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Simplified Modeling Method of Hybrid Structure of RC and Wood

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Summary

In Japan, construction of wood structures for public buildings is encouraged for environmental reason. For example, low-rise large wood building sometimes involves core parts which are usually reinforced concrete structures. This kind of structures is described as "horizontal hybrid structure". However, it is difficult to evaluate seismic force distribution and stress between wood parts and RC parts. In this paper, basic vibration properties and simplified modeling method of horizontal hybrid structure were discussed. Based on the model, practical formulae of amplitude and distribution of seismic force acting on wood parts and equivalent length of floor diaphragm to evaluate moment at connection between wood part and RC part were also proposed. The accuracy was demonstrated by comparison with earthquake response analyses, and they gave close agreement with them.

Keywords: Hybrid structure of wood and RC; Vibration analysis; Seismic force distribution.

1. Introduction

In Japan, construction of wood structures for public buildings is encouraged for environmental reason. Wood is eco-friendly material in terms of reduction of carbon-dioxide emission through construction activity. Nowadays, large wood buildings are built with the help of hybrid structure. For example, low-rise large wood building sometimes involves core parts which are usually reinforced concrete(RC) structure as shown in Fig. 1. This kind of hybrid structures is described as "horizontal hybrid structure" in this paper. It has advantages in not only seismic resistance but fire resistance.

There are two types of seismic design method for horizontal hybrid structures. The one is supposed to divide the structure by expansion joint at the connection between wood parts and RC parts. Wood parts and RC parts can be independently designed. Although it is no longer hybrid structure in terms of seismic resistance, the design is easily understandable.

Another design method takes advantage of high seismic resistance of RC parts when designing wood parts. By transferring seismic force on wood parts to RC parts, seismic resistant element such as shear walls can be reduced. Therefore, wide open space without columns and walls can be realized. However, the seismic design is quite difficult because wood buildings generally have flexible floor diaphragms and show complicated seismic force distribution. Previously, earthquake response analysis was necessary to simulate the seismic behavior. It requires time and effort, and it is not suitable situation to promote construction of wood buildings.

In this paper, basic vibration properties and simplified modeling method of horizontal hybrid structure are discussed. Based on the model, practical formulae of amplitude and distribution of seismic force acting on wood parts and equivalent length of floor diaphragm to evaluate moment at connection between wood parts and RC parts are also proposed. The accuracy is demonstrated by comparison with earthquake response analyses.

2. Prototype of Hybrid Structure

Architectural Institute of Japan provided a prototype of hybrid structures of wood and RC in order to show the procedure of structural design[1]. Fig. 2 is the typical floor plan. It intends three-story school building having RC core parts from X4 to X5 and from X11 to X12.

Wood parts consist of glued-laminated timber's framing and plywood sheathing walls and floor diaphragms. Connections between timber beams and RC parts have only axial and shear resistance because it is generally difficult to realize moment resisting joints by wood structures. Therefore, moment of floor diaphragms must be resisted by a couple of axial forces of timber beams, and especially tensile force of timber beams should be precisely evaluated.

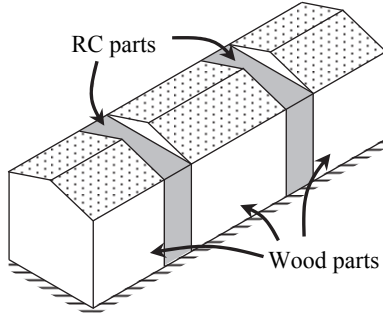


Fig. 1: An example of hybrid structure

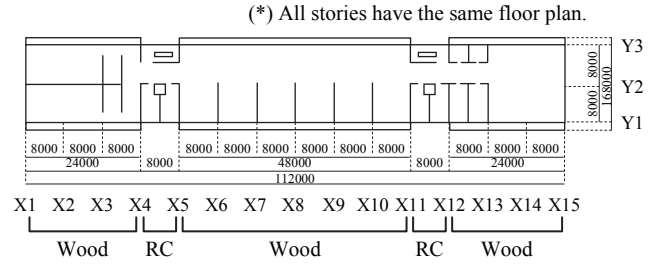


Fig. 2: Floor plan of prototype building[1]

3. Vibration Properties

3.1 Model

Eigen value analysis is carried out using simple shear spring model as shown in Fig. 3. Shear walls and floor diaphragms of wood parts are modelled by vertical and horizontal shear springs, respectively. RC parts are similarly modelled. The effect of torsion is supposed to be negligible. Properties of each element are shown in Table 1. Total weight of wood parts and RC parts are 12,653kN and 18,436kN, respectively.

