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A Hyperpath-based Network Generalized Extreme-value Model for Route Choice under Uncertainties

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Abstract

Previous route choice studies treated uncertainties as randomness; however, it is argued that other uncertainties exist beyond random effects. As a general modeling framework for route choice under uncertainties, this paper presents a model of route choice that incorporates hyperpath and network generalized extreme-value-based link choice models. Accounting for the travel time uncertainty, numerical studies of specified models within the proposed framework are conducted. The modeling framework may be helpful in various research contexts dealing with both randomness and other non-probabilistic uncertainties that cannot be exactly perceived.

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Keywords: hyperpath; network GEV model; route choice; uncertainties

1. Introduction

1.1. The short story of route choice studies

Route choice is one of the most important issues studied in travel behavior and transportation network analyses. The comprehensive review by Prato (2009) summarizes the status quo and the future research directions of route choice studies. As Prato (2009) argued, route choice models evolved, to a certain extent, with the enhancement of random utility-based discrete choice models (Ben-Akiva and Lerman, 1985). Although the multinomial logit (MNL) model has been widely used in practice due to its simplicity, it cannot capture the complex correlation structure among overlapped routes, due to the property of independence from irrelevant alternatives (IIA). The probit model (e.g., Daganzo and Sheffi 1997; Yai et al. 1998) or error component (mixed) logit model (e.g., Frejinger and Bierlaire 2007) could accommodate such a correlation structure; however, exhaustive Monte Carlo simulations would be required to compute the choice probabilities. Therefore, the closed-form choice model, particularly the family of generalized extreme-value (GEV) models (McFadden, 1978), would also be attractive. Nested logit (NL) models and cross-nested logit (CNL) models have helped to advance route choice studies, with application to traffic assignment (e.g., Vovsha and Behkor, 1998). The network GEV model (N-GEV) elaborates on this idea further (Bierlaire, 2002; Daly and

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Bierlaire, 2006). In the N-GEV model, a broad class of networks is used to generalize the use of trees to represent (cross) NL models in the network representation. At present, the N-GEV model would be the most general operational model in the GEV-based discrete choice family. As an application in the context of route choice, Papola and Marzano (2013) proposed a joint-network GEV model (JNG) to capture the complex correlation (network) structure using a closed-form formulation.

In route choice modeling and its application to assignment calculations, the means by which the set of alternative routes (a route choice set) is constructed is particularly important, regardless of the approach (i.e., explicit versus implicit). Bovy (2009) provided a synthetic review of route choice modeling, set mainly from behavioral perspectives. There are two camps of route choice modeling studies. The first camp generates multiple routes as a choice set, whether deterministic or probabilistic, by explicitly using methods such as deterministic approaches (e.g., K-shortest paths of Eppstein (1998) or the labeling approach of Ben-Akiva et al. (1984)) or probabilistic generation of a set (e.g., Cascetta and Papola, 2001), and then applies the discrete choice model for the given set of route alternatives. Despite these methodological developments, new models (e.g., Freginger et al., 2009; Flötteröd and Bierlaire, 2013) continue to be developed; however, the generation of a route choice set tends to be arbitrary and is still open to debate. The second camp follows the approach outlined in seminal work by Dial (1971), albeit developed for stochastic network loading in traffic assignments that proposed avoiding explicit path enumeration. Dial (1971) defined so-called "efficient paths" involving a forward pass from the origin based on the proportions determined using the sequential MNL model, and working backward from the destination. In line with this approach, several alternative approaches to Dial's method were developed by Bell (1995) and Akamatsu (1996), among others. Both Bell (1995) and Akamatsu (1996) argued that the efficient path may ignore realistic paths that were actually attractive to drivers but were assigned zero flow. Instead, they proposed alternative methods based on the more general idea of a link-based Markov decision process. In this approach, the choice set was not limited to efficient paths but included any path that resulted in a sub-camp of the second camp that also required no path enumeration. Recently, Fosgerau et al. (2013) suggested the recursive logit model (also known as the sequential logit model) for the unrestricted (infinite) path set, along with the notion of "link size", referring to the concept of path size (Ben-Akiva and Bierlaire, 1999). In the same year, Papola and Marzano (2013) proposed the JNG model to allow implicit route enumeration using a Dial-like algorithm. Hara and Akamatsu (2012) formulated a stochastic user equilibrium (SUE) problem using the N-GEV route choice model and proposed an efficient solution algorithm without explicit route enumeration.

Route choice for transit passengers and transit assignment have proven more complicated than private vehicle networks due to transit frequencies, which make the route choice behavior more complex. Because public transportation usually operates according to schedules or predetermined service frequencies, the total travel time may vary due to the change in the waiting times. Transit assignment, as it relates to common bus-line problems (Chriqui and Robillard, 1975), provides insight into solving the transit route choice problem by suggesting a probabilistic framework in calculating the expected travel time including waiting time at transit stops. Based on this framework, the frequency-based transit assignment was established (Nguyen and Pallottino, 1988; Spiess and Florian, 1989), as well as the concept of "hyperpath" (Nguyen and Pallottino, 1988), which has since become quite popular in frequency-based transit assignments. According to the well-known assignment algorithm of Spiess and Florian (1989), a hyperpath is generated with a backward pass, followed by assignment of the proportions to the optimal strategy hyperpath with a forward pass; this approach is quite similar to Dial's algorithm. Both algorithms generate the choice set implicitly, but they have different rules in judging attractive/efficient links. Dial's rule is intuitive, while that of Spiess and Florian is based on the concept of optimal strategy.

Some studies in transit assignments have revealed the importance of discrete choice models. Nguyen et al. (1998) incorporated a Dial-like sequential logit model into the hyperpath model. Florian and Constantin (2012) modified their logit choices in strategy transit assignments by including short walks to access attractive transit paths. With the accommodation of discrete choice models, the optimal strategy models for transit networks become more behaviorally realistic. Schmöcker et al. (2013) built a two-level model in which the upper level is constrained by logit choice, while the lower level is constrained by the frequency property; the model parameters are estimated using smart card data.

1.2. Route choice under uncertainty

Although route choice behavior under some stochastic conditions of the drivers' environment, such as travel-time variability, can be empirically modeled, in line with the discrete choice framework (e.g., de Palma and Picard, 2005), the error term of random utility generally stands for randomness only and can be generally represented by some parametric probability distribution. However, several other uncertainties in traffic conditions (e.g., traffic incidents or network disruption) may not necessarily be modeled by a specific probability distribution and these are classified as non-probabilistic uncertainties.

