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Design of FIR Digital Filters Using Estimates of Error Function over CSD Coefficient Space

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SUMMARY This paper proposes an algorithm for the design of FIR digital filters whose coefficients have CSD representations. The total number of nonzero digits is specified. A set of filters whose frequency responses have less than or equal to a given Chebyshev error have their coefficients in a convex polyhedron in the Euclid space. The proposed algorithm searches points where a coefficient is maximum or minimum in the convex polyhedron by using linear programing. These points are connected with the origin to make a convex cone. Then the algorithm evaluates CSD points near these edges of the cone. Moving along these edges means the scaling of frequency responses. The point where the frequency response is the best among all the candidates under the condition of specified total number of nonzero digits is selected as the solution. Several techniques are used to reduce the calculation time. Design examples show that the proposed method can design better frequency responses than the conventional methods.

key words: *FIR digital filter, CSD representation, optimization*

1. Introduction

Linear phase finite impulse response (FIR) filters are widely used in digital signal processing. They can be realized as a multiplierless wired logic using adders and shifters. When the specified transition band width is narrow, FIR filters need a large number of taps and longer wordlength of coefficients. Then the hardware size of such FIR filters becomes large. To reduce the hardware size the canonic signed digit (CSD) [1] code is often used to represent the coefficients rather than the normal binary system, and the total number of those nonzero digits is sometimes limited. The CSD code uses $0, 1$ and -1 in each digit. The implementation of -1 as a coefficient digit is as simple as that of 1, and the use of -1 can reduce the total number of nonzero digits or the complexity of a filter. In this case, however, the design of filters becomes difficult, because representable numbers distribute non-uniformly.

Many algorithms have been proposed for this design problem [1]–[7]. The mixed integer linear programing (MILP) [2] method can find an FIR filter whose frequency response is optimal in the minimax sense. The MILP method, however, needs enormous computational cost, and has a restriction that the maximum number of nonzero digits for each coefficient is fixed. The hardware size of a filter of a given order depends mainly on the total number of nonzero digits in the filter, and not on how the nonzero digits are allocated to the coefficients. Then such restriction is not reasonable when the hardware size is specified.

Later Li et al. proposed a polynomial time algorithm in which only the total number of nonzero digits of all coefficients is specified [6]. That method utilizes the freedom of selecting the number of nonzero digits in each coefficient to yield superior frequency responses compared to the MILP method. Li's method, however, only minimizes the l^{∞} norm of the difference between the CSD coefficients and the coefficients obtained by using the famous McClellan and Parks program [8], and so the frequency responses, which are not evaluated, are not good enough.

Then the authors have proposed a design method [7], which can design better frequency responses than those by Li's method. The computational time in that method is acceptable but not short. This paper presents a new design method which is superior to our previous method in both computational time and resultant frequency responses. Many examples show the superiority of the proposed method compared to Li's method and our previous method.

2. CSD Representation

A filter coefficient a is represented as sum and difference of powers of two, given by

$$a = \sum_{k=1}^{M} s_k 2^{-k}, \tag{1}$$

where $s_k \in \{-1, 0, 1\}$ and M is the number of ternary digits. There are several representations for a given a. Among them, the representation in which the number of nonzero digits becomes minimum is called a CSD [1] representation. A CSD representation is unique, and any nonzero digits are not adjacent. Consider that the wordlength M is given as 7 and the most significant digit (MSD) is 2^{-1}, for example. Among the M-digit numbers, the maximum and the minimum numbers are

$$0.9921875 = 1111111_{(2)}, \tag{2}$$

$$-0.9921875 = {*}{*}{*}{*}{*}{*}{*}_{(2)}, \tag{3}$$

respectively, where $*$ represents -1. Among the M-digit

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CSD numbers, however, the maximum and the minimum numbers are

$$0.6640625 = 1010101_{(2)}, \quad (4)$$

$$-0.6640625 = *0*0*0*_{(2)}, \quad (5)$$

respectively. The M-digit numbers a in the following ranges cannot be represented as the standard M-digit CSD representations:

$$0.6640625 < a \leqq 0.9921875, \quad (6)$$

$$-0.9921875 < a \leqq -0.6640625. \quad (7)$$

An M-digit number in the ranges shown in Eqs. (6) and (7) can be represented by a standard $M+1$ digit CSD code whose MSD is 2^0. Instead of increasing the wordlength the representable range of M-digit CSD numbers is extended. The following representations of the M-digit numbers in the ranges shown in Eqs. (6) and (7) are also defined in this paper as the M-digit CSD representations.