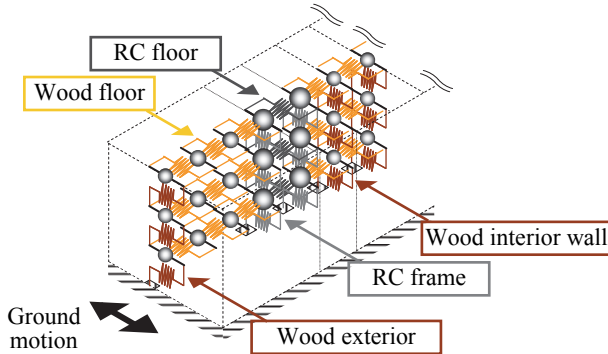


Fig. 3: Vibration analysis model

Table 1: Stiffness of shear springs (Unit: kN/cm)

	Wood		RC	
	Lateral	Vertical	Lateral	Vertical
Vertical	Floor	135	Floor	28200
	Interior wall	97.2	3F frame	27430
	Exterior wall	256.2	2F frame	40550
			1F frame	54170

3.2 Result of Eigen Value Analysis

The characteristics of important vibration modes are discussed here. Although total forty five modes can be obtained, two pairs of “wood part dominant modes” and “RC part dominant modes” are focused on. Figs. 4(a)and(b) show wood part dominant 1st and 2nd modes, and Figs. 4(c)and(d) show RC part dominant 1st and 2nd modes. Since the model is symmetric, Fig. 4 illustrates superposition of participation vectors of a few modes based on the modal shapes' and the natural periods' similarity.

We found that wood parts and RC parts do not act in the same modes. Additionally, natural periods of wood and RC part dominant modes are quite different. Therefore, these modes are not likely to be coupled.

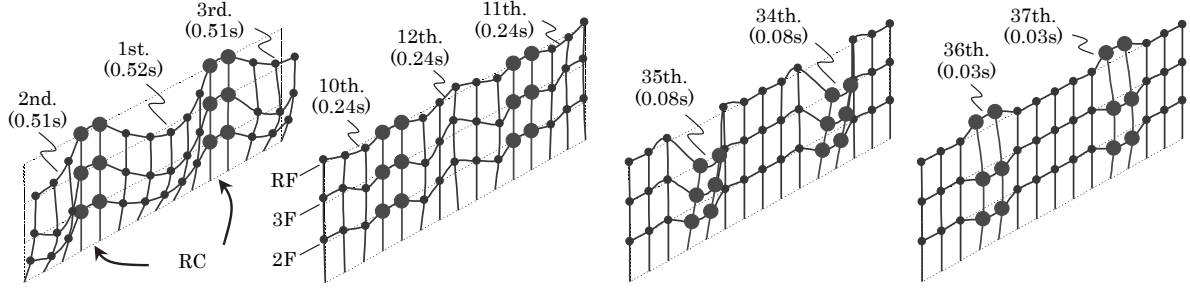


Fig. 4: Dominant participation vectors and natural periods ((a)Wood part dominant 1st. mode, (b)Wood part dominant 2nd. Mode, (c)RC part dominant 1st. mode and (d)RC part dominant 2nd. mode)

4. Theory

4.1 Simplified Model

In this chapter, two types of simplified models are proposed in order to calculate the response of wood part structure. Based on the result in the last chapter, the followings are assumed; RC parts are infinitely rigid, and floor diaphragms can be modelled by uniform shear bars. The difference between type1 and type 2 is arrangement of shear walls.

4.1.1 Uniform shear panel considering its out of plane deformation (Type 1)

If shear walls are distributed at small intervals like X5 to X11(Fig. 2), they are able to be modeled by continuous shear springs. Therefore, the structure is idealized by uniform shear panel considering its out of plane deformation having different shear moduli in two directions(Fig. 5, Fig. 6).

