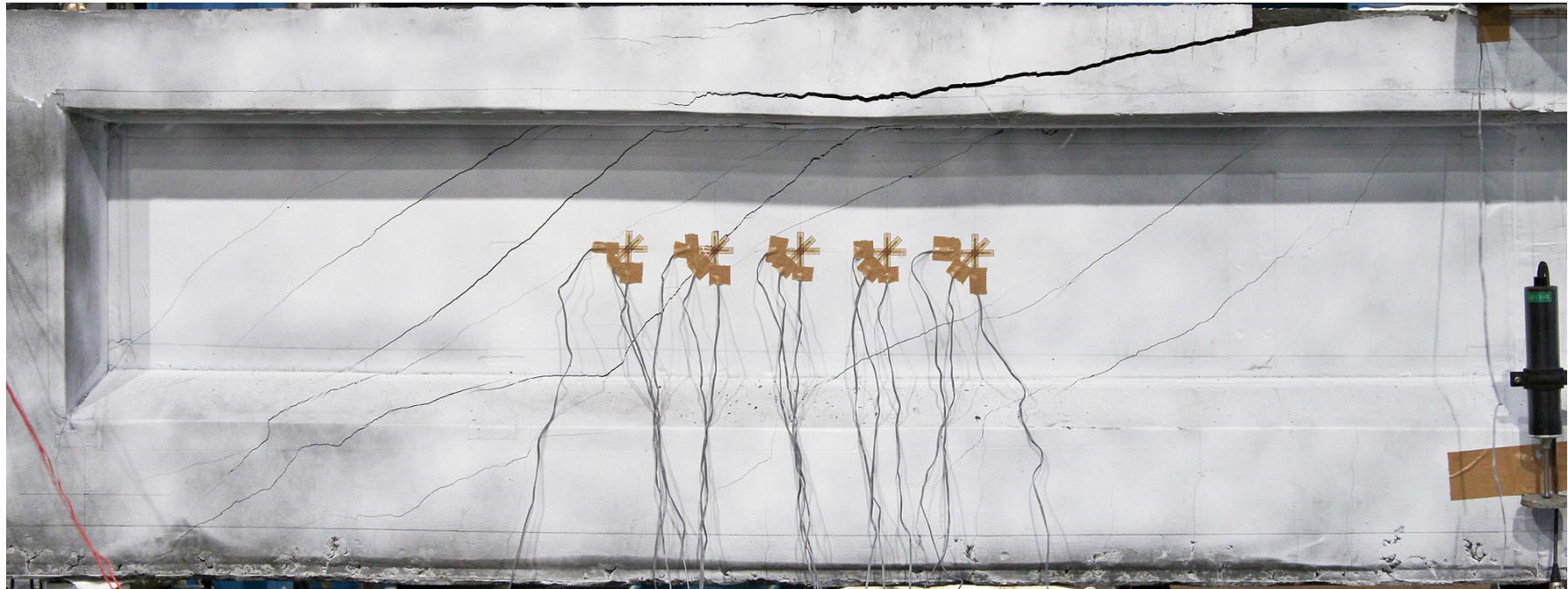
Crack pattern at the peak load: C-s100L



Crack pattern at the peak load: C-s300-II



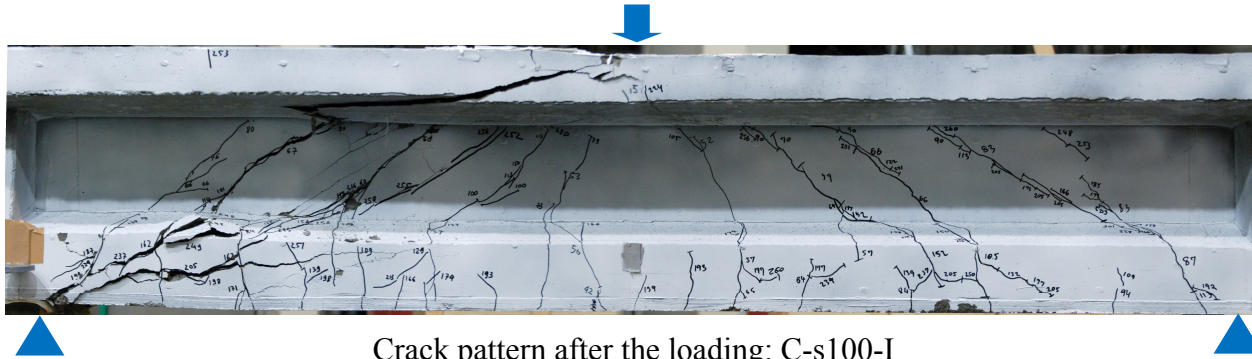
Crack pattern at the peak load: C-s370-II(13)



