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A CONSTITUTIVE RULE FOR VISCOELASTIC MATERIAL CONSIDERING HEAT CONDUCTION AND HEAT TRANSFER

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Abstract: This paper proposes a nonlinear model for VE damper subjected to a long duration cyclic load. The temperature rise and distribution of VE materials during cyclical loadings can be predicted by using a one dimensional heat transfer equation. Significant heat conduction and transfer, for some instance appears to influence the temperature rise, and this effect is included in the analytical model. The constitutive rule using fractional derivative considers VE material softening due to the temperature rise and sensitivity against excitation frequency. Excellent performance of proposed model is demonstrated through comparison with the long duration loading experimental results.

1. INTRODUCTION

It has been recognized that viscoelastic (VE) dampers have significant advantage of being effective against not only the wind but also the earthquake. First application of VE dampers to reduce building vibration was The World Trade Center in New York (Mahmoodi 1969). Since then, the VE dampers have been applied to buildings to reduce vibration due to earthquake and/or wind for many years (Nielsen et al. 1994).

It is known that VE materials have sensitivity against the frequency, temperature, and strain. The VE damper dissipates energy through shear deformation within the VE material during cycle loading, causing temperature-rise of the material. Consequently, the VE material becomes softer, and its energy dissipation capacity decreases during the loading. In order to simulate such nonlinear behavior, the temperature distribution of the VE material must be estimated accurately.

The nonlinear model was proposed earlier, by focusing on the effect of temperature-rise (Kasai et al. 1993, 2002, 2003). Since then, such model was adopted by Shen & Soong (1995), Huang et al. (2001). However, these models did not considered the heat conduction and heat transfer, which are still reasonable in case of short duration loading like a typical earthquake. However, in case of long duration loading such as long period earthquake or wind force, heat conduction and heat transfer must be considered.

In this paper, a nonlinear model for VE damper subjected to a long duration cyclic load is proposed. The constitutive rule of this model uses the fractional time-derivative of stresses and strain in order to accurately simulate sensitivities against the temperature and excitation frequency, and additionally considers VE material softening due to the temperature-rise. The present model also includes the effect of heat conduction and heat transfer using a one-dimensional heat transfer equation, and simulates the temperature distribution within the VE materials during cyclic loading. To validate performance of the proposed model, experiment applying sinusoidal displacement of almost one hour is conducted. Three-dimensional finite element analysis is also performed to demonstrate excellent accuracy of the proposed model

2. FRACTIONAL DERIVATIVE MODELS

2.1 Concept of Long Duration Fractional Derivative Model

Figures 1 and 2 contrasts the distributions of temperature, stiffness, and strain along thickness (Z-) direction of the VE damper, between the model for short duration loading like a typical earthquake and that for long duration loading such as long period earthquake or wind force. From now on, they will be called as "short duration model" and "long duration model", respectively.

The short duration model considers only the effect of temperature-rise caused by dissipation of energy. Due to the small heat conductivity of the VE material, heat conduction from VE material to steel plate and heat transfer to air are very slow, compared with the short duration of the loading. Therefore, the distributions of temperature, strain and stress are considered to be uniform within the VE material, and such a model has been found sufficiently accurate when typical earthquake loading is considered (Kasai et al. 1993).

In contrast, the long duration model considers one-dimensional heat conduction and transfer in the thickness direction. The area of VE material exposed to the air is small in comparison with such on area of steel plates that are bonded to the VE materials. Therefore, analysis assumes that the heat generated inside the VE material moves toward the steel plates, and finally transmitted to the air from the steel plates. Accordingly, strain and temperature are expressed by $\gamma(t, z)$ and $\theta(t, z)$, respectively, as the function of time "t" and in the thickness direction coordinate "z". Based on equilibrium, the VE material shear stress is assumed to be constant throughout the thickness.

