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Article / Book Information

Title	Yield-aware mask assignment using positive semidefinite relaxation in LELECUT triple patterning
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Citation(English)	Proc. SPIE 9427, Design-Process-Technology Co-optimization for Manufacturability IX, 94270B, , , 1-9
/Pub. date	2015, 3
DOI	http://dx.doi.org/10.1117/12.2085285
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Yield-aware Mask Assignment using Positive Semidefinite Relaxation in LELECUT Triple Patterning

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ABSTRACT

LELECUT type triple patterning lithography is one of the most promising techniques in the next generation lithography. To prevent yield loss caused by overlay error, LELECUT mask assignment which is tolerant to overlay error is desired. In this paper, we propose a method that obtains an LELECUT assignment which is tolerant to overlay error. The proposed method uses positive semidefinite relaxation and randomized rounding technique. In our method, the cost function that takes the length of boundary of features determined by the cut mask into account is introduced.

Keywords: Triple Patterning, LELECUT, Design for Manufacturability, Positive Semidefinite Relaxation

1. INTRODUCTION

Multiple patterning technique enables us to fabricate small features without using advanced technologies such as extreme ultra violet (EUV) lithography. Triple patterning lithography (TPL) is one of the most promising techniques in 14 nm logic node and beyond. In order to realize a target pattern, various types of techniques including design for manufacturability such as LELE type double patterning lithograph,[1–7] LELELE type TPL,[8–18] LELE-CUT type TPL,[19,20] and side wall process,[21] are used in addition to a basic litho-etch process with optimized mask. These techniques are summarized in.[22,23]

Sidewall process[21] forms a wall feature with unique width so that it surrounds the prefabricated polygon. The sidewall process which is used in self-aligned double patterning enables us to fabricate finer pattern pitch by combining a slimming process, but the variety of target patterns that can be fabricated is limited.

Two types of TPL technologies are often discussed in literature. In LELELE, litho-etch process is repeated three times. However, it is difficult to achieve high yield due to native conflict and overlay problems. In LELECUT, the third mask called cut (or trim) process removes a part of a fabricated pattern. It is used to improve the quality of fabricated patterns as well as to enhance the flexibility of layout. However, it has overlay problems and lithographical limitations. In order to prevent yield loss caused by overlay error as much as possible, LELECUT mask assignment which is tolerant to overlay error is desired.

To our best knowledge, two LELECUT mask assignment methods have been proposed. In,[19] LELECUT mask assignment problem is formulated as an integer linear programming problem. Although it minimizes the weighted summation of the number of conflicts and stitches, the effect of cuts on layout quality is not taken into account. In,[20] LELECUT mask assignment problem is solved by positive semidefinite relaxation. Although it minimizes the weighted summation of the number of conflicts, stitches, and polygons in the cut mask, the yield of obtained layout is also not discussed. Fig. 1 shows mask assignments in LELECUT. A target pattern is shown in Fig. 1 (a). The layouts obtained by two LELECUT mask assignments which are represented by blue, magenta, and cut masks without overlay error are shown in Fig. 1 (b) and Fig. 1 (d). These mask assignments have no

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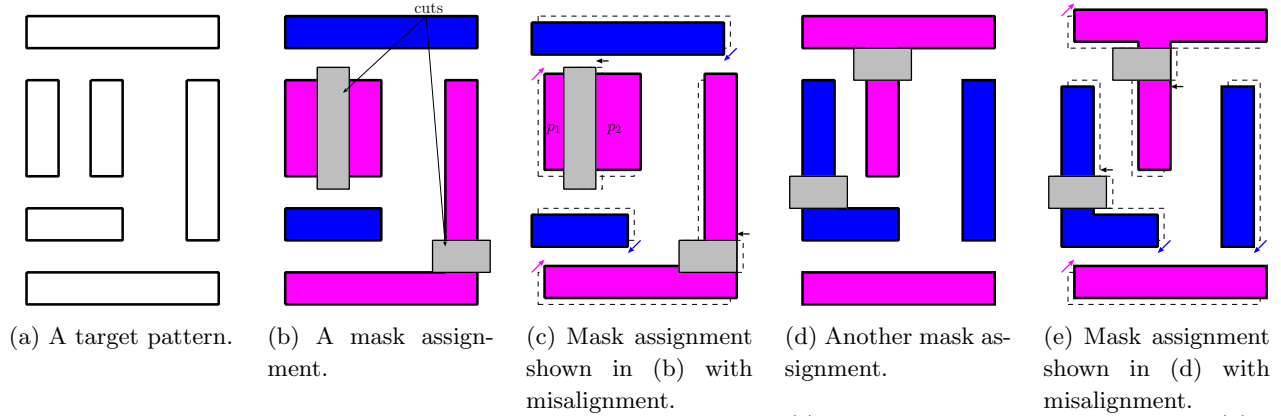| (a) A target pattern. | (b) A mask assignment. | (c) Mask assignment shown in (b) with misalignment. | (d) Another mask assignment. | (e) Mask assignment shown in (d) with misalignment. |

