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論 文 要 旨

THESIS SUMMARY

専攻： Department of	基礎物理学 専攻	申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of （ Science ）
学生氏名： Student's Name	Locke, Christopher Barry	指導教員（主）： Academic Advisor(main)	伊藤克司
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要旨（英文 800 語程度）
Thesis Summary (approx.800 English Words)

The ODE/IM correspondence is a mysterious relation that has been found between some ordinary differential equations (ODE) spectral problems and eigenvalues of certain operators of an integrable model (IM). It was first proposed by Dorey and Tateo 1999, where they discovered an interesting relationship between a certain Schrodinger-type ordinary differential equation with anharmonic potential and the conformal limit of a certain two-dimensional quantum integrable model. Since then, there have been many examples of this link between classical and quantum integrable models.

It turned out that this observation was just a first hint of a broader ODE/IM correspondence. One important generalization to simple Lie algebras of type ABCD was studied in Dorey et al. 2007. In this work they found (pseudo-)ODE spectral problems that correspond to a integrable vertex model in the conformal field theory limit. By using a set of functional relations called the ϕ-system between a collection of functions $\phi^{(1)}$, $\phi^{(2)}$, …, $\phi^{(r)}$ where r is the rank of the Lie algebra, which are uniquely defined solutions of (pseudo-)ordinary differential equations related to the Lie algebra, they show that certain connection coefficients $Q^{(a)}$ satisfy Bethe ansatz equations.

Another discovery was the existence of ODE/IM correspondence for massive models. The work of Lukyanov-Zamolodchikov 2010 studied the ODE/IM correspondence for the massive sine(h)-Gordon model and found that spectral determinants of a modified form of the classical sinh-Gordon model coincide with the Q-functions of the quantum sine-Gordon model, the affine Toda field theory for algebra $A^{(1)}{}_1$ (see also Dorey et al. 2013 for the classical Tzitzéica-Bullough-Dodd equation, $A^{(2)}{}_2$-type affine Toda field theory).

In this thesis, we accomplish two main things. First, in the work Ito-CL 2014 we looked at ABCDG-type affine Lie algebras and found that the (pseudo-)ordinary differential equation associated with $g^\vee$ affine Toda field equation was the same as that of Dorey et al. 2007 for simple Lie algebra g after taking the conformal limit. It was also shown that for ABCD-cases that connection coefficients for subdominant solutions to the linear problem associated with the affine Toda field equation should correspond to the vacuum eigenvalues of Q-operators for g-type quantum integrable models since they obey the appropriate Bethe ansatz equations. This was an important step forward in the study of the ODE/IM correspondence as it was the first work to look at more general affine Toda field equations and show that they also fall under the massive ODE/IM correspondence. The fact that the (pseudo-)ODEs that come from a modified affine Toda field equation reduce to those of previous works on spectral (pseudo-)ODEs connected to integrable vertex models in the conformal limit was a novel discovery.

備考 ： 論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。
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（博士課程）
Doctoral Program

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Secondly, in the work of Ito-CL 2015, we investigated the ϕ-system of Dorey et al. 2007 and show how it also holds in the massive case for subdominant solutions to the linear problem associated to a modified affine Toda field equation for simply-laced and twisted affine Lie algebras. It was noted that each function $\phi^{(a)}$ is naturally associated to a node of the Dynkin diagram, but the exact nature of this link was not clear. One of our discoveries was that these $\phi^{(a)}$ functions have a clear interpretation as the top component of the subdominant vector solution $\Psi^{(a)}$ to the linear problem associated with the modified affine Toda field equation in the fundamental representation of highest weight ω_a (see also Masoero-Raimondo-Valeri 2015 for simultaneous work for the simply laced Lie algebras in the massless case). Also, these $\Psi^{(a)}$ vectors are uniquely defined subdominant solutions to the linear problem, whose behavior is entirely determined by the eigenvalue of a certain element Λ_+ in g with largest real part and its eigenvector in the given representation. Study of the eigenvalues of Λ_+ for each fundamental representation then leads one naturally to the ϕ-system itself. This ϕ-system leads naturally to functional relations obeyed by the spectral determinants Q which have the same form as Bethe ansatz equations for a corresponding integrable model. This suggests that the ODE/IM correspondence should also exist for these more general massive affine Toda field equations.

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