The concept of hyperpath has also been applied to adaptive/policy-based routing (Miller-Hooks, 2001; Gao and Chabini, 2006; and Bell, 2009, among others). However, here, we focused on studies that dealt with uncertainties with the hyperpath methodology because hyperpath results are rooted in uncertainties. These uncertainties are interpreted differently in different studies (e.g., waiting time for transit lines (Spiess and Florian, 1989), stochastic time-dependent link travel time (Miller-Hooks, 2001; Gao and Chabini, 2006), and exposure to potential maximum delays (Bell, 2009; Bell et al., 2012). However, the behavioral basis remains weak; thus, the link proportion results may be unrealistic.

1.3. Road map from previous studies to this study

In an attempt to integrate the above-mentioned points, this paper proposes a novel model for general-purpose route choice under uncertainties, incorporating both randomness and non-probabilistic uncertainties. Based on Dial (1971) and Spiess and Florian (1989), Nguyen et al. (1998) integrates the hyperpath with the sequential logit choice model. As an improvement of Dial (1971), Hara and Akamatsu (2012) and Papola and Marzano (2013) apply the N-GEV choice model to efficient paths. By integrating previous studies, this study presents a new hyperpath-based N-GEV model. The road map of the development of route choice models from previous studies to the present study is shown in Figure 1.

In this study, a hyperpath-based N-GEV model was developed to analyze route choice under general uncertainties. On finding the optimal strategy under uncertainties, the model was modified using the hyperpath concept, which has been widely developed in the frequency-based transit assignment literature but can be extended to risk-averse route navigation of private vehicles as well. The GEV-based choice model was incorporated when making link choices from the hyperpath at decision nodes (e.g., intersections), similar to the N-GEV model. In the proposed model, probabilistic randomness and non-probabilistic uncertainties that affect route choices (e.g., potential travel delay or network disruption risks) were evaluated as a whole, and the behavioral implications of complex network correlation structures were considered.

With the integration of optimal strategies and an N-GEV-type link-choice formula, the proposed model can be used to describe route choice behavior under various uncertainties (e.g., travel time variability, traffic incidents, or network disruption) more flexibly. The basic model described in this paper relied on historical observations of uncertainties for empirical calibration of the model; however, it should be possible to extend the model to dynamic, real-time cases in future research.

The rest of the paper is organized as follows. Section 2 proposes the hyperpath-based N-GEV route choice model. Section 3 describes the algorithm used to determine the optimal strategy, along with the N-GEV proportions. Nu-

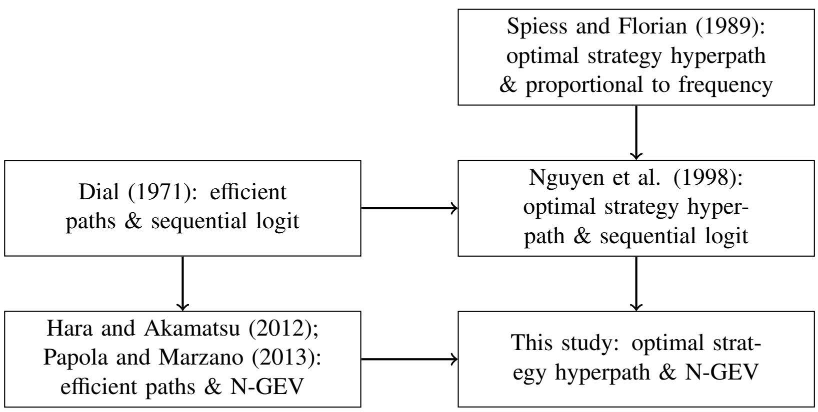

Fig. 1. Road map from previous studies to this study

merical simulations are discussed in Section 4 using an 8×8 grid network. Conclusions are presented in Section 5.

2. Hyperpath-based Network Generalized Extreme Value (N-GEV) Route Choice Model

2.1. Model setup

Nomenclature

$a(i, j) \equiv a$	link a from tail node i to head node j
$i(a) \equiv i,\ j(a) \equiv j$	tail and head nodes of link a
$H_{r,s} \equiv H$	strategic hyperpath, namely path strategies given an origin and destination pair
I^H	nodes involved in H
A^H	links involved in H
A_i^-, A_i^+	incoming and outgoing links of node i
A_i^{+H}	hyperpath (attractive) links going out of node i
V_a	deterministic link-specific utility of traversal through link a
V_i	deterministic node-specific utility of traversal through node i, which is associated with exposure to downstream non-probabilistic uncertainties
V_{is}	deterministic utility of a strategy using node i to reach destination s
$p_a,\ p_i$	marginal link choice probability and respective node choice probability
$P_{a\|i} \equiv P_a$	conditional probability of selecting link a when reaching node i
α_{a_{n-1},a_n}	non-negative degree of membership of a_{n-1} to a_n, $\sum_{a_{n-1}\in A^-_{i(a_n)}} \alpha_{a_{n-1},a_n} = 1$
θ_{a_n}	non-negative homogeneity parameter associated with link a_n

Consider a driver driving from the origin r to the destination s. Each link is associated with a generalized link utility, $V_a + \varepsilon_a + \overset{\approx}{V}_a$ where V_a represents the deterministic utility, ε_a represents the GEV distributed random/probabilistic utility, and $\overset{\approx}{V}_a$ is non-probabilistic (e.g., the risk of the network's being disrupted due to earthquake or variability in travel times). Drivers make their choices based on the perceived utility $V_a + \varepsilon_a$, which is in accordance with conventional random utility-based discrete choice models. At each decision node (e.g., intersections) i, drivers are supposed to evaluate the downstream Non-Probabilistic Uncertainties (NPU). NPU refers to the utilities of some uncertainties which are quantitative but their distributions are unknown or cannot be precisely perceived. For example, the utility of potential maximum delay in Bell (2012) can be viewed as NPU. We further assume that drivers evaluate downstream NPU in a short-sighted way (Fonzone et al., 2012) that only the utility of node i is associated with downstream NPU of attractive links A_i^{+H}. Thus, the expected utility e_i (the exposure to the downstream NPU) at node i is represented as

$$e_i = \sum_{k\in A_i^{+H}} P_k \cdot \overset{\approx}{V}_k \tag{1}$$

where P_k is the probability of selecting link k. The utility of node i is assumed to be a function of the exposure to the downstream NPU and the number of alternative links $|A_i^{+H}|$. For example, a possible definition of node utility could be:

$$V_i = \frac{\sum_{k\in A_i^{+H}} P_k \cdot \overset{\approx}{V}_k}{|A_i^{+H}|} \tag{2}$$

Thus, more attractive links would help to increase the node utility. Imagine a traveler facing only one link, with both the utility of NPU and the utility for the node being equal to -10. If there are two links with a utility of -10, with the choice probability of 0.5 for both links, then the utility for the node according to Eq. (2) becomes (-10 × 0.5 - 10× 0.5) / 2 = -5. This implies that even though the uncertainty exposure is the same, additional alternatives would reduce the risk of being limited to one choice.