Figure 1. Mask assignments in LELECUT. Mask assignment shown in (c) has higher yield than that shown in (d).

conflicts, no stitches, and the number of polygons in the cut mask is same. The layouts of them with overlay error in which blue, magenta, and cut masks move to the lower left, the upper right, and the left, respectively, are shown in Fig. 1 (c) and Fig. 1 (e), respectively. The former is expected to have lower yield than the latter since a longer dimension of features such as p_1 and p_2 is determined by the cut mask and is affected directly by the overlay error. The length of a boundary of a feature that is determined by the cut mask should be small enough to prevent the yield loss caused by overlay error.

In this paper, we propose a method that obtains an LELECUT assignment which is tolerant to overlay error. The proposed method is an enhancement of the method proposed in[20] and uses positive semidefinite relaxation and randomized rounding technique. In our method, the cost function that takes the length of boundary of features determined by the cut mask into account is introduced to obtain an overlay tolerant LELECUT assignment.

2. PRELIMINARIES

2.1 Problem Definition

Let $P = \{p_1, p_2, \ldots, p_n\}$ be the set of polygons in a target pattern. A polygon may represent a polygon decomposed by given stitch candidates. A stitch edge is defined between two polygons if and only if two polygons are decomposed by a stitch candidate. A polygon conflict edge is defined between two polygons if and only if two polygons are too close to assign the same mask. A cut candidate is defined between two polygons connected by a polygon conflict edge if and only if they can be cut by the cut mask when they are assigned to the same mask. A cut candidate c has a cost $l(c)$ which is defined by the length of boundary between polygons in the target pattern and the cut candidate. A cut candidate may not be independent of other cut candidates. A cut conflict edge is defined between two cut candidates if and only if they cannot be used simultaneously. Note that even if the distance between two cut candidates is not long enough, they can be used simultaneously if they can be merged into one. A cut conflict edge is not defined between two cut candidates if the distance between them is long enough or if they can be merged into one without affecting the critical dimension of pattern. Let S, C_P, T, and C_T be the set of stitch edges, the set of polygon conflict edges, the set of cut candidates, and the set of cut conflict edges, respectively. Note that both the set of stitch edges S and the set of polygon conflict edges C_P are families of unordered pairs of polygons in P, the set of cut candidates T is a sub-set of the set of polygon conflict edges C_P, and the set of cut conflict edges C_T is a family of unordered pairs of cut candidates in T.

Fig. 2 shows an example of problem. In this example, $P = \{p_1, p_2, p_3, p_4\}$, $S = \{\{p_3, p_4\}\}$, $C_P = T = \{c_1, c_2, c_3, c_4, c_5\}$, where $c_1 = \{p_1, p_3\}$, $c_2 = \{p_1, p_4\}$, $c_3 = \{p_1, p_2\}$, $c_4 = \{p_2, p_3\}$, and $c_5 = \{p_2, p_4\}$ with costs $l(c_1) = 2$, $l(c_2) = 2$, $l(c_3) = 1$, $l(c_4) = 2$, and $l(c_5) = 2$. The set of cut conflict edges is given by $C_T = \{\{c_1, c_2\}, \{c_1, c_3\}, \{c_2, c_3\}, \{c_3, c_4\}, \{c_3, c_5\}, \{c_4, c_5\}\}$.

In this paper, a polygon is assigned to one of two masks except the cut mask. The problem of finding an assignment of polygons, and/or two-coloring, is essentially equivalent to a maximum cut problem. We employ a
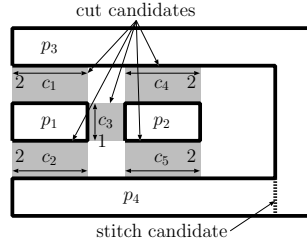
Figure 2. Polygons with stitch and cut candidates.

