**PAPER** *Special Section on High-Speed Analog Circuits and Signal Processing*

# Design and Implementation of High-Speed and High-$Q$ Active Bandpass Filters with Reduced Sensitivity to Integrator Nonideality

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**SUMMARY** An integrator is quite a suitable active element for high-speed filters. The effect of its excess phase shifts, however, is severe in the case of high-$Q$ filter realization. The deterioration due to the excess phase shifts cannot be avoided when only integrators are used as frequency-dependent elements like in leapfrog realization. This paper describes a design of second-order high-speed and high-$Q$ filters with low sensitivity to excess phase shifts of integrators by adding a passive $RC$ circuit. The proposed method can drastically reduce the effect due to the undesirable pole of an integrator, which is the cause of the excess phase shifts, compared to conventional filters using only integrators. As an example, a fourth-order bandpass filter with 5-MHz center frequency and $Q=25$ is implemented by the proposed method on a monolithic chip. The results obtained here show quite good agreement with the theoretical values. This demonstrates effectiveness of the proposed method and feasibility of high-speed and high-$Q$ filters on a monolithic chip.

***key words:*** *active filters, integrators, pole sensitivity*

## 1. Introduction

Recently, the integration of high-speed and high-$Q$ active $RC$ filters becomes quite important, because they have many applications, such as mobile communication, radio communication, substitution of SAW filters used in TV sets and so on. Implementations of high-speed filters often use integrators which have decreasing gain with the increase of frequencies. This feature makes the structure of integrators simple and their frequency characteristics excellent.(1) Although an integrator is suitable as an active element for the realization of high-speed filters, its excess phase shifts deteriorate filter performance, especially in the case of high-$Q$ filter realization. This deterioration cannot be avoided when only integrators are used as frequency-dependent elements like in leapfrog realization.(2)

This paper describes a design of second-order high-speed and high-$Q$ filters with low sensitivity to excess phase shifts of integrators by adding a passive $RC$ circuit. The effect due to the excess phase shifts is evaluated on the basis of the pole sensitivity, which is not a function of a frequency. This paper will present the theoretical minimum of the pole sensitivity in the case of infinite-$Q$ filters and propose a design of a high-$Q$ filter having the prescribed pole sensitivity more than the theoretical minimum. As an example, a fourth-order bandpass filter with 5-MHz center frequency and $Q=25$ is implemented by the proposed method on a monolithic chip. The proposed filter is compared with the leapfrog filter, which is a typical integrator-based realization, by SPICE simulation.

## 2. Preliminaries

### 2.1 Frequency Characteristics of Integrators

The transfer function $G_0(s)$ of an ideal integrator is given by

$$G_0(s)=\frac{\Omega_0}{s} \tag{1}$$

where $s$ and $\Omega_0$ are the complex angular frequency and the unity-gain angular frequency, respectively. The transfer function $G(s)$ of an actual integrator deviates from the above equation because of an excess phase shift. In the following discussions, the excess phase shift is modeled using an additional pole of the transfer function, i.e.,

$$G(s)=\frac{\Omega_0}{s(1+s/\omega_C)}. \tag{2}$$

For convenience $\omega_C$ is called the pole of $G(s)$, which will introduce no confusion.

### 2.2 Definition of Pole Sensitivity

The transfer function $T(s)$ of a filter usually has

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many poles. One of them is designated as $p_0$. The pole sensitivity $S_x^{p_0}$ with respect to an element value $x$ is defined as Ref. (3)

$$S_x^{p_0}=x\frac{\partial p_0}{\partial x}. \tag{3}$$

Since $T(s)$ is a bilinear function of $x$, the denominator $D(s)$ of $T(s)$ can be expressed by

$$D(s)=A(s)+xB(s), \tag{4}$$

where $A(s)$ and $B(s)$ are polynomials of $s$. By using these polynomials $A(s)$ and $B(s)$, Eq. (3) can be rewritten in the form

$$S_x^{p_0}=-x\frac{B(p_0)}{\left.\frac{\partial D(s)}{\partial s}\right|_{s=p_0}}=\frac{A(p_0)}{\left.\frac{\partial D(s)}{\partial s}\right|_{s=p_0}}. \tag{5}$$

The absolute value of the pole sensitivity shows how far the pole moves owing to the deviation of $x$ from its nominal value. If the absolute value of the pole sensitivity is small enough, the errors of the frequency characteristics from the designed ones become negligible.

