論文 / 著書情報
Article / Book Information

Title	2-D Variable FIR Filters Using 3-D Prototype Filters
Authors	Toshiyuki YOSHIDA, AKINORI NISHIHARA, N. Fujii
出典 / Citation	IEICE Trans. Fundamentals, Vol. E77, No. 9, pp. 1568-1572
発行日 / Pub. date	1994, 9
URL	http://search.ieice.org/
権利情報 / Copyright	本著作物の著作権は電子情報通信学会に帰属します。 Copyright (c) 1994 Institute of Electronics, Information and Communication Engineers.

LETTER

2-D Variable FIR Filters Using 3-D Prototype Filters

Toshiyuki YOSHIDA[†], **Akinori NISHIHARA**[†] *and* **Nobuo FUJII**[†], *Members*

SUMMARY This paper discusses a new design method for 2-D variable FIR digital filters, which is an extension of our previous work for 1-D case. The method uses a 3-D prototype FIR filter whose cross-sections correspond to the desired characteristics of 2-D variable FIR filters. A 2-D variable-angle FIR fan filter is given as a design example.

key words: *digital signal processing, multidimensional signal processing, linear FIR digital filters and variable filters*

1. Introduction

Variable digital filters (VDFs) are often required in two-dimensional (2-D) signal processing as well as one-dimensional (1-D) counterpart, some of whose applications are discussed in Refs. [1] and [2]. A number of approaches have been proposed for 1-D VDFs [1]–[6], but only a few for 2-D VDFs [2], [7], [8]. These approaches are mainly classified into two categories.

One of them changes a part of filter parameters by using, for example, a suitable transformation [3]–[5], [8]. This provides easy variability, but often involves some problems as discussed in the introduction of Ref. [6]. The other approach changes all the filter coefficients such that the desired filter characteristics can be obtained [1], [2], [6]. In general, this approach realizes better characteristics than the first one. However, the band edge frequencies and the maximum deviation in the passband and stopband cannot be controlled or prescribed in the design procedures as pointed out again in Ref. [6].

The authors have proposed a novel design method for 1-D FIR VDFs using 2-D prototype filters, which is based on the latter approach [6]. The method makes it possible to completely control the band edge frequencies and the maximum deviations. This paper extends the above design method to 2-D case and discusses the performance of an example designed by the method.

Section 2 explains the proposed design method. In Sect. 3, a 2-D variable-angle FIR fan filter will be designed as an example, and its performance will be compared with that of other designs. Finally, Sect. 4 concludes this paper.

2. Design of 2-D Variable FIR Filters

2.1 Principle of the Design Method

Consider, for example, the 3-D octantally-symmetric zero-phase conical FIR filter shown in Fig. 1. Note from Fig. 1 that the cross section at $\omega_3 = 2\pi k$ corresponds to 2-D circularly-symmetric lowpass characteristics. Therefore, by changing the intersection $\omega_3 = 2\pi k$ in the range $0 \leqq k \leqq 0.5$, the diameter of the passband can be varied. The design of 2-D FIR VDFs here is based on this idea.

Now assume that the 3-D prototype filter is a $(2N_1+1)\times(2N_2+1)\times(2N_3+1)$-tap octantally-symmetric zero-phase FIR filter. Its transfer function is written as

$$\begin{aligned} H(z_1,z_2,z_3) = & \sum_{n_1=-N_1}^{N_1} \sum_{n_2=-N_2}^{N_2} \sum_{n_3=-N_3}^{N_3} \\ & h(n_1,n_2,n_3)\, z_1^{-n_1} z_2^{-n_2} z_3^{-n_3}\,, \qquad (1) \\ h(n_1,n_2,n_3) = & \; h(-n_1,-n_2,-n_3) \\ & (0 \leqq n_1 \leqq N_1, 0 \leqq n_2 \leqq N_2, 0 \leqq n_3 \leqq N_3)\,, \qquad (2) \end{aligned}$$

where $h(n_1,n_2,n_3)$ represents the impulse response of the filter.