Furthermore, both link utilities and node utilities are assumed to be additive in the network level so that a sum can be attained as the trip utility. Route choice then becomes the result of a sequential decision-making process in which link choices are made consecutively. In the decision process, a rational driver is supposed to choose the strategy that maximizes the utility achieved from his / her trip.

2.2. Two fundamental rules

Consider a trip utility represented by the sum of the deterministic link and node utilities, $\sum_{a\in A} p_a V_a + \sum_{i\in I} p_i V_i$, which is a generalization of that presented by Marcotte and Nguyen (1998) that considers link and node traversal cost (disutility). A trip-maker with a given origin and destination pair takes the optimal strategy that seeks the shortest hyperpath maximizing the trip utility. By imposing the Markov transition property from nodes to links such that $p_a = P_{a|i(a)} \cdot p_{i(a)}$, a basic mathematical programming model for the optimal strategy problem can be established as follows:

$$\max \sum_{a\in A} p_a V_a + \sum_{i\in I} p_i V_i$$
$$s.t. \begin{cases} \sum\limits_{a\in A_i^+} P_a - \sum\limits_{a\in A_i^-} P_a = \begin{cases} -1, & \text{if } i = s \\ 1, & \text{if } i = r \\ 0, & \text{others} \end{cases} \\ p_a = P_{a|i(a)} \cdot p_{i(a)}, \quad a \in A_i^+ \\ p_i = 1, \quad i \in \{r, s\} \end{cases} \tag{3}$$

The first constraint ensures the in-out dependency, the second represents the Markov link transition probability, and the third constraint represents the end point (origin and destination) constraints.

The solution to this problem can be reduced to two questions:

(1) Which links should be considered as alternatives in the optimal strategy?
(2) What are the optimal link choice probabilities? These two questions are answered by the following hyperpath rule (Section 2.3) and the GEV based N-GEV link choice rule (Section 2.4), respectively.

2.3. Hyperpath rule

The concept of hyperpath originates from hypergraph theory. In hypergraphs, a hyperarc is allowed to connect to more than two vertices, unlike standard graphs. In general graphs, a hyperarc can be deemed as an attractive link set at a node. Hyperpath is a sub-hypergraph connected by hyperarcs from the source to the target. There are possibly infinite hyperpaths in general transportation networks given an origin-destination pair. Supposing that there are many possible strategies to make out of all hyperpaths, we may naturally have the notion of the shortest hyperpath corresponding to the optimal strategy. Essentially a collection of paths (with many common links) as hyperpath is, it is also suggested to understand hyperpath as a potentially optimal link set in the context of route choices (Fonzone et al., 2012). The concept of hyperpath has been explicitly or implicitly applied in transportation fields, such as frequency-based transit assignment (Nguyen and Pallottino, 1988; Spiess and Florian, 1989), time-dependent least-expected time path problems (Miller-Hooks, 2001), bush-based traffic assignment (Dial 1971; Dial, 2006; Bar-Gera, 2002), and robust route guidance with uncertain networks (Bell, 2009; Bell et al., 2012).

In the route choice context, the hyperpath rule is sometimes used to determine the choice set, whether it is implicit or explicit. For example, Dial (1971) defined an intuitive rule to find efficient paths, in which the hyperpath is explicitly given; in this case, the link choice probability can be attained easily by accessing each link in the order of incremental weights. In contrast, for an optimal strategy, the hyperpath is interdependent with the specification of link choice probability such that the problem becomes more complicated.

Now we define the strategy utility from node i to the destination s based on the condition that link a is utilized. This can be represented by

$$V_{is|a} = V_a + V^*_{js} \tag{4}$$

Then the utility of the optimal strategy is then represented by the maximal strategy utility as

$$V_{is}^{*} = \max_{k \in A_i^+} V_{is|k} \tag{5}$$

Imagine an intermediate state of the strategy utility V'_{is} before reaching the optimum, if taking another link can improve the strategy utility, then the link should be considered attractive. In other words, the link is considered potentially optimal (Fonzone et al., 2012). This rule is called hyperpath rule and is formulated as

$$V_{is|a} = V_a + V_{js}^{*} > V'_{is} \Rightarrow a \in A_i^{+H} \tag{6}$$

Note that the hyperpath rule is an sufficient condition to determine the strategies that are potentially optimal instead of optimal strategy (shortest hyperpath). Optimal strategy hyperpath is actually a subset of hyperpath. According to Nguyen and Pallottino (1989), there are three characteristics of a hyperpath H:

(1) As a graph, H is acyclic.
(2) H has an unique origin and an unique destination.
(3) Every node $i \in I^H$ and every link $a \in A^H$ belong to at least one path in H.

2.4. N-GEV link choice rule

The N-GEV model for route choice was specified as JNG in Papola and Marzano (2013). Figure 2 shows an example of a JNG choice network representation for a route choice from Node 1 to Node 9 in a nine-node Manhattan network. The physical network (left) can be converted into the CNL-type tree network (right), by which the N-GEV-type link choice can be incorporated.

From this, the GEV-based link choice rule can be established. Given the JNG generating function G_{a_n} (Papola and Marzano, 2013) associated with link a_n, where n denotes the choice set level, the following behaviroral assumption regarding link choice must be satisfied:

$$P_{a_n|a_{n-1}} \propto \alpha_{a_{n-1},a_n} G_{a_n}^{\theta_{a_n}/\theta_{a_{n-1}}}, \forall a \in A_i^{+H} \tag{7}$$

where α_{a_{n-1},a_n} is the degree of overlapping of choice alternative a_{n-1} to a_n (a_{n-1} and a_n are in different nests) and should be non-negative. The JNG generating function is recursively given by

$$G_{a_n} = e^{V_{a_n}/\theta_{a_n}} \sum_{a_{n+1} \in A^{+H}} \alpha_{a_n,a_{n+1}} G_{a_{n+1}}^{\theta_{a_{n-1}}/\theta_{a_n}} \tag{8}$$

