論文 / 著書情報
Article / Book Information

Title	A Design Method of Variable FIR Filters using Multi-Dimensional Fulters
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Citation	IEICE Transactions on Fundamentals., Vol. E75, No. 8, pp. 964-971
Pub. date	1992, 8
URL	http://search.ieice.org/
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IEICE TRANS. FUNDAMENTALS, VOL. E75-A, NO. 8 AUGUST 1992

PAPER *Special Section on the 6th Digital Signal Processing Symposium*

A Design Method of Variable FIR Filters Using Multi-Dimensional Filters

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SUMMARY This paper proposes a new design method of variable FIR digital filters. The method uses a multi-dimensional linearphase FIR filter as a prototype. The principle of the proposed method is based on the fact that the cross-sectional characteristics of a 2-D filter along with a line vary if the intersection of this line is changed. The filter characteristics can be varied by recalculating all the filter coefficients from proposed equations, which leads to an advantage that the variable range is very wide. Another advantage is that the passband and stopband deviations are completely predetermined in the design procedures and that the passband edge can be accurately settled to a desired frequency while keeping the transition band width unchanged. First the proposed design method is explained and the effect of the transition band of 2-D filters is discussed. Then the calculation cost required in updating the filter coefficients are considered. Finally two design examples are presented and the proposed method is compared with the existing one, which shows the usefulness of our method.

key words: *digital signal processing, FIR filters, variable filters, multi-dimensional filters*

1. Introduction

In many applications of digital signal processing, it is often desired to vary the filter characteristics during its operation. Several approaches have been proposed to realize such variable digital filters[(1)–(13)], which are mainly classified into two categories.

One possible approach is to change only a few parameters in the filter to get the desired filter characteristics, which is well suitable for hardware implementation. A typical realization of this approach is to replace each of the delay elements in the filter, i.e. z^{-1}, with an all-pass subnetwork whose characteristics can be varied by tuning a single parameter[(1),(2)]. This method, however, involves some problems that delay-free loops are introduced when applied to direct-form IIR filters and that the linear-phase property is lost when applied to FIR filters[(3),(4)]. Some modifications make it possible to realize variable IIR filters[(5)–(8)] while variable FIR filters having linear-phase property cannot be realized with this method. Oppenheim et al. have proposed an alternative transformation for FIR filters which preserves the linear-phase property[(3)] and further investigations have been made with this method[(9)–(12)]. Their method, however, increases the transition bandwidth of the variable FIR filter, which results in a severe restriction of the range over which the characteristics can be varied[(9)].

As suggested in Ref. (3), an alternative approach is to change all of the filter parameters in such a way that the desired characteristics can be obtained. Reference (3) also points out that this approach requires the ability to vary a number of parameters and that filter coefficients are generally a complicated function of the filter cutoff frequency, which makes this approach unpractical in hardware implementation. Yet this approach will be very useful when the variable filter is implemented by software on a Digital Signal Processor (DSP) and the coefficients are represented by a simple function. Based on such a background, Jarske et al. have proposed a simple function representing the coefficients of FIR filters and applied this function to variable FIR filters[(13)]. Their method, however, is lacking in the theoretical background because their function basically depends upon the experimental results obtained by designing a great number of FIR filters. Furthermore the passband and stopband edges cannot be set to the desired frequency and the deviations in the both bands cannot be predicted in the design procedures.

In this paper we propose a new design method of variable FIR filters employing the latter approach, which uses a multidimensional filter as a prototype. The method have the following useful advantages.

1. The wide-range variation is possible.
2. The band edge frequencies and the deviations are thoroughly controlled in the design procedure.
3. The method is applicable to 2-D variable FIR filters.

In Sect. 2, the principle of the proposed method will be discussed and the detailed design method will be explained. In Sect. 3, two design examples will be shown and the results will be compared with Jarske's method. Section 4 will give concluding remarks.

2. Design Method of Variable FIR Filters

2. 1 Principle of the Proposed Method

The principle of the proposed method is quite

Manuscript received December 25, 1991.
Manuscript revised April 3, 1992.

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simple. Consider, for example, a quadrantally symmetric 2-D filters with a hexagonal passband as shown in Fig. 1. The cross-sectional amplitude response along with a line $\omega_2=2\pi k$ exhibits lowpass characteristics, which suggests that by varying the parameter $k=k_1, k_2, k_3, \cdots$, a variable 1-D lowpass filter can be obtained.

