

論文 / 著書情報
Article / Book Information

題目(和文)	粒状体上の表面基礎の支持力に関する研究
Title(English)	On the bearing capacity of surface foundation on granular media
著者(和文)	上野勝利
Author(English)	
出典(和文)	学位:博士(工学), 学位授与機関:東京工業大学, 報告番号:乙第3546号, 授与年月日:2001年7月31日, 学位の種別:論文博士, 審査員:
Citation(English)	Degree:Doctor (Engineering), Conferring organization: Tokyo Institute of Technology, Report number:乙第3546号, Conferred date:2001/7/31, Degree Type:Thesis doctor, Examiner:
学位種別(和文)	博士論文
Type(English)	Doctoral Thesis

G2001
U

On the bearing capacity of surface foundation on granular media

Katsutoshi Ueno

March 1, 2001

Contents

1	Introduction	17
1.1	Background	17
1.2	Brief review of previous studies	18
1.2.1	Problems related to the theme	18
1.2.2	Method to account for size effects	21
1.3	Objectives and scope	22
1.4	Composition of the present thesis	24
2	Materials	31
2.1	General	31
2.2	Overview of materials	31
2.2.1	Natural sand	31
2.2.2	Graded sand	32
2.2.3	Artificial materials	33
2.3	Physical properties	34
2.3.1	Soil indices	34
2.3.2	Angularity	34
2.4	Stress dependency of shear strength	35
3	Apparatus and measurement	51
3.1	General	51
3.2	Centrifuge apparatus	51
3.3	Setup for loading test	53
3.3.1	Soil container	53
3.3.2	Loading system	53

3.3.3	Measurements and devices	53
3.4	CCIP method: measuring method of displacement field for sand by image processing	55
3.4.1	Review on measurement of displacement in model tests	55
3.4.2	Development of a displacement field measuring technique	56
3.4.3	Evaluation of accuracy of CC Method	59
3.4.4	Example of application	62
3.5	Summary	62
4	Influence of boundary and initial conditions	89
4.1	General	89
4.1.1	Influence of boundary friction	89
4.1.2	Influence of repetitious centrifuge operation	91
4.2	Friction between sand and container	91
4.2.1	Direct shear tests	91
4.2.2	Materials	92
4.2.3	Results	92
4.3	Stress distribution in a container	93
4.3.1	Influence of friction	94
4.3.2	Influence of centrifuge accelerating operation	98
4.4	Loading Test	100
4.4.1	Test procedure	100
4.4.2	Test results	100
4.4.3	Discussion	102
4.5	Summary	103
4.5.1	On friction and lubrication	103
4.5.2	On initial condition	103
5	Particle versus shear band	135
5.1	General	135
5.2	Introduction	135
5.3	Material used	137
5.4	Test apparatus	137

5.5	Test procedure	138
5.5.1	Specimen preparation	138
5.5.2	Shearing test specimen	138
5.5.3	Test program	138
5.6	Deformation pattern and formation of shear band	139
5.7	Definition of the thickness of shear band	140
5.8	Test results and discussion	141
5.8.1	General remarks	141
5.8.2	Influences of depth of specimen	141
5.8.3	Influence of relative density	142
5.8.4	Influence of mean grain size and grain shape	142
5.9	Summary	143
6	Size effects on bearing capacity	171
6.1	General	171
6.2	Procedure of loading test	172
6.3	Results	173
6.4	Discussions	174
6.4.1	Size effects in load intensity - settlement curves	174
6.4.2	Size effects in settlement at failure	175
6.4.3	Size effects in subgrade reaction factor	175
6.4.4	Size effects in ultimate bearing capacity	176
6.5	Summary	177
7	Analysis and design	201
7.1	General	201
7.2	Introduction	202
7.3	Development of prediction method M-I	205
7.3.1	Extended slip line method incorporating stress level effects	205
7.3.2	Numerical Calculation of Bearing Capacity	207
7.3.3	Discussion on calculation results and derivation of simplified prediction method M-I	208
7.4	Development of prediction method M-II	210