The cross-sectional characteristics $H(\omega_1,\omega_2)$ for $\omega_3 = 2\pi k$ can be calculated as

$$H(\omega_1,\omega_2) = g(0,0) + 2\sum_{n_1=1}^{N_1} +g(n_1,0)\cos(n_1\omega_1)$$

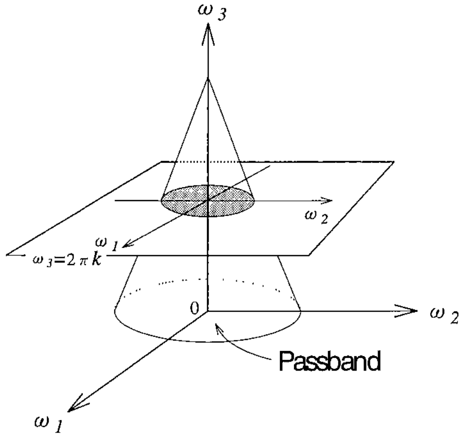


Fig. 1 3-D prototype filter and 2-D variable FIR filter.

Manuscript received December 17, 1993.
Manuscript revised May 9, 1994.

[†] The authors are with the Faculty of Engineering, Tokyo Institute of Technology, Tokyo, 152 Japan.

$$+2\sum_{n_2=1}^{N_2} g(0,n_2)\cos(n_2\omega_2)$$
$$+4\sum_{n_1=1}^{N_1}\sum_{n_2=1}^{N_2} g(n_1,n_2)\cos(n_1\omega_1)\cos(n_2\omega_2)\,, \quad (3)$$

where

$$g(i,j) = h(i,j,0) + 2\sum_{n_3=1}^{N_3} h(i,j,n_3)\cos(2\pi n_3 k)\,. \quad (4)$$

Equation (3) indicates that the filter characteristics can be varied by calculating all the independent impulse response coefficients $g(i,j)$ according to Eq. (4) for the given parameter k. Thus, by suitably designing 3-D prototype FIR filters, 2-D FIR VDFs can be easily obtained. It should be noted that the linear-phase (causal) FIR filter, if required for real-time operation, can be easily obtained by suitably shifting the impulse response in Eq. (1).

Since the calculations of Eq. (3) involve no approximation, the maximum amplitude deviations and the transition band widths of the resulting 2-D FIR VDF never exceed the corresponding values of the 3-D prototype. Therefore, all of these values can be controlled precisely at the design procedures of the 3-D prototype.

2.2 Updating the Filter Characteristics and the Required Cost

Equation (4) can be rewritten as

$$g(i,j) = h(i,j,0) + \sum_{n_3=1}^{N_3} 2h(i,j,n_3) \times T_{n_3}(K)\,, \quad (5)$$

where K is defined as $K = \cos(2\pi k)$ and $T_n(x)$ represents the n-th order Chebyshev polynomial of the first kind. Equation (5) simplifies the recalculation of the independent impulse response coefficients as shown in Ref. [6]. The total amount of computations in a single update requires

$$N_{Mul} = (N_1+1)(N_2+1)N_3 + 2(N_3-1) \quad (6)$$

multiplications and

$$N_{Add} = (N_1+1)(N_2+1)N_3 + (N_3-1) \quad (7)$$

additions at most.

The direct form realization of $(2N_1+1)\times(2N_2+1)$-tap 2-D quadrantally-symmetric FIR filter requires $(N_1+1)(N_2+1)$ multiplications and about $(2N_1+1)(2N_2+1)$ additions for the calculation of a single output point. Therefore, it is concluded that in updating the filter characteristics, the proposed 2-D VDF requires approximately N_3 times the calculation cost for a single output point.

2.3 Design of 3-D Prototype FIR Filters

The next problem is how to design the 3-D prototype FIR filter. It is assumed that the lengths N_1 and N_2, and the shape of the passband and stopband $PB^2(k)$ and $SB^2(k)$, respectively, are given as the specifications for the required 2-D variable FIR filter.

