論文 / 著書情報
Article / Book Information

題目(和文)	分数量子 Hall 状態の 1 次元描像と関連するスピン鎖
Title(English)	One-dimensional description of fractional quantum Hall states and related spin chains
著者(和文)	汪正元
Author(English)	Zhengyuan Wang
出典(和文)	学位:博士(理学), 学位授与機関:東京工業大学, 報告番号:甲第10057号, 授与年月日:2016年3月26日, 学位の種別:課程博士, 審査員:村上 修一,斎藤 晋,藤澤 利正,西田 祐介,笹本 智弘
Citation(English)	Degree:Doctor (Science), Conferring organization: Tokyo Institute of Technology, Report number:甲第10057号, Conferred date:2016/3/26, Degree Type:Course doctor, Examiner:,,,,
学位種別(和文)	博士論文
Category(English)	Doctoral Thesis
種別(和文)	要約
Type(English)	Outline

論 文 概 要

THESIS SUMMARY AND OUTLINE

One-dimensional description of fractional quantum Hall states and related spin chains

(分数量子 Hall 状態の 1 次元描像と関連するスピン鎖)

Department of Physics, Tokyo Institute of Technology
Zheng-Yuan Wang

In this thesis, we study fractional quantum Hall (FQH) states with thin (but finite) torus geometry, where the original two-dimensional system in a magnetic field can be regarded as a one-dimensional lattice model with short-range interactions. Based on this point of view, we discuss exactly solvable one-dimensional lattice models describing the filling factor $\nu = 1/q$ Laughlin states. Using matrix product method, the physical properties of those states are obtained analytically. We further discuss the relationship between the hierarchy of FQH states and the Haldane conjecture for quantum spin chains, by mapping the $m = 1$ Jain states to effective spin chains. We also discuss the $\nu = 1/2$ FQH states in a system with Landau level mixing. The one-dimensional lattice model has a ladder structure, and the Hamiltonian can be rewritten with projection operators by converting the 2-body interaction into 3-body one.

We review the standard theory for the FQH problem. This theory well describes the FQH states as incompressible quantum fluids based on the variational wave functions. However, the supposed wave functions are written in a first-quantized form, so that there are few methods to understand the physical origin and the physics of the states. We also review some important works based on the one-dimensional framework for the FQH states with torus geometry. In the TT (Tao-Thouless a.k.a. thin-tours) limit, the model is exactly solvable, and the eigenstates can be derived. Nevertheless phase transitions occur upon deformation of the torus, in even denominator states.

We then introduce models with an exact ground state describing the $\nu = 1/q$ FQH states on a torus. We show the method to construct the ground state of the $\nu = 1/3$ model firstly, and then extend this argument to the $\nu = 1/q$ Laughlin states. We also show that the interaction of this model has the same degree of freedom as that of the pseudo potential for the Laughlin wave function, and one can naturally derive general properties of the Laughlin wave function such as the Z_2 classification and the relation between fermions and bosons. Thus our model gives a simple reference model to describe the $\nu = 1/q$ Laughlin states.

We introduced the matrix-product method to obtain the exact ground states. The physical quantities such as correlation functions, string order parameters, entanglement entropy, and excitation spectra are calculated analytically using the matrix product method. The obtained ground states have many properties similar to Laughlin states. For example, they have the common feature that the charge density functions tend to be uniform with the torus becoming isotropic. We can conclude that our solvable model captures physical features of the $\nu = 1/q$ FQH system. We show that the $m = 1$ Jain's series, with the filling factor $\nu = p/(2p+1)$ and$\nu = (p+1)/(2p+1)$, can be mapped to $S = 1$ effective spin chains with p sites in a unit cell by considering the dominant hopping term. By numerically analyzing, we point out that these effective spin chains have similar properties to those of $S = p$ integer Heisenberg chains with Haldane gaps which decrease as S is increased. These results give one of the explanations regarding the relationship between the hierarchy of FQH states and Haldane conjecture on the quantum spin chains.

Finally, we introduce a 1D representation of the FQH systems with Landau level mixing. We show the 2-body interactions in a $\nu = 1/2$ FQH system can be converted into 3-body interactions in each unit cell, which correspond the 3-body Haldane pseudo-potential for the even-denominator FQH states. These interactions can be written into projection operators in the Hamiltonian, so that we can find the exact ground state of this system in the thin torus.