### 2. 3 Transfer Functions of Filters Using One Integrator

A schematic filter structure using one integrator and a three-terminal passive RC circuit is illustrated in Fig. 1. In the case of a zeroth- or first-order RC circuit, this structure cannot realize arbitrary second-order transfer functions, even though the effect of $\omega_C$ is taken into account, as explained in Appendix. Thus, a circuit having a second-order transfer function is used for the passive RC part in this paper.

The transfer functions $T_D(s)$ from the node 2 to the node 3 with the node 1 grounded and $T_N(s)$ from the node 1 to the node 3 with the node 2 grounded are generally given by

$$T_D(s)=\frac{as^2+b(\sigma_1+\sigma_2)s+c\sigma_1\sigma_2}{(s+\sigma_1)(s+\sigma_2)} \tag{6}$$

and

$$T_N(s)=\frac{N(s)}{(s+\sigma_1)(s+\sigma_2)}, \tag{7}$$

where $N(s)$ is a second-order polynomial of $s$, and $\sigma_i$ $(i=1, 2)$ represents a real pole of the passive $RC$ circuit, and $a$, $b$ and $c$ are constants.

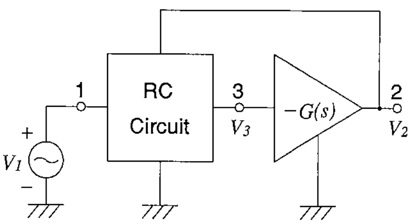


Fig. 1 Schematic filter structure.

For simplicity the following new variables

$$x=\frac{\sigma_1+\sigma_2}{2} \tag{8}$$

$$y=\sqrt{\sigma_1\sigma_2} \tag{9}$$

are introduced, where $x$ is always greater than or equal to $y$. $x$ becomes equal to $y$ if and only if $\sigma_1$ is equal to $\sigma_2$. $T_D(s)$ can be rewritten in the form

$$T_D(s)=\frac{as^2+2bxs+cy^2}{s^2+2xs+y^2}. \tag{10}$$

Practically important are third-order passive circuits having degenerated second-order transfer functions, because they can realize higher-$Q$ transfer functions than second-order circuits. When a third-order circuit is used, the $Q$ of the RC circuit becomes

$$Q_{RC}=\frac{y}{2x}\leqq\frac{1}{2}. \tag{11}$$

$Q_{RC}$ explicitly gives the necessary condition for $T_D(s)$ to be a transfer function of a third-order passive $RC$ circuit as

$$0\leqq a\leqq 1 \tag{12}$$

$$-Q_{RC}\leqq b\leqq Q_{RC}+1 \tag{13}$$

$$0\leqq c\leqq 1. \tag{14}$$

On the other hand, the overall transfer function $T(s)$ from the node 1 to the node 2 in Fig. 1 can be expressed in terms of $T_D(s)$, $T_N(s)$ and $G(s)$ as

$$T(s)=-\frac{G(s)T_N(s)}{1+G(s)T_D(s)} \tag{15}$$

$$=-\frac{\Omega_0\omega_C N(s)}{D(s)}, \tag{16}$$

where the input impedance and output impedance of the integrator are assumed to be infinite and zero, respectively, and $D(s)$ is

$$D(s)=s(s+\omega_C)(s^2+2xs+y^2)+\Omega_0\omega_C(as^2+2bxs+cy^2). \tag{17}$$

## 3. Minimization of the Pole Sensitivity with Respect to $\omega_C$

### 3. 1 Approximation

Since the transfer function $T(s)$ is fourth order, it can be rewritten in the form

$$T(s)=H_0\frac{N(s)}{D_0(s)\cdot D_p(s)}, \tag{18}$$

where $H_0$ is the gain factor of the filter shown in Fig.

1 and $D_0(s)$ and $D_p(s)$ are

$$D_0(s)=s^2+2k_0s+\omega_0^2 \tag{19}$$

and

$$D_p(s)=s^2+2k_ps+\omega_p^2. \tag{20}$$

A simple way to approximate $T(s)$ to the desired frequency characteristics of a second-order filter as closely as possible is the design only using one of $D_0(s)$ and $D_p(s)$ as a denominator of the transfer function while reducing the effect of the other as much as possible. For example, if $\omega_p$ is

$$\omega_p \gg \omega_0, \tag{21}$$

the effect of $D_p(s)$ becomes negligible at least in the passband. In the following discussion, a second-order filter will be designed so that the relationship between $\omega_p$ and $\omega_0$ satisfies Eq.(21).