Now assume that the filter shown in Fig. 1 is a 2-D quadrantally symmetric FIR filter with $N_1\times N_2$ taps (both N_1 and N_2 are odd numbers). Since the following discussion is easily applied to an even-tap FIR filter by a slight modification, we concentrate our discussion on a 2-D FIR filter with odd taps here. The transfer function of such a filter is written as

$$H(z_1, z_2)=\sum_{n_1=-(N_1-1)/2}^{(N_1-1)/2}\sum_{n_2=-(N_2-1)/2}^{(N_2-1)/2} h(n_1, n_2) z_1^{-n_1} z_2^{-n_2} \tag{1}$$

$$h(i,j)=h(-i,j)=h(i,-j)=h(-i,-j), \tag{2}$$

where $h(i,j)$ denotes the impulse response of the filter[14]. By setting $z_1=e^{j\omega_1}$ and $z_2=e^{j\omega_2}$, the frequency response of the filter is calculated as

$$\begin{aligned}H(e^{j\omega_1}, e^{j\omega_2}) &= h(0,0) \\ &+2\sum_{n_1=1}^{(N_1-1)/2} h(n_1,0)\cos(n_1\omega_1) \\ &+2\sum_{n_2=1}^{(N_2-1)/2} h(0,n_2)\cos(n_2\omega_2) \\ &+4\sum_{n_1=1}^{(N_1-1)/2}\sum_{n_2=1}^{(N_2-1)/2} h(n_1,n_2) \\ &\cdot\cos(n_1\omega_1)\cos(n_2\omega_2).\end{aligned} \tag{3}$$

The cross-sectional frequency response $H(e^{j\omega})$ along with $\omega_2=2\pi k$ is obtained by substituting $\omega_1=\omega$ and $\omega_2=2\pi k$, which leads to

$$\begin{aligned}H(e^{j\omega}) &=\left\{h(0,0)+2\sum_{n_2=1}^{(N_2-1)/2} h(0,n_2)\cos(2\pi n_2 k)\right\} \\ &+2\sum_{n_1=1}^{(N_1-1)/2}\left\{h(n_1,0)+2\sum_{n_2=1}^{(N_2-1)/2} h(n_1,n_2)\right. \\ &\left.\cdot\cos(2\pi n_2 k)\right\}\cos(n_1\omega).\end{aligned} \tag{4}$$

Let

$$g(i)\equiv h(i,0)+2\sum_{j=1}^{(N_2-1)/2} h(i,j)\cos(2\pi jk)$$
$$\left(i=-\frac{N_1-1}{2}\sim\frac{N_1-1}{2}\right), \tag{5}$$

then it is found that Eq. (4) represents the frequency response of a 1-D linear-phase FIR filter whose impulse response is given by Eq. (5). Therefore a variable 1-D linear-phase FIR filter can be obtained by the following procedures. First a 2-D hexagonal FIR filter is designed in such a way that the variable 1-D FIR filter derived from it would satisfy the prescribed specifications, and the designed impulse response are stored. Then the impulse response coefficients of the variable 1-D FIR filter are calculated from the stored 2-D impulse response for the given parameter k by using Eq. (5).

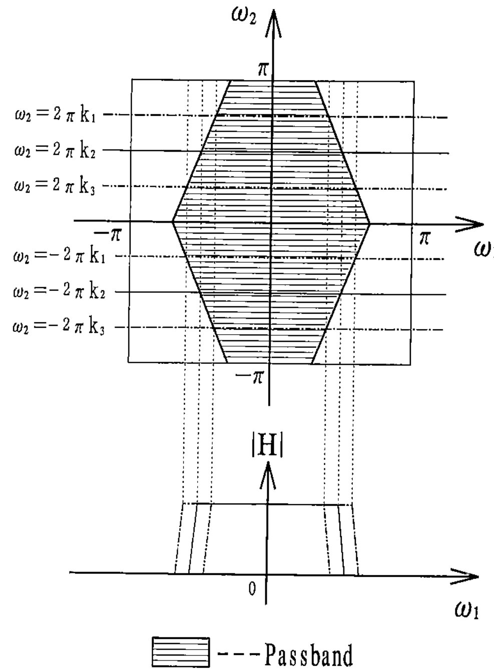

Fig. 1 Principle of the proposed method.