Note that $G_{a_n} = e^{V_{a_n}/\theta_{a_n}}$ for $A_{j(a_n)}^{+H} = \emptyset$. Thus, the link choice probability within A_i^{+H} can be written as

$$P_{a_n|i(a_n)} = P_{a_n|a_{n-1}} = \frac{\alpha_{a_{n-1},a_n} G_a^{\theta_{a_n}/\theta_{a_{n-1}}}}{\sum_{k \in A_{i(k_{n-1})}^{+H}} \alpha_{k_{n-1},k_n} G_k^{\theta_{k_n}/\theta_{k_{n-1}}}} \tag{9}$$

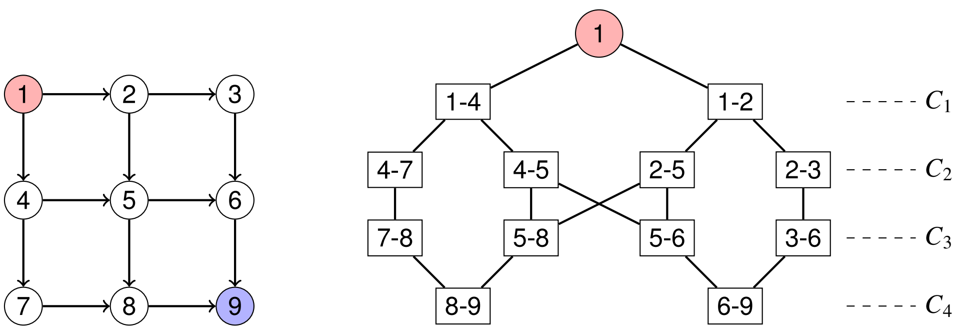

Fig. 2. Example of a joint network-generalized extreme value (JNG) route choice network (right) representation for a nine-node Manhattan network (left): route choice from Node 1 to Node 9 is discretized as four sequential levels of link choices (C_1 - C_4)

The path choice δ_{rs}, that is, the sequences of consecutive links from the origin node r to the destination node s, is the result of sequential link choices. As the GEV properties propagate across the network, according to the GEV-inheritance theorems (Daly and Bierlaire, 2006), the corresponding path choice probability can be written as

$$p_{\delta_{rs}} = p_r \cdot P_{a_1|r} \cdot P_{a_2|j(a_2)} \cdots P_{a_m|j(a_{m-1})} \cdot p_s = \prod_{a_n \in \delta_{rs}} P_{a_n|j(a_{n-1})} \tag{10}$$

Thus, for a given path δ_{rs}, as $j(a_{n-1}) = i(a_n)$, we have

$$p_{\delta_{rs}} = \prod_{a_n \in \delta_{rs}} P_{a_n|i(a_n)} = \prod_{a_n \in \delta_{rs}} \frac{\alpha_{a_{n-1},a_n} G_a^{\theta_{a_n}/\theta_{a_{n-1}}}}{\sum_{k \in A^{+H}_{i(k_{n-1})}} \alpha_{k_{n-1},k_n} G_k^{\theta_{k_n}/\theta_{k_{n-1}}}} \tag{11}$$

In this way, the N-GEV model allows the route choice problem to be converted into a sequential link choice problem while maintaining the GEV property. The hyperpath-based N-GEV constrains the choice set within the optimal strategy choice set (links of the shortest hyperpath) instead of infinite choice set (all links).

3. Solution Algorithm to the Hyperpath-based N-GEV Model

3.1. Generalized Bellman equation for optimal strategy

Now that we have the link choice rule, let us focus on the decision-making process at decision node i. The utility of a strategy for a Markov decision process (Puterman, 2005) can, in general, be represented by

$$V_{is}^h = \begin{cases} 0, & \text{if } i = s \\ V_i + \sum_{a \in A_i^+} P_{a|i(a)} \cdot (V_a + V_{js}^h), & \text{others} \end{cases} \tag{12}$$

where V_{is}^h represents the utility of a hyperpath h. In our case, strategic decision making happens only within the links involved in the hyperpath that are identified as attractive. Unattractive links is also allowed in case there is any better solution than strategic decision. The corresponding strategic hyperpath utility is given by:

$$V_{is}^h = \begin{cases} 0, & \text{if } i = s \\ V_i + \sum_{a \in A_i^{+H}} P_{a|i(a)} \cdot (V_a + V_{js}^h), & \text{if } i \neq s \text{ and } i \in I^H \\ V_a + V_{js}^h, & \text{others} \end{cases} \tag{13}$$

Consequently, the corresponding generalized Bellman equation for optimal strategy hyparpath h^* is given by

$$V_{is}^{h^*} = \begin{cases} 0, & \text{if } i = s \\ \max_{a \in A_i^{+H}} \left[V_i + \sum_{a \in A_i^{+H}} P_{a|i(a)}(V_a + V_{js}^{h^*}) \right], & \text{if } i \neq s \text{ and } i \in I^H \\ \max_{a \in A_i^+} (V_a + V_{js}^{h^*}), & \text{others} \end{cases} \tag{14}$$

3.2. The modified SF^{di} algorithm

A greedy-type algorithm, which resembles the Spiess-Florian algorithm with directed node search and admissive node potentials (SF$^{\text{di}}$) proposed by Ma et al. (2013), can be used to solve for the hyperpath, together with GEV-correspondent link-choice probabilities. In order to be in line with the original SF$^{\text{di}}$ algorithm, we let V denote the disutilities instead of utilities in Algorithm 1 and the algorithm actually seeks for a minimization of disutility, which is equivalent to solve the original problem that maximizes the utility. Additionally, if the strategy disutilities have a lower bound (denoted by $\underline{V_{is}^*}$), then the lower bound can be used as admissive node potentials to speed up the

Algorithm 1 SF$^{\mathrm{di}}$ algorithm with GEV link choices (A^o and A^c denote open and closed links. For simplicity, let $a \equiv a_n, \theta_i \equiv \theta_{a_{n-1}}, \theta_j \equiv \theta_{a_n}, \alpha_{i,j} \equiv \alpha_{a_{n-1},a_n}$)