For simplicity $\omega_0$ is normalized to 1[rad/sec] and the comparison of Eq.(17) and Eqs.(19) and (20) gives

$$2k_0+2k_p=2x+\omega_C, \tag{22}$$

$$1+\omega_p^2+4k_0k_p=\omega_C(2x+\Omega_0 a)+y^2, \tag{23}$$

$$2k_0\omega_p^2+2k_p=\omega_C(2x\Omega_0 b+y^2), \tag{24}$$

and

$$\omega_p^2=\Omega_0\omega_C c y^2. \tag{25}$$

To simplify the above equations further, the desired $Q_0$ is assumed to be infinite, i.e.,

$$k_0=\frac{1}{2Q_0}=0. \tag{26}$$

Although this assumption is relevant only in the case of realization of a high-$Q$ filter, such a filter still has many applications. By using this assumption, Eqs.(22) and (23) become

$$2k_p=2x+\omega_C \tag{27}$$

and

$$\omega_p^2=2x\omega_C+y^2-1+\Omega_0\omega_C a. \tag{28}$$

### 3.2 Pole Sensitivity

From Eq.(19) the desired poles $p_0$ and $\overline{p_0}$, which is the conjugate of $p_0$, become

$$p_0=-k_0+j\sqrt{1-k_0^2} \tag{29}$$

$$\overline{p_0}=-k_0-j\sqrt{1-k_0^2}. \tag{30}$$

From the definition given in Eq.(5), the pole sensitivity of $p_0$ to $\omega_C$ can be obtained as

$$|S_{\omega_C}^{p_0}|=\left|\frac{p_0^2}{p_0-\overline{p_0}}\right|\left|\frac{p_0^2+2xp_0+y^2}{p_0^2+2k_pp_0+\omega_p^2}\right|$$

$$=\frac{1}{2\sqrt{1-k_0^2}}\cdot\sqrt{\frac{4(x-k_0)(x-k_0y^2)+(y^2-1)^2}{4(k_p-k_0)(k_p-k_0\omega_p^2)+(\omega_p^2-1)^2}}. \tag{31}$$

The assumption of Eq.(26) simplifies the above equation to

$$|S_{\omega_C}^{p_0}|\cong\frac{1}{2}\sqrt{\frac{4x^2+(y^2-1)^2}{4k_p^2+(\omega_p^2-1)^2}}=\frac{1}{2}\sqrt{F}. \tag{32}$$

Consequently, if the function $F$ becomes small, then the effect of $\omega_C$ will be reduced. It should be noted that the increase of $\omega_p$ is effective not only for reducing the effect of $D_p(s)$ as discussed previously but also for reducing the pole sensitivity with respect to $\omega_C$.

### 3.3 Minimization of $S_{\omega_C}^{p_0}$

Since the range of $a$ is limited by Eq.(12), $\omega_p$ given in Eq.(28) is also restricted as

$$2x\omega_C+y^2-1\leqq\omega_p^2\leqq 2x\omega_C+y^2-1+\Omega_0\omega_C. \tag{33}$$

Equations (14) and (25) also limit the range of $\omega_p$ as

$$0\leqq\omega_p^2\leqq\Omega_0\omega_C y^2. \tag{34}$$

$\omega_p$ must be in both ranges given in the above two equations. If $x$ and $y$ satisfy

$$2x\omega_C+y^2-1\leqq\Omega_0\omega_C y^2, \tag{35}$$

$\omega_p$ is in both ranges. Under this condition $\omega_p$ is determined to be equal to one of the upper limits which is lower than the other so as to make $\omega_p$ as large as possible. If $2x\omega_C$ is less than or equal to $(\Omega_0\omega_C-1)(y^2-1)$, then $\omega_p$ is chosen as

$$\omega_p^2=2x\omega_C+y^2-1+\Omega_0\omega_C, \tag{36}$$

and otherwise

$$\omega_p^2=\Omega_0\omega_C y^2. \tag{37}$$

While $y$ is desired to be equal to 1 in order to reduce the value of the function $F$, $y$ is desired to be as large as possible in order to increase the value of $\omega_p$. From this fact, the range of $y$ is determined as

$$1\leqq y. \tag{38}$$

The minimum value of the function $F$ of $x$ and $y$ can be obtained under the conditions given in Eqs.(36) and (38) or Eqs.(37) and (38). The detailed derivation of the minimum value is given in Ref.(4).