2.2 Considerations on the Actual Prototype 2-D FIR Filters

A transition band always exists in the actual FIR filter whose support of the impulse response is finite.

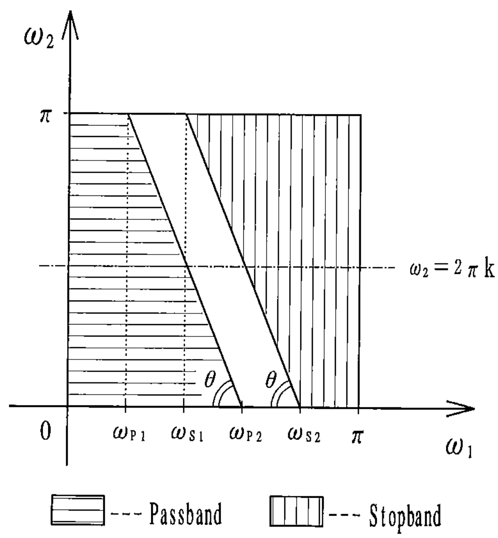

Fig. 2 Actual 2-D filters.

Here we consider the design of the prototype 2-D FIR filter which satisfies the given specification of the resultant variable 1-D FIR filter. The effect of the transition band of the prototype 2-D filter is also considered. Figure 2 illustrates an actual filter characteristics for the filter in Fig. 1, in which only the first quadrant is shown. Assume that the passband and stopband satisfies

$$\text{passband}: |\omega_1| \tan\theta + |\omega_2| \leqq \omega_{P2} \tan\theta \qquad (6)$$

$$\text{stopband}: |\omega_1| \tan\theta + |\omega_2| \geqq \omega_{S2} \tan\theta \qquad (7)$$

$$\theta = \tan^{-1}\left(\frac{\pi}{\omega_{P2}-\omega_{P1}}\right) = \tan^{-1}\left(\frac{\pi}{\omega_{S2}-\omega_{S1}}\right). \qquad (8)$$

The variation of k from $k=0$ through $k=0.5$ changes the passband edge ω_P and stopband edge ω_S from ω_{P1} through ω_{P2} and from ω_{S1} through ω_{S2}, respectively, in which the transition band width never be altered because Eq.(8) is always satisfied. Since the passband and stopband edges can be varied linearly with respect to the parameter k, a selection of

$$k = \frac{(\omega_{P2}-\omega_P)\tan\theta}{2\pi} \qquad (9)$$

construct a 1-D FIR filter whose passband edge lies on ω_P accurately.

Conversely, consider the design of the prototype 2-D FIR filter when the variable range $[\omega_{P1}, \omega_{P2}]$, the transition band width $\Delta\omega$, and the passband and stopband deviations are given as the specification for the 1-D variable filter. First, the angle θ in Fig. 2 is determined from

$$\theta = \tan^{-1}\left(\frac{\pi}{\omega_{P2}-\omega_{P1}}\right) \qquad (10)$$

and the stop band edges ω_{S1}, ω_{S2} are calculated as

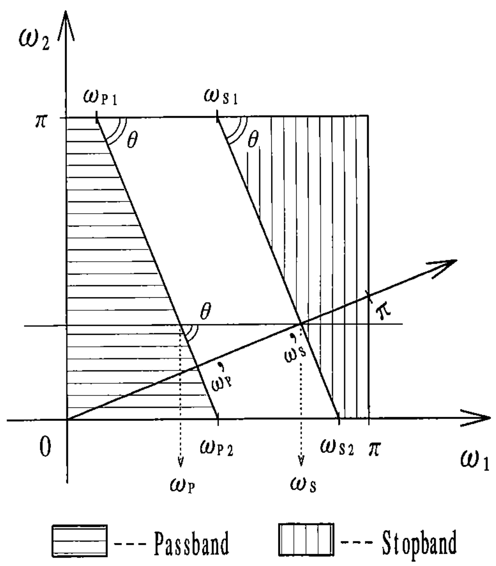

Fig. 3 Transition band of 2-D filters.

$$\omega_{Si} = \omega_{Pi} + \Delta\omega \quad (i=1, 2). \qquad (11)$$

Then the prototype 2-D filter is designed which satisfies the given deviations and Eqs.(10) and (11). It is easily understood that the maximum deviations of the variable 1-D filter never exceed those of the corresponding prototype 2-D filter.

