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# A SIMPLIFIED LONG DURATION MODEL FOR ANALYSIS OF VISCOELASTIC DAMPERS CONSIDERING HEAT TRANSFER

構造一振動

Viscoelastic Damper, Long Duration Excitation, Uniform Strain Distribution, Fractional Derivative, Heat Transfer

# 1. Introduction

#### 1.1 Background

One of the widely-used devices to control vibration of structures is viscoelastic damper (Fig. 1). It works by dissipating energy induced by the seismic or wind excitations. In the process of energy dissipation, heat is generated within the viscoelastic (VE) material, affecting its dynamic mechanical properties. The heat will flow through the material and will then be dispersed to the surrounding air by convection (Fig. 2).

A time-history analysis model based on fractional time-derivatives of strain and stress was formulated by Kasai et al <sup>1)</sup>. It considered the heat generation and consequent softening of the VE material. The model is accurate enough specifically for short duration excitation, during which heat convection is negligible, and will be called as the "short duration model".

#### 1.2 Long Duration Model

On the other hand, for longer duration loading such as wind, heat conduction and convection (Fig. 2) greatly affect the temperature distribution in the damper.



Kasai et al in 2006 <sup>2)</sup> conducted steady-state heat transfer analysis by using three-dimensional (3-D) finite



Fig. 2. Heat generation, transfer, and convection of VE damper

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elements for the VE damper experimented under long duration loading applying harmonic displacement (Fig. 1). The analysis accurately simulated the distributions of temperature as well as corresponding stiffness and damping in the VE material at the steady-state. The temperature distribution is found to be uniform within the VE material except for the thickness direction.

Based on the findings, Kasai et al proposed the "long duration model", the time-history model combining the simplified one-dimensional (1-D) heat transfer analysis with the viscoelastic constitutive rule<sup>2)</sup>. The model predicts lower temperature and less shear strain near the interface with the steel plate, and predicts higher temperature and more strain at inner locations, (Fig. 2). As Fig. 3 shows, however, analytically obtained deformed shape of the VE material are almost straight, suggesting approximately uniform strain.

The present study therefore proposes a simplified long duration (SLD) model idealizing the shear strain to be uniform in the VE material, in contrast to the original and detailed long duration (DLD) model. The former is much more computationally-efficient than the latter.



Fig. 3. Deformed shape of VE material<sup>2)</sup>

#### 2. Formulation

# 2.1 Damper Stress and Strain

Fig. 4a shows a VE damper model subjected to dynamic loading, where  $F_d^{(n)}$  and  $u_d^{(n)}$  are damper force and deformation at time step *n*. By equilibrium, stress  $\tau^{(n)}$  is uniform along the thickness direction. For node *j* of the VE material (Fig. 4b), the constitutive equation of the DLD model was given as <sup>2</sup>:

$$\tau^{(n)} + a_j^{(n)} D^{\alpha} \tau^{(n)} = G \cdot \left[ \gamma_j^{(n)} + b_j^{(n)} D^{\alpha} \gamma_j^{(n)} \right]$$
(1)

where G = static shear modulus (N/cm<sup>2</sup>);  $\tau^{(n)}$  = shear stress (N/cm<sup>2</sup>);  $\gamma_j^{(n)}$  = shear strain; and  $a_j^{(n)}$  and  $b_j^{(n)}$  = temperature-dependent constants at node *j*, and;  $D^{\alpha}$  = fractional derivative operator<sup>1), 2)</sup> of order  $\alpha$ .

As for the SLD model, the above shear strain  $\gamma_j^{(n)}$  is approximated by uniform strain  $\gamma^{(n)}$ . This also implies that  $a_j^{(n)}$  and  $b_j^{(n)}$  are approximated by uniform values  $a^{(n)}$  and  $b^{(n)}$ , respectively. Thus, one obtains the constitutive equation like that of the short duration model (1993) mentioned earlier, i.e.,

$$\tau^{(n)} + a^{(n)} D^{\alpha} \tau^{(n)} = G \cdot [\gamma^{(n)} + b^{(n)} D^{\alpha} \gamma^{(n)}]$$
(2)

where for the given damper displacement  $u_d^{(n)}$  and thickness *H* of the VE material,

$$\gamma^{(n)} = u_d^{(n)} / H \tag{3}$$

Also, the  $a^{(n)}$  and  $b^{(n)}$  vary due to the change of the temperature as follows:

$$a^{(n)} = a_{ref} \cdot \lambda^{(n)} \quad , \qquad b^{(n)} = b_{ref} \cdot \lambda^{(n)} \tag{4}$$

where  $a_{ref}$  and  $b_{ref}$  = values at reference temperature  $\theta_{ref}$ , and  $\lambda^{(n)}$  = shift factor defined below:

$$\lambda^{(n)} = \exp \frac{-P_1(\bar{\theta}^{(n)} - \theta_{ref})}{\left(P_2 + \bar{\theta}^{(n)} - \theta_{ref}\right)}$$
(5)



Note that  $\bar{\theta}^{(n)}$  is the average of the nodal temperature  $\theta_i^{(n)}$  to be explained in Sec. 2.2.

Considering step-by-step integration scheme

$$\tau^{(n)} + \frac{a^{(n)}}{(\Delta t)^{\alpha}} \sum_{i=0}^{N} w^{(i)} \tau^{(n-i)} = G \cdot \left[ \gamma^{(n)} + \frac{b^{(n)}}{(\Delta t)^{\alpha}} \sum_{i=0}^{N} w^{(i)} \gamma^{(n-i)} \right]$$
(6)

where  $\Delta t$  =time step size,  $w^{(i)}$  =weight function<sup>1) 2)</sup>, and N =number of integration steps. Substituting Eq. 3 into Eq. 6, and rearranging the terms, the stress at time step can then be calculated by Eq. 7.

$$\tau^{(n)} = \frac{\frac{u_d^{(n)}}{H}GB + Gb^{(n)}\sum_{i=1}^N w^i \gamma^{(n-i)} - a^{(n)}\sum_{i=1}^N w^i \tau^{(n-i)}}{A}$$
(7)

where  $A = [(\Delta t)^{\alpha} + a^{(n)}w^{(0)}]$  and  $B = [(\Delta t)^{\alpha} + b^{(n)}w^{(0)}]$ . Damper force  $F_d^{(n)}$  can then be calculated by

$$F_d^{(n)} = \tau^{(n)} \cdot A_v \tag{8}$$

where  $A_{v}$  = shear area of the VE material.

### 2.2 Temperature Rise and Heat Transfer

Since the strain distribution is considered to be uniform, the energy dissipated at any node in the VE material is also uniform at each time step, i.e.

$$\Delta E_d^{(n)} = \frac{(\tau^{(n)} + \tau^{(n-1)})(\gamma^{(n)} - \gamma^{(n-1)})}{2} \tag{9}$$

Then the temperature rise due to dissipated energy will also be uniform for each element, and

$$\Delta \theta^{(n)} = \frac{\Delta E_d^{(n)}}{s\rho} \tag{10}$$

where s and  $\rho$  = specific heat capacity (N·cm/kg/°C) and mass density (kg/cm<sup>3</sup>) of the VE material. Eqs. 9 and 10 are similar those used in the short duration model<sup>1)</sup>.

The temperature of the VE damper at time step n + 1 at node *j*, however, is not necessarily uniform, since it depends on the temperature rise due to energy dissipated  $\Delta \theta^{(n)}$  and the temperature  $\hat{\theta}_j^{(n)}$  rises or falls due to the transfer of heat via conduction and convection, i.e.

$$\theta_i^{(n+1)} = \hat{\theta}_i^{(n)} + \Delta \theta^{(n)} \tag{11}$$

where  $\hat{\theta}_{j}^{(n)}$  is calculated by the 1-D heat transfer analysis method used for the DLD model<sup>2)</sup>.

# 3. Implementation

The experimental results and analytical results by using the DLD model (2006) will be compared with the analytical prediction by the proposed SLD model.

#### 3.1 Damper Properties

The viscoelastic material used was a 3M-ISD110 type with dimensions W = 3.76 cm , L = 5.08 cm , and H = 1.33 cm (Fig. 5). Thickness of steel plates  $d_s = 0.48$  cm. Total shear area  $A_v = 38.17$  cm<sup>2</sup>.