$$a = s_1 s_2 \cdots s_{M(2)} \quad (8)$$

where

$$s_1, \cdots, s_{M'} = \begin{cases} 1 & a > 0 \\ -1 & a < 0 \end{cases}, \quad 1 \leqq M' \leqq M \quad (9)$$

and

$$s_i s_{i+1} = 0, \; i = M', \cdots, M-1. \quad (10)$$

In the above M-digit representation, the number of nonzero digits of the expression $s_{M'} s_{M'+1} \cdots s_M$, where any nonzero digits are not adjacent, becomes minimum among all the representations equivalent to $s_{M'} s_{M'+1} \cdots s_M$. But $s_1, \cdots, s_{M'}$ is a sequence of nonzero digits of a kind. Unless an additional digit s_0 is introduced, the representation $s_1, \cdots, s_{M'}$ is unique. Therefore the M-digit CSD representations shown in Eqs. (8)–(10), which include a sequence of nonzero digits as $s_1 \cdots s_{M'}$, have the minimum number of nonzero digits among all equivalent M-digit representations. Those CSD representations can be obtained from the binary representations in the same way as the standard CSD representations can.

3. Evaluation of Frequency Response

To improve the frequency response, the passband gain is not given as a fixed value. For the evaluation of the designed frequency response $H(\omega)$, the error function $e(\omega)$ is defined as

$$e(\omega) = \frac{W(\omega)|H(\omega) - qD(\omega)|}{q}, \quad (11)$$

where $D(\omega)$ is the ideal response which has the passband gain 1.0 and $W(\omega)$ is a weighting function and $q \in \boldsymbol{R}$ is the passband gain. q is substituted for $1/k$ in Eq. (11). Then

$$e(\omega) = W(\omega)|kH(\omega) - D(\omega)| \quad (12)$$

is obtained. The maximum value of Eq. (12) is defined as

$$\hat{e} = \max_{0 \leqq \omega \leqq \pi} e(\omega). \quad (13)$$

In the proposed method, k is selected so that the value of Eq. (13) is minimized. In other words, the design of the filter is formulated as an optimization problem

$$\min_k \max_{i,j} \{W(\omega)|kH_j^i(\omega) - D(\omega)|\}, \quad (14)$$

where $H_1^i(\omega)$ and $H_2^i(\omega)$ are the maximum and minimum values of $H(\omega)$ in the band i.

4. Algorithm to Optimize the Filter Coefficients

4.1 Outline of the Proposed Method

A linear phase FIR digital filter of a length $2N$ or $2N-1$ has N independent coefficients due to the symmetry. Let filter coefficients be $\boldsymbol{x} = [x_1 \; x_2 \; \cdots \; x_N]^T$, and the frequency response $H(\omega)$ is written as

$$H(\omega) = \boldsymbol{P}(\omega)^T \boldsymbol{x} \quad (15)$$

where $\boldsymbol{P}(\omega)$ is given as

$$\boldsymbol{P}(\omega) = \begin{cases} [1 \; 2\cos(\omega T) \; 2\cos(2\omega T) \\ \quad \cdots \; 2\cos((N-1)\omega T)]^T & \text{Type I,} \\ [2\cos(\frac{1}{2}\omega T) \; 2\cos(\frac{3}{2}\omega T) \\ \quad \cdots \; 2\cos((N-\frac{1}{2})\omega T)]^T & \text{Type II.} \end{cases} \quad (16)$$

Now δ_R is defined as the error value of the frequency response optimized by using McClellan and Parks program [8]. And Δ is defined as $\Delta = \delta_R + \delta, \delta \geqq 0$. If the error of frequency response as Eq. (13) for $q = 1.0$ is smaller than Δ, the filter coefficients $\boldsymbol{x}$ satisfy

$$\boldsymbol{A}\boldsymbol{x} \leqq \boldsymbol{d}, \quad (17)$$

where

$$\boldsymbol{d} = [\; \Delta + W(\omega_1)D(\omega_1) \quad \Delta - W(\omega_1)D(\omega_1) \\ \cdots \quad \Delta - W(\omega_p)D(\omega_p) \;]^T, \quad (18)$$

$$\boldsymbol{A} = [\; W(\omega_1)\boldsymbol{P}(\omega_1) \quad -W(\omega_1)\boldsymbol{P}(\omega_1) \\ \cdots \quad -W(\omega_p)\boldsymbol{P}(\omega_p) \;]^T. \quad (19)$$

p is the number of sampling frequencies from 0 to π. In the N-dimensional Euclid space, the point $\boldsymbol{x}$ is in a convex polyhedron as Eq. (17).