The 1-D variable FIR filter designed by the above procedure is compared with the optimal FIR filter satisfying the same specification to derive a simple relation between the variable range and the filter length. Consider an optimal prototype 2-D filter with $N_1 \times N_2$ tap ($N_1 \geqq N_2$) shown in Fig. 3, from which an N_1-tap variable 1-D FIR filter is derived. In order to simplify the following discussion, we now assume that the steepest cross-sectional characteristics shown in Fig. 3 is roughly equivalent to the characteristics of the N_1-tap optimal linear-phase FIR filter. Under such an assumption, the filter length of the variable 1-D FIR filter N_1 is compared with that of the optimal FIR filter N_{opt}. Since the filter length of a linear-phase FIR filter is nearly inversely proportional to the transition width[15], a relation

$$\frac{N_1}{N_{\text{opt}}} \cong \frac{\Delta\omega}{\Delta\omega'} = \frac{1}{\sin\theta} \qquad (12)$$

holds for the transition width of the variable 1-D FIR filter $\Delta\omega = \omega_S - \omega_P$ and that of the steepest cross-sectional characteristics $\Delta\omega' = \omega'_S - \omega'_P$. A substitution of Eq.(8) into Eq.(12) leads to

$$\frac{N_1}{N_{\text{opt}}} \cong \sqrt{1+\left(\frac{\omega_{P2}-\omega_{P1}}{\pi}\right)^2}, \qquad (13)$$

which represents an important property that the filter length increases as the wider variable range is required. Note that the case $\omega_{P2}-\omega_{P1}=\pi$ makes the ratio N_1/N_{opt} maximum, i.e. $N_1/N_{\text{opt}} = \sqrt{2}$.

2.3 Calculation Cost for Updating the filter Coefficients

As shown in Sect. 2, the filter characteristics can be varied by calculating all the impulse response coefficients from Eq.(5). Equation (5), however, is a rather complicated function because the values of $\cos(2\pi kn)$ should be calculated. Using

$$K = \cos(2\pi k) \qquad (14)$$

and j-th order Chebyshev polynomial, Eq.(5) reduces to

$$g(i) = h(i, 0) + 2\sum_{j=1}^{(N_2-1)/2} h(i, j)\, T_j(K), \qquad (15)$$

which is quite a simple expression. It is sufficient to calculate the impulse response coefficients $g(i)$ only for $0 \leqq i \leqq (N_1-1)/2$ because the impulse response is symmetric with respect to $i=0$. To calculate the coefficients, first the values of $T_n(K)$ $(1 \leqq n \leqq (N_2-1)/2$ are calculated to store and then the coefficients $g(i)$ are obtained from Eq.(15) using the stored values. With the recurrence formula for the Chebyshev polynomials

$$T_0(x)=1 \tag{16}$$

$$T_1(x)=x \tag{17}$$

$$T_n(x)=2xT_{n-1}(x)-T_{n-2}(x), \tag{18}$$

$T_n(K)$ $(1 \leqq n \leqq (N_2-1)/2)$ can be calculated recursively, which requires at most (N_2-3) additions and $(N_2-3)/2$ multiplications. On the other hand the calculations of Eq.(15) for $0 \leqq i \leqq (N_1-1)/2$ require $(N_1+1)(N_2-1)/4$ additions and $(N_1+1)(N_2-1)/4$ multiplications. Consequently the total amount of the calculation in a single update is

$$N_{\mathrm{Mul}}=\frac{N_1N_2-N_1+5N_2-13}{4} \tag{19}$$

multiplications and

$$N_{\mathrm{Add}}=\frac{N_1N_2-N_1+3N_2-7}{4} \tag{20}$$

additions at most. Equations (19) and (20) assume that in Eq.(14) the value K is given instead of the parameter k, otherwise Eq.(14) should be calculated by an adequate method.

3. Design Examples and Comparison with Jarske's Method

In this section two 1-D variable FIR filters are designed to show the ability of the proposed method. Then the designed filters are compared with a variable filter proposed by Jarske et al.