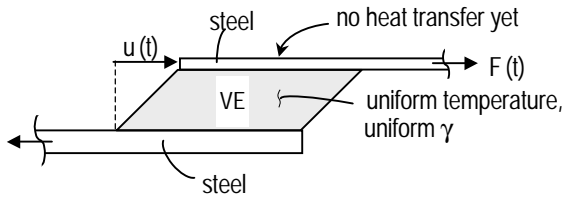


Figure 1 Short Duration Model

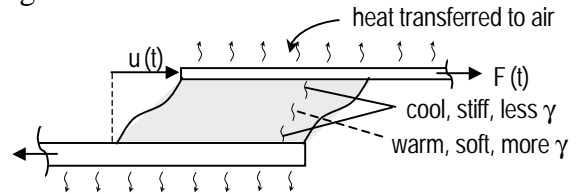


Figure 2 Long Duration Model

2.2 Short Duration Model (Past Fractional Derivative Model)

The shear stress $\tau(t)$ and strain $\gamma(t)$ of the VE damper are related to the force $F(t)$ and deformation $u(t)$ of the damper as follows:

$$\tau(t) = F(t)/A_s, \quad \gamma(t) = u(t)/d \quad (1)$$

where, A_s = shear area, and d = thickness of the VE damper.

From now on, we explain a method to obtain $F(t)$ for a given $u(t)$. Kasai et al. (1993) proposed the following shear stress–strain constitutive rule of the VE material as the short duration model.

$$\tau(t) + aD^\alpha \tau(t) = G[\gamma(t) + bD^\alpha \gamma(t)] \quad (2)$$

where $D^\alpha = d^\alpha/dt^\alpha$ = fractional derivative operator, α = order of the fractional derivative, a and b = parameters of constitutive rule, G = static shear modulus. Coefficients a and b are calculated by

$$a = a_{ref} \lambda^\alpha, \quad b = b_{ref} \lambda^\alpha, \quad \lambda = \exp[-p_1(\theta - \theta_{ref}) / (p_2 + \theta - \theta_{ref})] \quad (3a-c)$$

where a_{ref} and b_{ref} = constants at reference temperature θ_{ref} , λ = shifting factor, and p_1 and p_2 = constants to model temperature-frequency equivalency of the VE material.

Numerical integration is used to simulate the stress – strain value at the n -th step of analysis (Eq.4).

$$\tau^{(n)} + \frac{a^{(n)}}{\Delta t^\alpha} \sum_{i=0}^N w^{(i)} \tau^{(n-i)} = G \left[\gamma^{(n)} + \frac{b^{(n)}}{\Delta t^\alpha} \sum_{i=0}^N w^{(i)} \gamma^{(n-i)} \right] \quad (4)$$

where, i = time step number, Δt = time step size, $w^{(i)}$ = weight function, and N = window size (Kasai et

al. 2001, 2002, 2003). Next step temperature $\theta^{(n+1)}$ is calculated from the every dissipated up to the n -th step as follows (Kasai et al. 1993):

$$\theta^{(n+1)} = \theta^{(n)} + \frac{(\tau^{(n)} + \tau^{(n-1)})(\gamma^{(n)} - \gamma^{(n-1)})}{2s\rho} \quad (5)$$

where, s = specific heat, ρ = mass density of the VE material.

2.3 Formulation of Long Duration Model

Under the long duration loading, temperature and strain inside the VE material depend on the location " z " and time " t ". Therefore, Eqs. (1) ~ (3) should be modified to Eqs. (6) ~ (8), respectively.

$$\tau(t) + a(z)D^\alpha \tau(t) = G[\gamma(z, t) + b(z)D^\alpha \gamma(z, t)] \quad (6)$$

$$a(z) = a_{ref}(\lambda(z))^\alpha, \quad b(z) = b_{ref}(\lambda(z))^\alpha \quad (7a, b)$$

$$\lambda(z) = \exp[-p_1(\theta(z, t) - \theta_{ref}) / (p_2 + \theta(z, t) - \theta_{ref})] \quad (8)$$

By integrating the fractional derivatives of Eq. (6), the shear stress $\tau^{(n)}$ and strain $\gamma_j^{(n)}$ can be obtained at n step,

$$\tau^{(n)} + \frac{a_j^{(n)}}{\Delta t^\alpha} \sum_{i=0}^N w^{(i)} \tau^{(n-i)} = G \left[\gamma_j^{(n)} + \frac{b_j^{(n)}}{\Delta t^\alpha} \sum_{i=0}^N w^{(i)} \gamma_j^{(n-i)} \right] \quad (9)$$

where j = the index denoting the location ($j = 0 \sim m$), and m = number of sampling points in the VE material in the thickness (Z direction).