$\{-1, 1\}$ formulation represented by function $x : P \to \{-1, 1\}$, which is introduced by Goemans and Williamson in[24] for a max cut problem. This formulation naturally yields a positive semidefinite relaxation appearing in a later section. For two-coloring x, the set of polygon conflict edges that connect the same color polygons is represented by

$$C_P(x) \overset{\text{def.}}{=} \{\{p, q\} \in C_P \mid x(p) = x(q)\}.$$

Similarly, the set of stitch candidates that connect the different color polygons is represented by

$$S(x) \overset{\text{def.}}{=} \{\{p, q\} \in S \mid x(p) \neq x(q)\}.$$

The set of feasible cuts for two-coloring x is a subset of $C_P(x) \cap T$ and an independent set of cut graph (T, C_T). The problem we consider is given as follows:

P1: minimize

$$\alpha_1 |C_P(x) \setminus T'| + \alpha_2 |S(x)| + \alpha_3 \sum_{c \in T'} l(c)$$

$$= \alpha_1 |C_P(x)| + \alpha_2 |S(x)| - \alpha_1 \sum_{c \in T'} 1 + \alpha_3 \sum_{c \in T'} l(c)$$

$$= \alpha_1 |C_P(x)| + \alpha_2 |S(x)| + \sum_{c \in T'} (\alpha_3 l(c) - \alpha_1)$$

subject to

- $x(p) \in \{-1, 1\}$ $(\forall p \in P)$,
- $T' \subseteq C_P(x) \cap T$,
- T' is an independent set of cut graph (T, C_T).

In this formulation, the weighted sum of the number of unresolved conflict edges, the number of caused stitches, and the total cost of used cut candidates is minimized. In the following, we assume that $\alpha_1 \geq \alpha_2 \geq 0$ and $\alpha_1 \geq \alpha_3 l(c) \geq 0$. According to this assumption, the total cost of used cut candidates is minimized under the condition that the number of conflict edges that connect the same polygons is minimized and the number of used cut candidates is maximized.

Fig. 3 shows examples of mask assignments. The mask assignment shown in Fig. 3 (a) has two cuts with total cost 4 and one stitch. On the other hand, that shown in Fig. 3 (b) has one cut with total cost 1. Obviously, the mask assignment shown in Fig. 3 (b) is better than that shown in Fig. 3 (a).

2.2 Maximum Independent Set with Minimum Total Cost Problem

For a given two-coloring $x : P \to \{-1, 1\}$, the problem P1 is equivalent to a maximum independent set with minimum total cost problem MISMTCP1 since we assume $\alpha_1 \geq \alpha_3 l(c)$.

(a) Two cuts with total cost 4 and one stitch.　　　(b) One cut with total cost 1.

Figure 3. Mask assignments for layout in Fig. 2.

Step 1: Formulate a positive semidefinite relaxation SDP-L.

Step 2: Solve SDP-L by SDP solver.

Step 3: Obtain a mask assignment by randomized rounding algorithm with iterative improvement.

Figure 4. Outline of the proposed method.

MISMTCP1: maximize

$$\sum_{c \in T'} (1 - \alpha l(c))$$

subject to

- $\alpha l(c) < 1$,
- $T' \subseteq C_P(x) \cap T$,
- T' is an independent set of cut graph (T, C_T).

A maximum independent set with minimum total cost problem is known to be NP-hard. In this paper, the following 0-1 integer linear programming MISMTCP2 is formulated by introducing 0-1 variable $y(c)$.

MISMTCP2: maximize

$$\sum_{c \in T} y(c) \cdot (1 - \alpha l(c))$$

subject to

$$y(c) \in \{0, 1\} \qquad (\forall c \in T), \tag{1}$$

$$y(c) + y(c') \leq 1 \qquad (\forall \{c, c'\} \in C_T), \tag{2}$$

$$y(c) = 0 \qquad (\forall c \in T \setminus C_P(x)). \tag{3}$$

3. POSITIVE SEMIDEFINITE RELAXATION

3.1 Outline of Proposed Method

The outline of the proposed method is shown in Fig. 4. The proposed method is based on the method proposed in.[20] In the proposed method, a positive semidefinite relaxation of P1 called SDP-L is formulated and a mask assignment is obtained from an optimum solution of the relaxation by randomized rounding technique with iterative improvement. It is well-known that a positive semidefinite programming problem can be solved by interior point methods in polynomial time.