First the passband PB^3 and the stopband SB^3 for the 3-D prototype are determined as

$$PB^3 = \bigcup_k \Big\{(\omega_1,\omega_2,\omega_3)\,|\,\omega_3 = 2\pi k,$$
$$(\omega_1,\omega_2) \in PB^2(k),\ 0 \leqq \omega_1,\omega_2,\omega_3 \leqq \pi\Big\}, \quad (8)$$
$$SB^3 = \bigcup_k \Big\{(\omega_1,\omega_2,\omega_3)\,|\,\omega_3 = 2\pi k,$$
$$(\omega_1,\omega_2) \in SB^2(k),\ 0 \leqq \omega_1,\omega_2,\omega_3 \leqq \pi\Big\}. \quad (9)$$

The filter lengths N_1 and N_2 are determined from the given specifications of the 2-D variable FIR filter. N_3 can be set arbitrary, but should be chosen carefully because there is a certain tradeoff between the calculation cost of Eq. (5) and the characteristics of the resulting 2-D VDFs.

Having been determined the specifications of the 3-D prototype FIR filter, it should be designed by an excellent method because the designed 3-D prototype directly affects the performance of the resulting 2-D VDF. Since the linear programming approach [9] yields optimal 3-D FIR filters, it is most suitable for such an application.

3. Design Example and Comparisons

3.1 Design Example – A Variable-Angle FIR Fan Filter –

The filter designed as an example here is the variable-angle FIR fan filter shown in Fig. 2, where the passband angle θ can be varied in the range $\theta_1 \geqq \theta \geqq \theta_2$ while the transition width $\Delta\omega$ is kept constant.

From Fig. 2, the passband $PB^2(k)$ is given as a function of k by

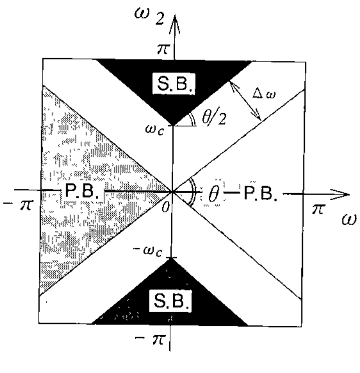
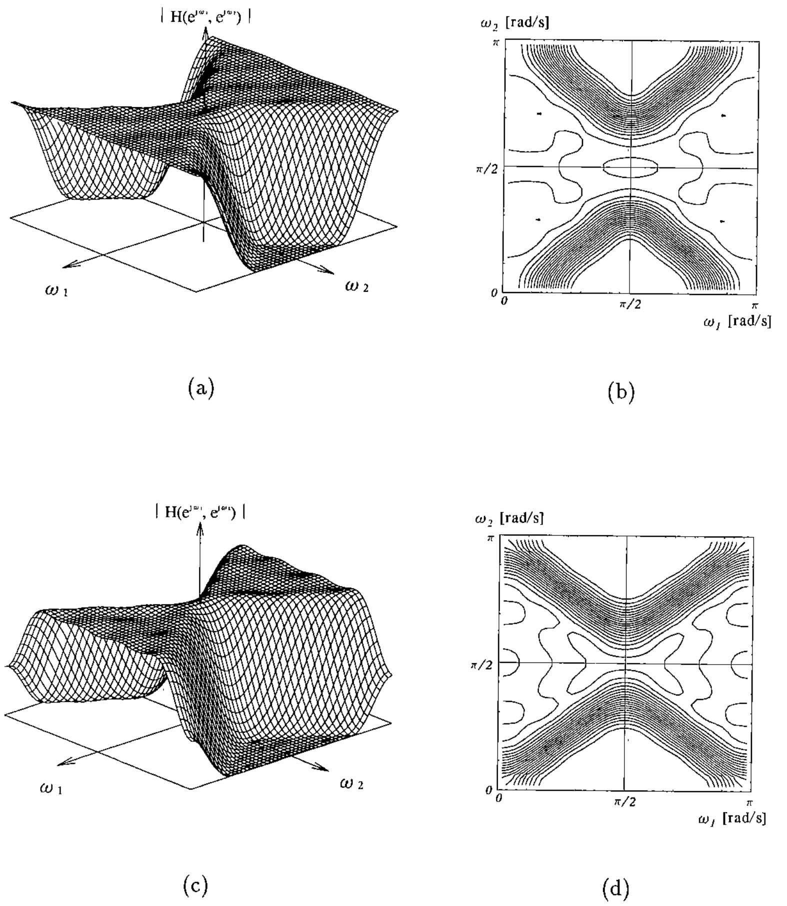


Fig. 2 Specifications of the variable fan filter.



