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論文 / 著書情報
Article / Book Information

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Title(English)	Stochastic Heat Diffusion Modelling and the Discrete Laplace-Beltrami Operator on Non-Uniformly Gridded Spaces
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Category(English)	Doctoral Thesis
種別(和文)	論文要旨
Type(English)	Summary

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（博士課程）

Doctoral Program

論 文 要 旨

THESIS SUMMARY

専攻： Department of 情報環境学 専攻	申請学位（専攻分野）： Academic Degree Requested 博士 Doctor of (Philosophy)
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要旨（英文 800 語程度）

Thesis Summary（approx.800 English Words ）

The heat equation is a useful model in various scientific contexts. Analytical solutions to the heat equation may be difficult to obtain due to the complexity of the partial differential equations in many engineering applications.

In this thesis we introduce a new stochastic framework for the approximation of the diffusion of heat. Our main interest lies in the fundamental connection between the random walk of particles on non-uniformly spaced nodes, the discrete Laplace-Beltrami operator, and the diffusion of heat. Specifically, the random walk is driven by a continuous-time Markov chain and we show how the entries of the infinitesimal generator of the Markov chain, also called the transition rates, have to be chosen such that the random variable of the Markov chain has the same moments as the probability density function of a Brownian particle, which is the solution to the heat equation. Therefore, in this framework, we characterize the random movement of heat particles on a microscopic level such that we can obtain an approximation of the heat distribution on a macroscopic level by using the Kolmogorov forward equation, where the Kolmogorov forward equation is an ordinary differential equation which can be solved deterministically.

In the dissertation we first review current deterministic and stochastic frameworks for the approximation of diffusion processes and state our research goals and motivation. Furthermore, we pose questions which we investigate in the dissertation and give the mathematical preliminaries and definitions for the description of the deterministic diffusion of heat and the continuous-time Markov chain.

We then develop our basic framework for the infinite straight line by the technique for showing that the first two moments of the random variable of the continuous-time Markov chain are equal to the first two moments of probability density function of a Brownian particle, or in other words, the Gaussian distribution with a certain mean and variance. To illustrate the results we conduct stochastic particle simulations on exemplary grids.

We then extend our framework for two-dimensional lattices. In two dimensions we find limitations regarding the placement of nodes. This is due to the space discretization that does not allow for particle movement according to the natural phenomenon of heat diffusion, which takes place isotropically to all directions, if the nodes are inappropriately chosen. Therefore we show how to obtain the transition rates for some classes of feasible lattices and illustrate these results with stochastic particle simulations.

To increase the number of classes of lattices which can be used with our framework, we introduce smallest unit-based lattices. We show how to subdivide a lattice into smallest units and how to find the transition rates on lattices that have, for instance, missing edges. We discuss the implications of this result and again provide an example.

After considering the two-dimensional case we investigate the approximation of the diffusion of heat on closed curves. Important extensions are circular boundary conditions and the consideration of the Laplace-Beltrami operator, which becomes necessary since now spaces with non-zero curvature are dealt with. In addition, we introduce notions and tools used in the field of directional statistics. In that case trigonometric moments of the random variable of the Markov chain are utilized, as well as other tools provided by directional statistics. We show how transition rates have to be chosen, and how the transition rates for the closed curve are related to the transition rates for the infinite straight line. As before, we show stochastic simulations to illustrate our results.

After considering one-dimensional manifolds with non-zero curvature, we take a look at a two-dimensional manifold with non-zero curvature, the sphere. The main obstacle when trying to characterize the infinitesimal generator for the Markov chain on the sphere is the definition of a two-dimensional lattice with zero curvature that can be wrapped around the sphere in a rigorous manner. We discuss this problem and then show an empirical approach to the approximation of heat on the sphere.

Finally, we summarize the results of each chapter and the dissertation as a whole. We draw concluding remarks and give directions for future research. In summary, we considered the microscopic random movement of heat particles and showed in stochastic simulations how a multitude of particles diffuses, where each particle is modelled with continuous-time Markov chain whose infinitesimal generator is characterized in a certain way. The mean distribution of those particle positions delivers a good approximation of the distribution of heat. This has been shown by comparing the mean and variance of the distribution of particles with the mean and variance of the Gaussian distribution.

The comparison between the mean distribution of particle positions and the analytical heat distribution takes place on a macroscopic level. This comparison can be viewed as the comparison between the solution to the Kolmogorov forward equation and the analytical heat distribution. Hence, we find the elements of the infinitesimal generator which is used in the Kolmogorov forward equation, such that we can deterministically calculate the distribution of heat.

備考 ： 論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。

Note : Thesis Summary should be submitted in either a copy of 2000 Japanese Characters and 300 Words (English) or 1copy of 800 Words (English).