3.1 Example 1

A diamond-shaped filter belongs to hexagonal filters as a special case that $\theta=45°$. A diamond-shaped prototype 2-D filter makes it possible to vary the cutoff frequency of the corresponding 1-D variable filter in the full range of the baseband. Another advantage of such a filter is the symmetry with respect to not only $\omega_1=\omega_2=0$ but also $\omega_1=\pm\omega_2$, which simplifies the design of the prototype 2-D filter. In Example 1, a 1-D variable filter is designed from a diamond-shaped prototype filter with the following specifications.

Specifications :

Passband $|\omega_1|+|\omega_2| \leqq 0.50 \times 2\pi$[rad/s]

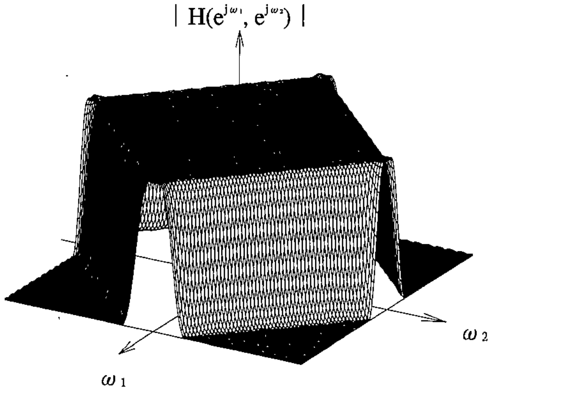

Fig. 4 Perspective plot of 27×27 diamond-shaped FIR filter used as a prototype filter in Example 1.

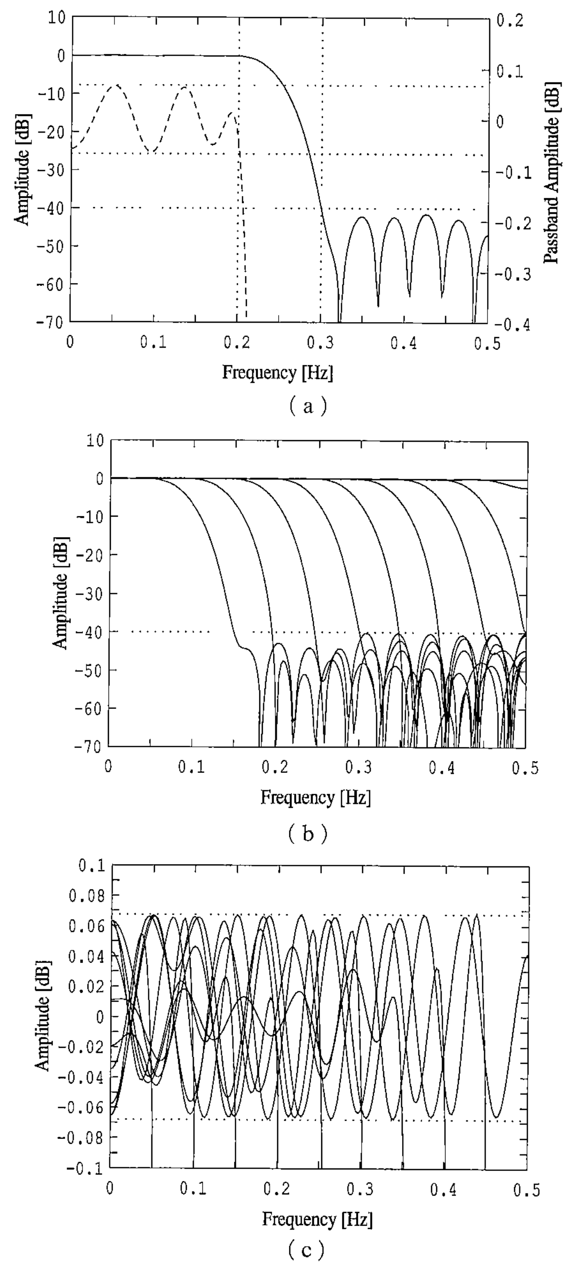

Fig. 5 Frequency response of the variable 1-D FIR filter in Example 1. (a) Frequency response for $k=0.3$ ($f_p=0.2$ [Hz], $f_s=0.3$ [Hz]). (b) Frequency response for k varying from 0.0 to 0.45 in steps of 0.05. (c) Passband characteristics of (b).