Next if an arbitrary passband gain q is given for the evaluation of the point $\boldsymbol{x}$, from Eq. (11) $\boldsymbol{x}$ satisfies

$$\frac{W(\omega_i)|\boldsymbol{P}(\omega_i)^T \boldsymbol{x} - qD(\omega_i)|}{q} \leqq \Delta \quad (20)$$

where $i = 1, \cdots, p$. By using Eqs. (18) and (19), Eq. (20) can be written as

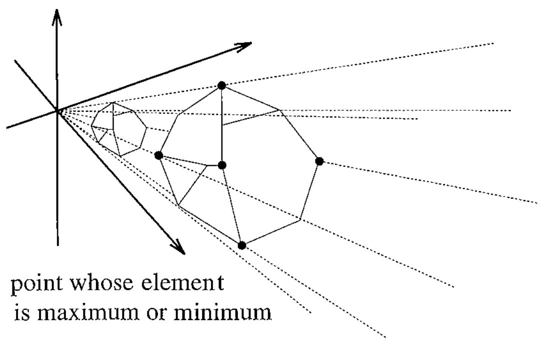


Fig. 1 A convex polyhedron and cone in the 3-dimensional space.

$$A(\frac{x}{q}) \leqq d. \tag{21}$$

Equation (21) shows that the point x, where the error is below Δ, is obtained by scaling a point in the original convex polyhedron of Eq. (17). Accordingly by changing the scaling factor the points x form a convex cone shown in Fig. 1.

Now $\overline{x}^i$ and $\underline{x}^i$ are respectively defined as the points whose element x_i is maximum or minimum among all the points in the convex polyhedron of Eq. (17). The algorithm finds $2N$ points $\overline{x}^i$ and $\underline{x}^i$, $i = 1, \cdots, N$. These points are regarded to approximate the convex polyhedron. Therefore the convex cone can be approximated by the $2N$ half-lines $t\overline{x}^i$ and $t\underline{x}^i$ where $t \geqq 0$. Among the CSD points near by those half-lines, the proposed method evaluates the frequency responses of the CSD points where the total number of nonzero digits satisfies the specification.

Next consider the convex polyhedrons as Eq. (17) where Δ is given as

$$\Delta = \delta_R + n\delta \tag{22}$$

where $0 \leqq n \leqq n_{max}$, and n and n_{max} are positive integers. If n is decreased from n_{max} to 0, the convex polyhedron gradually shrinks. Finally the convex polyhedron shrinks to be a single point at $n = 0$. This point corresponds to the point whose elements, namely, filter coefficients are obtained by using Remez exchange algorithm. The proposed algorithm searches for the CSD points near all the half-lines that approximate the convex cones as Eq. (21) where Δ is varied as Eq. (22). The proposed method selects the minimum error CSD point as the solution.

4.2 Determination of the Vertices

The problem of finding x whose element x_m is maximum or minimum subject to Eq. (17), becomes equivalent to the following linear programing (LP) problem:

$$\text{Minimize } z = c^T x, \tag{23}$$

$$\text{Subject to } Ax \leqq d, \tag{24}$$

where

$$c = [\ c_1 \ \cdots \ c_j \ \cdots \ c_N\ ]^T, \tag{25}$$

$$c_j = \begin{cases} \mp 1 & j = m \\ 0 & \text{otherwise} \end{cases}. \tag{26}$$

In Eq. (26), if c_m is the positive number, the minimum x_m is obtained by solving the LP problem. If c_m is negative, the maximum x_m is obtained. In the proposed method, the dual problem to the prime problem given by Eqs. (23) and (24) is solved in order to reduce the computational time, because the number of sampling points p is generally much larger than the number of independent coefficients N.

The change of Δ as Eq. (22) corresponds to the following change of the vector d:

$$d = d_R + nd_\delta \tag{27}$$

where

$$d_R = [\ \delta_R \ \cdots \ \delta_R\ ]^T, \tag{28}$$

$$d_\delta = [\ \delta \ \cdots \ \delta\ ]^T \tag{29}$$

and $n = 0, \cdots, n_{max}$ in Eqs. (23) and (24). Namely the change of Δ means the modification of the objective function in its dual problem. Therefore the algorithm preserves the optimum solution and basic inverse matrix obtained by solving Eqs. (23) and (24) for the original d. By using the preserved matrix and solution, the algorithm solves Eqs. (23) and (24) where the vector d is modified, and obtains the new optimum solution and basic inverse matrix.