By rearranging and manipulating about $\gamma_j^{(n)}$ of Eq. (9), $\gamma_j^{(n)}$ is written as

$$\gamma_j^{(n)} = \frac{\tau^{(n)}(\Delta t^\alpha + a_j^{(n)}w^{(0)}) + \tilde{A}_j^{(n)} - \tilde{B}_j^{(n)}}{G(\Delta t^\alpha + b_j^{(n)}w^{(0)})} \quad (10)$$

where,

$$\tilde{A}_j^{(n)} = a_j^{(n)} \sum_{i=1}^N w^{(i)} \tau^{(n-i)}, \quad \tilde{B}_j^{(n)} = Gb_j^{(n)} \sum_{i=1}^N w^{(i)} \gamma_j^{(n-i)} \quad (11a, b)$$

As the length of per unit element divided into m is expressed by h_j ($= z_j - z_{j-1}$; See Figure 4) and when n step displacement of the damper $u_d^{(n)}$ is known, compatibility condition between $u_d^{(n)}$ and $\gamma_j^{(n)}$ can be given by Eq. (12).

$$u_d^{(n)} = \frac{1}{2} \left[\sum_{j=0}^{m-1} h_{j+1} \gamma_j^{(n)} + \sum_{j=1}^m h_j \gamma_j^{(n)} \right] \quad (12)$$

By substituting Eq. (10) into Eq. (12), then $\tau^{(n)}$ is expressed by Eq. (13).

$$\tau^{(n)} = \frac{2Gu_d^{(n)} - \sum_{j=0}^{m-1} \frac{h_{j+1}(\tilde{A}_j^{(n)} - \tilde{B}_j^{(n)})}{\Delta t^\alpha + b_j^{(n)}w^{(0)}} - \sum_{j=1}^m \frac{h_j(\tilde{A}_j^{(n)} - \tilde{B}_j^{(n)})}{\Delta t^\alpha + b_j^{(n)}w^{(0)}}}{\sum_{j=0}^{m-1} \frac{h_{j+1}(\Delta t^\alpha + a_j^{(n)}w^{(0)})}{\Delta t^\alpha + b_j^{(n)}w^{(0)}} + \sum_{j=1}^m \frac{h_j(\Delta t^\alpha + a_j^{(n)}w^{(0)})}{\Delta t^\alpha + b_j^{(n)}w^{(0)}}} \quad (13)$$

After obtaining $\tau^{(n)}$, it can be re-substituted into Eq. (10) to determine $\gamma_j^{(n)}$ at any location j .

In order to predict the temperature distribution of VE damper, the following heat conduction

3. LONG DURATION TEST AND ANALYSIS RESULTS

3.1 Test Piece Summary and Measurement Method

To understand characteristic of VE damper under the long duration loading and to validate about excellent performance of the long duration model, the long loading experiment of an individual VE damper is carried out. The damper is made of an acrylic polymer VE material (3M Corp., ISD 111). The two VE material laminations are bonded between outer steel plate and inner steel plate. The total area of two VE material laminations is 38.171 cm^2 , and the thickness of one lamination is 1.328cm. The thickness of one steel plate is 0.476 cm.

Sinusoidal displacement of 0.664 cm peak amplitude $u_{d \max}$ (50% shear strain) and 3.0 seconds period are used in this study. The duration of sinusoidal wave is 3000 seconds (i.e., 1000 cycles). After the excitation, damper temperature decrease is monitored up to 5000 seconds. Ambient temperature is maintained at $24 \text{ }^\circ\text{C}$ for 5000 seconds.