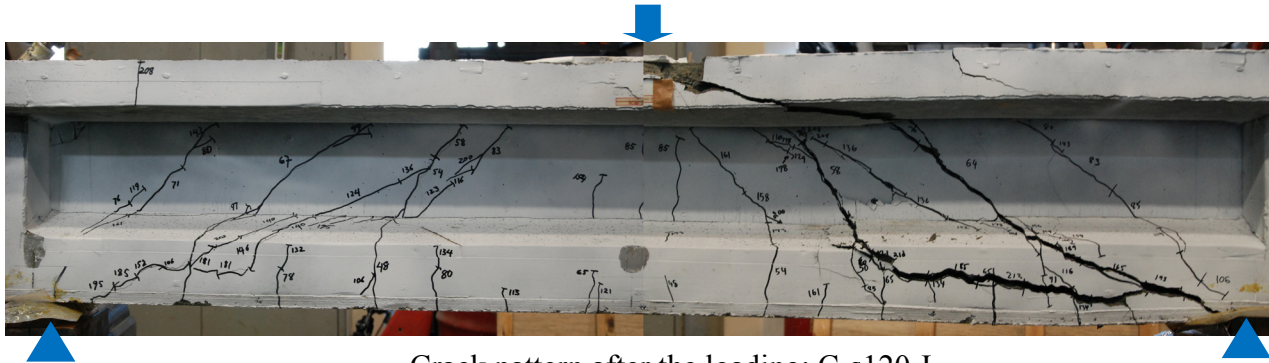
Crack pattern at the peak load: C-s370-II(16)



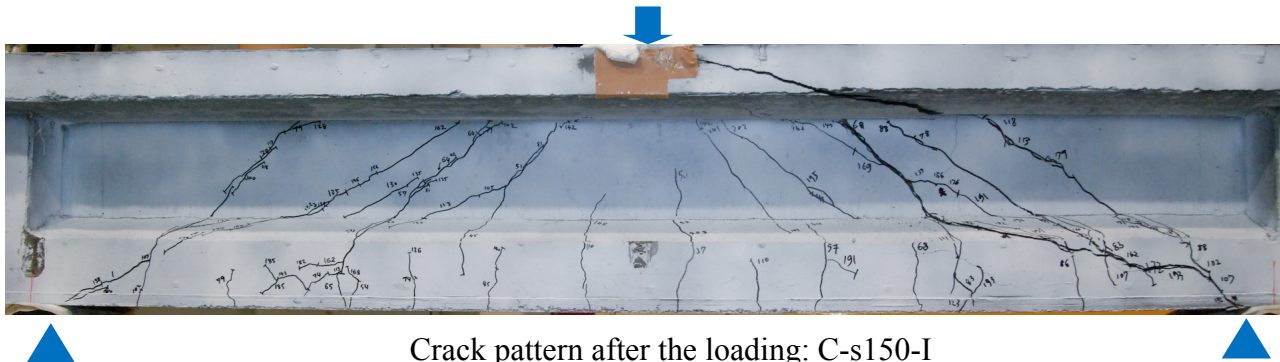
Crack pattern at the peak load: C-s230-I