Displacement u is expressed by the following partial differential equation.

$$\rho \frac{\partial^2 u}{\partial t^2} = G_x \frac{\partial^2 u}{\partial x^2} + G_y \frac{\partial^2 u}{\partial y^2} \quad (1)$$

Where, ρ = specific gravity of the body, G_x , G_y = shear moduli in x - and y -direction, respectively. Eq.(1) is a kind of equation of wave motion. However, the body has anisotropy. The eigen value problem can be solved by using general solution of Helmholtz equation.

$$u(x, y, t) = \sum_{m,n=1}^{\infty} \left(A_{mn} \sin \frac{(2m-1)\pi x}{2L_f} \sin \frac{(2n-1)\pi y}{2H} \right) \sin \omega_{mn} t \quad (2)$$

$$\omega_{mn} = \sqrt{\frac{(2m-1)^2 G_x / L_f + (2n-1)^2 G_y / H}{\rho}} \quad (3)$$

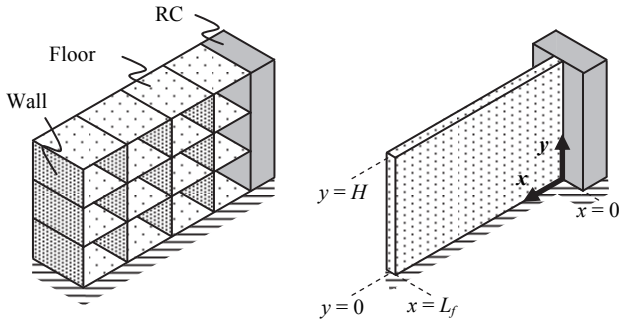
Since the continuous element is just idealization, the thickness of the body can be assumed to be unit. G_x and G_y are calculated as follows.

$$G_x = \frac{\sum k_f}{H}, \quad G_y = \frac{\sum k_w}{L_f} \quad (4a,b) \quad \bar{K}_f = \frac{l_f \sum k_f}{H} = G_x l_f, \quad \bar{K}_w = \frac{h \sum k_w}{L_f} = G_y h \quad (5a,b)$$

k_w , k_f = stiffness of a shear wall and a floor diaphragm of unit span, respectively. h = height of a story, l_f = length of a floor diaphragm connecting unit span.

In this research, 1st mode is focused on because the structure have short natural period and it belongs to constant acceleration region in response spectrum. Therefore, the modal shape and natural circular frequency are expressed as follows.

$$u(x, y) = u_{\max} \sin \frac{\pi x}{2L_f} \sin \frac{\pi y}{2H} \quad (6) \quad \omega_1 = \sqrt{\frac{G_x / L_f + G_y / H}{\rho}} \quad (7)$$



(a): Original model

(b): Idealization to uniform shear panel

Fig. 5: Model of type 1

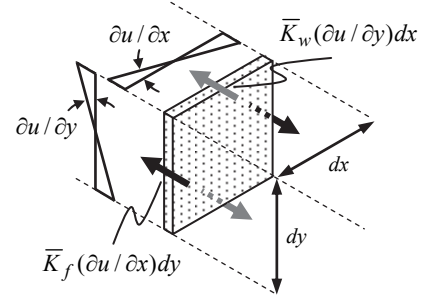


Fig. 6: Uniform shear panel having out-of-plane stiffness in two direction

4.1.2 Simple spring model (Type 2)

If shear walls are distributed only at the edge of the building like X1 to X4 and X12 to X15(Fig.2), the behavior is clearly different from the one of type 1(Fig. 7). Although, cantilever uniform shear beam model supported by elastic spring at the end can be applied, it is likely to be difficult to obtain a practical solution. Therefore, simplified solution method is proposed.