Table 1 Comparison between the variable FIR filter in Example 1 and the optimal FIR filter.

	Number of Taps	Passband Deviation	Stopband Deviation
Variable FIR Filter in Example 1	27	0.0078	0.01
Optimal FIR Filter	21	0.0128	0.01
	22	0.0056	0.01

Stopband $|\omega_1|+|\omega_2| \geqq 0.60 \times 2\pi$[rad/s]
Length 27×27 [tap]
Stopband Attenuation 40 [dB]

Among numbers of excellent design methods for 2-D linear-phase FIR filters[14], a linear programming method[16] is used to design the prototype 2-D filter. Figure 4 shows the perspective plot of the designed 2-D prototype filter. The maximum deviations in the passband and stopband are 0.0078 and 0.0100, respectively. The corresponding 1-D variable filter constructed from Fig. 4 is a 27-tap linear-phase FIR filter with the transition band width 0.10 Hz. Figure 5 gives the frequency response of the variable 1-D FIR filter, where (a) shows the response for $k=0.3$ ($f_p=0.2$ [Hz], $f_s=0.3$ [Hz]), (b) shows the responses for k varying from 0.0 to 0.45 in steps of 0.05, and (c) shows the passband characteristics of (b). The dashed lines in Fig. 5 represent the maximum passband and stopband deviations of the prototype 2-D filter. It should be noted that the variable filter never exceeds the maximum deviations of the prototype filter.

The variable range of this filter is very wide as shown before, while Eq. (13) suggests that about $\sqrt{2}$ times taps are required compared with the optimal FIR filter which realizes the similar characteristics. Table 1 summarizes the performance of the proposed variable filter and that of the optimal FIR filter[17], where the passband and stopband edges are kept at 0.2 Hz and 0.3 Hz, respectively. Table 1 shows that 22-tap optimal FIR filter is comparable with the proposed 27-tap variable FIR filter while from Eq. (13) the corresponding filter length is calculated as $27/\sqrt{2} \cong 19.1$, from which the usefulness of Eq. (13) is verified to some extent.

From Eqs. (19) and (20), 220 multiplications and 194 additions are required in updating the filter coefficients, which are approximately equivalent to 8 times of the calculations required during a 27-tap direct-form FIR filter operates.

3.2 Example 2

It is observed from Eq. (13) that the restriction of the variable range leads to the better filter characteristics. In Example 2 a hexagonal shaped filter is used as a prototype filter with the following specifications.

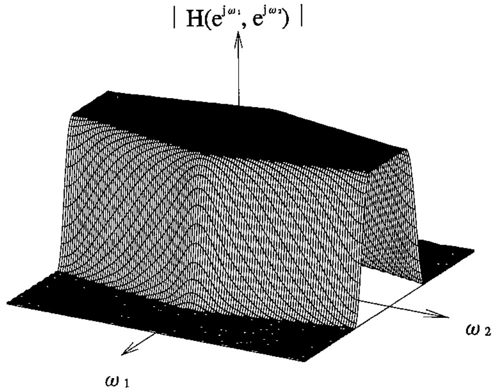

Fig. 6 Perspective plot of 27×17 hexagonal shaped FIR filter used as a prototype filter in Example 2.

Specifications:
Passband $5|\omega_1|+|\omega_2| \leqq 1.00 \times 2\pi$ [rad/s]
Stopband $5|\omega_1|+|\omega_2| \geqq 1.50 \times 2\pi$ [rad/s]
Length 27×17 [tap]
Stopband Attenuation 40 [dB]

Figure 6 shows the perspective plot of the prototype filter designed also by the linear programming method, where the passband and stopband deviations are 0.00194 and 0.0100, respectively. Figure 7 gives the frequency response of the variable 1-D FIR filter, where (a) shows the response for $k=0.0$ ($f_p=0.2$ [Hz], $f_s=0.3$ [Hz]), (b) shows the responses for k varying from 0.0 to 0.50 in steps of 0.10, and (c) shows the passband characteristics of (b). The passband characteristics is far better than that of the filter in Example 1.

The comparison with the optimal FIR filter is again summarized in Table 2, where the passband and stopband edges are 0.20 Hz and 0.30 Hz, respectively. This filter requires 140 multiplications and 119 additions to update the filter coefficients, which are much less than those of the filter in Example 1.