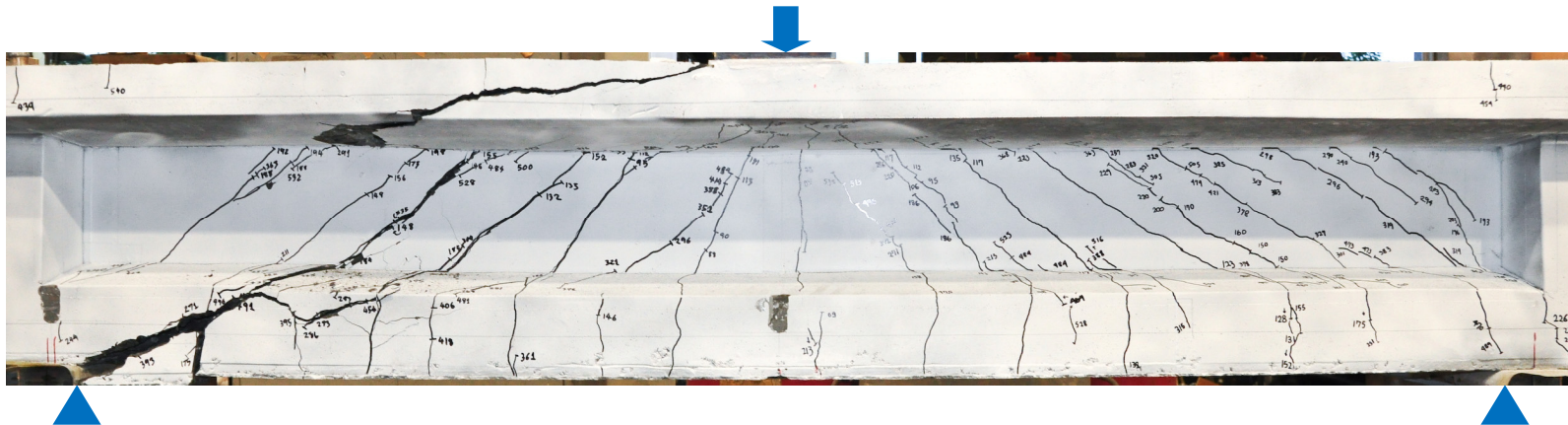
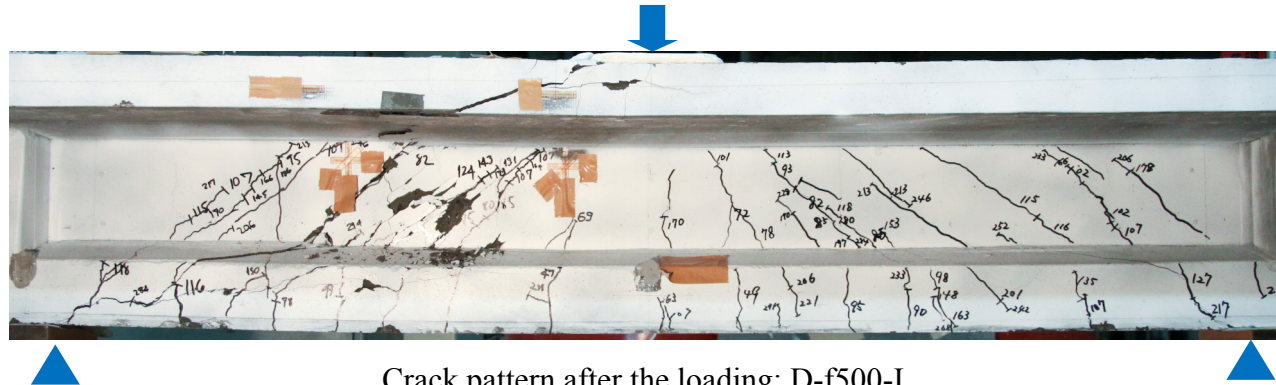
Crack pattern after the loading: C-s100-I