(a) (b)

(c) (d)

Fig. 3 Frequency response of the designed variable fan filters. (a) Perspective plot for $k = 0.15$ ($\theta = 82.3°$). (b) Contour plot for $k = 0.15$ ($\theta = 82.3°$). (c) Perspective plot for $k = 0.40$ ($\theta = 67.0°$). (d) Contour plot for $k = 0.40$ ($\theta = 67.0°$).

$$PB^2(k) = \Big\{(\omega_1,\omega_2)\,|\,\omega_2 \leqq a(k)\,\omega_1, \quad 0 \leqq \omega_1,\omega_2 \leqq \pi\Big\}, \qquad (10)$$

where $a(k)$ determines the passband angle θ, which should satisfy $a(0) = \tan\theta_1/2$ and $a(0.5) = \tan\theta_2/2$. In this example, the linear function

$$a(k) = \tan\frac{\theta_1}{2} - 2\left(\tan\frac{\theta_1}{2} - \tan\frac{\theta_2}{2}\right) k \qquad (11)$$

is used for $a(k)$, which leads to

$$\theta(k) = 2\tan^{-1}\left(\tan\frac{\theta_1}{2} - 2\left(\tan\frac{\theta_1}{2} - \tan\frac{\theta_2}{2}\right) k\right). \qquad (12)$$

Similarly, the stopband SB^2 is given as

$$SB^2(k) = \Big\{(\omega_1,\omega_2)\,|\,(\omega_2 - \omega_c) \leqq a(k)\,\omega_1, \quad 0 \leqq \omega_1,\omega_2 \leqq \pi\Big\}, \qquad (13)$$

where ω_c represents the stopband edge on the axis ω_2 in Fig. 2. In order to keep the transition width $\Delta\omega$ constant, ω_c have to be altered as

$$\omega_c = \Delta\omega\sqrt{1 + a^2(k)} \qquad (14)$$

in accordance with the change of the parameter k.

The actual specifications for the variable-angle fan filter are as follows;

Filter length	:	$9 \times 9 \quad (N_1 = N_2 = 4)$
Transition width	:	$\Delta\omega = 2\pi \times 0.24$ [rad/s]
Range of θ	:	$90° \geqq \theta \geqq 60°$
Max. stopband dev.	:	$\geqq$ 40dB

First, the 3-D prototype FIR filter with $N_3 = 4$ ($9\times9\times9$ taps) is designed by using Eqs. (11)–(14). The linear programming technique[9] is employed as discussed in Sect. 2.3, which results in the filter with the maximum passband and stopband deviations 0.0141 and 0.00996, respectively. Then, by calculating the filter coefficients by Eq. (5) for the given k, the design procedure completes. Figure 3 shows the frequency response of the designed filter for $k = 0.15$ ($\theta = 82.3°$) and $k = 0.40$ ($\theta = 67.0°$).

Table 1 Comparison of the variable fan filters.

θ [deg]		Proposed	Method 1	Method 2
82.3	Passband Error	0.0141	0.0142	0.0675
	Stopband Error	0.00996	0.0152	0.0100
67.0	Passband Error	0.0141	0.0277	0.0665
	Stopband Error	0.00996	0.0401	0.0100

3.2 Comparison with Other Methods

In order to compare our method with others, two methods, which are referred to as Methods 1 and 2 in this section, are employed for the design of the 9 × 9-tap variable FIR fan filter with the same specifications as in Sect. 3.1.

Method 1 is a general design method for 2-D VDFs proposed by Zarour and Fahmy [2]. The method begins with designing a set of 2-D filters reasonably spaced in the given range. Then, by using a curve fitting technique, the filter coefficients are expressed by a set of functions obtained from the designed filters. Equation (5) with $N_3 = 4$ is employed for the curve fitting technique, whose recalculation cost is the same as that of the proposed method. This requires five sets of 2-D FIR fan filters with suitable θ as the sampled points for Eq. (5), which are again designed by the linear programming technique. Table 1 summarizes the resulting maximum deviations in the passband and the stopband for $\theta = 82.3°$ and $67.0°$.