At first, global behavior is simulated by a spring model shown in Fig.8. The model is simple three degrees of freedom system. Static lateral force $\mathbf{f}^{(1)}$ is applied to the model, and the static displacement mode \mathbf{u} is obtained as follows.

$$\mathbf{u} = \{u_1 \quad u_2 \quad u_3\}^T = \mathbf{k}^{-1} \mathbf{f}^{(1)} \quad (8)$$

$$\mathbf{k} = \begin{bmatrix} K_{w1} + K_{w2} + K_{f1} & -K_{w2} & 0 \\ K_{w2} + K_{w3} + K_{f2} & -K_{w3} & \\ \text{sym.} & K_{w3} + K_{f3} & \end{bmatrix} \quad (9)$$

$$\mathbf{f}^{(1)} = \{f_1^{(1)} \quad f_2^{(1)} \quad f_3^{(1)}\}^T \quad (10)$$

Where, \mathbf{k} = stiffness matrix of the spring model. K_{wi} , K_{fi} = stiffness of shear walls and floor diaphragms of i -th floor and are expressed as follows.

$$K_{wi} = k_w, \quad K_{fi} = (l_f / L_f) k_f \quad (11a,b)$$

Static force distribution $\mathbf{f}^{(1)}$ is determined according to the method mentioned in 4.3. The above \mathbf{u} is assumed to be expressed by the combination of $\mathbf{u}^{(1)}$ and $\mathbf{u}^{(2)}$. They are simulated by two virtual conditions without shear walls. $\mathbf{u}^{(1)}$ is displacement mode of the virtual model subjected to $\mathbf{f}^{(1)}$, and $\mathbf{u}^{(2)}$ is displacement mode subjected to reaction forces of shear walls as concentrated force at the edge of floor diaphragm(Fig. 9). As a result, the following equations are obtained.

$$\mathbf{u} = \mathbf{u}^{(1)} + \mathbf{u}^{(2)} = \{u_1^{(1)} \quad u_2^{(1)} \quad u_3^{(1)}\}^T + \{u_1^{(2)} \quad u_2^{(2)} \quad u_3^{(2)}\}^T \quad (12)$$

$$Q_1 = K_{w1} u_1 \quad Q_2 = K_{w2} (u_2 - u_1) \quad Q_3 = K_{w3} (u_3 - u_2) \quad (13a-c)$$

$$u_1^{(1)} = f_1^{(1)} / K_{f1} \quad u_2^{(1)} = f_2^{(1)} / K_{f2} \quad u_3^{(1)} = f_3^{(1)} / K_{f3} \quad (14a-c)$$

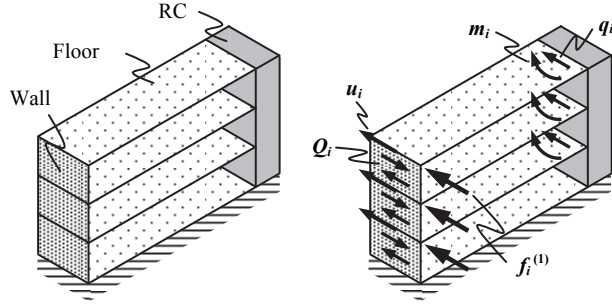
$$u_1^{(2)} = (Q_2 - Q_1) / K_{f1} \quad u_2^{(2)} = (Q_3 - Q_2) / K_{f2} \quad u_3^{(2)} = -Q_3 / K_{f3} \quad (15a-c)$$

Q_i is shear force of shear wall of i -th floor. Since $\mathbf{u}^{(1)}$ and $\mathbf{u}^{(2)}$ are assumed to be sine and linear distribution as shown in Fig. 10, shear force of floor diaphragm of i -th floor q_i is expressed as follows.

$$q_i = \max \left[K_{fi} \left(\frac{\pi}{2} u_i^{(1)} + u_i^{(2)} \right) \quad K_{fi} |u_i^{(2)}| \right] \quad (16)$$

4.2 Amplitude of Seismic Force on Wood Parts

Generally, amplitude of seismic force which is approximated by lateral static force is characterized



(a): Original model (b): Deformation and force
Fig. 7: Model of type2

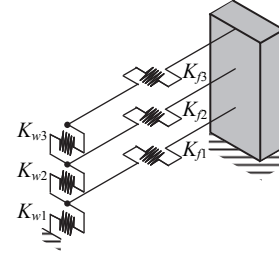
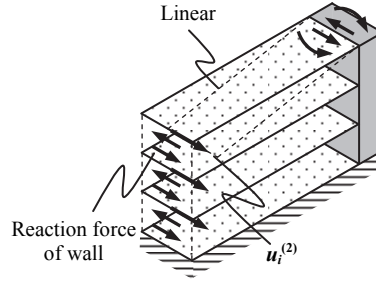
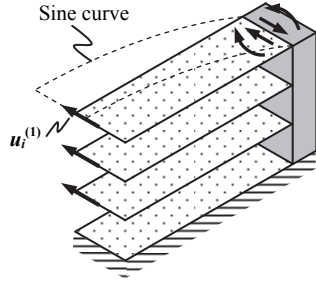