When the value of $F$ is minimum, $x$ and $y$ must satisfy

$$x=y \tag{39}$$

and

$$2x\omega_C=(\Omega_0\omega_C-1)(y^2-1). \tag{40}$$

On the assumption that $\omega_C \gg 1$[rad/sec], $x$ and $y$ become

$$x=y=\frac{1+\sqrt{1+\Omega_0^2}}{\Omega_0} \tag{41}$$

If $x$ and $y$ satisfy Eq.(41), the upper limits given by Eqs.(33) and (34) become the same. This means that the coefficients $a$ and $c$ of the transfer function $T_D(s)$ must be 1. Although only a bandpass filter can be realized under this restriction, the other types of filters can also be realized if the pole sensitivity is determined as a larger value than its minimum.

By using the assumption that $\omega_C \gg 1$[rad/sec], the minimum value of $F$ becomes

$$F \cong \frac{1}{\omega_C}\frac{(x^2+1)}{\Omega_0^2x^4+1}$$
$$=\frac{4}{\Omega_0^2\omega_C^2}\frac{\Omega_0^4+3\Omega_0^2+2+2(1+\Omega_0^2)\sqrt{1+\Omega_0^2}}{\Omega_0^4+9\Omega_0^2+8+4(1+\Omega_0^2)\sqrt{1+\Omega_0^2}}. \tag{42}$$

Thus, the theoretical minimum value of the pole sensitivity when $Q$ is infinite is given by

$$|S_{\omega_C}^{p_0}|=\frac{1}{2}\sqrt{F}$$
$$=\frac{1}{\Omega_0\omega_C}\sqrt{\frac{\Omega_0^4+3\Omega_0^2+2+2(1+\Omega_0^2)\sqrt{1+\Omega_0^2}}{\Omega_0^4+9\Omega_0^2+8+4(1+\Omega_0^2)\sqrt{1+\Omega_0^2}}}. \tag{43}$$

In order for a passive circuit part to be realizable, Eq.(13) must also be satisfied, which restricts the range of the unity gain frequency $\Omega_0$ of the integrator part as

$$\sqrt{2} \leqq \Omega_0. \tag{44}$$

The derivation of the above equation is also shown in Appendix.

### 3. 4 Sensitivity with Respect to $\Omega_0$

The condition for minimizing the pole sensitivity with respect to $\Omega_0$ can be easily derived as follows. The pole sensitivity to $\Omega_0$ is given by

$$|S_{\Omega_0}^{p_0}|=\frac{|p_0||p_0+\omega_C||p_0^2+2xp_0+y^2|}{|p_0-\bar{p}_0||p_0^2+2k_pp_0+\omega_p^2|}. \tag{45}$$

Since $\omega_C$ is much larger than $p_0$, the above equation can be approximated as

$$|S_{\Omega_0}^{p_0}| \cong \frac{|p_0||\omega_C||p_0^2+2xp_0+y^2|}{|p_0-\bar{p}_0||p_0^2+2k_pp_0+\omega_p^2|}. \tag{46}$$

On the other hand the pole sensitivity to $\omega_C$ is given by

$$|S_{\omega_C}^{p_0}|=\frac{|p_0|^2|p_0^2+2xp_0+y^2|}{|p_0-\bar{p}_0||p_0^2+2k_pp_0+\omega_p^2|}. \tag{47}$$

The substitution of Eq.(47) into Eq.(46) gives

$$|S_{\Omega_0}^{p_0}| \cong \frac{|\omega_C|}{|p_0|}|S_{\omega_C}^{p_0}|. \tag{48}$$

This equation shows the proportional relationship between the pole sensitivities with respect to $\Omega_0$ and $\omega_C$, i.e., the condition for minimizing the pole sensitivity to $\omega_C$ also minimizes the pole sensitivity to $\Omega_0$. This condition is consistent with the result given in Ref.(5) which discusses the pole sensitivity of a gain-bandwidth product of an operational amplifier but assumes a gain having only a single pole at D.C. like an ideal integrator.

## 4. Design and Experiments

### 4. 1 Design of a Bandpass Filter

It is assumed that $\Omega_0$ and $\omega_C$ is given in advance. These values can be obtained from a simulator like SPICE after an integrator is designed. For example, the phase difference $\theta_D$ between the ideal transfer function and the actual transfer function of an integrator at a certain angular frequency $\omega$ gives $\omega_C$ as

$$\omega_C=\frac{\omega}{\tan\theta_D} \tag{49}$$

The specification gives the desired $Q_0$ and $\omega_0$. Realization of the theoretical minimum pole sensitivity to $\omega_C$ requires infinite distribution of element values. Thus, the value above the minimum has to be chosen in order to employ elements with reasonable distribution. This choice makes additional freedom. The seven parameters $x$, $y$, $a$, $b$, $c$, $\omega_p$ and $k_p$ should be determined by the use of the five equations Eqs.(22) to (25), Eq.(31) and appropriate conditions. Such kind of conditions will be discussed in the next section.