■ **Initialization**
$V'_{is} \leftarrow \infty, \forall i \in I - \{s\};\ V'_{ss} \leftarrow 0;\ W_i \leftarrow 0, \forall i \in I; p_i \leftarrow 0, i \in I - \{r\};\ p_r \leftarrow 1;\ A_i^{+H} \leftarrow \emptyset$;
$j \leftarrow s, A^o \leftarrow \emptyset;\ p_a \leftarrow 0, P_{a|i(a)} \leftarrow 0, \forall a \in A$;
■ **Optimal strategy calculation (backward pass)**
while $V'_{js} + V_a > V'_{rs}$ **and** $A^o \neq \emptyset$ **do**
  for each link $a \in A_j^-$ **do**
    if $a \notin A^o$ **then**
      $A^o \leftarrow A^o + \{a\}$;
    end if
  end for
  $a \leftarrow \mathrm{argmin}_{a \in A^o - A^c}(V_a + V'_{js}) + \underline{V^*_{is}}$;
  $j \leftarrow i(a);\ A^c \leftarrow A^c + \{a\}; A^o \leftarrow A^o - \{a\};\ V^*_{js} \leftarrow V'_{js}$
  if $V'_{is} > V_a + V^*_{js}$ **and** $a \in A^o - A^c$ **then**
    $A_{i(a)}^{+H} \leftarrow A_{i(a)}^{+H} + a$; *// Hyperpath rule*
    if $j = s$ **then**
      $W_a \leftarrow \exp(V_a/\theta_i)$;
    else
      $W_a \leftarrow \exp(V_a/\theta_i) \cdot (\alpha_{i,j} W_{j(a)})^{\theta_j/\theta_i}$ *// GEV inheritance*
    end if
    $P_{a|i(a)} \leftarrow \frac{W_a}{W_i + W_a};\ \ W_i \leftarrow W_i + W_a$; *// Link choice rule*
    if $V'_{is} > \sum_{k \in A_i^{+H}} P_{k|i(k)} \cdot (V_k + V_{js}) + V_i$ **then**
      $V'_{is} \leftarrow \sum_{k \in A_i^{+H}} P_{k|i(k)} \cdot (V_k + V_{js}) + V_i$ *// Bellman Equation for attractive links*
    end if
  else
    $V'_{is} \leftarrow V_a + V_{js}$ *// Bellman Equation for unattractive links*
  end if
end while
■ **Proportion calculation (forward pass)**
for each $a \in A^H$ with the decreasing order of $V^*_{js} + V_a + \underline{V^*_{is}}$ **do**
  $p_a \leftarrow P_{a|i(a)} \cdot p_{i(a)}; p_{j(a)} \leftarrow p_{j(a)} + p_a$; *// One-step Markov transition*
end for

calculation. For example, by setting a physical possible maximum travel speeds, the least possible arrival time can be calculated with street distances. If the travel time is taken as the disutility, an admissive node potential is the least possible travel time which can be calculated by shortest path algorithms. The corresponding pseudo code is shown as Algorithm 1. The performance is the same as the original SF$^{\mathrm{di}}$ algorithm. The details are shown in Ma et al. (2013). Algorithm 1 illustrates the algorithm structure for the proposed framework leaving V_i and V_a to be specified according to application contexts and modelers. As an example, the specifications referred to in Eq. (2), Eq. (15) and Eq. (16) are used in the numerical studies, which can be considered as possible specifications for dealing with travel time uncertainties.

4. Numerical Studies

The model and algorithm described in Section 3 provide a general framework for modeling route choice under uncertainties with a GEV-based link-choice rule. Next, we explain the simple numerical results with the proposed model. Simple utility functions and parameter settings were used in the numerical studies. Because Papola and Marzano (2013) compared N-GEV with many other route choice models on a four-link small network, the numerical studies focused on the comparison of two hyperpath-based models: the inverse proportionality model (Bell, 2009) and the N-GEV model from this paper. More appropriate specification/selection of the form of link deterministic utility, the degree of membership, and homogeneity parameters are beyond the scope of this paper.

4.1. Specifications of utility functions

In Bell (2009), although the link choice rule is simply assumed inversely proportional to the potential maximum delay d_a (deemed as non-probabilistic disutility) and seeks the minimization of expected trip travel time. When making decisions at intersections, the risk aversion can be viewed as a Logit model with a special risk-averse utility $-\ln d_a$, which is a special case of Constant Relative Risk Aversion (CRRA) with the Arrow-Pratt index of risk aversion equals -1 (de Palma and Picard, 2005). In this sense, the Logit link choice results in $P_a \propto e^{-\ln d_a} = 1/d_a$.

Appropriate specification of risk aversion in the model obviously requires more discussion and is beyond this paper. In the numerical studies, we assume the travel time lower bound $t_{a,\min}$ and upper bound $t_{a,\max}$ of link a are known (e.g., 5 and 95 percentile travel times). The difference of the bounds corresponds to d_a. We considers travelers percept link travel times pessimistically and the link utility is slightly different from Bell (2009) and is set as:

$$V_a = -t_{a,\max} \tag{15}$$

Also, we simply set the utility of uncertainties as negative potential maximum delay such that

$$\overset{\approx}{V}_a = -(t_{a,\max} - t_{a,\min}) \tag{16}$$

As for V_i, we stick to Eq. (2) that is mentioned in Section 2.1.

4.2. Parameter settings for N-GEV model

As for the allocation parameters $\alpha_{a_n,a_{n-1}}$ and the error-scale parameters θ_{a_n} in the N-GEV model for route choices, Hara and Akamatsu (2012) used the following simple specification:

$$\alpha_{a_n,a_{n-1}} \equiv \alpha_i = \frac{1}{|E_i^-|} \tag{17}$$

$$\theta_{a_n} \equiv \theta_j = \frac{D_{j,s}}{D_{r,j}} \tag{18}$$

where $|E_i^-|$ denotes the number of links that belong to efficient paths connected to node i, and $D_{r,i}$ denotes the least distance between node r and node i.

In contrast, Papola and Marzano (2013) suggest the following specifications:

$$\theta_{a_n} \equiv \theta_j = \frac{\sqrt{6\gamma D_{j,s}}}{\pi} \tag{19}$$

$$\alpha_{a_n,a_{n-1}} \equiv \alpha_{i,j} = \frac{N_{r,i}/(D_{r,i} + D_{i,j})}{\sum_{k\in A_j^-} N_{r,i}/(D_{r,i(k)} + D_{i(k),j})} \tag{20}$$

where $N_{r,i}$ denotes the number of paths connecting i for the origin node r, and γ is the proportionality coefficient of variance of path utility U_{δ_k}, which is assumed to be proportional to the least distance of δ_k:

$$\sigma^2(U_{\delta_k}) = \gamma D_{r,s} \tag{21}$$

There is still no global agreement on the specification of N-GEV parameters and more empirical studies are required for the best parameter settings. Besides, unlike Dial's efficient paths, the hyperpath is not a prior so that it is not easy to set $\alpha_{i,j}$ in advance. How to set the N-GEV parameters is beyond this paper. Without incurring more complexities, in the numerical studies simple settings are used by following Papola and Marzano (2013).