4.3 Filter Coefficient Roundoff

Even though the half-lines whose initial point is the origin and which cross the vertices of the convex polyhedron are obtained, CSD points are, in general, not on the half-line because of their discrete nature. It is therefore necessary to examine frequency responses at CSD points near the half-line.

When the wordlength is specified, coefficient vector space is divided into regions such that any point in each region is rounded off to a single CSD point in that region. The algorithm successively examines regions which the half-line crosses or passes very near. Now let the least significant digit (LSD) of a filter coefficient be 2^{-l}. The procedure is as follows:

1. Let initial and terminal coefficients on the half-line be $x^1 = [x_1^1 \cdots x_N^1]^T$ and $x' = [x_1' \cdots x_N']^T$ respectively. The determinations of those initial and terminal coefficients will be explained in the next subsection. Let the direction vector along the half-line be $v = [v_1 \cdots v_N]^T$, $|v| = 1$. Iteration counter is set $m := 1$.
2. Consider two CSD numbers adjacent to x_i^m. Let

the larger one be $\lceil x_i^m \rceil$, and the other $\lfloor x_i^m \rfloor$. Then the following $\langle x_i^m \rangle$ is determined such that

$$\langle x_i^m \rangle := \begin{cases} \dfrac{\lceil x_i^m \rceil + x_i^m}{2} & v_i > 0 \\ \dfrac{\lfloor x_i^m \rfloor + x_i^m}{2} & v_i < 0 \end{cases}. \tag{30}$$

$\langle x_i^m \rangle$ is the boundaries of the region in which any points are rounded off to $\boldsymbol{x}^m$.

Now $\boldsymbol{x}^{m+1} := \boldsymbol{x}^m$.

3. Define t_i by

$$t_i := \frac{\langle x_i^m \rangle - x_i^1}{v_i}, \tag{31}$$

which is the distance between the initial point $\boldsymbol{x}^1$ and the intersecting point of the direction vector and each boundary plane.

4. Define $k(j)$ such that $t_{k(j)}$ is the j-th smallest value in all t_i.

5. Determine the set K of $k(j)$ such that the difference between $t_{k(j+1)}$ and $t_{k(j)}$ is smaller than 2^{-l-1}. $K(k(1) \in K)$ satisfies

$$K = \{k(j+1) \mid t_{k(j+1)} - t_{k(j)} < 2^{-l-1}, \quad k(j) \in K, j = 1.\cdots, N-1\}. \tag{32}$$

6. Determine all sets of arbitrary selections of $k(j)$ out of all $k(j) \in K$. Assign

$$x_{k(j)}^{m+1} := \begin{cases} \lceil x_{k(j)}^m \rceil & v_{k(j)} > 0 \\ \lfloor x_{k(j)}^m \rfloor & v_{k(j)} < 0 \end{cases}, \tag{33}$$

where each $k(j)$ belongs to a set of selections. Then determine all $\boldsymbol{x}^{m+1}$ by Eq. (33) in respect to all sets of the selections of $k(j)$. Among all $\boldsymbol{x}^{m+1}$, evaluate the frequency response of $\boldsymbol{x}^{m+1}$ which has the total number of nonzero digits within the specification.

7. Assign $x_{k(j)}^{m+1}$, all $k(j) \in K$ as Eq. (33). If $x_{k(1)}^{m+1} \geq x'_{k(1)}$, then stop. Otherwise let $m := m+1$ and go to Step 2.

The smallest magnitude t_i is defined as $t_{k(1)}$. Hence the direction vector crosses the region including the point $\boldsymbol{x}^{m+1}$, whose element $x_{k(1)}^{m+1}$ is assigned as Eq. (33), after crossing the region including the $\boldsymbol{x}^m$. From the fact that each difference between $t_{k(i)}$ and $t_{k(j)} (\forall k(i), \forall k(j) \in K)$ is very small, the algorithm also defines the points whose element $x_{k(j)}^{m+1} (\forall k(j) \in K)$ is assigned by the two coefficients $x_{k(j)}^m$ and $\lceil x_{k(j)}^m \rceil$ or $\lfloor x_{k(j)}^m \rfloor$ as the points near by the direction vector. Figure 2 shows the determination of the regions which the direction vector passes very near in case of $N = 2$. In this figure the point $\boldsymbol{x}^m$ is assumed to be the point 2. In case that the direction vector is A, $t_{k(2)} - t_{k(1)}$ is obtained as