7.4.1	Finite element method	210
7.4.2	Numerical Calculation of Bearing Capacity	210
7.4.3	Stress range in the ground beneath footing	211
7.4.4	Proposal of a prediction method M-II for cohesive and cohesionless soils	212
7.4.5	Verification with FEM results	213
7.5	Verification of the proposed prediction methods	213
7.6	Summary	217
8	Conclusions	245
8.1	Development of a measuring method of displacement field for sand by image processing	245
8.2	Influence of boundary and initial conditions	246
8.2.1	On friction and lubrication	246
8.2.2	On initial condition	247
8.3	Particles and shear bands	247
8.4	Size effects on bearing characteristics	248
8.5	Proposal of design methods	249

List of Figures

1.1	Variation of bearing capacity factor with foundation width after de Beer (1965)	27
1.2	Particle size effect (after Siddiquee <i>et al.</i> , 1999)	27
1.3	Comparison of computed and measured value of $Q_d/\gamma B^3$ (after Shiraishi 1990).	28
1.4	Mohr's stress circles at failure and failure envelopes obtained from conventional drained triaxial compression tests on.	29
1.5	Composition of the present thesis	30
2.1	Grain size distribution curves of Inagi Sand (IN, INS and INL)	40
2.2	Grain size distribution curves of other natural sands.	40
2.3	Grain size distribution curves of graded sands used in shear band tests in Chapter 5.	41
2.4	Definition of <i>angularity</i> in plan view of a particle.	44
2.5	Mohr's stress circles at failure	46
2.6	Failure criterion for cohesionless soils with confining stress dependency of shear strength.	47
2.7	Failure criterion for cohesive soils with confining stress dependency of shear strength.	48
2.8	Determination of shear strength of granular materials from laboratory element tests with the aid of the proposed confining stress dependent failure criterion.	49
2.9	Dependency of shear strength of Toyoura Sand on relative density.	50
3.1	Centrifuge facility at Utsunomiya University.	64
3.2	Balancing beam type centrifuge at Utsunomiya University.	66
3.3	Typical container and lubrication of its inner walls.	67
3.4	Setup for loading test with circular footing	68
3.5	Setup for loading test with strip footing	69

3.6	A LVDT measuring surface settlement.	70
3.7	Calibration results of the LVDT used	72
3.8	Design diagram of the load cell for loading tests with strip footing in centrifuge (capacity is 9.8kN)	73
3.9	Calibration results of a type BLC-01t load cell	75
3.10	Design diagram of BLC-10k load cell for loading tests with strip footing under the gravity acceleration field (capacity is 98N)	75
3.11	Calibration results of a type BLC-10k load cell	76
3.12	Earth pressure cell	77
3.13	A flowchart of displacement field measurement process by image analysis . . .	79
3.14	Flowchart of pattern recognition by CC method	80
3.15	A schematic diagram of IP method	81
3.16	Deformation examined	83
3.17	The results of numerical experiments	84
3.18	Distribution of accumulated maximum shear strain γ_{max} ($s = 0 - 6\text{mm}$). . . .	86
3.19	Distribution of maximum shear strain increment $\Delta\gamma_{max}$	87
4.1	Influence of boundary friction on bearing characteristics (after Nishida <i>et al.</i> (1996)).	105
4.2	Influence of wall friction.	105
4.3	Direct shear apparatus to measure friction between sand and interface.	106
4.4	Comparison of friction resistance between lubricated and non-lubricated inter- faces obtained for Toyoura Sand.	107
4.5	Comparison of friction resistance between lubricated and non-lubricated inter- faces obtained for Toyoura Sand.	108
4.6	Frictional resistance on lubricated interface.	109
4.7	Set-up for EPNI and EPLI tests.	115
4.8	Earth pressure cells and their locations in a container.	115
4.9	Comparison between predicted and measured vertical pressure σ_v in non-lubricated container ($2b=15\text{cm}$).	116
4.10	Comparison between predicted and measured vertical pressure σ_v in lubricated container ($2b=15\text{cm}$).	117