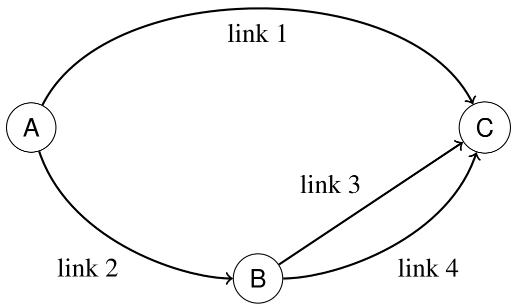

Fig. 3. A three-node toy network with travel time correlations

4.3. Toy network results

To illustrate the difference among models, experiments are conducted on a toy network with 3 nodes and 4 links (shown in Figure 3). There are 3 possible paths from node A to node C:

- path 1 (node A → link 1 → node C)
- path 2 (node A → link 2 → node B → link 3 → node C)
- path 3 (node A → link 2 → node B → link 4 → node C)

The test scenarios are created as follows: first we would like to see the results for a scenario that the possible paths are equally attractive (Table 1); then the potential maximum delay of link 4 is increased to double (Table 2); finally the potential maximum delay is increased to 6 times (Table 3) of that in the first scenario.

Table 1. Marginal probabilities of scenario I (texts in bold indicate the changing values)

$t_{\min}, t_{\max}$	link 1 10, 20	link 2 $10-k$, $20-2k$	link 3 k, $2k$	link 4 k, $\boldsymbol{2k}$	path 1 10, 20	path 2 10, 20	path 3 10, 20
			Scenario I (a): $\boldsymbol{k=2}$				
IP	0.4444	0.5556	0.2778	0.2778	0.4444	0.2778	0.2778
SL	0.3333	0.6667	0.3333	0.3333	0.3333	0.3333	0.3333
NG	0. 4231	0.5769	0.2884	0.2884	0.4231	0.2884	0.2884
			Scenario I (b): $\boldsymbol{k=8}$				
IP	0.1667	0.8333	0.4167	0.4167	0.1667	0.4167	0.4167
SL	0.3333	0.6667	0.3333	0.3333	0.6667	0.3333	0.3333
NG	0.3498	0.6502	0.3251	0.3251	0.645	0.1775	0.1775

A variable k is used to represent link travel times to introduce the link correlations. For each scenario, k is set to two different values to scale the correlations. The increase of k means less overlap between path 2 and 3 and thus less correlations. Three models with different link choice rules are analyzed: Inverse Proportionality (IP), Sequential Logit (SL) and Network GEV (NG). For NG model parameters, we stick to the settings recommended by Papola and Mazarno (2013) in Eq. (19) and Eq. (20).

Comparing scenario I (Table 1) with scenario II (Table 2), IP is obviously more sensitive to the changes in $t_{\max}$ than SL and NG. On the contrary, benefited from the discrete choice model, SL and NG are less sensitive. The inheritance property of SL and NG, which enable drivers to make far-sighted decision, also accounts for the sensitivity. In other words, IP drivers would definitely try to avoid links with large potential delay while SL and NG drivers would think about the traffic situations. If to take a link with large potential delay may actually results in less travel time for remaining links, SL and NG drivers would feel comfortable to consider the link attractive.

Similar to Dials approach (Dial, 1971), SL results in the Logit route choice results. At the same time, since the utility function also considers the potential maximum delays (Eq. (15)), SL can also react on the changes in

Table 2. Marginal probabilities of scenario II

$t_{\min}, t_{\max}$	link 1 10, 20	link 2 $10-k$, $20-2k$	link 3 k, $2k$	link 4 k, $\mathbf{3k}$	path 1 10, 20	path 2 10, 20	path 3 10, $20+k$
			Scenario II (a): $\boldsymbol{k = 2}$				
IP	0.4444	0.5556	0.3704	0.1852	0.4444	0.3704	0.1852
SL	0.4683	0.5317	0.4683	0.0634	0.4683	0.4683	0.0634
NG	0.4960	0.5040	0.4863	0.0177	0.5040	0.4863	0.0177
			Scenario II (b): $\boldsymbol{k = 8}$				
IP	0.1667	0.8333	0.5556	0.2778	0.1667	0.5556	0.2778
SL	0.4999	0.5001	0.4999	0.0002	0.4999	0.4999	0.0002
NG	0.4997	0.5003	0.4996	0.0007	0.4997	0.4996	0.0007

Table 3. Marginal probabilities of scenario III

$t_{\min}, t_{\max}$	link 1 10, 20	link 2 $10-k$, $20-2k$	link 3 k, $2k$	link 4 k, $\mathbf{7k}$	path 1 10, 20	path 2 10, 20	path 3 10, $20+5k$
			Scenario III (a): $\boldsymbol{k = 2}$				
IP	0.4444	0.5556	0.4762	0.0794	0.5556	0.4762	0.0794
SL	0.5000	0.5000	0.5000	0	0.5000	0.5000	0
NG	0.5000	0.5000	0.5000	0	0.5000	0.5000	0
			Scenario III (b): $\boldsymbol{k = 8}$				
IP	0.1667	0.8333	0.7143	0.1190	0.8333	0.7143	0.1190
SL	0.5000	0.5000	0.5000	0	0.5000	0.5000	0
NG	0. 5000	0.5000	0.5000	0	0.5000	0.5000	0

uncertainties. In scenario I (Table 1) when each route is evenly attractive in both minimum and maximum travel times, SL results in equal proportions for each route. In scenario III (Table 3), due to the increase of uncertainties on link 4, SL only sets the equal proportion to paths 1 and 2 since path 3 turns out to be unattractive.

All scenarios show NG penalizes correlated routes. As the increase of k, the choice probabilities of paths 2 and 3 tends to decrease. Compared with Dial-based N-GEV approach, according to the network uncertainties, the choice set may be different. For example, as the increase of the delay to as large as 6 times of the minimum travel time on link 4 (Table 3), it becomes no more attractive.

4.4. Small-grid network results

Numerical tests comparing the inverse proportionality hyperpath (Bell, 2009) with the N-GEV hyperpath were conducted on the same network used by Bell (2009)[1]. We used the same origin/destination (OD) pair of Bell's test scenario, that is, the travel from Node 1 to Node 37. For simplicity, the experiments on Bell's network did not take link correlations into account and thus we focused on sequential logit as a special case of N-GEV such that $\alpha_{i,j} = 1, \theta_i = \theta_j = 1, \forall i, j \in I$.