On the other hand, Method 2 is specific to 2-D variable fan filters, which is based on the McClellan transformation [8]. This method changes the passband angle θ by recalculating the filter coefficients of both the transformation filter and its 1-D prototype. Although, for real-time variation, the 1-D prototype should be designed by a 1-D VDF technique, here, the Remez exchange algorithm is employed for simplicity. Table 1 again summarizes the maximum deviations for $\theta = 82.3°$ and $67.0°$. Note, however, that in this case the stopband of the designed filter is smaller than that of Fig. 2 because the proposed McClellan transformation heavily distorts the equiamplitude lines around $\Omega = \pi$ (see Fig. 3 of Ref. [8]).

Compared with Methods 1 and 2, the proposed design achieves the lowest deviations. Method 1 gives relatively good results, but the maximum deviations for the given θ cannot be known prior to the change of θ because, in this case, the frequency response is approximated indirectly through the curve fitting of the filter coefficients. The maximum deviations of the proposed method is readily known from that of the 3-D prototype, which is the most outstanding advantage as compared with the other two method.

On the other hand, in the design procedure, the proposed method requires hundreds or thousands times of calculation cost as compared with the other two methods owing maily to the design of 3-D prototype filters. For example, the design of the 9 × 9 × 9 prototype filter takes roughly an hour of calculations on a vectorized processor. From the practical viewpoint, however, this problem is not so serious because a single effort of design is sufficient in order to satisfy the given specifications of a required filter.

4. Conclusion

This paper has described a design method for 2-D variable FIR filters. The method has the advantages that the band edge frequencies and the maximum deviations are completely controllable at the design procedures and that by designing suitable prototype filters, various types of variable filters can be designed, e.g., 2-D variable-direction fan filters. The variable-angle fan filter designed as an example shows that the proposed method realizes the best characteristics. On the other hand, our method requires relatively large amount of recalculation cost for the filter coefficients compared with the McClellan transformation method. Therefore, it is concluded that the proposed method is well suitable for the application where the change of the filter characteristics is required only occasionally.

Acknowledgement

The authors would like to thank Prof. S. Takagi of Tokyo Institute of Technology for his helpful discussions. They are also grateful to Mr. K. Shinsako of NEC Corporation for his support in computer programming while in the same institute.

References

[1] Jarske, P., Neuvo, Y. and Mitra, S.K., "A simple approach to the design of linear phase FIR digital filters with variable characteristics," *Signal Processing*, vol.14, no.4, pp.313–326, Jun. 1988.

[2] Zarour, R. and Fahmy, M.M.,"A design technique for variable two-dimensional recursive digital filters," *Signal Processing*, vol.17, pp.175–182, 1989.

[3] Schüessler, W. and Winkelnkemper, W., "Variable digital filters," *Arch. Electr. Übertr.*, vol.24, pp.524–525, 1970.

[4] Oppenheim, A.V., Mecklenbräuker, W.F.G. and Mersereau, R.M., "Variable cutoff linear phase digital filters," *IEEE Trans. Circuits Syst.*, vol.CAS–23, no.4, pp.199–203, Apr. 1976.

[5] Mitra, S.K., Neuvo, Y. and Roivainen, H., "Design

of recursive digital filters with variable characteristics," *Int. J. Cir. Theory and Appl.*, vol.18, no.2, pp.107–119, Mar.–Apr. 1990.

[6] Yoshida, T., Nishihara, A. and Fujii, N., "A design method of variable FIR filters using multi-dimensional filters," *IEICE Trans. Fundamentals*, vol.E75-A, no.8, Aug. 1992.

[7] Pendergrass, N.A., Mitra, S.K. and Jury, E.I., "Spectral transformations for two-dimensional digital filters," *IEEE Trans. Circuits Syst.*, vol.23, no.1, pp.26–35, Jan. 1976.

[8] Hasegawa, M., Sekiguchi, T. and Takahashi, S., "2D FIR fan filters with variable passband support based on spectral transformation," *IEICE Technical Report*, CAS89-158, Mar. 1990.

[9] Higuchi, T., Ohki, M. and Kawamata, M., "Optimal design of three-dimensional FIR digital filters — Design method by linear programming and reduction of computations by exploiting symmetries," *Trans. IEICE*, vol.J70-A, no.7, pp.1042–1050, Jul. 1988.