4.11	Force acting on a small soil element in a container.	118
4.12	Predicted vertical stress distributions in non-lubricated container under the gravity acceleration field.	119
4.13	Predicted vertical stress distributions in non-lubricated container under the gravity acceleration field.	120
4.14	Predicted vertical stress distributions in non-lubricated container with a width of $2b = 15\text{cm}$ under various acceleration fields.	121
4.15	Predicted vertical stress distributions in lubricated container with a width of $2b = 15\text{cm}$ under various acceleration fields.	122
4.16	Predicted vertical stress distributions under a surcharge of 100kPa	123
4.17	Set-up for C tests.	124
4.18	Settlement in C tests.	125
4.19	Void ratio change versus centrifuge operation cycle obtained from C tests. . . .	125
4.20	Void ratio under $100g$	126
4.21	K_0 obtained from C1-test: (a) accelerating sequence, (b) deaccelerating sequence.	126
4.22	Test set-up for LN, LC and LO tests.	127
4.23	Sand surface settlement due to centrifuge operation.	127
4.24	$e - \log_{10} \sigma_v$ relationships obtained from LC tests.	129
4.25	Oedometer test results.	130
4.26	Comparison of LN-test results and LC-test results.	130
4.27	Comparison of LN-test results and LO-test results.	131
4.28	Effect of over-compression on subgrade reaction factor K_s	132
4.29	Determination of ultimate bearing capacity q_u and subgrade reaction factor K_s .	132
4.30	Influence of K_0 on normalized ultimate bearing capacity $2q_u/\gamma B$	133
5.1	Test apparatus for observing shear bands with Box I.	145
5.2	Test apparatus for observing shear bands with Box I.	146
5.3	Illustration of shear deformation and definition of thickness of shear zone D_d and shear bands D_b	148
5.3	Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width= 100mm , 1div.= 10%).	152

5.3	Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=100mm, 1div.=10%).	153
5.4	Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=150mm, 1div.=10%).	154
5.4	Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=150mm, 1div.=10%).	155
5.5	Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=200mm, 1div.=10%).	156
5.5	Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=200mm, 1div.=10%).	157
5.6	Displacement around a shear band obtained by CCIP method.	160
5.7	Width of shear zone D_d observed on the specimens of grass beads with depth $d = 80\text{mm}$ and relative density $D_r = 70\%$	161
5.8	Thickness of shear band D_b observed on the specimens of grass beads with depth $d = 80\text{mm}$ and relative density $D_r = 70\%$	162
5.9	Influence of the depth of specimen d on the thickness shear zone D_d	163
5.10	Relationship between relative depth of specimen d/D_{50} and thickness shear zone D_d	164
5.11	Influence of the depth of test specimen d on the thickness of shear band D_b	165
5.12	Influence of relative density D_r on the width of shear zone D_d	165
5.13	Influence of relative density D_r on the thickness of shear band D_b	166
5.14	Relationship between width of shear zone D_d and mean grain size D_{50} ; specimen depth d of 40mm and relative density D_r of 70%.	167
5.15	Positive correlation of the thickness of shear band D_b with mean grain size D_{50}	168
5.16	Dependency of shear band thickness ratio (D_b/D_{50}) on; (a) angularity A_{2D} , (b) ($e_{max} - e_{min}$).	169
6.1	Reduction of bearing capacity factor N_γ with increasing footing breadth (see Table 7.1).	179
6.2	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B obtained from Series TC.	183

6.3	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B obtained from Series GS conducted under 1g.	185
6.4	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B obtained from Series GS conducted under 50g.	185
6.5	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 12.5g obtained from Series INSS-1.	187
6.6	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 25g obtained from Series INSS-2.	187
6.7	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 37.5g obtained from Series INSS-3.	188
6.8	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 50g obtained from Series INSS-4.	188
6.9	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 12.5g obtained from Series INLS-1.	190
6.10	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 25g obtained from Series INLS-2.	190
6.11	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 37.5g obtained from Series INLS-3.	191
6.12	Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 50g obtained from Series INLS-4.	191
6.13	Size effects in load intensity vs. settlement curves (Series TC).	193
6.14	Comparison of $2q/\gamma B$ vs. s/B relationships obtained under 1g and 50g.	193
6.15	Size effects in load intensity vs. settlement curves (Series INSS).	194
6.16	Size effects in load intensity vs. settlement curves (Series INLS).	194
6.17	Changes in settlement at failure normalized by footing breadth S_f/B with increasing normalized footing breadth $\gamma B/\sigma_a$	195
6.18	Changes in subgrade reaction factor K_i and K_f with increasing normalized footing breadth $\gamma B/\sigma_a$ (continued).	196
6.18	Changes in subgrade reaction factor K_i and K_f with increasing normalized footing breadth $\gamma B/\sigma_a$	197
6.19	Changes in normalized bearing capacity $N_\gamma = 2q_u/\gamma B$ with increasing normalized footing breadth $\gamma B/\sigma_a$ (continued).	198