As shown in Figure 5, the temperature of VE damper is measured at the four locations; (A) air side of outer steel plate, (B) 1/4 thickness point of VE material, (C) 1/2 thickness point of VE material, and (D) center of inner steel plate.

The sampling frequency for measuring the damper deformation and damper force are 20 Hz. However, because of the capacity of the measuring instrument, it is impossible to measure for 3000 seconds continuously. Hence, the measurement duration of one time is 60 seconds, and that is repeated 13 times at 240 seconds interval.

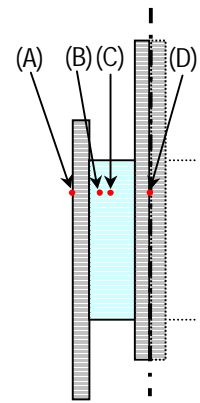


Figure 5
Measurement
Points

3.2 Parameter of Long Duration Model

Parameters of the long duration model are indicated in Table 1. The heat transfer factors of the outer steel plate side $\alpha_{c,out}$ and the inner steel plate side $\alpha_{c,in}$ are $0.911 \text{ N / (sec cm }^\circ\text{C)}$ and $0.613 \text{ N / (sec cm }^\circ\text{C)}$ respectively. After the motion, convection factor of the inner steel plate side $\alpha_{c,in}$ is reduced to $0.438 \text{ N / (sec cm }^\circ\text{C)}$ since the plate dose not move.

The analysis using the long duration model is done under the same conditions as experiment. From beginning to 3000 seconds, deformation of the sinusoidal wave whose period is 3.0 seconds and amplitude is 0.664 cm are given. After that, the only heat transfer analysis is done until 5000 seconds. External temperature of the outer steel plate and the inner steel plate are $24 \text{ }^\circ\text{C}$ always.

Table 1 Parameters of Long Duration Model

VE Material		Steel Plate	
$G \text{ (N/cm}^2\text{)}$	6.5158	$s \rho \text{ (N/cm}^2\text{ }^\circ\text{C)}$	363.79
α	0.609	$\kappa \text{ (N/sec }^\circ\text{C)}$	43.128
a_{ref}	0.0115	$\alpha_{c,out} \text{ (N/sec cm }^\circ\text{C)}$	0.911
b_{ref}	21.157	$\alpha_{c,in} \text{ (N/sec cm }^\circ\text{C)}$	0.613
$s \rho \text{ (N/cm}^2\text{ }^\circ\text{C)}$	193.97		0.438
$\theta_{ref} \text{ (}^\circ\text{C)}$	0.2	$H_s \text{ (Outer)}$	0.476
p_1	19.5	(cm)	(4Elem.)
p_2	80.2	$H_s \text{ (Inner)}$	0.238
$\kappa \text{ (N/sec }^\circ\text{C)}$	0.188	(cm)	(2Elem.)
$A_d \text{ (cm}^2\text{)}$	38.171		
$H_d \text{ (cm)}$	1.328 (12 Elem.)		

$$1 \text{ N / sec cm }^\circ\text{C} = 100 \text{ W / m}^2\text{ }^\circ\text{C}$$

3.3 Comparison between Test Results and Analysis Results

Figure 6 shows that the time history of temperature at the each measurement point (See Figure 5) with the experimental and analytical results. The temperature distributions inside the VE damper obtained from experiment and analysis are shown in Figure 7. From test results in Figure 6 and 7, it is recognized that the change of temperature is different depending upon the position of the damper. Figure 6 shows that temperature rise becomes sluggish and temperature is constant at each measurement points after 1000 seconds. In case of under the long duration loading such as this study, as effect of the heat conduction to the steel plates, temperature of VE damper becomes the steady state at a certain time, and temperature does not continue to rise even if the VE damper have been vibrating.

After 3000 seconds when the vibration finished, temperature of each point rapidly decreases until 24 °C because the internal heat generation inside the VE materials stops.

Good agreements of temperature rise and distribution between test and analysis are shown in Figure 6 and 7, respectively.