Figure 4 shows that the N-GEV (specified as sequential logit) route choice tended to split upstream evenly because the link travel times, as well as the remaining travel times, were similar. In contrast, the inverse proportionality model resulted in much more preferences for links with lower potential maximum delay.

With the original input of Bell's network, the hyperpaths for both models remained the same. This was also the case after increasing the potential maximum delay of the link from 0.868 to 1.868 (Figure 5). However, the inverse proportionality model reacted as expected, with the probability decreasing from 0.0952 to 0.0598 for the changed link, whereas the N-GEV route choice proportions remained the same (equal to 0.2315). This is due to the characteristic differences between these two models; the inverse proportionality model makes a shortsighted behavior assumption, whereas the N-GEV route choice takes the remaining routes into account.

[1] The test network data (ESRI shape files) can be downloaded at the author's website (*https://sites.google.com/site/tonny2vblog/home/data.tar.gz*).

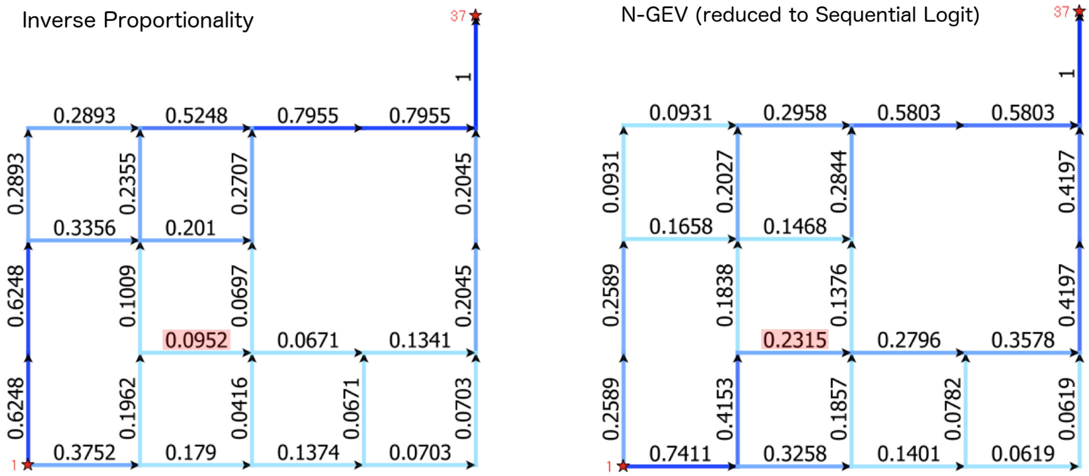

Fig. 4. Hyperpath results with original input from Bell (2009), the numbers are marginal link choice probabilities.

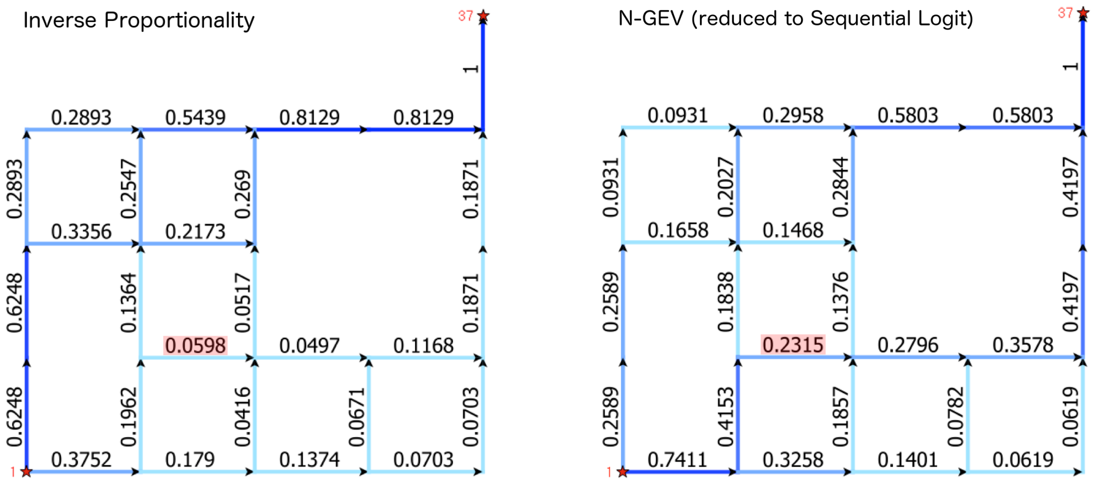

Fig. 5. Hyperpath results after increasing d_a of a link (id = 9) in the shaded area from 0.868 to 1.868

In Figure 6, by increasing the potential maximum delay to 2.868, the link choice probability inverse proportionality model still provided the same hyperpath and decreased the choice probability of the changed link. In contrast, the hyperpath of the N-GEV route choice changed and avoided using the changed link. It turns out that the inverse proportionality model follows the risk-averse choice rule, allowing additional alternatives to be considered. Also, it shows that the hyperpaths for the two models are not necessarily the same.

To compare the accuracy of the path probabilities, we changed seven links while keeping both travel time and potential maximum delay the same. Figure 7 shows the results from the inverse proportionality model and the N-GEV model. Because the links in the rectangular area have the same properties, the routes shown in Figure 8 should have the same choice probability. For the inverse proportionality model, the route choice possibilities include the following:

$$p_1 = 0.6248 \times 0.3124/(0.3124 + 0.3124) = 0.3124$$
$$p_2 = 0.6248 \times 0.3124/(0.3124 + 0.3124) \times 0.2325/(0.2325 + 0.2325) \times 0.1024/(0.1024 + 0.1301) \approx 0.0688$$
$$p_3 = 0.6248 \times 0.3124/(0.3124 + 0.3124) \times 0.2325/(0.2325 + 0.2325) = 0.1562$$