6.19	Changes in normalized bearing capacity $N_\gamma = 2q_u/\gamma B$ with increasing normalized footing breadth $\gamma B/\sigma_a$	199
6.20	Determination of shear strength of ground materials from laboratory element tests with an aid of the proposed confining stress dependent failure criterion. .	200
7.1	Definition of stress condition in a soil element; (a) element in $x - z$ coordinate system, (b) principal stress and slip lines S_1 and S_2 , (c) Mohr's stress circle. .	219
7.2	The relationships between calculated normalized bearing capacity $2q_u/\gamma B$ and the angle of trapped rigid wedge.	220
7.3	Plastic regions and slip lines under a strip footing calculated with extended slip line method with $\phi_a^*=30\text{deg.}$ and surcharge $q_s=1.96\text{kPa}$; (a) with $\alpha = 0.0$, (b) with $\alpha = 0.15$	221
7.4	Convergence of solution for zero surcharge surface strip footings.	222
7.5	Calculated relationships between $N_\gamma (= 2q_u/\gamma B)$ and γB	223
7.6	Relationship of reference bearing capacity factor N_γ^* with reference secant internal friction angle ϕ_a^*	224
7.7	Relationship of reference base pressure p^* with reference secant internal friction angle ϕ_a^*	224
7.8	Relationship of stress level effect factor β with reference secant internal friction angle ϕ_a^*	225
7.9	Explanation of excessive shear stress.	226
7.10	Geometry of numerical model ground and footing used for the calculation by FEM.	227
7.11	Typical calculation results by FEM. (b) circular footings.	228
7.12	Comparisons of bearing capacity factors obtained from FEM analysis and calculated from Terzaghi and Peck (1967) and Caquot and Kérisel(1953).	229
7.13	Ultimate bearing capacity q_u under various conditions.	230
7.14	Histograms for distribution of effective mean principal stress in failed elements. .	231
7.15	Conceptual illustration of size effect on bearing capacity related to confining stress dependency of shear strength of ground.	232
7.16	Illustration of determination method of c and ϕ ; (a) in $\tau_m - \sigma_m$ plane, (b) in $\tau - \sigma$ plane.	233

7.17 Sensitivity of calculated ultimate bearing capacities for strip footing with respect to parameters α and β	234
7.18 Comparisons between FEM analysis and proposed method.	235
7.19 Comparison of estimated and measured ultimate bearing capacities for Series TS; strip footing.	236
7.20 Comparison of estimated and measured ultimate bearing capacities for Series TC; circular footing.	236
7.21 Comparison of estimated and measured ultimate bearing capacities for Series IC; circular footing.	237
7.22 Comparison of estimated and measured ultimate bearing capacities for Series GS; strip footing.	237
7.23 Comparison of estimated and measured ultimate bearing capacities for Series SQ; square footing.	238

List of Tables

2.1	Granular materials used.	39
2.2	Soil testing for indices.	39
2.3	Indices of materials used in loading tests (Chapter 4, 6) and direct shear tests for wall friction (Chapter 3).	41
2.4	Physical properties of sand used in shear band tests in Chapter 5.	42
2.5	Physical properties of artificial materials used in shear band tests in Chapter 5.	43
2.6	Shear strength parameters for materials used in Chapter 7.	50
3.1	Specification of the centrifuge.	67
3.2	Design specification of the load cells	78
3.3	Physical property of graded silica sand	83
3.4	Test conditions of the numerical experiment	84
3.5	Relationship between size of referential image and limitation of deformations (error within 2pixels)	84
3.6	Test condition	85
4.1	Properties of sand.	110
4.2	Direct shear tests using Toyoura Sand ($D_r = 80\%$) with changing thickness of latex membrane t	110
4.3	Direct shear tests using Toyoura Sand with changing relative density D_r	111
4.4	Direct shear tests using Toyoura Sand ($D_r = 80\%$) with various test conditions.	112
4.5	Direct shear tests using Inagi Sand ($D_r = 60\%$) with different normal pressure p	112
4.6	Direct shear tests using Inagi Sand ($D_r = 80\%$) with different normal pressure p	113
4.7	Friction parameters determined by direct shear tests.	113
4.8	Test conditions	114
4.9	Test conditions (C tests).	124