### 4. 2 Simulation

As an example, a sixth-order bandpass filter with 0.2 relative bandwidth is realized by the proposed method and the leapfrog method,[2] respectively. The proposed method uses a cascade connection to realize the sixth-order bandpass filter while the sixth-order leapfrog filter is derived from a doubly-terminated third-order lowpass filter through a lowpass-to-bandpass transformation. Its center frequency is normalized to 1 [Hz]. Figure 2 shows a model of an

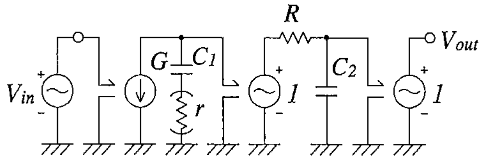



Fig. 2 Integrator model for simulation.

Table 1 Pole sensitivity and element values (pole sensitivities are normalized.)

| Order | section1 | section2 | section3 |
|---|---|---|---|
| $f_0$[Hz] | 0.908 | 1.000 | 1.101 |
| $Q_0$ | 20.33 | 10.12 | 20.33 |
| $min\|S^{p_0}_{\omega_C}\|$ | 0.00694 | 0.00829 | 0.00986 |
| $\|S^{p_0}_{\omega_C}\|$ | 0.00902 | 0.0108 | 0.0128 |
| $R_1[\Omega]$ | 1.000 | 1.000 | 1.000 |
| $R_2[\Omega]$ | 2.455 | 2.237 | 2.370 |
| $R_3[\Omega]$ | 15.64 | 7.806 | 14.58 |
| $R_4[\Omega]$ | 0.7397 | 0.7966 | 0.7101 |
| $C_1$[F] | 0.1363 | 0.1205 | 0.1061 |
| $C_2$[F] | 0.05554 | 0.05387 | 0.04477 |
| $C_3$[F] | 0.008719 | 0.01544 | 0.007278 |
| $C_4$[F] | 0.1843 | 0.1513 | 0.1494 |

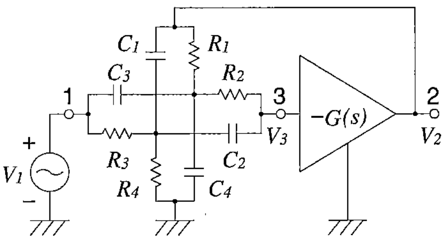



Fig. 3 Proposed second-order bandpass filter.

integrator used in the simulation. The element values are designed for the integrator to have $\Omega_0=5$ [Hz] and $\omega_C=20$ [Hz]. The resistor $r$ in Fig.2 is used only for the leapfrog method in order to compensate for the frequency characteristics.

The above specification gives the desired $Q_0$ and $\omega_0$ shown in Table 1. In the case of realization of a bandpass-type frequency characteristic, the numerator $N(s)$ of a transfer function of each second-order section must be just $s$. From Eqs.(5) and (6), it is better to use the following conditions

$$a=1 \tag{50}$$

and

$$c=1, \tag{51}$$

because these conditions require a fewer number of passive elements and reasonable passive value distribution.

The other five parameters are obtained by the use of Eqs.(22) to (25) and Eq.(31). The negative parameter $b$ can be realized using a twin-$T$ circuit. Figure 3 gives a second-order bandpass filter configuration. Table 1 also shows the theoretical minimum pole sensitivities, the prescribed pole sensitivities and the calculated element values used in Fig.3.

The frequency characteristics of the two filters are shown by the solid lines in Figs.4 and 5. The dashed lines and the dash-dotted lines in both figures stand for the frequency characteristics when $\omega_C$'s uniformly deviate from the nominal value by +30% and −30%,

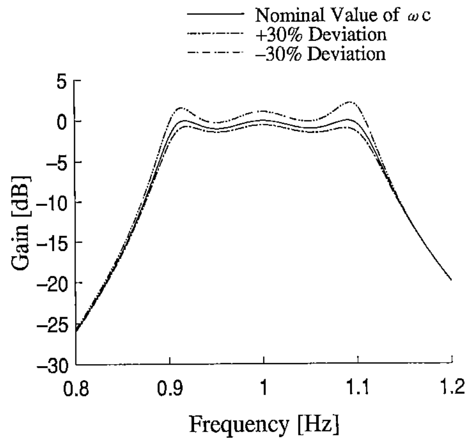



Fig. 4 Simulation results (proposed filter).

