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論文 / 著書情報
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種別(和文)	論文要旨
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（博士課程）
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論 文 要 旨

THESIS SUMMARY

専攻： Department of	数学 専攻	申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of (理学)
学生氏名： Student's Name	坪井成文	指導教員（主）： Academic Advisor(main)	川平友規
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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

Let $D(r)$ $(0 < r \leq +\infty)$ denote the open disk in the complex plane with its center 0 and radius r. Let $f(z) = 1 + c_1 z^{n_1} + c_2 z^{n_2} + \cdots$ be a power series convergent in $D(R)$ with gaps, i.e. the strictly increasing sequence $\{n_k\}$ of positive integers diverges rapidly as k approaches to infinity. The study of value distribution of power series with gaps has a long history. Let $f(z)$ be an entire function. Fejér (1908) proved that if the sum of inverses of n_k converges, then $f(z)$ assumes every finite complex value at least once. A strictly increasing sequence n_k of positive integers with this condition is called a Fejér gap sequence. Biernacki (1927) improved this theorem: $f(z)$ with Fejér gaps has no finite Picard exceptional value, i.e. $f(z)$ assumes every finite complex value a infinitely often. Then detailed studies of value distribution of gap series have been done in terms of Nevanlinna theory.

We introduce the notations of Nevanlinna theory. Let $f(z)$ be analytic in $D(R)$. We define the characteristic function $T(r, f)$ $(0 \leq r < R)$ by the average over the interval $[0,\ 2\pi)$ of $\log^+ |f(re^{i\theta})|$, where $\log^+ x$ is the maximum of $\log x$ and 0. We define the proximity function $m(r,a)$ $(0 \leq r < R)$ by the average over the interval $[0,\ 2\pi)$ of $\log^+ 1/|f(re^{i\theta}) - a|$. If $T(r, f)$ diverges as r approaches to R, then the defect $\delta(a, f)$ of $f(z)$ at a is defined by the limit infimum of $m(r,a)/T(r, f)$ with r approaching to R. If a finite complex number a satisfies $\delta(a, f) > 0$, then a is called a finite defective value of $f(z)$.

Let $n(r,a)$ be the number of a-point of $f(z)$ in the open disk $D(r)$ counting with multiplicity. We define the counting function $N(r,a)$, $0 \leq r < R$ by the Lebesgue integral of $n(t,a)/t$ from $t = 0$ to $t = r$. The first main theorem of Nevanlinna states that $T(r, f) = m(r,a) + N(r,a) + O(1)$. It has to be mentioned particularly that Murai (1983) showed that an entire function $f(z)$ with Fejér gaps has no finite defective value, i.e. the Nevanlinna defect $\delta(a, f)$ of $f(z)$ vanishes for arbitrary finite complex number a. Since there are, of course, many entire functions having finite defective value whose Taylor expansions are not Fejér gap series (e.g. the exponential function), the problems of value distribution of entire functions with gaps were solved in a sence.

We shall be concerned with only the case where the radius of convergence of $f(z)$ equals 1. Unlike the case of entire functions, no relationship between the value distribution of $f(z)$ in the unit disk $D(1)$ and Fejér gap conditon has been ever known. However, if $\{n_k\}$ satisfies $n_{k+1}/n_k \geq q$ for some q larger than 1, then several results about the value distribution of $f(z)$ have been established. A sequence $\{n_k\}$ of positive integers satisfying this condition is called an Hadamard gap sequence. It is obvious that an Hadamard gap sequence is a Fejér gap sequence. Hadamard (1892) proved that $f(z)$ with Hadamard gaps whose convergent radius is 1 has the unit circle as its natural boundary. Fuchs (1967) proved that if an analytic function $f(z)$ in the unit disk with Hadamard gaps satisfies (A): the limit supremum of c_k is positive, then $f(z)$ assumes zero infinitely often. Murai (1980) improved this theorem: under the same conditions, the Nevanlinna defect $\delta(0, f)$ of $f(z)$ at 0 vanishes. More precisely he showed that if (and only if) (B): the square sum of moduli of coefficients c_k diverges, then the Nevanlinna characteristic function $T(r, f)$ diverges as r approaches to 1 and if we assume (A), then the proximity function $m(r,0)$ is bounded as r approaches to 1 through a suitable sequence of r. Remark that these results yield that $f(z)$ with Hadamard gaps and (A) has no finite defective value, that is, $\delta(a, f)$ vanishes for arbitrary finite complex number a. (See Corollary of this paper.)

Now we turn to consider the case where (C): the limit c_k is zero. Murai (1981) also showed that if an analytic function $f(z)$ in the unit disk with Hadamard gaps and condition (C) is unbounded in the unit disk, then $f(z)$ assumes zero infinitely often. It is well known (Sidon, 1927) that such $f(z)$ is unbounded in the unit disk if and only if (D): the sum of moduli of coefficients c_k diverges. Therefore it is natural to ask whether for $f(z)$ with Hadamard gaps, condition (B) and (C), $\delta(0, f)$ equals zero holds or not. (Note that the conditions (B) and (C) imply (D), and the radius of convergence of of $f(z)$

with Hadamard gaps, condition (B) and (C) must be 1.) We shall study this problem and show a sufficient condition for $\delta(0,f)$ to vanish in the present paper. In particular, our main theorem will show that if the square sum of moduli of the coefficients c_k has sufficiently rapid growth, then $\delta(0,f)$ vanishes.

Here is a brief outline of our proof of this theorem. Main tools for our proof are the central limit theorem for Hadamard gap series, an analogue of Poisson-Jensen formula for sectors, BMO norm inequality for Hadamard gap series and an operator introduced by Littlewood and Offord. First we construct a sequence $\{R_l\}$ of radii for the function $f(z)$ such that near R_l we can estimate the derivative of $f(z)$ and apply the Littlewood-Offord operator. Next we show that the measure of the set of points θ such that the modulus of $f(R_l e^{i\theta})$ is smaller than 1 is very small. Note that on the complement of this set $\log^+ 1/|f(re^{i\theta})|$ is zero and this estimate will be proved by using the central limit theorem. We represent this set as a finite disjoint union of closed intervals I_j and consider the sectors whose arcs are I_j. Applying an analogue of Poisson-Jensen formula for sectors to these, BMO norm inequality for Hadamard gap series and Littlewood-Offord operator yield that the average over the interval I_j of $\log^+ 1/|f(R_l e^{i\theta})|$ is dominated by $T(R_l,f)$. Therefore the central limit theorem implies that $m(R_l,0)$ is of small order of $T(R_l,f)$ as l approaches to infinity. This proves our theorem.

備考 : 論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。

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