3.3 Comparison with Jarske's Method

Jarske et al. have derived a simple function which represents the filter coefficients $h(n)$ with respect to the 6 dB cutoff frequency ω_c[13]. Their function is written as

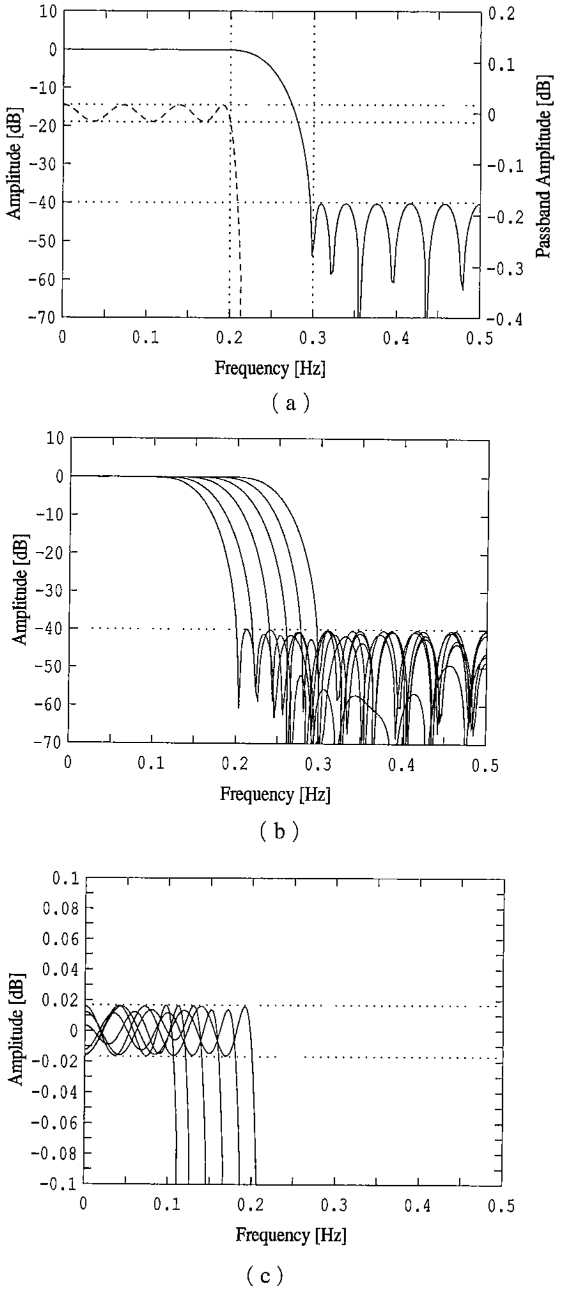

Fig. 7 Frequency response of the variable 1-D FIR filter in Example 2. (a) Frequency response for $k=0.0$ ($f_p=0.2$ [Hz], $f_s=0.3$ [Hz]). (b) Frequency response for k varying from 0.0 to 0.50 in steps of 0.10. (c) Passband characteristics of (b).