Fig. 8: Simplified analytical model of type2



(a): Forced vibration (b): Statically subjected to reaction force of walls

Fig. 9: Two virtual conditions of type2 without shear walls

by response base shear force of a structure subjected to an earthquake. However, two different modes contribute to base shear force in the case of horizontal hybrid structure. Therefore, base shear force and responses of wood part elements are not able to be simulated at the same time by static analysis.

Since the SRSS(square root of the sum of the squares) combination of seismic forces acting on wood parts and RC parts seems to be total base shear force, the seismic force on wood parts ${}_wQ_1$ when the corresponding base shear force fits the target value is expressed as follows.

$${}_wQ_1 = C_0 {}_wM \frac{({}_wM + {}_cM)}{\sqrt{({}_wM)^2 + ({}_cM)^2}} \frac{0.816}{p} \quad (17)$$

Where, ${}_wM$, ${}_cM$ = total weight of wood parts and RC parts, respectively. C_0 = target base shear coefficient of the whole structure. 0.816 in right side member is the factor to consider the difference of base shear coefficient between single-degree-of-freedom system and multi-degree-of-freedom system. p is the calibration factor prescribed by Japanese Building Standard Law to fit the base shear coefficient according to the number of stories. In this case, the number of stories being three, $p = 0.9$ is used[2].

Eq. 1 is based on the assumption that wood part dominant 1st mode and RC part dominant 1st mode belong to constant pseudo acceleration region even though it is not always satisfied. However, the above assumption is necessary for short-period structure because of syntony effect between seismic force and ground acceleration[3].

4.3 Distribution of Seismic Force on Wood Parts

Vertical distribution of seismic force on wood parts is discussed here. " A_i distribution" well-known in Japan is vertical distribution factor of seismic shear coefficient of i -th story: story shear force divided by weight above i -th story. The original formula is expressed as follows[4].

$$A_i = 1 + \left(\frac{1}{\sqrt{\alpha_i}} - \alpha_i \right) \frac{2T}{1 + 3T}, \quad \alpha_i = \frac{\sum_{j=i}^N W_j}{\sum_{j=1}^N W_j} \quad (18a,b)$$

Where, α_i = normalized weight of i -th story, W_j = weight of i -th story, N = the number of stories. T

is natural period of 1st mode for design which is expressed as a function of building height. Generally, $T = 0.03H$ for wood or steel structure or $T = 0.02H$ for RC structure (H =Total height) are used.

This formula originally deals with normal multi-story buildings, especially uniform shear bar model. However, horizontal hybrid structures of wood and RC are not likely to meet the precondition. Modified A_i distribution is discussed to simulate the actual phenomenon.

Based on the discussion in 4.1.1, seismic force above i -th story ${}_wQ_i$ is expressed as follows.

$${}_wQ_i = {}_wQ_1 \frac{\sin \pi h(i-0.5)/(2H)}{\sin 0.5\pi h/(2H)} \quad (19)$$

In order to fit the situation of discretized model(Fig. 3) to the continuous one's(Fig. 5(b)), shear stiffness of floor diaphragm of 3rd floor must be a half of the others while they are likely to be the same in actual structures. This fatal error of discretized model becomes remarkable in lower structures. Therefore, a simple modification method is proposed here.