4.10	Loading test on Toyoura sand (LN, LC, LO tests).	128
6.1	Testing conditions of loading tests of footings quoted in the present study.	180
6.2	Conditions of loading tests conducted by the author.	180
6.3	Loading test on Toyoura Sand (Series TC).	181
6.4	Loading test on glass beads (Series GS).	181
6.5	Loading test on Inagi Sand (IN-250-105, Series INSS).	182
6.6	Loading test on Inagi Sand (IN-840-420, Series INLS).	182
6.7	Test results obtained from Series TC.	184
6.8	Test results obtained from Series GS.	186
6.9	Test results obtained from Series INSS.	189
6.10	Test results obtained from Series INLS.	192
6.11	Size effects of ultimate bearing capacity.	200
7.1	Conditions of performed parametric FEM analysis.	239
7.2	Stress ranges of interest for determining shear strength constants c and ϕ , for strip footing and circular footing.	240
7.3	Loading tests used for evaluation	240
7.4	Parameters for method M-I.	241
7.5	Predictions obtained form M-II	242
7.6	Prediction and its comparison for Series BS	243

List of Photographs

2.1	Particle shape of the materials used.	45
3.2	Centrifuge facility and a loading jack.	65
3.3	Loading jack with five divided strip footings for loading test under gravity acceleration field	70
3.4	A LVDT and its bracket made of aluminum alloy.	71
3.5	A laser displacement transducer.	73
3.6	Divided footing and five BLC-01t load cells.	74
3.7	Divided strip footing with five BLC-10k load cells	76
3.8	Original image used in numerical experiments(500×500pixels, 51mm×51mm, approximate resolution is 0.1mm/pixel)	82
3.9	Sand with colored particles in soil container	83
3.10	Loading tests	85
5.11	Test apparatus for observing shear bands with Box II.	147
5.12	Observed shear deformation of the specimen of material CA-300-250 with depth of $d = 40\text{mm}$ and relative density of $D_r = 70\%$	149
5.12	Observed shear deformation of the specimen of material CA-300-250 with depth of $d = 40\text{mm}$ and relative density of $D_r = 70\%$	150
5.12	Observed shear deformation of the specimen of material CA-300-250 with depth of $d = 40\text{mm}$ and relative density of $D_r = 70\%$	151
5.13	Example of photographs to measure the thickness of shear band D_b taken for a specimen made of CA-177-150 with depth of $d=40\text{mm}$ and $s = 0\text{mm}$; (a)close- up photograph, (b)microscopic photograph.	158
5.14	Example of photographs to measure the thickness of shear band D_b taken for a specimen made of CA-177-150 with depth of $d=40\text{mm}$ and $s = 10\text{mm}$; (a)close- up photograph, (b)microscopic photograph.	159

Chapter 1

Introduction

1.1 Background

Terzaghi's bearing capacity formula has been widely used for the calculation of ultimate bearing capacity of footings (Terzaghi and Peck, 1967):

$$q_u = S_c c N_c + S_\gamma \frac{1}{2} \gamma B N_\gamma + S_q \gamma D_f N_q \quad (1.1)$$

where q_u is the ultimate bearing capacity. S_c, S_γ and S_q are shape factors. N_c, N_γ and N_q are bearing capacity factors. c is the cohesion. γ is the unit weight of soil. B and D_f are the width and the embedded depth of footing, respectively. Commonly used bearing capacity formulae employ bearing capacity factors N_c and N_q due to Prandtl (1921) and Reissner (1924), respectively, which are based on rigid-perfectly plastic theory.

For the surface footings on cohesionless sand ground, the above equation simply becomes

$$q_u = S_\gamma \frac{1}{2} \gamma B N_\gamma \quad (1.2)$$

where q_u is ultimate bearing capacity and B is footing breadth, respectively. Parameters S_γ and N_γ are shape factor and bearing capacity factor, respectively; S_γ is unity for strip footing and 0.6 for circular footings, and N_γ is a function of the internal friction angle.

Vesic (1973) recommended to use the bearing capacity factor proposed by Caquot and Kérisel (1953) which is approximated by

$$N_\gamma = 2(N_q + 1) \tan \phi = 2 \left\{ \exp(\pi \tan \phi) \tan \left(\frac{\pi}{4} + \frac{\phi}{2} \right) + 1 \right\} \tan \phi \quad (1.3)$$

Several formulae have been proposed for N_γ by many researchers as cited by Zadoroga (1994).