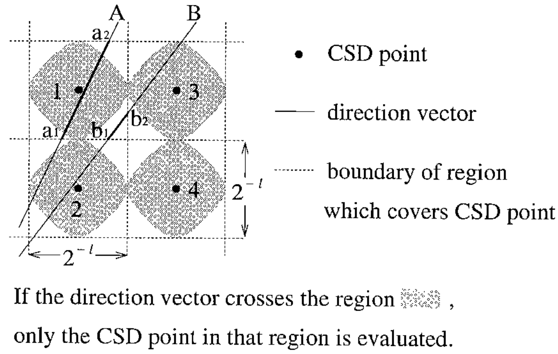


Fig. 2 Regions near by the direction vector for $N = 2$.

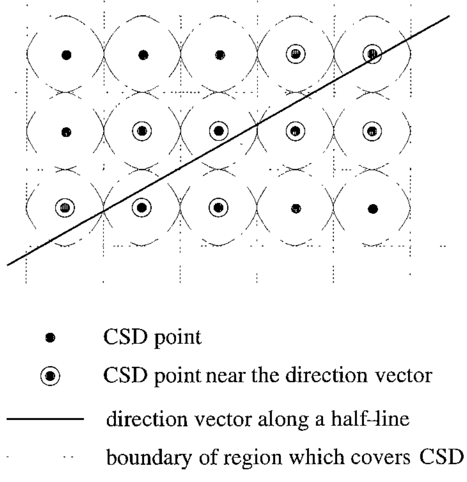


Fig. 3 CSD points near by the direction vector.

$$t_{k(2)} - t_{k(1)} = \overline{\mathrm{a_1 a_2}} \geqq 2^{-l-1} \tag{34}$$

from Fig. 2. So the algorithm regards only the point 1 as $\boldsymbol{x}^{m+1}$, and evaluates the error value of that point. If the direction vector is B, however, $t_{k(2)} - t_{k(1)}$ is given as

$$t_{k(2)} - t_{k(1)} = \overline{\mathrm{b_1 b_2}} \leqq 2^{-l-1} \tag{35}$$

from Fig. 2. Then the algorithm regards all the point 1, 3 and 4 as $\boldsymbol{x}^{m+1}$. If the total number of nonzero digits of each point satisfies the specification, the algorithm evaluates the frequency response of each point.

Figure 3 shows the way that CSD coefficients are selected for an example $N = 2$.

4.4 Speed up of Frequency Response Evaluations

The change of the filter coefficients $\boldsymbol{x}$ on the half-line, which crosses the origin and the vertex, only scales the gain coefficient in the frequency response without the variation of the normalized frequency response. Thus the normalized frequency responses at the CSD points near the half-line resemble one another. From this fact, it is possible to reduce the calculating time by using the following processes as the successive estimation of the CSD points near by the half-line.

1. Initially assign $e_b := \infty$.
2. Calculate the frequency response $\tilde{H}(\tilde{\omega})$ at an initial point on the half-line. Find all $\tilde{\omega}_i$ where $\tilde{H}(\tilde{\omega}_i)$ become the local maxima and minima.
3. Applying the method in the above subsection, find a CSD point $\boldsymbol{x}$ near the half-line where the number of nonzero digits is less than or equal to the specified value. If the evaluations on the half-line finish, then determine an initial point on another half-line and go to Step 2.
4. Calculate the values $H(\tilde{\omega}_i)$ at the found CSD point $\boldsymbol{x}$ in Step 3, where $\tilde{\omega}_i$ are the frequencies found in Step 2.
5. Among the values $H(\tilde{\omega}_i)$, choose the frequencies where the values of $H(\tilde{\omega}_i)$ are the maximum and minimum in each band.
6. Find the extrema of $H(\omega)$ such that the frequencies are closest to the frequencies obtained in Step 5. Then evaluate the error value $\hat{e}$ from the determined extrema according to Eq. (14).
7. If $e_b < \hat{e}$, then go to Step 3. Otherwise evaluate the frequency response all over the frequency, and let that error value be $\hat{e}$.
8. If $e_b < \hat{e}$ then go to Step 3. Otherwise assign $e_b := \hat{e}$ and $\boldsymbol{x}_b := \boldsymbol{x}$. Go to Step 3.