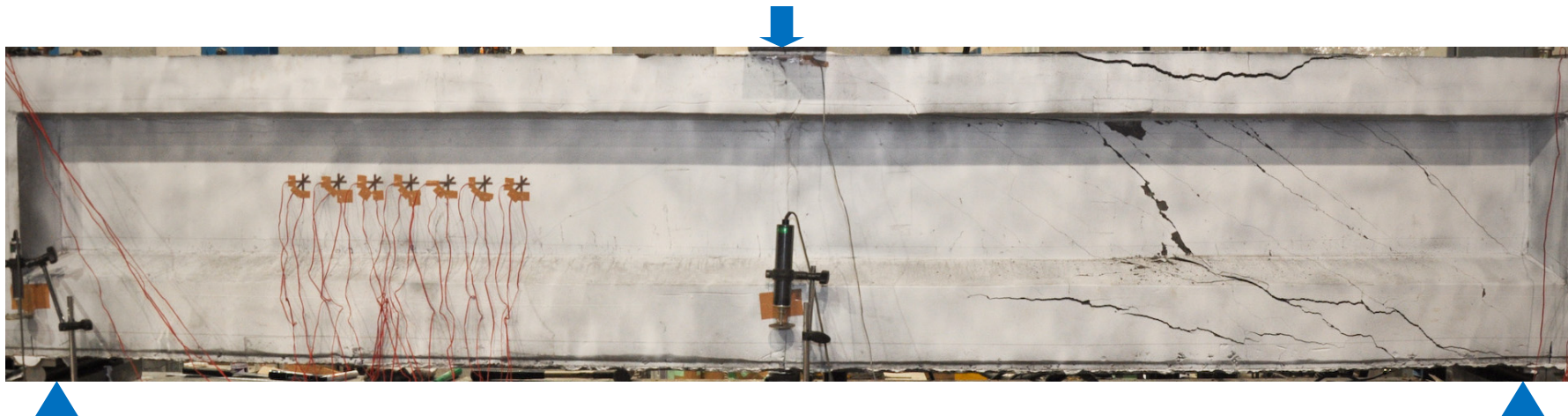


Crack pattern after the loading: C-s120-I

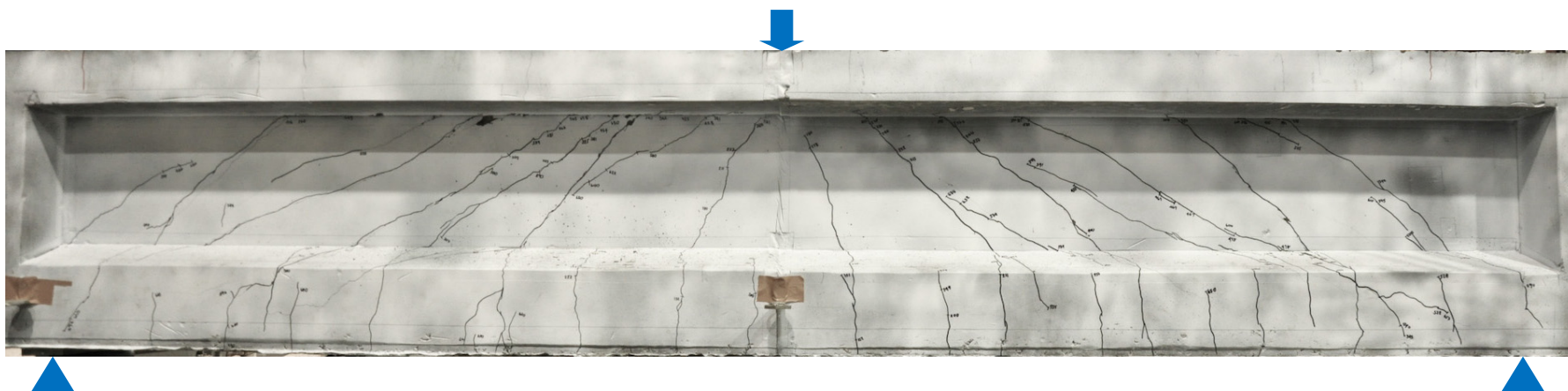


Crack pattern after the loading: C-s150-I

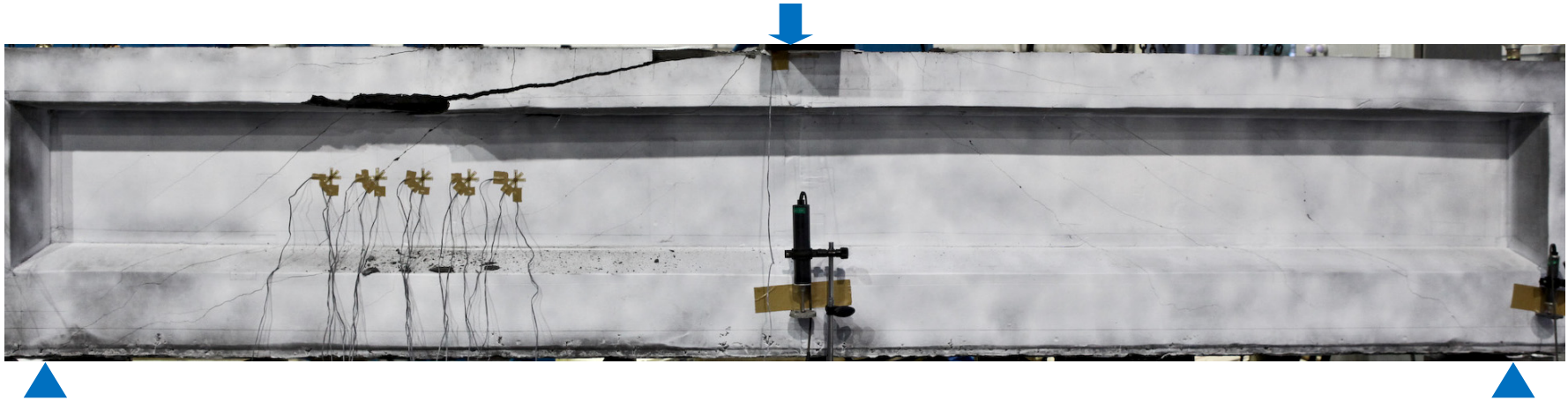