Shear stiffness of floor diaphragm of the top floor is twice compared to the idealized model. The floor diaphragm of top floor can bear more lateral force and the surplus stiffness apparently has an effect of reducing mass of the top story. Therefore, seismic force above i -th story ${}_wQ'_i$ using modified normalized weight of i -th story α'_i is proposed as follows.

$$\begin{aligned} \alpha'_i &= \frac{\sum_{j=1}^{N-1} W_j + W_N(0.5G_x/G_y + 1)/(G_x/G_y + 1)}{\sum_{j=1}^{N-1} W_j + W_N(0.5G_x/G_y + 1)/(G_x/G_y + 1) + 0.5W_1} \quad (\text{if } i \neq N) \\ &= \frac{W_N(0.5G_x/G_y + 1)/(G_x/G_y + 1)}{\sum_{j=1}^{N-1} W_j + W_N(0.5G_x/G_y + 1)/(G_x/G_y + 1) + 0.5W_1} \quad (\text{if } i = N) \end{aligned} \quad (20)$$

$${}_wQ'_i = {}_wQ_1 \frac{\sin \pi \alpha'_i / 2}{\sin \pi \alpha'_1 / 2} \quad (21)$$

In Eq. 20, $(0.5G_x/G_y + 1)/(G_x/G_y + 1)$ is multiplied by W_N as a reduction factor of mass of the top floor based on the stiffness balance between shear walls and floor diaphragm. A member $0.5W_1$ is also modification part to deal with the error of discretized model. Finally, modified A'_i is obtained as follows.

$$A'_i = \frac{{}_wQ'_i}{\alpha'_i} = \frac{{}_wQ_1}{\alpha'_1} \frac{\sin \pi \alpha'_i / 2}{\sin \pi \alpha'_1 / 2} \quad (22)$$

Although this formula originally intends a structure of type 1, it is extended to type 2. The applicability will be discussed in the next chapter.

4.4 Equivalent Length of Floor Diaphragm

In normal multi-story structures, equivalent height of a building is defined to evaluate overturning moment, and it is usually $0.67H$ (H =total height) by assuming linear vibration mode. Similarly, equivalent length of floor diaphragm is defined to evaluate moment at connection between wood parts and RC parts(Fig. 10).

Since displacement mode in horizontal plane becomes sine curve in type 1, equivalent length of floor diaphragm of i -th floor $\bar{l}_{fi}(=m_i/q_i)$ is expressed as follows.

$$\bar{l}_{fi} = \pi / 2L_f = 0.64L_f \quad (23)$$

In type 2, displacement mode in horizontal plane is evaluated as a combination of sine and linear curves. \bar{l}_{fi} is expressed by the combination of $u_i^{(1)}$ and $u_i^{(2)}$ as follows.

$$\bar{l}_{fi} = \frac{u_1^{(1)} + u_1^{(2)}}{(\pi/2)u_1^{(1)} + u_1^{(2)}} L_f \quad (24)$$

5. Comparison with Earthquake Response Analysis

5.1 Model and Input Motion

Earthquake response analysis is conducted using the vibration model as shown in Fig. 3. All spring elements are elastic. Damping matrix of the model is constructed so that damping ratio of all vibration modes become 5%. The model having the properties shown in Table 1 is named "basic model" which is the prototype provided by AIJ. In addition, models having half and twice stiffness of wooden floor diaphragm are analyzed. They are described as "floor*0.5" and "floor*2.0", respectively.

Four artificial earthquakes having a phase property of Elcentro-NS, Taft-EW, Hachinohe-NS and JMA Kobe-NS earthquake are generated[5]. Response spectra of the input motions are shown in Fig. 11. It has constant pseudo acceleration region from 0.16 to 1 second, and the intensity is 0.2 times of gravity acceleration.

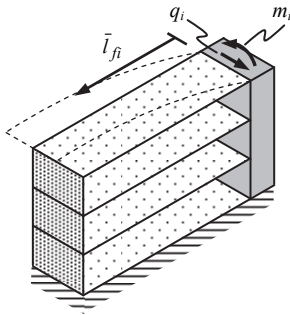


Fig. 10: Equivalent length of floor

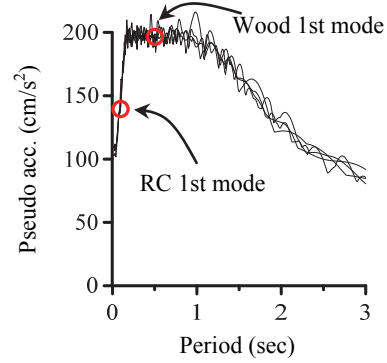


Fig. 11: Response spectra of earthquake (5%damping)

5.2 Result and Discussion

Comparison of natural period of 1st mode is presented in Table 2. Estimations by Eq. 7 give a little shorter results compared to analytical results. However, the differences are less than 10%. Although Eq. 7 was derived from a theory of type 1, it seems to be applicable to type 2.