After the evaluations on all the half-lines, the algorithm selects $\boldsymbol{x}_b$ as the solution.

Both the initial and the terminal points on the half-lines are given so that the wordlength may be most effectively utilized. Now let a point $\boldsymbol{x}$ be a vertex on a convex polyhedron which has the passband gain of 1.0. In all the filter coefficients, that is, the elements of $\boldsymbol{x}$, let x_{max} be the maximum absolute value. For instance, the wordlength is given as 9 with the MSD 2^{-1}. Among the positive CSD numbers whose MSD is nonzero, the minimum and the maximum numbers are respectively written as

$$10{*}0{*}0{*}0{*}_{(2)} = 0.333984375, \tag{36}$$
$$111111111_{(2)} = 0.998046875. \tag{37}$$

From Eqs. (36) and (37), the initial and the terminal points can be obtained as

$$\text{initial point}: \frac{0.333984375}{|x_{max}|}\boldsymbol{x}, \tag{38}$$
$$\text{terminal point}: \frac{0.998046875}{|x_{max}|}\boldsymbol{x}. \tag{39}$$

The gain coefficient normalizes the error of the frequency response according to Eq. (14). Therefore there are many good CSD points in a space where a gain coefficient is large. However the number of nonzero digits of CSD points in such a space becomes large. So the algorithm evaluates the CSD points not near the whole segment between the initial point and the terminal point as Eqs. (38) and (39) on the half-line but near a 60 percent of the segment on the side of the terminal point. If the algorithm cannot find any CSD points where the number of nonzero digits satisfies the specification near the 60 percent of the segment, the algorithm evaluates the CSD points near the remaining segment that is the 40 percent of the whole segment.

5. Parameters for the Algorithm

The variables n_{max} and δ are explained. The algorithm searches near edges of n_{max} convex cones. Large value of n_{max} increases the possibility to find a better solution at the expense of computational time. When the LSD is given as 2^{-l}, the difference of two CSD numbers is 2^{-l}. δ is the step size for the shrinking convex polyhedron, so that δ should satisfy the following condition:

$$\delta \leqq 2^{-l-1}. \tag{40}$$

Here e_b is defined as the error of the best point among all the CSD points which have been previously evaluated during the optimization. Then the algorithm need not calculate any vertices on the convex polyhedron that contains the point where the error is equal to or lager than e_b. During the optimization, if δ and n_{max} satisfy the inequality

$$\delta n_{max} \geqq e_b, \tag{41}$$

the algorithm assigns δ as

$$\delta = \frac{e_b}{n_{max}}, \tag{42}$$

and continues the optimization by using the modified δ. In the proposed method, 2^{-l-2} is given as the initial value of δ.

6. Design Examples

Three types of specifications are designed and compared with MILP algorithm, Li's method and our previous method. Example 1 uses the same specifications as Li used in [6]. The other two examples use different weights of the passband and the stopband. The common specifications are: passband 0.0 – 0.15, stopband 0.25 – 0.5, wordlength 9, the total number of nonzero digits $2N$, filter length 27 – 43. Passband and stopband weights are 1 : 1, 1 : 10 and 10 : 1 in design examples 1, 2 and 3. In case of MILP, two nonzero digits are given to each coefficient.

Figures 4, 5 and 6, respectively, show the results of the proposed method together with those of MILP,

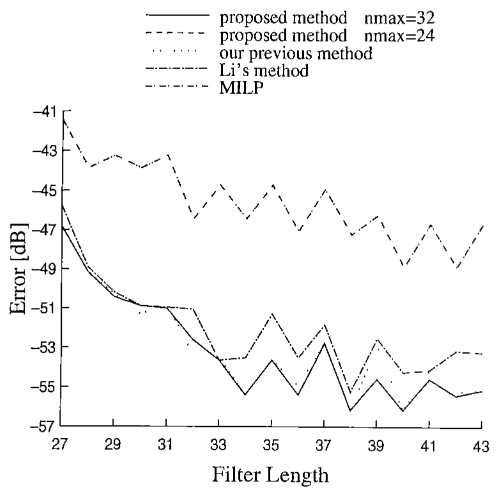


Fig. 4 Error in design example 1.