AS early as Golder (1941), it was observed that bearing capacity does not, in general, increase linearly with footing width B as predicted in Eq. (1.2) and Eq. (1.3). Bearing capacity factor N_γ is independent of footing breadth B in the original meaning of the formulae shown in Eqs. (1.2, 1.3); however, the bearing capacity factor N_γ calculated back from the loading tests of footings on sand grounds reduces with increasing footing size. This observation was termed a scale effect and was reviewed and discussed by De Beer (1963) and De Beer (1965); see Fig. 1.1. This observation has also been made in more recent studies (Ovesen, 1975; Yamaguchi *et al.*, 1976, 1977; Kutter *et al.*, 1988; Okahara *et al.*, 1988; Kusakabe *et al.*, 1991, 1992; Ueno *et al.*, 1994).

The consequence of the scale effect is that bearing capacity factors N_γ determined from small-scale tests will generally be aggressive when extrapolated to field-size footings. Therefore, excessively large safety factors are often used in practical design. While Terzaghi's bearing capacity formula has remarkably contributed to the practical design of foundations, the lack of proper consideration on the scale effects brings the over-design and caused a swell of construction cost.

1.2 Brief review of previous studies

1.2.1 Problems related to the theme

De Beer (1965) proposed two reasons for the size effects:

- the friction angle ϕ is reduced by higher pressure occurring with larger foundations (stress dependency);
- mobilization of shear strength, i.e, progressive failure.

Tatsuoka *et al.* (1991) and Siddiquee *et al.* (1999) pointed out third factor constituting the size effects, that is, effects of the ratio of footing breadth with respect to particle size of ground materials, based on the experimental and analytical investigations on the size effects, as shown in Fig. 1.2. They call the phenomenon particle size effect.

1.2.1.1 Stress dependency

As shown by Bishop (1966), the stress-strain-dilatancy behavior of sand varies remarkably with confining stress. The mechanical properties of sands under high confining pressure were investigated by Bishop and Clugh (1968) and Miura *et al.* (1979, 1979). They reported large compressibility and a notable reduction of internal friction angle due to particle crushing and negative dilatancy, which occur under high pressure. On the other hand, the mechanical properties of sands under extremely low confining pressure were investigated by Ponce and Bell (1971), Tatsuoka and Fukushima (1984), Tatsuoka *et al.* (1986).

Maeda and Miura (1999) investigated the confining stress dependency of mechanical properties of sands extensively; deformation-failure behaviors of about 80 granular materials were observed in conventional drained triaxial compression tests under variable effective confining stresses. Additionally they showed that all the tested cohesionless materials exhibit a notable confining stress dependency of σ_m^* and the linear relationships can be found in $\tan \phi^*$ vs. σ_m^* plots. **Figures 1.4(a-c)** show the results of a series of conventional triaxial compression tests on three different granular materials. While the Mohr-Coulomb failure criterion assumes straight failure envelopes as shown with dashed lines in these figures, actual failure envelopes for the materials are upper convex curves. While Toyoura sand in **Fig. 1.4(a)** is cohesionless, Shirohata scoria and Mori volcanic ash soil in **Figs. 1.4(b, c)** have notable cohesion in undisturbed state; all the tested materials show marked nonlinearity of envelope.

Size effects are due in part to the nonlinear shape of the soil failure envelope, which results in a secant measure of the friction angle that decreases with increasing mean normal effective stress. Confining stress dependency of the shear strength of ground material is one of the most important factors in controlling the size effect of surface footings; even for cohesionless materials, ϕ is not constant and reduces with an increase of confining stress. If a footing becomes larger, the location of the failure surface becomes deeper; the depth is approximately proportional to the breadth of footing B . Additionally, in an identical foundation ground made of non-cohesive soil, bearing capacity q_u could be approximately in proportion to the value of γB , as long as the intensity of the confining stress dependency of shear strength is low. Therefore, it can be expected that the stress range to be interested in in the foundation ground beneath the footing should be in proportion to the value of γB . Higher stress leads to a lower angle of internal friction ϕ , which yields a smaller bearing capacity factor N_γ . Consequently,

N_γ does decrease with increasing γB ; this is a major mechanism of size effects related to stress level effect on shear strength.