Figure 8 shows the distribution of strain obtained from the analysis at a certain time when the peak strain of VE material at the center point occurs. The distortion strain at 0.75 seconds immediately after the starting almost is 50% at all positions of VE material. However, when analytical step have elapsed, the strain on this point increase because the temperature on this point increase (See Figure 7). So, the distributions inside the VE material under the long duration loading become uneven as shown in Figure 8.

Figure 9 shows the comparison of temperature rise between the results obtained from using the short duration model and using the long duration model. Since the short duration model is not considered the heat conduction and transfer, the temperature obtained from using the short duration model continues to increase when the duration time is long. Even if after the loading stops, the temperature of the short duration model not decreases like the long duration model.

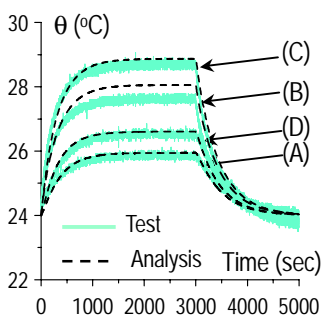


Figure 6 Temperature Time History

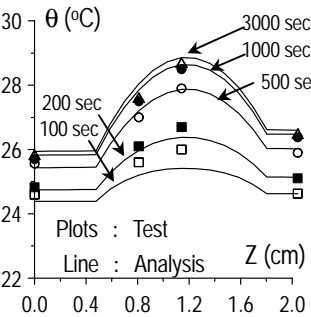


Figure 7 Temperature Distribution

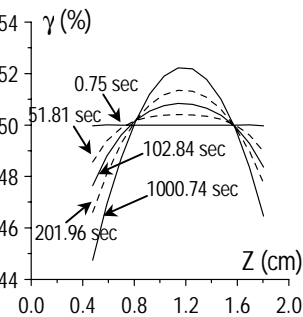


Figure 8 Strain Distribution

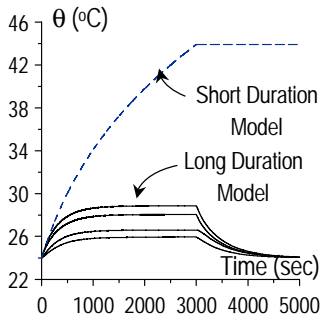


Figure 9 Temperature Time History

Figure 10 (a) shows the hysteresis obtained from the test result, and Figure 10 (b) and (c) express obtained from the analysis using the long duration model and the short duration model, respectively. Obviously, Figure 10 (b) shows good agreement with Figure 10 (a), but not Figure 10 (c) erroneously significant material softening due to the overestimated temperature (see Figure 9).

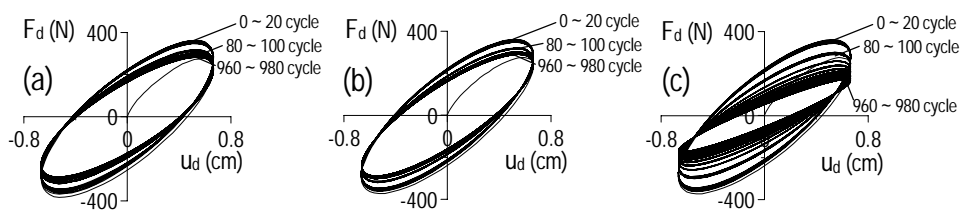


Figure 10 Comparison of Hysteresis Loop:
(a) Test, (b) Long Duration Model, and (c) Short Duration Model

4. THREE - DIMENSIONAL FINITE ELEMENT ANALYSIS OF VE DAMPER

4.1 Object of Three-Dimensional Finite Element Method Analysis and Analytical Technique

Because of convenience, the long duration model proposed in this study is done heat transfer analysis as one-dimensional problem of thickness direction. Object of this chapter is to examine one-dimensional problem, which is assumed in chapter 2, by comparing with results of the three-dimensional finite element method (3D-FEM) analysis.

