T2R2 東京科学大学 リサーチリポジトリ
Science Tokyo Research Repository

論文 / 著書情報
Article / Book Information

題目(和文)	コバノフ型絡み目不変量とホモトピー的量子場の理論
Title(English)	Khovanov type link invariant and homotopy quantum field theory
著者(和文)	田神慶士
Author(English)	Keiji Tagami
出典(和文)	学位:博士（理学）, 学位授与機関:東京工業大学, 報告番号:甲第9701号, 授与年月日:2015年3月26日, 学位の種別:課程博士, 審査員:ｶｰﾙﾏﾝ　ﾀﾏｼｭ,遠藤　久顕,小島　定吉,服部　俊昭,村山　光孝
Citation(English)	Degree:, Conferring organization: Tokyo Institute of Technology, Report number:甲第9701号, Conferred date:2015/3/26, Degree Type:Course doctor, Examiner:,,,,
学位種別(和文)	博士論文
Category(English)	Doctoral Thesis
種別(和文)	論文要旨
Type(English)	Summary

Powered by T2R2 (Science Tokyo Research Repository)

（博士課程）
Doctoral Program

論 文 要 旨

THESIS SUMMARY

専攻： Department of	数学	専攻	申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of	（　理学　）
学生氏名： Student's Name	田神　慶士		指導教員（主）： Academic Advisor(main)	カールマン　タマシュ	
			指導教員（副）： Academic Advisor(sub)		

要旨（英文800語程度）
Thesis Summary（approx.800 English Words ）

A *link* is an embedding of a disjoint union of finitely many circles into the 3-sphere. A *link diagram* is a projection of a link onto the 2-sphere whose all multiple points are transverse double points and each double point has an over/under information. In Knot Theory, we often use *link invariants* in order to study geometric properties of links. A link invariant is a map from the set of links to a set and if two links are ambient isotopic, the images of them under the map are the same. In this thesis, we study a knot invariant which is called “Khovanov homology".

For each link diagram, Khovanov defined a graded chain complex. He proved that its homology is a link invariant and its graded Euler characteristic is equal to the Jones polynomial. It is called the Khovanov homology. Later, Bar-Natan gave a geometric chain complex for Khovanov homology and explained Khovanov homology by using an oriented (1+1)-dimensional topological quantum field theory (TQFT), where an oriented (d+1)- dimensional TQFT assigns a module to each oriented closed d-dimensional manifold and assigns a homomorphism of modules to each oriented (d + 1)-dimensional cobordism, satisfying certain axioms. By using Bar-Natan's complex and an unoriented (1+1)-dimensional TQFT, several extensions of the Khovanov homology are discovered. For example, Turaev and Turner constructed a link homology for each stable equivalence class of link diagrams on surfaces. Manturov and Tubbenhauer, independently, also gave other extensions of the Khovanov complex to virtual links, that is stable equivalence classes. Asaeda, Przytycki and Sikora also constructed link homologies for link diagrams on surfaces.

On the other hand, there is an extension of TQFT. Turaev defined the concept of homotopy quantum field theories (HQFTs) with target X, where X is a connected topological space with a base point. An oriented HQFT assigns a module and a map of modules to each “oriented X-manifold" and “oriented X-cobordism", respectively. An oriented X-manifold is a pair of an oriented manifold with some base points and a continuous map from the manifold to X. An oriented X-cobordism is a pair of an oriented cobordism with pointed boundaries and a continuous map from the cobordism to X. An unoriented HQFT is defined analogously.

In this thesis, the author studied two topics, one is computations of Khovanov homology and geometric properties of links, another is homotopy quantum field theory and an extension of Khovanov homology.

In Chapters 3 and 4, we compute the maximal homological degree of the Khovanov homology for some link. In particular, we compute the maximal homological degree of Khovanov homology of a cable link in Chapter 3, and we consider an asymptotic behavior of the Khovanov homology under twisting in Chapter 4. The maximal homological degree of the Khovanov homology of a link gives a lower bound of the minimal positive crossing number of the link. Hence, it seems that the computations estimate the positivity of the links. In Chapter 5, we compute the 0-th homological term of the Khovanov homology for almost positive links, where an almost positive link is a non-positive link represented by a link diagram which has exactly one negative crossing. As a corollary of the computation, we determine the Rasmussen invariants of almost positive knots. Moreover, we describe a difference between positive knots and almost positive knots. In fact, although all positive knots are homogeneous, the author proved that all almost positive knots are non-homogeneous. In Chapters 6 and 7, we construct an extension of the Khovanov homology by utilizing HQFTs. In Chapter 6, we find a bijection between the set of algebras which satisfy certain conditions and the set of unoriented (1 + 1)-dimensional HQFTs. By this result, we can construct some examples of HQFTs. In Chapter 7, we construct a geometric complex in a cobordism category for each link diagram on a surface and prove that its homotopy class is an invariant of links in a line bundle over the surface. Since an HQFT is regarded as a functor from a cobordism category to the category of vector spaces, by applying an HQFT (which satisfies some conditions) to our geometric complex, we obtain a homology invariant of links in a line bundle over the surface. In particular, if we use an example of HQFT given by Chapter 7, we obtain an extension of the Khovanov homology.

備考 ： 論文要旨は、和文2000字と英文300語を1部ずつ提出するか、もしくは英文800語を1部提出してください。
Note : Thesis Summary should be submitted in either a copy of 2000 Japanese Characters and 300 Words (English) or 1copy of 800 Words (English).

注意：論文要旨は、東工大リサーチリポジトリ（T2R2）にてインターネット公表されますので、公表可能な範囲の内容で作成してください。
Attention: Thesis Summary will be published on Tokyo Tech Research Repository Website (T2R2).