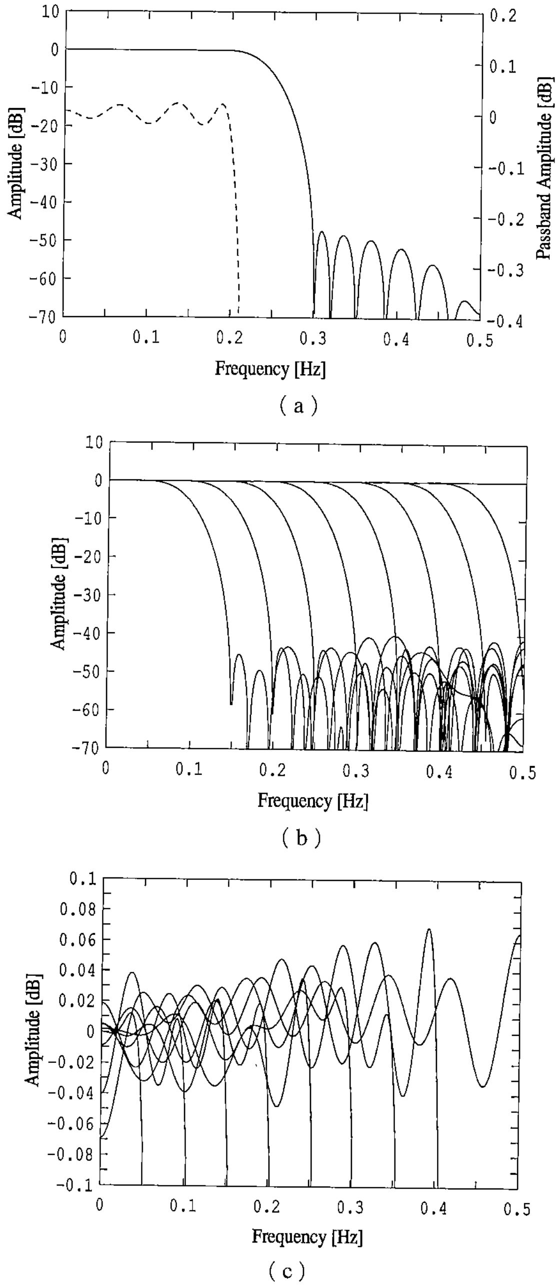

Fig. 8 Frequency response of the variable 1-D FIR filter by Jarske's method (a) Frequency response characterized by $f_c=0.25$ [Hz] (b) Frequency response for f_c varying from 0.10 to 0.50 in steps of 0.05. (c) Passband characteristics of (b).

Table 2 Comparison between the variable FIR filter in Example 2 and the optimal FIR filter.

	Number of Taps	Passband Deviation	Stopband Deviation
Variable FIR Filter in Example 2	27	0.00194	0.01
Optimal FIR Filter	24	0.00428	0.01
	25	0.00168	0.01

$$h_0(n) = \begin{cases} c(n)\,\omega_c + d(n) & \text{for } n=0 \\ c(n)\sin(\omega_c n) + d(n)\cos(\omega_c n) & \\ \quad \text{for } 1 \leqq |n| \leqq N, & \end{cases} \tag{21}$$

where the filter length is $2N+1$. $c(n)$, $d(n)$ $(1 \leqq |n| \leqq N)$ in Eq. (21) are constants determined from the coefficients of two optimal lowpass filters having different 6 dB cutoff frequencies. Using Eq. (21) a variable cutoff lowpass filter of length 27 is designed so as to obtain the frequency response similar to that of the filters in Example 1 and 2. Note that the filter is designed by trial and error to satisfy 40 dB attenuation and 0.1 Hz transition width because neither the stopband attenuation nor transition width are controlled by their method. Figure 8 shows the frequency response of the designed filter, where (a) is characterized by the 6 dB cutoff frequency $f_c=0.25$ [Hz], (b) shows the response for f_c varying from 0.1 Hz to 0.5 Hz in steps of 0.05 Hz, and (c) is the passband characteristics of (b).

Comparing Fig. 8 with Figs. 5 and 7, the following are observed. When a very wide variable range is required, the performance of these two methods are comparable. In such a case, the advantage of our method is found in the complete controllability of the band edge frequencies and the deviations in both bands. When the required variable range is rather narrow as in Example 2, the advantage of our method is trivial. It is, however, considered that the calculation cost of our method is higher than that of their method. The above discussion shows that our method is suitable for applications where the update of the filter coefficients is required only occasionally and the precise control of the filter characteristics is indispensable.

4. Concluding Remarks

In this paper we have proposed a new design method of variable FIR filters which uses a multidimensional FIR filter as a prototype. The proposed method has several advantages that the filter characteristics can be varied over a very wide range and that the passband and stopband deviations as well as the band edge frequencies can be completely predetermined in the design procedures. On the other hand the method involves some problems that the necessity of a wide-range variation increases the filter length required to satisfy the specification and that the amount of the calculation required in updating the filter coefficients is slightly large. Two variable filters have been presented as design examples and they are compared with the variable filter proposed by Jarske et al., to point out the features of our method.

In this paper discussed a construction of variable lowpass filters while the other types of filters, such as HPF and BPF, can be obtained by designing the corresponding 2-D prototype filters. Furthermore, this method is applicable to 2-D variable filters, in which 2-D variable filters are derived from 3-D prototype filters. The design of such filters is left for future work.

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