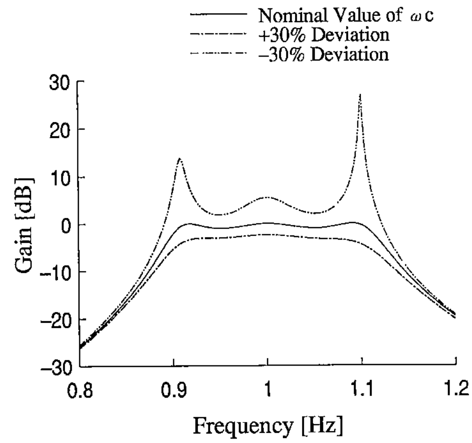


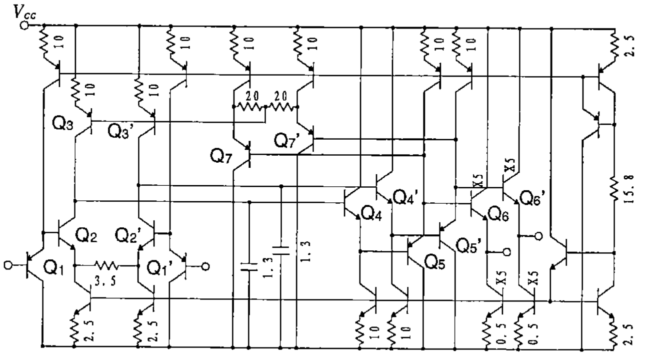

Fig. 5 Simulation results (leapfrog filter).

respectively. It is quite clear that the proposed filter is much less sensitive to the deviation of $\omega_C$'s.

It should be noted that, if it is assumed that filters are realized by only integrators as frequency-dependent elements and by ideal elements except integrators like the above simulations, the uniform deviation of $\omega_C$'s of integrators gives exactly the same effect on any integrator-based filters. This is because the complex angular frequency $s$ is just replaced by $s(1+s/\omega_C)$ from Eqs.(1) and (2). By noting that this replacement corresponds to an extended frequency scaling of a filter, it is easily understood that the effect of the uniform deviation of $\omega_C$'s is independent of a filter topology. Therefore, the above simulation results are valid not only for the leapfrog filter but also for the other integrator-based filters.

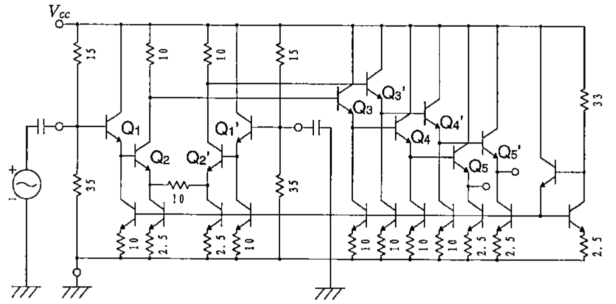
## 4. 3 Experiments

In order to demonstrate the validity of the proposed method, a fourth-order bandpass filter is implemented by a standard 3 $\mu$m bipolar process which can produce npn transistors with 5-GHz $f_T$ and vertical pnp transistors with 1-GHz $f_T$. The filter is



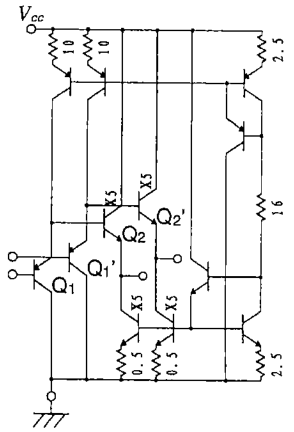

R's in [kΩ], C's in [pF]

Fig. 6 Newly developed integrator.




R's in [kΩ]

Fig. 7 Input circuit.




R's in [kΩ]

Fig. 8 Output circuit.

designed to have 5-MHz center frequency, 2-dB passband ripple and 0.1 relative bandwidth. This specification requires the cascade of two second-order filter having $Q_0$ equal to 25. In this specification the parameter $b$ becomes negative as in the previous section. The use of a twin-T circuit, however, would seriously deteriorate the filter performance, because the floating capacitors have large parasitic capacitances on a monolithic chip. The use of a balanced-type configuration can avoid this problem. A balanced-type integrator designed for the filter is depicted in Fig.

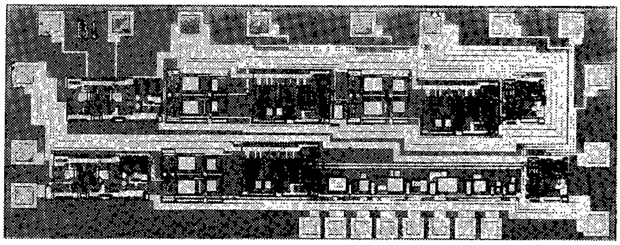


Fig. 9 Photomicrograph.

