論文 / 著書情報
Article / Book Information

題目(和文)	平均場模型の量子アニーリングにおける量子相転移の制御
Title(English)	Control of quantum phase transitions in mean-field models for quantum annealing
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出典(和文)	学位:博士(理学), 学位授与機関:東京工業大学, 報告番号:甲第10061号, 授与年月日:2016年3月26日, 学位の種別:課程博士, 審査員:西森 秀稔,斎藤 晋,古賀 昌久,西田 祐介,竹内 一将
Citation(English)	Degree:Doctor (Science), Conferring organization: Tokyo Institute of Technology, Report number:甲第10061号, Conferred date:2016/3/26, Degree Type:Course doctor, Examiner:,,,,
学位種別(和文)	博士論文
Category(English)	Doctoral Thesis
種別(和文)	論文要旨
Type(English)	Summary

（博士課程）
Doctoral Program

論 文 要 旨

THESIS SUMMARY

専攻： Department of	物性物理学　専攻	申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of　（　理学　）
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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

We study statistical-mechanical properties of mean-field models to reveal an effect of various types of quantum fluctuations against the efficiency of quantum annealing (QA).

Quantum annealing is a quantum computation model to solve combinatorial optimization problems by taking advantage of quantum fluctuations. Since combinatorial optimization problems can be expressed as finding global minimum of a scalar function (cost function) with binary variables, the problems are equivalent to searching the ground state of Ising spin systems. Quantum annealing can find the ground states through the simulation of quantum system that is governed by the Schrödinger equation. The Hamiltonian of the system consists of two parts: problem part and driver part. The problem part corresponds to cost function. The driver part must not commute with the problem part, and induce quantum fluctuations into the system. A central issue of QA is to reduce the running time needed to solve the problems. It has been reported that the conventional QA using the transverse-field term as the driver Hamiltonian requires exponentially-long time for certain problems such as the ferromagnetic p-spin model.

Quantum phase transition is closely related to the efficiency of QA. According to the finite-size scaling theory, systems that undergo a second-order quantum phase transition have polynomial decay of the minimum energy gap in the vicinity of the phase transition point. It is known that the required running time of QA is evaluated by the inverse square of the minimum energy gap. Since the running time of QA increases at most polynomially, QA is efficient for the problem. On the other hand, systems with first-order phase transition typically have the minimum gap that decays exponentially. Hence, QA fails to find the ground state in a reasonable time except special cases. We can thus estimate the efficiency of QA by analyzing the degree of quantum phase transition of the system.

A solution to avoid the difficulty mentioned above is to use a degree of freedom of QA. Although many studies adopt the transverse-field term as a driver part of the Hamiltonian, there is no restriction to use the term. The present dissertation shows that first-order quantum phase transition can be avoided by using the degree of freedom.

We first show that the first-order quantum phase transition in the ferromagnetic p-spin model can be avoided by using quantum fluctuations induced by transverse antiferromagnetic interactions. We analyze the phase diagram of the model by using the mean-field analysis. The results indicate that the first-order quantum phase transition changes to the second-order quantum phase transition thanks to the antiferromagnetic term. In addition, we calculate the minimum energy gap of the model. The results show that the running time of QA with the antiferromagnetic term increase at most polynomially.

We next study the quantum Hopfield model to check if the antiferromagnetic term is effective for random spin systems. We adopt the Hopfield model as a random spin system because the model is expected to cover a wide range of randomness. We use the mean-field analysis again, and the replica method to deal with the randomness. The results indicate that the antiferromagnetic term is also effective for certain random spin systems.

Finally, we investigate an alternative way to avoid first-order quantum phase transitions by using the Wajnflasz-Pick model. The classical Wajnflasz-Pick model is known as a model whose phase transition can be controlled by adjusting the orders of degeneracy of the upper spin state and the lower spin state. Through the mean-field analysis of the quantum Wajnflasz-Pick model, we find that the order of quantum phase transition of the model can be also controlled by the orders. Furthermore, we show that the order of quantum phase transition depends on a transition factor between the degenerate states which is considered as a degree of freedom of quantum systems.