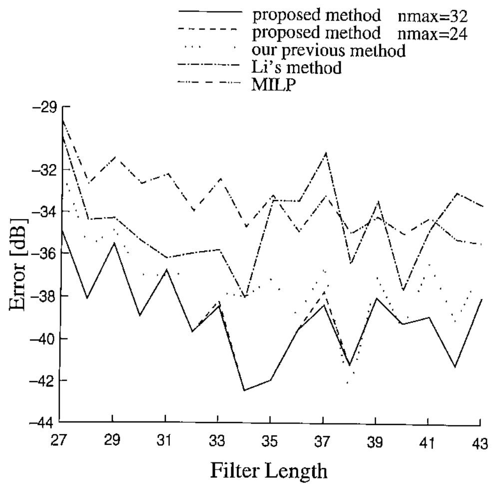


Fig. 5 Error in design example 2.

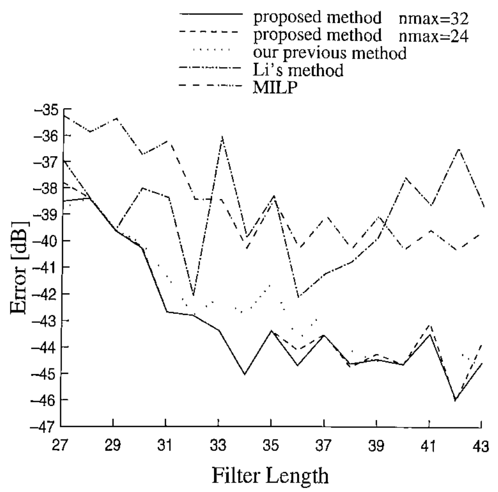
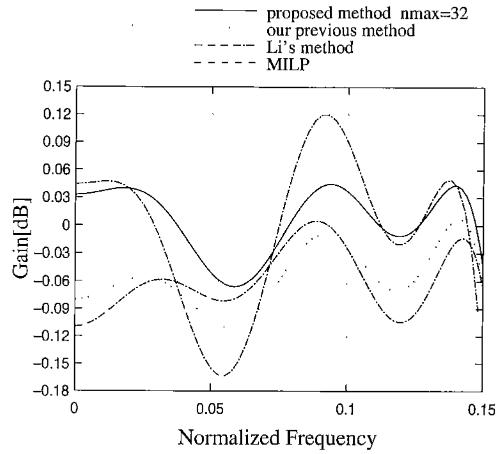
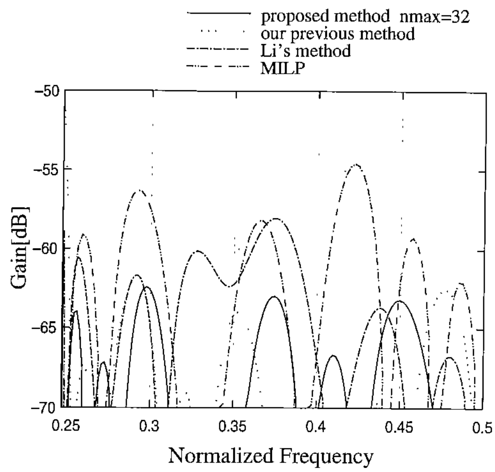


Fig. 6 Error in design example 3.

Li and our previous method for three design examples. To show the typical results n_{max} are chosen as 24 and



(a) Frequency responses in the passband



(b) Frequency responses in the stopband

Fig. 7 Frequency responses for 34-tap FIR filters of design example 2.

32. In these figures the error $\hat{e}$ defined in Eq. (13) is shown in dB. In design example 1 the error value $\hat{e}$ is equivalent to the normalized peak ripple. In the design example 2 for filter length 34, the optimized frequency responses using the proposed method and the other two methods are shown in Figs. 7 (a) and 7(b). In Figs. 7 (a) and 7(b) all the frequency responses are normalized so as to minimize the error $\hat{e}$.

From these figures, the frequency responses designed by the proposed method are far better than those designed by Li's method, and better than our previous method for almost all the specifications.

