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## Doctoral Thesis Outline

Fitting geometric models to image data is a fundamental operation in a great variety of computer vision applications, including 3D reconstruction, robot localization, and augmented reality. The core challenge lies in the fact that the practically available data are usually corrupted by noise, or even outliers that drastically violate the model under consideration. As a result, tremendous endeavors have been poured into two types of fitting tasks, i.e., the robustness-oriented estimation whose primary goal is to suppress or remove outliers, and the accuracy-oriented estimation in which one needs to estimate a model as accurate as possible by using noisy and redundant inliers.

In either task, it is conventional to minimize (or maximize) a properly defined cost function and extract its minimizer (or maximizer) as the estimation result. Unfortunately, the resulting mathematical optimization problems are usually nonconvex, whose globally optimal solution is difficult to retrieve.

Rather than being satisfied with a locally optimal solution, the primary objective of this dissertation is to deterministically retrieve the global optimum of a nonconvex optimization problem, arising from a series of important geometric fitting problems, such as homography estimation, fundamental matrix estimation and the perspective-n-point pose estimation problem, in the context of either the robustness- or the accuracy-oriented estimation. Three typical deterministic global optimization techniques, branch-and-bound, convex relaxation and polynomial system solving, are systematically investigated and carefully adapted so as to take advantage of certain problem-specific structures. The organization of this dissertation and the main contributions are summarized as follows.

Chapter 1, *Introduction*, describes the role of geometric fitting problems in single-view and two-view geometric computer vision, followed by the challenges of nonconvexity inherent in the optimization problems arising from either the robustness- or accuracy-oriented estimation. Three typical deterministic global optimization techniques are briefly reviewed, together with their respective characteristics summarized.

Chapter 2, *Feasible Linear Subsystem Maximization*, addresses the aspect of the robustness-oriented estimation. It is shown how the robust linear model estimation

problem can be interpreted in the sense of feasible subsystem maximization and deterministically solved by using a branch-and-bound algorithm. The key contribution is to introduce the piecewise linear relaxation scheme for square terms into computer vision. The tight relaxation and natural variable bounds in the proposed formulation lead to an algorithm with superior numerical stability and computational efficiency to the existing global optimization methods. Although existing RANSAC-type methods and robust norm based methods are generally more efficient, the proposed algorithm is capable of retrieving the global optimum, that is, the feasible subsystem with maximum cardinality.

Chapter 3, *Fundamental Matrix Estimation – Geometric Criteria*, starts to consider the accuracy-oriented estimation by using noisy and redundant features, in which outliers, if any, have been properly removed via any robust method. It is shown that the Sampson distance and the point to epipolar line distance (P2ELD) can be deterministically minimized by using the branch-and-bound technique. The novelty lies in a simple but effective denominator linearization scheme, on the basis of which the nonconvexity can be transferred into a few square, bilinear and trilinear terms, independent of the number of feature correspondences. The convex and concave envelope of a trilinear term is introduced and utilized in computer vision tasks for the first time. These proposed techniques enable the first feasible algorithm for the Sampson and the P2ELD distance minimization with guaranteed global optimality. Experimental results have also revealed that the widely used state-of-the-art locally optimal methods indeed suffer from the risk of being trapped into local minimum.

Chapter 4, *Fundamental Matrix Estimation – Algebraic Criterion*, uses instead the algebraic distance criterion and focuses primarily on the challenges arising from the rank-2 constraint of a fundamental matrix, which can be regarded as a substantial augmentation of the well-known normalized eight-point algorithm. It has been revealed that the fundamental matrix estimation problem is equivalent to a three-variable six-degree homogeneous ratio problem. The primary contribution is to develop a sum-of-square convex relaxation to the homogeneous ratio problem, as well as a generalized eigenvalue factorization method for its de-homogenized formulation on the basis of a hidden-variable resultant polynomial system solving technique. The proposed algorithm, taking advantage of the simplicity of algebraic distance criterion, is much faster than the algorithm in the previous chapter.

Experiment results have verified the superiority of the proposed method over existing algebraic distance based algorithms in terms of computational efficiency, numerical stability, as well as its advantages when used to initialize geometric distance based algorithms.

Chapter 5, *Perspective-n-Point Pose Problem*, is devoted to the perspective-n-point pose estimation problem of a calibrated perspective camera, also known as the PnP problem. The main idea is to minimize a properly designed algebraic distance function and extract all its stationary points via the Gröbner basis technique. The contribution lies in a simple but elegant formulation of the PnP problem into an unconstrained functional minimization problem, whose first-order optimality condition assumes two-fold symmetry, a property that can be utilized by a Gröbner basis technique to improve the solving speed and numerical stability. The proposed solution is the first non-iterative  $O(n)$  algorithm that is fast, globally optimal and generally applicable. Experiment results have also demonstrated that, in terms of accuracy, our proposed solution is definitely better than the state-of-the-art methods, and even comparable with the reprojection error minimization method.

Chapter 6, *Conclusion and Future Works*, concludes the works in this dissertation and briefly mentions possible future trends.

To sum up, the presented results in this dissertation have justified the necessity and feasibility of deterministic global optimization in various geometric fitting problems, enriched the family of global optimization techniques and, most importantly, drive forward the state-of-the-art for some important geometric model fitting problems.