3.2 Our Semidefinite Relaxation

In this subsection, we introduce a positive semidefinite relaxation of P1. Our relaxation is an enhancement of the formulation proposed by Goemans and Williamson[24] for a max cut problem to handle a maximum independent set with minimum total cost.

First, we represent the objective function of P1 as a linear function. An arbitrary two-coloring $x : P \to \{-1, 1\}$, satisfies: $x(p) = x(q) \leftrightarrow x(p)x(q) = 1$ and $x(p) \neq x(q) \leftrightarrow x(p)x(q) = -1$. By using these properties, $|C_P(x)|$ and $|S(x)|$ are represented in terms of x as follows:

$$|C_P(x)| = \sum_{\{p,q\}\in C_P} \left(\frac{x(p)x(q)}{2} + \frac{1}{2} \right)$$

$$|S(x)| = \sum_{\{p,q\}\in S} \left(-\frac{x(p)x(q)}{2} + \frac{1}{2} \right).$$

Let X be the $n \times n$ matrix whose (p, q)-th element is $X_{pq} = x(p)x(q)$ $(\forall p, \forall q \in P)$. Let C and S be matrixes that represent polygon conflicts and stitch candidates, respectively. That is, C and S are the $n \times n$ symmetric matrixes, and

$$C_{pq} = \begin{cases} \frac{1}{4} & (\{p,q\} \in C_P), \\ 0 & (\{p,q\} \notin C_P), \end{cases} \quad S_{pq} = \begin{cases} \frac{1}{4} & (\{u,v\} \in S), \\ 0 & (\{u,v\} \notin S), \end{cases}$$

respectively. Then, we have

$$|C_P(x)| = \sum_{\{p,q\}\in C_P} \left(\frac{x(p)x(q)}{2} + \frac{1}{2} \right) = C \bullet X + \text{const},$$

$$|S(x)| = \sum_{\{p,q\}\in S} \left(-\frac{x(p)x(q)}{2} + \frac{1}{2} \right) = -S \bullet X + \text{const},$$

where $M \bullet M'$ is defined as $\sum_i \sum_j M_{ij}M'_{ij}$ for square matrixes M and M' of the same size. Let $y(c)$ be a 0-1 variable for a cut candidate c. c is assigned to the cut mask, if and only if $y(c) = 1$. Then, we have

$$\sum_{c\in T'} (\alpha_3 l(c) - \alpha_1) = \sum_{c\in T} y(c) \cdot (\alpha_3 l(c) - \alpha_1).$$

Therefore, the objective function of P1 is represented as

$$\alpha_1(C \bullet X) - \alpha_2(S \bullet X) + \sum_{c\in T} y(c) \cdot (\alpha_3 l(c) - \alpha_1) + \text{const}. \tag{4}$$

Note that C and S are constant matrixes.

Next, constraints of P1 are represented as linear functions in terms of X and y. Constraint (3) in MISMTCP2 is represented as

$$0 \leq y(c) \leq \begin{cases} 1 & (\text{if } c \in T \cap C_P(x)), \\ 0 & (\text{if } c \in T \setminus C_P(x)). \end{cases}$$

Then, constraint (3) is represented by using two-coloring x as follows:

$$0 \leq y(c) \leq \frac{x(p)x(q)}{2} + \frac{1}{2} \qquad (\forall c = \{p, q\} \in T).$$

The representations of other constraints are straightforward, and P1 is represented as follows:

P2: minimize

$$\alpha_1(C \bullet X) - \alpha_2(S \bullet X) + \sum_{c \in T} y(c) \cdot (\alpha_3 l(c) - \alpha_1)$$

subject to

$$
\begin{aligned}
X_{pq} = x(p)x(q) \qquad & (\forall (p,q) \in P^2), & (5) \\
x(p) \in \{-1, 1\} \qquad & (\forall p \in P), & (6) \\
y(c) \in \{0, 1\} \qquad & (\forall c \in T), & (7) \\
y(c) + y(c') \le 1 \qquad & (\forall \{c, c'\} \in C_T), \\
0 \le y(c) \le \frac{x(p)x(q)}{2} + \frac{1}{2} \qquad & (\forall c = \{p, q\} \in T).
\end{aligned}
$$

The objective function of P2 is obtained from Eq. (4) by removing constant term. Note that $X_{pp} = 1$ for all $p \in P$.