3D-FEM done in this study is carried out alternately static analysis and heat transfer analysis without dynamic analysis. So, the variation of temperature and stiffness of VE damper having the

temperature dependency are simulated. This method is expressed as follows.

Firstly, initial temperature of VE damper is set. The Young's modulus of VE material E'_j is calculated from Eq. (25). Then G'_j in Eq. (26) is expressed by Eq. (26) (Kasai et al. 2001). Where, ν = Poisson's ratio.

$$E'_j = 2G'_j(1+\nu) \quad , \quad G'_j = G \frac{1+a_j b_j \omega^{2\alpha} + (a_j + b_j) \omega^\alpha \cos(\alpha\pi/2)}{1+a_j^2 \omega^{2\alpha} + 2a_j \omega^\alpha \cos(\alpha\pi/2)} \quad (25),(26)$$

As next step, the inner steel plate is moved the damper maximum deformation $u_{d \max}$, and static analysis is carried out

The dissipation energy $W_{d,j}$, which is dissipated by the element j of VE material in one cycle of steady state, is calculated from Eq. (27). In Eq. (28), η_j is expressed by Eq. (28). Where $W_{s,j}$ = strain energy of element j obtained from the static analysis results.

$$W_{d,j} = 2\pi\eta_j W_{s,j} \quad , \quad \eta_j = \frac{(-a_j + b_j) \omega^\alpha \sin(\alpha\pi/2)}{1+a_j b_j \omega^{2\alpha} + (a_j + b_j) \omega^\alpha \cos(\alpha\pi/2)} \quad (27),(28)$$

The heat generated per unit volume \dot{Q}_j is expressed by Eq. (29)

$$\dot{Q}_j = W_{d,j} / (T \cdot V_j) \quad (29)$$

Where, T = period of 1 cycle, V_j = volume of element j . The unsteady state heat transfer analysis is done for T seconds using the heat generated per unit volume in Eq. (28). After that, the Yong's modulus of each point is updated using temperature obtained from the heat transfer analysis results (Eq. (25)), and static analysis is done again.

The above process is repeated to simulate the temperature distribution inside the VE damper and the decrease of stiffness of VE material.

4.2 Three Dimensional Analytical Model

The 3D-FEM model is shown in Figure 11. This model is symmetric with on the center of inner plate, therefore in this study, 3D-FEM is carried out by using half of this model. Three dimensional coupled temperature – displacement elements (ABAQUS Ver. 6.4) are utilized in this model. The parameters for 3D-FEM are indicated in Table 2. The heat transfer factor of inner side of VE material and steel plate are $30 \text{ W} / \text{m}^2 \text{ }^\circ\text{C}$, and outer side of VE and steel are $30 \text{ W} / \text{m}^2 \text{ }^\circ\text{C}$. Because of the three dimensional effect, the heat transfer of 3D-FEM model become lower than the long duration model.

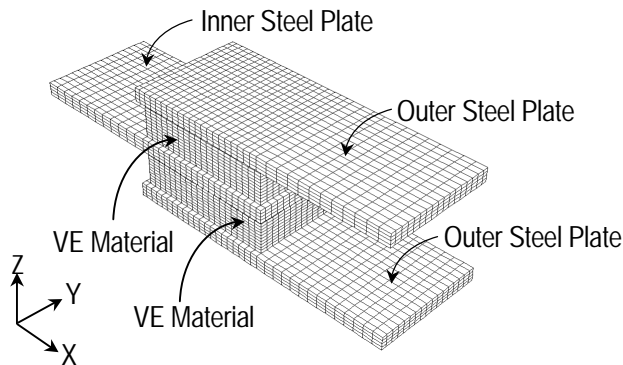


Figure 11 Three Dimensional FEM Model

Table 2 Parameters of 3D-FEM Model

VE Material		Steel Plate	
κ (W/m $^\circ\text{C}$)	0.188	κ (W/m $^\circ\text{C}$)	43.128
s (J/kg $^\circ\text{C}$)	1939.7	s (J/kg $^\circ\text{C}$)	466.3
ρ (kg/m 3)	1000	ρ (kg/m 3)	7801
$\alpha_{c,out}$ (W/m 2 $^\circ\text{C}$)	20	$\alpha_{c,out}$ (W/m 2 $^\circ\text{C}$)	20
$\alpha_{c,in}$ (W/m 2 $^\circ\text{C}$)	30	$\alpha_{c,in}$ (W/m 2 $^\circ\text{C}$)	30
ν	0.47	ν	0.3
		E (N/m 2)	2.05×10^{11}