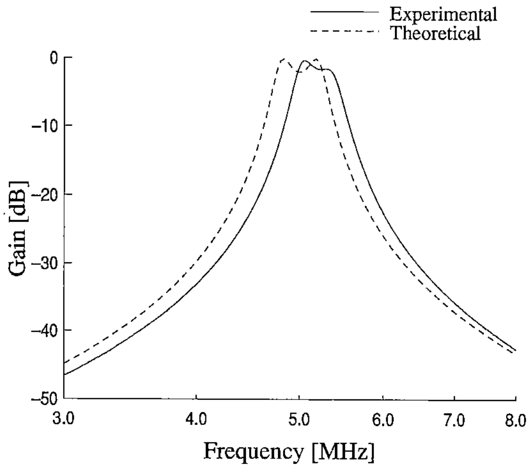



Fig. 10 Experimental results.

6. This integrator requires a single 5-V supply and has a 46.65-MHz unity-gain frequency and a pole at 1.015 GHz, which are obtained by SPICE simulations. Figures 7 and 8 show an input circuit for converting unbalanced-type input signals to balanced-type input signals and an output circuit for reducing the effect of a load. A photomicrograph of the chip on which the fourth-order filter is implemented is shown in Fig.9. The chip size is 3.0 mm × 1.0 mm. This chip includes not only the input and output circuits but also another second-order bandpass filter which is designed to have 5-MHz center frequency and $Q=25$.

Figure 10 shows the experimental results. The results are quite consistent with the theoretical values except the center frequency shift due to the low absolute accuracy of passive elements on a monolithic chip. The difference between layouts of passive $RC$ parts in the two second-order sections reduces the relative accuracy of passive elements and causes a slight degradation at higher frequencies in the passband. The fourth-order filter including the input and output circuits consumes about 25 mW.

Experimental results of the second-order bandpass filter on the same chip shows $Q$ of the filter is about 21. This fact also demonstrates the validity of the proposed method.

## 5. Conclusions

This paper has proposed a method to implement active RC filters using integrators. The proposed method can drastically reduce the effect due to the undesirable pole $\omega_C$ of an integrator compared to conventional filters using only integrators, for example, leapfrog filters. The simulation showed validity of the proposed method. Furthermore, monolithic implementation of the fourth-order filter designed by the method demonstrated feasibility of high-speed and high-$Q$ filters on a monolithic chip. This method assumes that $Q$ is infinite when a theoretical minimum of a pole sensitivity is derived. This assumption confine the application of the method to a bandpass filter realization, because lowpass and highpass filters do not have such high $Q$'s. How to design such low-$Q$ filters still remains as a problem. And also, since the proposed method uses not only integrators but also a passive $RC$ circuit, this makes it difficult to control frequency characteristics. The design of tunable filters by the proposed method requires further investigations.

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## Appendix A: Realization Using Zeroth and First-Order RC Circuits

A zeroth-order RC circuit is realized just by resistors. Therefore, the transfer functions $T_D(s)$ and $T_N(s)$ in Fig.1 become

$$T_D(s)=k_D \tag{A·1}$$

and

$$T_N(s)=k_N, \tag{A·2}$$

where $k_D$ and $k_N$ are constants. The substitution of these equtions into Eq.(15) yields

$$T(s)=\frac{\Omega_0 k_N}{s(1+s/\omega_C)+\Omega_0 k_D}. \tag{A·3}$$

As a conclusion, the transfer function $T(s)$ must be of lowpass type.

In the case of a first-order $RC$ circuit, the transfer functions $T_D(s)$ and $T_N(s)$ become

$$T_D(s)=\frac{a_1 s+b_1\sigma}{s+\sigma} \tag{A·4}$$

and

$$T_N(s)=\frac{N_1(s)}{s+\sigma}, \tag{A·5}$$

where $a_1$ and $b_1$ are constants, $N_1(s)$ is a first-order polynomial of $s$, and $\sigma$ is a real pole of the $RC$ circuit. Since these functions originate from the passive $RC$ circuit, the constants $a_1$ and $b_1$ and the real pole $\sigma$ must satisfy

$$0\leqq a_1\leqq 1, \tag{A·6}$$

$$0\leqq b_1\leqq 1 \tag{A·7}$$

and

$$0<\sigma. \tag{A·8}$$

The substitution of Eqs.(A·4) and (A·5) into Eq.(15) gives the overall transfer function $T(s)$ as

