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種別(和文)	論文要旨
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論 文 要 旨

THESIS SUMMARY

専攻： Department of	数学　専攻	申請学位（専攻分野）： Academic Degree Requested	博士　（　理学　） Doctor of
学生氏名： Student's Name	山木　大輔	指導教員（主）： Academic Advisor(main)	志賀　啓成
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要旨（英文 800 語程度）

Thesis Summary (approx.800 English Words)In this thesis, we study the relations between holomorphic 1-forms and holomorphic 1-cochains on closed Riemann surfaces with triangulations. In 2008, Scott Wilson applied the combinatorial Hodge theory to closed Riemann surfaces with triangulations and then defined holomorphic 1-cochains. Holomorphic 1-cochains satisfy Riemann's bi-linear relations and this property implies that for a triangulation, holomorphic 1-cochains define a unique element in the Siegel upper half space, which is called the combinatorial period matrix. Then Wilson showed that the combinatorial period matrix of a triangulation converges to the (conformal) period matrix as the mesh of the triangulation tends to zero. This implies that combinatorial period matrices provide an approximation of (conformal) period matrices. However Wilson did not study the relations holomorphic 1-forms and holomorphic 1-cochains explicitly.

First, to consider the relations, we show a further relation between combinatorial period matrices and (conformal) period matrices as follows. Although Wilson showed that combinatorial period matrices converge to (conformal) period matrices, we study a relation between them for a fixed triangulation. Then we show that for a fixed triangulation, there exists a matrix equation which includes the combinatorial period matrix and the (conformal) period matrix in the fourth chapter.

Next, using Riemann's bi-linear relations, we define an isomorphism from holomorphic 1-forms to holomorphic 1-cochains. As the main result of this thesis, we show that holomorphic 1-cochains provide an approximation of holomorphic 1-cochains, that is, we prove that the Lˆ2-differences between holomorphic 1-forms and holomorphic 1-cochains converge to 0 as the mesh of a triangulation tends to zero in the fifth chapter. To show this, we prove three theorems as follows. In the first theorem, for a holomorphic 1-form, we estimate the upper bound of the differences between holomorphic 1-forms and holomorphic 1-cochains. This estimation is proved by using the matrix equation which includes the combinatorial period matrix and the (conformal) period matrix and the upper bound is given by the diagonal elements of the imaginary parts of combinatorial period matrices and the eigenvalues of holomorphic 1-cochains for the combinatorial Hodge star operator, which is defined by Wilson in 2007. Therefore we show that the upper bound converges to zero, as the mesh of a triangulation tends to zero. Namely, we need to verify that all diagonal elements of the imaginary part of the combinatorial period matrix converge to zero, or all eigenvalues of holomorphic 1-cochains converge to 1. However, since both combinatorial period matrices and (conformal) period matrices lie in the Siegel upper half space, all diagonal elements of the imaginary part of the combinatorial period matrix do not converge to zero. Thus we need to show that the eigenvalues of holomorphic 1-cochains converge to 1. Then we may verify that for closed Riemann surface of genus 1, the eigenvalues are equal to 1 for all triangulations in the second theorem, and for genus>1, the eigenvalues converge to 1 in the third theorem.

This thesis is organized by five chapters as follows. The first chapter is the preface. In the second chapter, we explain the fundamental of Riemann surfaces. Especially, we recall the fundamental properties of holomorphic 1-forms on closed Riemann surfaces.

In the third chapter, we recall the construction of a combinatorial Hodge theory. This combinatorial theory was constructed on closed Riemannian manifolds with triangulations as follows. In 1945, Eckamann showed that an inner product on cochains of a triangulation of a closed Riemannian manifold defines harmonic 1-cochains and give the Hodge decompositions of cochains. Note that the decompositions of cochains depend upon the choice of inner products on cochains. Dodziuk and Patodi studied the relations between differential forms and cochains. To study the relations, they employed the Whitney map and the de Rham map which are the important tools of this combinatorial theory. Then Dodziuk and Patodi showed that using these two maps, for a smooth differential form, there exists a cochain which provides an approximation of the smooth differential form. In 2007, Wilson defined a combinatorial star operator on cochains which is analogue to the Hodge star operator on smooth differential forms. Also, Wilson showed that this combinatorial star operator on cochains converges to the Hodge star operator. Finally, we extend the combinatorial Hodge theory to complex-valued cochains on closed Riemann surfaces with triangulations and define holomorphic 1-cochains.

In the last two chapters, we prove the main results.