4.3 Results of Three Dimensional Finite Element Method Analysis

The 3D-FEM analysis is carried out until 3000 seconds. The comparison between test results and 3D-FEM analysis results are shown in Figure 12 (a) temperature, (b) storage stiffness K'_d . FEM analysis results good agree with results of tests, so this method can express accurately temperature distribution inside the VE damper and the change of the stiffness of VE material.

The temperature distributions of VE damper obtained by 3D-FEM are shown in Figure 13 (a) ~ (c). The distribution of Z - axis have different temperature on each point, however, the temperature of X - axis and Y - axis are uniform distribution excluding the extremely portion of the surface. So, it is recognized that the assumption under the long duration model which temperature distribution inside the VE damper can be express by using one - dimensional problem.

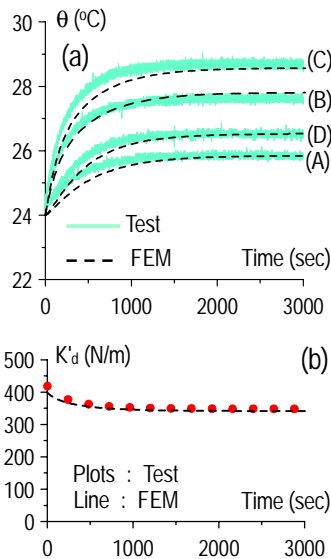


Figure 12 Comparison with Test and FEM:

(a) Temperature, and (b) K'_d

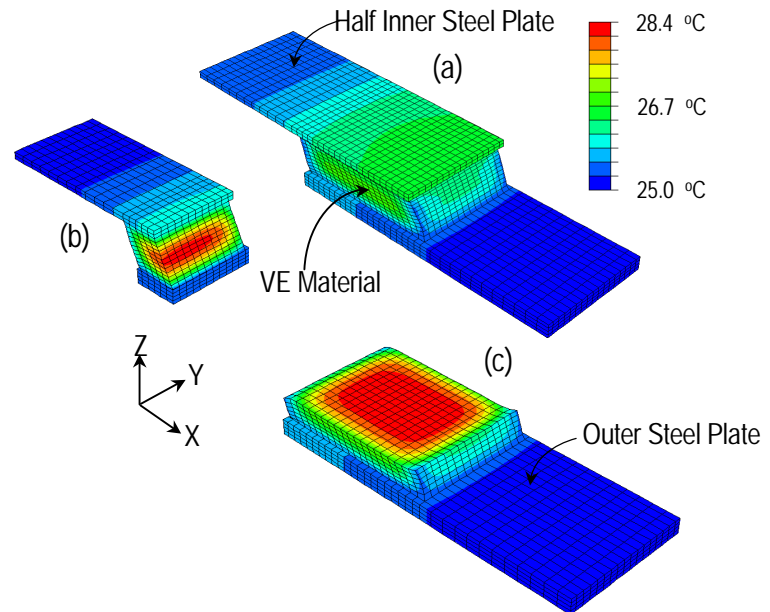


Figure 13 Results of 3D-FEM Analysis at 3000 seconds:
(a) Half VE damper , (b) Section of Measurement Points, and
(c) Section Point (C)

5. CONCLUSIONS

This paper has proposed a non-linear hysteresis model of VE damper considering heat conduction and heat transfer. In addition, the three dimensional finite element method analysis is carried out to validate about the assumption for the long duration model. This model may be used for design of the VE damper and / or the building having VE damper under the long period earthquake or wind force.

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