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論 文 要 旨

THESIS SUMMARY

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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

This work is divided in two parts. In the first part, we examine several issues related to ill-posedness in linear conic programming such as weak infeasibility, nonzero duality gaps and nonattainment of optimal values. Analysis of these issues is important not only from a theoretical perspective but also from a practical one, since the presence of these phenomena may cause well-known conic programming solvers to display strange behavior. In this work, we will analyze and, to some extent, fix these issues with techniques based on facial reduction. Among our contributions, we have a geometrical analysis of weakly infeasible problems which ends up revealing the number of directions needed to approach the underlying cone. Moreover, for unattained problems we show how to obtain solutions arbitrary close to optimality without repeatedly solving additional conic linear programs.

One of the main results of this part is a proof that although an arbitrary conic linear program can be very badly behaved indeed, if we suppose that we are able to solve "well-behaved" problems, we can "completely solve" any conic linear program. This is accomplished with a technique known as facial reduction and a finite sequence of auxiliary problems that are assuredly "well-behaved" even if the original problem is not. In order to model our hypothetical ability of solving well-behaved problems we introduce the notion of "interior point oracle", which is a conceptual machine that receives problems that are both primal and dual strongly feasible and outputs a pair of primal and dual optimal solutions. We note that this kind of assumption underlines a number of algorithms and software for conic optimization. The meaning of "completely solved" is explained in detail in the thesis, but it includes, among other things, the capacity of identifying whether the problem is feasible/infeasible, obtaining the optimal value and an optimal solution (if it exists). If the optimal value is finite but there is no optimal solution, "completely solving" also entails obtaining approximate optimal solutions.

Since our tool of choice is facial reduction, we also take a deep look at it. In a nutshell, facial reduction algorithms aim at finding the minimal face of the cone that contains the feasible region of the problem in question. In this thesis, we propose "FRA-Poly", a new facial reduction algorithm that it is able to exploit the existence of polyhedral faces in the lattice of faces of an arbitrary closed convex cone in order to find the minimal face in less reduction steps. FRA-poly, in opposition to the classical approach, only proceeds until a polyhedral face is reached and, then, jumps directly to the minimal face. This is accomplished by carefully tailored auxiliary problems. FRA-Poly is no worse than the classical approach and, under a mild condition, an upper bound for the number of reduction steps can be obtained that is smaller than the bound for the classical facial reduction approach. In particular, we show that for the doubly nonnegative cone, FRA-Poly ends after at a quantity of steps that is linear in the number of rows, while the classical analysis gives a bound that is quadratic.

In the second part of this thesis we take a look at the nonlinear semidefinite programming and we look at the prospects of transforming a nonlinear semidefinite program (NSDP) into a common nonlinear program (NLP) by introducing squared slack variables. By doing so, we transform a relatively hard positive semidefiniteness constraint into several equality constraints. This work is a natural continuation of the research line carried out by Fukuda and Fukushima on the usage of slack variables for nonlinear second order cone programming. This time, we show that

this provides an easy route for deriving second order optimality conditions for NSDPs. Furthermore, we show that those conditions are essentially equivalent to the ones derived from considerations based on variational analysis. In particular, we show that this approach produces "no-gap" conditions. We also present a detailed analysis of KKT points, constraint qualifications and optimality conditions for both formulations.

This inquiry also has a practical side. Since solvers for NSDPs are still scarce, it makes sense to consider the possibility of using NLP solvers, since they are more widely available. In this part, we also include a few preliminary computational results, where we take a few problems and compare the performance of solving them directly as NSDPs and solving them by using slack variables to transform them into NLPs. These experiments were carried with the PENNON solver, one of the few solvers out there that is able to handle both types of problems.

Finally, we conclude the thesis by examining future research directions for both themes discussed in this thesis.