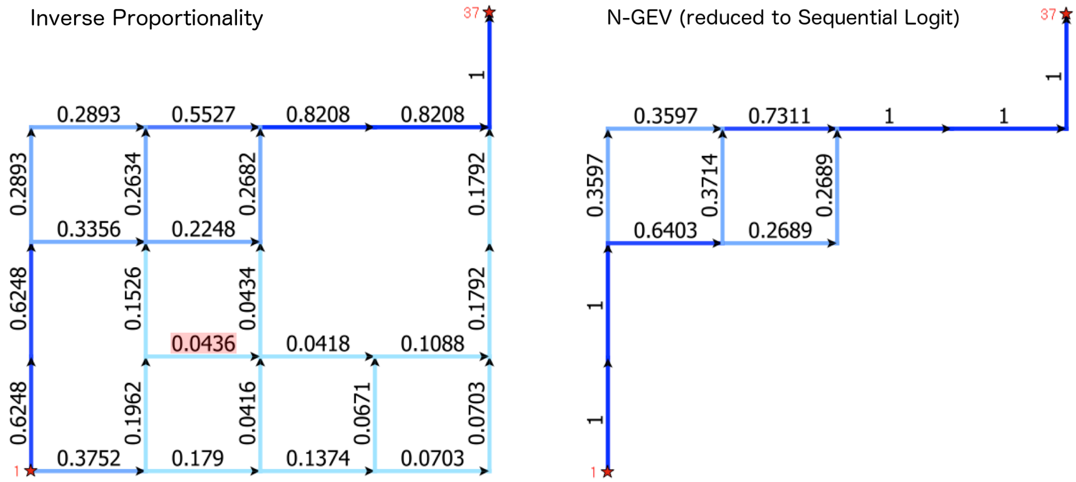

Fig. 6. Hyperpath results after increasing d_a of the link (id=9) in the shaded area from 1.868 to 2.868

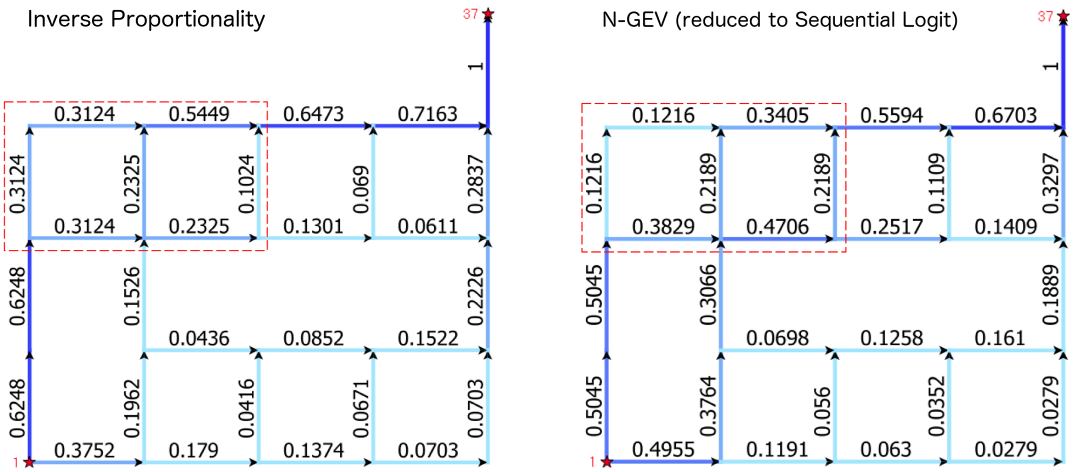

Fig. 7. Hyperpath results with further changes in a set of links within the red rectangle (id = 15, 16, 22, 23, 73, 74, 75) with $t_a = 1$, $d_a = 1$

For the N-GEV model, the route choice possibilities are given as

$$p'_1 = 0.5045 \times 0.1216/(0.1216 + 0.3829) \approx 0.1216$$
$$p'_2 = 0.5045 \times 0.3829/(0.1216 + 0.3829) \times 0.4706/(0.2189 + 0.4706) \times 0.2189/(0.2189 + 0.2517) \approx 0.1216$$
$$p'_3 = 0.5045 \times 0.3829/(0.1216 + 0.3829) \times 0.2189/(0.2189 + 0.4706) \approx 0.1216$$

As reflected in this simple numerical example, the N-GEV model is preferable to the inverse proportionality model in capturing the choice possibilities correctly.

5. Conclusions and Future Research Directions

By integrating the hyperpath and N-GEV methodologies, this paper presents a general initial framework based on previous studies for modeling route choice behavior under uncertainties and demonstrates the advantages of the proposed approach over the inverse proportionality hyperpath in producing more reasonable link choice possibilities

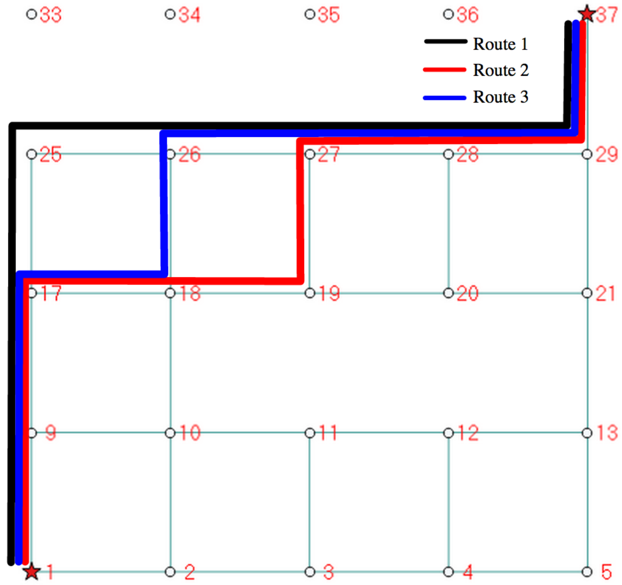

Fig. 8. Three alternative routes

as well as can capture link correlations. In addition to providing insight into dealing with route choices with uncertainties with hyperpath approaches, the added benefit of using the N-GEV framework is that the model can deal with randomness by way of utility maximization. The hyperpath takes uncertainty exposure into account in dealing with unperceived uncertainties beyond randomness. In specific contexts that have been developed, the uncertainty may result from different sources (e.g., uncertain wait time for transit lines or uncertain potential maximum delay on roads), and is likely to develop according to the severity of the conditions considered, for example, those involving hazardous material transportation, transportation against terrorists' attack, and evacuation transportation after disasters.

Although integration of the hyperpath and N-GEV is promising, a considerable amount of work remains. Firstly, the parameter settings should be investigated further; specifically, it is important to clarify how the degree of membership for a non-predetermined hyperpath is determined efficiently in large-scale networks. Iterative search initializing with sequential logit would be helpful, given that the hyperpath and the degree of membership should stabilize eventually. Secondly, the current model framework does not consider the reinforcement learning mechanism for individuals, which can be developed to a greater extent. Finally, empirical studies are necessary. Many unanswered questions remain; however, the route choice data collected by global positioning systems would be helpful with regard to parameter estimations and model adjustments.

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