Comparison of amplitude of seismic force is presented in Table 3. The results of earthquake response analysis are normalized so that the base shear coefficient becomes 0.2. Estimation by Eq. 17 with $C_0 = 0.2$ give close agreement with the ones of earthquake response analysis.

Comparison of vertical seismic shear coefficient distribution is presented in Table 4. While conventional A_i given by Eq. 18(a) show large value in upper stories, analytical A_i are not so high. They are apparently constant along the height. This is one of unique phenomena of horizontal hybrid structure, and modified A'_i estimated by Eq. 22 can simulate such a tendency.

Comparison of equivalent floor length ratio of each floor is presented in Table 5. Equivalent floor length \bar{l}_{fi} is normalized by L_y . While all \bar{l}_{fi} of type 1 are around 0.64, \bar{l}_{fi} of type 2 are totally smaller than the one of type 1 and clearly affected by floor stiffness and location of the floor. Estimation by Eq. 24 can simulate such a tendency.

6. Conclusions

The followings are findings of this paper.

- 1) Since wood parts and RC parts had quite different vibration properties, they did not act in the same vibration modes.

Table 2: Natural period of 1st mode

		Basic	Floor*0.5	Floor*2.0
Type 1	Analysis	0.52	0.63	0.41
	Evaluation	0.48	0.58	0.38
Type 2	Analysis	0.51	0.65	0.40
	Evaluation	0.48	0.58	0.38

Unit:second

Table 3: Seismic force on wood parts

	Basic	Floor*0.5	Floor*2.0
Analysis	1821	1743	1806
Evaluation	1808		

Unit:kN

Table 4: Vertical seismic shear coefficient distribution
(a):From X5 to X11 (Type1)

	Conventional A_i (*)	Basic		Floor*0.5		Floor*2.0	
		Modified	Analytical	Modified	Analytical	Modified	Analytical
		A'_i	A_i	A'_i	A_i	A'_i	A_i
A_3	1.65	1.01	1.17	1.11	1.24	0.91	1.15
A_2	1.22	1.17	1.13	1.19	1.16	1.16	1.32
A_1	1.00	1.00	1.00	1.00	1.00	1.00	1.00

(b):From X1 to X4 (Type2)

	Conventional A_i (*)	Basic		Floor*0.5		Floor*2.0	
		Modified	Analytical	Modified	Analytical	Modified	Analytical
		A'_i	A_i	A'_i	A_i	A'_i	A_i
A_3	1.60	1.00	0.96	1.10	0.87	0.91	0.98
A_2	1.21	1.16	1.11	1.18	1.08	1.15	1.12
A_1	1.00	1.00	1.00	1.00	1.00	1.00	1.00

(*) $T = 0.03H = 0.31$ second

Table 5: Equivalent floor length ratio of each floor

(a):From X5 to X11 (Type1)

	Basic		Floor*0.5		Floor*2.0	
	Evaluation	Analysis	Evaluation	Analysis	Evaluation	Analysis
3rd.	0.64	0.64	0.64	0.64	0.64	0.65
2nd.		0.63		0.63		0.67
1st.		0.63		0.60		0.67

(b):From X1 to X4 (Type2)

	Basic		Floor*0.5		Floor*2.0	
	Evaluation	Analysis	Evaluation	Analysis	Evaluation	Analysis
3rd.	0.56	0.59	0.46	0.49	0.62	0.68
2nd.	0.41	0.35	0.31	0.23	0.49	0.46
1st.	0.32	0.31	0.23	0.20	0.40	0.44

2) Simplified modeling method of horizontal hybrid structure were presented. If shear walls are distributed at small intervals, idealized uniform shear panel considering its out of plane deformation having different shear stiffness in two directions is applied. If shear walls are distributed only at the edge of the building, simple spring model and the two virtual conditions are applied.

3) Based on the model, practical formulae of fundamental period, amplitude and distribution of seismic force acting on wood parts and equivalent length of floor diaphragm to evaluate moment at connection between wood part and RC part were also proposed.

4) The accuracy of the formulae was demonstrated by comparison with earthquake response analyses, and they gave close agreement with them.

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