Since X in P2 is a positive semidefinite symmetric matrix, the problem P2 has a positive semidefinite programming relaxation as follows. Let $\mathcal{S}^n_+$ be the set of $n \times n$ positive semidefinite symmetric matrix. A positive semidefinite relaxation problem SDP-L is obtained from P2 by restricting X within $\mathcal{S}^n_+$ and ignoring 0-1 constraints for y, instead of constraints (5), (6), and (7).

SDP-L: minimize

$$\alpha_1(C \bullet X) - \alpha_2(S \bullet X) + \sum_{c \in T} y(c) \cdot (\alpha_3 l(c) - \alpha_1)$$

subject to

$$
\begin{aligned}
X_{pp} = 1 \qquad & (\forall p \in P), \\
y(c) + y(c') \le 1 \qquad & (\forall \{c, c'\} \in C_T), \\
0 \le y(c) \le \frac{1}{2} X_{pq} + \frac{1}{2} \qquad & (\forall c = \{p, q\} \in T), \\
X \in \mathcal{S}^n_+. &
\end{aligned}
$$

SDP-L is a positive semidefinite programming problem and can be solved by interior point methods in polynomial time.

3.3 Randomized Rounding for LELECUT

In this subsection, we propose a randomized rounding technique based on the *hyper-plane separation technique* proposed by Goemans and Wiiliamson,[24] which gives 0.878 approximation algorithm for a max cut problem. The randomized rounding technique is based on the method proposed in,[20] and the iterative improvement is applied as post-processing.

For any positive semidefinite symmetric matrix $X \in \mathcal{S}^n_+$, there exists a matrix Z satisfying $X = Z^\top Z$. This decomposition is called Cholesky decomposition.

We solve problem SDP-L by a SDP solver and obtain an optimal solution $(\widetilde{X}, \widetilde{y})$ for SDP-L. Let $\widetilde{Z}^\top \widetilde{Z}$ be the Cholesky decomposition of $\widetilde{X}$ and d be the number of rows of $\widetilde{Z}$. Here we note that columns of $\widetilde{Z}$ are indexed by polygons in P and the length of every column vector is equal to 1. For each polygon $p \in P$, vector $\widetilde{z}(p) \in \mathbb{R}^d$ denotes the corresponding column vector of $\widetilde{Z}$. Algorithm RR shown in Fig. 5 outputs a two coloring $\widetilde{x} : P \to \{-1, 1\}$ and the set $\widetilde{T}$ of cuts. The mask assignment is modified by the greedy iterative improvement so that better solutions are obtained. The quality of the obtained mask assignment depends on the generated random unit vector and the runtime of Algorithm RR is very small. Therefore, Algorithm RR is repeated appropriate times and the best mask assignment is output. During the repetition, we might choose a comfortable mask assignment as well.

Figure 5. Randomized Rounding Algorithm.

Table 1. Experimental results. The obtained mask assignments have no polygon conflicts and cut conflicts in ILP-#, ILP-L, SDP-#, and SDP-L Imp. The units of cost and time are [nm] and [s], respectively.

circuit	$|P|$	# Seg	$|C_P|$	# comp total	# comp target	$|T|$	ILP-# $|T'|$	ILP-# cost	ILP-# time	ILP-L $|T'|$	ILP-L cost	ILP-L time	SDP-# $|T'|$	SDP-# cost	SDP-# time	SDP-L $|T'|$	SDP-L $|C_P(x)|$	SDP-L cost	SDP-L time	SDP-L Imp $|T'|$	SDP-L Imp cost	SDP-L Imp time
c432	850	4918	540	414	1	12	1	40	0.01	1	40	0.01	1	40	0	1	0	40	0	1	40	0
c499	1491	9518	1489	502	50	278	58	2720	0.42	58	2720	0.45	58	6030	0.12	62	0	2940	0.12	58	2720	0.15
c880	1872	10666	1422	717	168	934	172	11720	2.46	198	10390	6.25	172	20590	0.44	202	2	11105	0.35	198	10390	0.51
c1355	2656	15246	1514	1328	76	514	90	9760	1.10	122	8320	6.10	90	13230	0.27	130	2	9635	0.17	122	8320	0.37
c1908	4191	24370	3141	1733	182	1462	211	37700	5.77	373	30410	24.34	211	41140	0.73	411	13	36600	0.43	373	30410	1.37
c2670	6371	37564	5802	2056	585	4298	686	76950	10.52	958	64710	38.98	686	101975	2.32	1069	66	84540	1.39	952	65775	5.26
c3540	8188	47244	6897	2896	775	4794	821	73560	11.06	1044	63445	29.61	821	111330	2.24	1090	11	72630	1.47	1044	63445	3.02
c5315	11498	68476	10097	3926	1193	7552	1259	108290	17.52	1572	93965	53.23	1259	162510	3.59	1647	13	106280	2.33	1572	93965	4.11
c6288	11605	64762	5602	6259	256	1282	256	10320	2.39	256	10320	2.33	256	29840	0.66	256	0	10240	0.45	256	10240	0.66
c7552	17167	99526	14027	6258	1448	9325	1587	162980	20.02	2122	138905	89.62	1587	221025	4.47	2243	23	159505	3.01	2122	138905	6.12
ave.							0.82	1.12	0.50	(1)	(1)	(1)	0.82	1.77	0.09	1.05		1.12	0.07	1.00	1.00	0.12