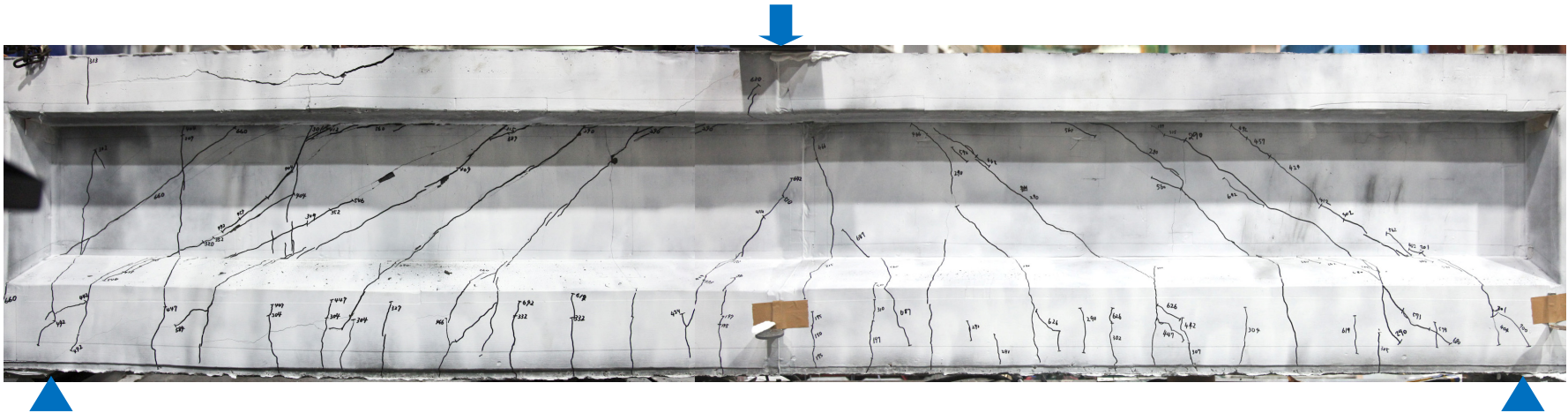
Crack pattern after the loading: C-s300-II



Crack pattern after the loading: C-s370-II(13)



Crack pattern after the loading: C-s370-II(16)



Crack pattern after the loading: C-s230-I