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Article / Book Information

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種別(和文)	論文要旨
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論 文 要 旨

THESIS SUMMARY

専攻： Department of	知能システム科学　専攻	申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of　（　理学　）
学生氏名： Student's Name	許　インイン	指導教員（主）： Academic Advisor(main)	樺島祥介
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要旨（英文 800 語程度）
Thesis Summary (approx.800 English Words)

A theory to connect the microscopic and the macroscopic worlds was needed after the discovery of the microscopic world, which gave birth to statistical mechanics. Statistical mechanics was the first foundational physical theory in which probabilistic concepts and probabilistic explanation played a fundamental role. Since the framework of statistical mechanics is general, its application domains are wide-ranging. Nowadays, it starts to come in to the world of information, which is abstract and different from the traditional physical object. The connection between microscopic and macroscopic behaviors is the core of the so-called "big data" research, which is also the essence of statistical mechanics. The common goal of physics and information science is to infer the behavior of the system from the observed data. Realizing the same essence, can crash down the barrier between different research areas and enhance the ability of the whole. This thesis is an example of how method developed in physics can empower the theory and algorithm in information science.

The 1-bit compressed sensing framework enables the recovery of a sparse vector from the sign information of each entry of its linear transformation. Discarding the amplitude information can significantly reduce the amount of data, which is highly beneficial in practical applications. For simplicity, we consider the case that the measuring matrix has i.i.d entries, and the measurements are noiseless.

First, we analyze the typical performance of an L1-norm based signal recovery scheme for the 1-bit compressed sensing using statistical mechanics methods. Signal recovery based on the L1-norm minimization is a standard approach in CS research. Unlike the normal CS scheme, however, the L1-based signal recovery cannot be formulated as a convex optimization problem, which makes practically performing it nontrivial. We have shown that the theoretical prediction of the performance of the L1-based scheme, which is obtained by the replica method under the replica symmetric (RS) ansatz, exhibits a fairly good accordance (in terms of MSE) with experimental results obtained using for an approximate signal recovery algorithm, RFPI. The replica symmetry of the RS solution turned out to be broken, however, which implies that there are many local optima for the optimization problem of the signal recovery. Our results suggest that the local optima, which can be searched by RFPI, yield similar values of MSE representing the potential performance limit of L1-based recovery scheme. We have also developed an approximate signal recovery algorithm utilizing the cavity method. Naive iterations of self-consistent equations derived directly from the cavity method hardly converge in most cases, which can be regarded as a consequence of the replica symmetry breaking. However, we have shown that modification of one equation in an appropriate manner, in conjunction with controlling a macroscopic variable in the outer loop, results in a fairly good signal recovery algorithm.

Compared with RFPI, the resultant algorithm is beneficial in that the number of tuning parameters is reduced from two to one. Numerical experiments have also shown that whenever the density of nonzero entries of the original signal is not considerably small the cavity-inspired algorithm performs as well as or better than RFPI (in terms of MSE) and has a lower computational cost.

To further develop the study of L1-norm based signal recovery scheme for 1-bit compressed sensing, we suggest a strategy that captures scale information by introducing a threshold parameter to the quantization process. Considering the most general situation, where no detailed prior knowledge of sparse signals is available, we employed the L1-norm minimization approach. By utilizing the replica method from statistical mechanics, the mean squared error behavior of reconstruction for standard i.i.d measurement matrix and i.i.d Bernoulli-Gaussian signal was derived in the large system size limit. We compared two design strategies for the elements of the threshold vector, which corresponded to setting a fixed or random value as threshold. Our analysis showed that the fixed threshold strategy achieve lower MSE than the random threshold strategy.

Another observation from the replica results was that there is an optimal threshold that minimizes MSE for a set of signal distributions and measurement ratios. However, in order to evaluate the optimal threshold, we need to know the prior distribution of the signal, which is not necessarily available in practical situations. Therefore, we shifted our focus to the relation between the optimal threshold and the distribution of the binary outputs, which can be empirically evaluated from signal measurements. The replica analysis indicated that the MSE is minimized when $P(y = +1)$ is set in the vicinity of 0.75 to 0.85 for a wide region of system parameters.

On the basis of this observation, an algorithm that adaptively tunes the threshold at each measurement in order to obtain $P(y = +1)$ close to our target value was proposed. Combined with versatile convex optimization algorithms, the adaptive thresholding algorithm offers a computationally feasible and widely applicable 1-bit CS scheme. Numerical experiments showed that it can yield nearly optimal performance, even when no detailed prior knowledge of sparse signals is available.

Besides L1-norm minimization, we present a Bayesian approach to signal reconstruction for 1-bit compressed sensing, which provides the optimal bound of the recovery performance of 1-bit measurements. Utilizing the replica method of statistical mechanics to analyze the typical performance, we show that the Bayesian approach enables better reconstruction than the L1-norm minimization approach, asymptotically saturating the performance obtained when the non-zero entries positions of the signal are known under an appropriate condition. We also test a message-passing algorithm for signal reconstruction on the basis of belief propagation. The results of numerical experiments are consistent with those of the theoretical analysis.