$|P|$	the number of polygons
# Seg	the number of line segments
$|C_P|$	the number of polygon conflict edges
total	the number of components in the conflict graph (P, C_P)
target	the number of components in the conflict graph (P, C_P) in which cuts must be inserted
$|T|$	the number of cut candidates in the target components
$|T'|$	the number of inserted cuts
$|C_P(x)|$	the number of conflicts in the obtained mask assignment
cost	the total cost of cuts in the obtained mask assignment
time	computational time
ave.	the average normalized by ILP-L

4. EXPERIMENTS

Our proposed mask assignment method is implemented by using a SDP solver and C++ language. We compare the following five methods. ILP-# and ILP-L are ILP formulations based on[19] which minimizes the number of cuts and minimizes the total cost of cuts, respectively. SDP-# is the positive semidefinite relaxation proposed in[20] which minimizes the number of cuts. SDP-L Imp and SDP-L are the proposed methods. SDP-L Imp applies the greedy iterative improvement as post-processing and SDP-L does not. The methods are executed on a Linux machine with 12 GB memory by using Intel core i7-3770 of 3.40 GHz. In our implementation, SDP problems are solved by SDPA 7.3.8[25] which is a free tool. On the other hand, ILP problems are solved by CPLEX 12.6.1[26] which is one of the most famous commercial ILP solvers. In these methods, a speedup technique in which the conflict graph $(P, S \cap C_P)$ is decomposed into connected components is adopted. This speedup technique is discussed in many previous studies.[1–3, 5, 6, 19, 20] In this experiment, we do not prepare stitch candidates to focus on observing the total cost, the number of cuts, and the number of conflicts. The parameters in objective functions are set to $\alpha_1 = 10^6$ and $\alpha_3 = 1$. $\alpha_1 = 10^6$ is much larger than cut costs. Algorithm RR is applied 100 times in SDP-#, SDP-L, and SDP-L Imp.

ISCAS benchmarks which were used in[3, 5] are used. The benchmarks are reproduced from the information given by authors of[3, 5] and from figures in,[3] though we could not obtain the same data. We followed the

parameters as in,[3, 5] where the minimum polygon space in a mask is 54 nm. If stitches are allowed to be inserted, the mask assignment without cuts is obtained. Therefore, we also do not insert stitches in this experiment. Table 1 shows the results. Note that the mask assignments obtained by all methods except SDP-L have no polygon conflicts and cut conflicts. Since the minimization of the total cost is added into the objective function of ILP-L, the total cost obtained by ILP-L is optimum. Similarly, since the minimization of cuts is added into the objective function of ILP-#, the number of cuts obtained by ILP-# is optimum. Although ILP-# and SDP-# obtain mask assignments with the minimum number of cuts, the total cost of the mask assignment obtained by them is larger than that by ILP-L since the minimization of cuts does not corresponds to that of the total cost. Although SDP-L is fast, it obtains mask assignments with conflicts. The total cost of the mask assignment obtained by SDP-L Imp is the same as that by ILP-L in nine circuits of ten circuits. Moreover, SDP-L Imp is much faster than ILP-L. Consequently, SDP-L Imp obtains optimum solutions in the shortest computational time in almost all circuits.

5. CONCLUSIONS

In this paper, we propose a fast LELECUT mask assignment method to be tolerant to overlay error. The proposed method applies a positive semidefinite relaxation. The experimental results show the efficiency and the validity of the proposed method. We will take the mask density balance, stitch direction, and etc. into account to improve the quality of the mask assignment in our future works.

ACKNOWLEDGMENTS

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