From Eq. (16) $\boldsymbol{P}(\omega)$ given in odd filter length is essentially different from that given in even filter length. Hence Figs. 4, 5 and 6 show natural results that minimized error values by all methods are not monotonically decreasing functions. The FIR filters obtained by all methods except for MILP may be the local optimum solutions. Therefore the error values of odd (even)

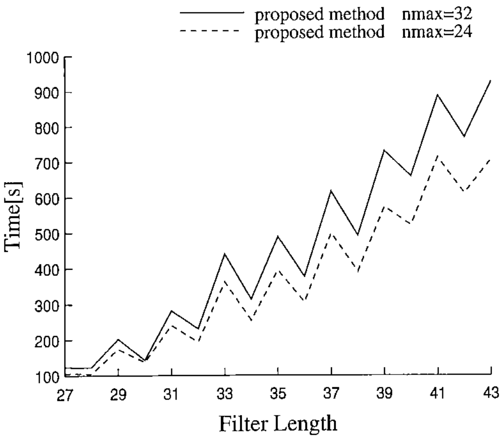


Fig. 8 Growth of computing time with increase of n_{max}.

length obtained by all methods except for MILP do not always monotonically decrease to the increase of filter length as shown in Figs. 4, 5 and 6. The design program was run on SONY NEWS5000SB, and the proposed method took 370 $[s]$ and 459 $[s]$ for $n_{max} = 24$ and 32, respectively, while our previous method 1104 $[s]$ and Li's method 9.12 $[s]$. The proposed method needs shorter time of the computation than our previous method. The growth of the computational time with the increase of n_{max} is not so much as shown in Fig. 8.

7. Conclusion

A new design method for the design of FIR filters whose coefficients have CSD representation has been proposed. Because of the non-uniform nature of CSD representable numbers, filter gain is not specified but determined as a result of the optimization to have the best shape of frequency response.

Filters whose frequency responses have less than or equal to a given Chebyshev error have their coefficients in a convex polyhedron in the coefficient space. A cone spanned by the polyhedrons with different gains is approximated by a simpler cone made from the points where coefficient values are either maximum or minimum in a polyhedron. The proposed method evaluates the CSD points near the edges of the cone. In this case the frequency responses are roughly the same (except for the gain) along the edge, and thus their evaluation can be simplified. The method successively searches the CSD points along the edges, i.e. changing the gain, and selects the best points as the solution.

Only the total number of nonzero digits is specified and each coefficient may have arbitrary number of nonzero digits, and from this nature the proposed method can design better filters compared to MILP. The proposed method is far better than Li's method. Especially when the ratio of weights in the passband and the stopband are different, the superiority of the proposed method is remarkable. This is probably because the difference between the weight ratio distorts the coefficient space. Li's method cannot cope with that distortion because of evaluating no frequency responses.

Our previous method searches the local area in the coefficient space. The proposed method approximates the error function in the full coefficient space, and thus can design the better filters than our previous method. The computational time is shortened too. By using larger n_{max}, a better filter may be designed at the expense of computational cost. The growth of computing time, however, is not so much.

References

[1] H. Samueli, "An improved search algorithm for the design of multiplierless FIR filters with powers-of-two coefficients," IEEE Trans. Circuits & Syst., vol.36, no.7, pp. 1044–1047, July 1989.

[2] Y.C. Lim and S.R. Parker, "FIR filter design over a discrete power-of-two coefficient space," IEEE Trans. Acoust., Speech, & Signal Process., vol.ASSP-31, no.3, pp.583–591, June 1983.

[3] Y.C. Lim, "Design of discrete-coefficient-value linear phase FIR filters with optimum normalized peak ripple magnitude," IEEE Trans. Circuits & Syst., vol.37, no.12, pp.1480–1486, Dec. 1990.

[4] N. Benvenuto, M. Marchesi, and A. Uncini, "Applications of simulated annealing for the design of special digital filters," IEEE Trans. Signal Process., vol.40, no.2, pp. 323–332, Feb. 1992.

[5] Q. Zhao and Y. Tadokoro, "A simple design of FIR filters with power-of-two coefficients," IEEE Trans. Circuits & Syst., vol.35, no.5, pp.566–570, May 1988.

[6] D. Li, J. Song, and Y.C. Lim, "A polynomial-time algorithm for designing digital filters with power-of-two coefficients," Proc. IEEE Int. Symp. Circuits & Syst., pp.84–87, May 1993.

[7] M. Yagyu, T. Yoshida, A. Nishihara, and N. Fujii, "Design of FIR digital filters with minimum weight representation," Proc. IEEE Int. Symp. Circuits & Syst., pp. 227–230, May 1995.

[8] T.W. Parks and J.H. McClellan, "Chebyshev approximation for nonrecursive digital filters with linear phase," IEEE Trans. Circuit Theory, vol.CT-19, pp.189–194, March 1972.

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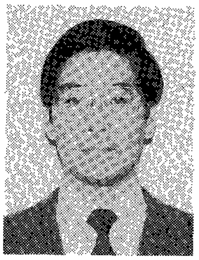

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