$$T(s)=-\frac{\Omega_0\omega_C N_1(s)}{D_1(s)}, \tag{A·9}$$

where

$$D_1(s)=s(s+\omega_C)(s+\sigma)+\Omega_0\omega_C(a_1 s+b_1\sigma). \tag{A·10}$$

The above polynomial is factored as

$$D_1(s)=(s^2+2k_{01}s+\omega_{01}^2)(s+\rho). \tag{A·11}$$

The comparison between Eqs.(A·10) and (A·11) leads to

$$\rho+2k_{01}=\omega_C+\sigma, \tag{A·12}$$

$$1+2k_{01}\rho=\omega_C(\sigma+\Omega_0 a_1), \tag{A·13}$$

and

$$\rho=\Omega_0\omega_C b_1\sigma, \tag{A·14}$$

where $\omega_{01}$ is normalized to 1 [rad/sec]. If $\rho$ is much greater than 1 [rad/sec], the factor $(s+\rho)$ can be neglected and the frequency characteristic is determined by the other factor $(s^2+2k_{01}s+\omega_{01}^2)$.

Similarly to the second-order $RC$ circuit case, it is assumed that the $Q$ is infinite, i.e., $k_{01}=0$. On the assumption, Eqs. (A・12) and (A・13) can be rewritten as

$$\rho=\omega_C+\sigma \tag{A・15}$$

and

$$\sigma=\frac{1-\Omega_0\omega_C a_1}{\omega_C}. \tag{A・16}$$

From Eqs. (A・14) and (A・15) $\sigma$ can be expressed by another form as

$$\sigma=\frac{\omega_C}{\Omega_0\omega_C b_1-1} \tag{A・17}$$

Since the constants $a_1$ and $b_1$ are restricted by Eqs. (A・6) and (A・7), $\sigma$ has to be in both the following ranges

$$0<\sigma\leqq\frac{1}{\omega_C} \tag{A・18}$$

and

$$\frac{1}{\Omega_0}<\sigma<\infty, \tag{A・19}$$

where $\Omega_0\omega_C$ is assumed to be much greater than 1. In order for $\sigma$ to be in both ranges, $\Omega_0$ and $\omega_C$ must satisfy

$$\Omega_0>\omega_C. \tag{A・20}$$

Generally $\omega_C$ is greater than $\Omega_0$ for an integrator. The design of an integrator satisfying the above inequality means to reduce its bandwidth. Therefore, the choice given in Eq. (A・20) is irrelevant when high-frequency filters are implemented.

## Appendix B: Condition for $\Omega_0$

From Eqs. (22) and (24), $b$ is approximately given by

$$b\cong\frac{2x+\omega_C(1-y^2)}{2x\Omega_0\omega_C}, \tag{A・21}$$

where $Q$ of a filter is assumed to be infinite as Eq. (26). The substitution of Eq. (A・21) into Eq. (13) gives

$$-\frac{y}{2x}\leqq\frac{2x+\omega_C(1-y^2)}{\Omega_0\omega_C} \tag{A・22}$$

and

$$\frac{2x+\omega_C(1-y^2)}{\Omega_0\omega_C}\leqq\frac{y}{2x}+1 \tag{A・23}$$

Equations (A・22) and (A・23) can be rewritten in the forms

$$\omega_C(y^2-\Omega_0 y-1)\leqq 2x \tag{A・24}$$

and

$$-\frac{1}{\Omega_0}(y^2-1)-y\leqq 2x \tag{A・25}$$

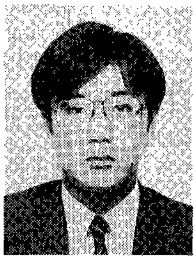
The latter is automatically satisfied because $x=y=(1+\sqrt{1+\Omega_0^2})/\Omega_0$. The former approximately becomes

$$\omega_C(y^2-1)\leqq x(2+\Omega_0\omega_C)\cong\Omega_0\omega_C x$$

$$\frac{2x}{\Omega_0}\leqq\Omega_0 x$$

$$\sqrt{2}\leqq\Omega_0 \tag{A・26}$$

where it is assumed that $x=y\cong\Omega_0(y^2-1)/2$ and that $\Omega_0\omega_C/2\gg1$.



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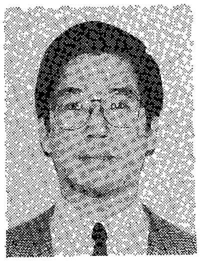

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