The VE material properties were provided by the manufacturer as follows:  $G = 6.519 \text{ N/cm}^2$ ;  $\alpha = 0.609$ ; at reference temperature  $\theta_{ref} = 0.2 \text{ °C}$ ,  $a_{ref} = 0.0115$  and  $b_{ref} = 21.157$ , and;  $P_1 = 19.5$  and  $P_2 = 80.2$ 

Additional properties such thermal conductivity  $\kappa$ , specific heat capacity *s*, and mass density  $\rho$  of steel plates and VE material are indicated in Table 1.

Furthermore, from the 3D finite element analysis (Sec 1.2), the heat transfer coefficients  $\alpha_{c,out}$  and  $\alpha_{c,in}$ , for the outer and inner plates, were 0.956 and 0.524 N/s/ cm/°C, respectively.



Fig. 5. Damper dimension

Table 1.	Material	Properties	of Steel	and V	Е
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	Steel	VE		
$\kappa$ (N/s/°C)	43.128	0.188		
s(N·cm/kg/°C)	46.63 x 10 <sup>3</sup>	$19.40 \ge 10^4$		
$\rho$ (kg/cm <sup>3</sup> )	7.80 x 10 <sup>-3</sup>	1.00 x 10 <sup>-3</sup>		
$1 \text{ N/s/cm/}^{\circ}\text{C} = 100 \text{ W/m}^{2}/^{\circ}$				

#### 3.2 Loading Conditions

Damper was subjected to harmonic displacement of peak value 0.66 cm (50% maximum strain level) at a frequency of 0.33 Hz. Loading duration was from t = 0 to 3000 s but analysis continued up to t = 5000 s. Ambient temperature for the test was 24°C.

#### 3.3 Damper Model

For SLD model, VE material was discretized into 16 elements, and outer and inner plates were divide in to 4

and 2 elements, respectively.

Temperature at points A, B, C and D (Fig5b) were investigated. Their locations are defined as: A at z = 0 (at outer plate surface); B at z = 0.48cm + 0.25*H*; C at z = 0.48cm + 0.50*H*, and; D at z = 2.29cm (center of inner plate).

#### 4. Results

#### 4.1 Temperature Rise

Fig. 6 compares temperatures predicted by DLD and SLD models. The latter agrees well with the former.

Noting that the mid-portion of the VE material has the greatest rise in temperature. By convection process, heat in the steel plates are transferred to the air, and with steel conducting heat much faster, the VE near the steel plates loses heat faster than the mid-portion. Thus, heat is accumulated more in the mid-portion.



#### 4.2 Stiffness

As shown by Fig. 7, the storage stiffness  $K'_d$  and loss stiffness  $K'_d$  of the SLD model also agrees well with those of the DLD model, indicating that the damper force and the stress of the two models are almost equal.

Both models were in steady-state when thermal equilibrium was reached (i.e. the amount of heat produced is equal to the heat transferred).



Fig. 7. (a) Storage stiffness and (b) Loss Stiffness

### 4.3 Strain Distribution and Dissipated Energy

The strain distribution of the DLD model varied along the thickness of the VE material (Fig. 8). Mid-portion (elements 12 & 16) had the highest strain while those nearest the steel plates (elements 5 & 20) had the lowest, respectively.

However, the strain varied from 44% to 52%, which are within  $\pm 12\%$  of the average strain of 50%. This could justify the idealization of uniform strain considered by the SLD model.

Fig. 9(a) shows the histories of the energy dissipated by the SLD and DLD models, respectively, and Fig. 9(b) shows those by element 5 (Fig. 8) where the largest discrepancy of the strain occurs between the two models.

However, the sum of energy at each element does not differ much for the two models. Both, DLD and SLD models show almost the same amount of total energy even where the maximum difference is expected (element 5).



#### 4.4 Hysteresis Loop

Fig. 10 shows that the hysteresis loops for the DLD and SLD models are almost identical. For  $t = 0 \sim 1000$  s, the damper was in transient-state as manifested by changing hysteresis. After long time (Figs 10c & d), the thermal equilibrium was reached and the VE damper was stable and had responded steadily (in steady-state). This is the greatest advantage of the long duration models since inclusion of heat transfer and convection in the analysis greatly defines the real behavior or VE damper when subjected to long duration loading.



Fig 10. Hysteresis loop at different time

# 5. Conclusion

The simplified long duration (SLD) model based on fractional time-derivative and idealized uniform strain distribution of the VE material were proposed for analysis of VE damper. The analytical results from the SLD and those from the DLD (detailed long duration) model showed high congruency.

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