1.2.1.2 Mobilization

Muhs offered the explanation of progressive failure to account for differences between experiment and prediction. As footing load is gradually increased, shearing strength is not simultaneously mobilized at all points on a potential slip surface. Shearing strength is first mobilized at points where shear strains are greatest, with strength mobilization progressing to other regions as shear strains develop and advance through the soil mass. The consequence of this phenomenon is that when the peak bearing capacity of the foundation is reached, only a percentage of the material along the slip plane may be contributing a shear strength that is dependent on the soil's peak friction angle, whereas the remaining regions contribute a shear strength dependent on a residual, constant volume, or critical state friction angle. Yamaguchi *et al.* (1976, 1977) presented results that indicate that progressive failure becomes more prominent with increasing footing width, or increasing soil confinement, for a soil of a given relative density.

1.2.1.3 Particle size effect

In granular media, a slip surface has in fact a certain thickness, which is a function of particle size among other factors (Scarpelli and Wood, 1982, Mühlhaus and Vardoulakis, 1987 Ueno *et al.*, 1994 and Yoshida *et al.*, 1994). The thickness of shear band dominates the relationships between the displacement and the amount of shear strain. Strain field induced in a failing soil mass depends on the shear banding pattern, which is a function of the ratio of the thickness of shear band or the particle size to the footing size.

The formation and existence of shear bands in soil mass play important roles in many failure problems of various types in soil mechanics. In soil mass shear bands develop near the instance of shear failure. After the formation of shear band, deformation is concentrated or localized along the shear bands and, therefore, the strain defined macroscopically for the overall granular media has no mechanical meanings. Consequently, the behavior associated with strain softening and/or progressive failure becomes evident (Oda and Furuto, 1982); this have been discussed in many geotechnical problems such as skin friction of friction piles (Uesugi *et al.*, 1988), stability of soil mass (Roscoe, 1970) and bearing capacities of foundation

ground (Tatsuoka *et al.*, 1991 and 1992).

The thickness of shear band is an important information to find engineering solutions regarding strain softening and progressive failure phenomena. The Finite Element analysis on the deformation of soil mass with shear bands by Desai and Zaman (1984), Tatsuoka *et al.* (1991), Miura *et al.* (1991) and Yoshida *et al.* (1994) showed that the thickness of shear band affects significantly the failure behavior of soil masses. In the advanced analysis methods where the mechanical behavior of shear band is modeled with theoretical background, the numerical value of shear band thickness are required as a primary input parameter; see e.g. Mühlhaus and Vardoulakis (1987), Vardoulakis and Graf (1982 and 1985) and Pietruszczak and Niu (1993).

1.2.2 Method to account for size effects

Although many advanced numerical calculation methods such as FEM were proposed by many researchers (e.g. Tatsuoka *et al.*, 1991 and Tanaka, 1992), the simplified calculation methods which incorporate the conventional bearing capacity formula is needed for practical design purposes. A possible method to incorporate the stress level effects into the bearing capacity formula is to find the proper values of shear strength parameters c and ϕ in accordance with the stress level beneath the footing; see e.g. Meyerhof (1950) and De Beer (1970). Some formulae to determine the representative stress level were proposed by Meyerhof (1950), De Beer (1970), Hettler and Gudehus (1988), Perkins (1995) and Perkins and Madson (1996). Kutter *et al.* (1988) verified de Beer's equation with a limited numbers of loading test results. The author, Ueno *et al.* (1998), proposed a method for the determination of representative stress level which is applicable to circular and strip surface footings on the ground with/without cohesion.

The other method to incorporate the stress level effects is the modification of the bearing capacity formula. Shiraishi (1990) deduced the following empirical equation from a series of in-situ loading tests on piles for the nonlinear relationship of bearing capacity coefficient N_γ with footing breadth B .

$$q_u = \frac{1}{2} S_\gamma \gamma B N_\gamma^* \left(\frac{B}{B^*} \right)^{-\beta} \quad (1.4)$$

where B^* is reference footing breadth; a parameter β represents the degree of stress level dependency. Shiraishi (1990) obtained the values of the parameters from the limited number

of experiments; B^* was set to be 140cm and β was 0.2. This empirical formula is supported by the measured data in centrifuge model loading tests on surface footing on sand grounds as shown in **Fig. 1.3**. This modification of the bearing capacity formula is promising for its simplicity; however, the mechanical meaning of the parameters appearing in the formula is not clear.

In this study the mechanical meaning of the modified bearing capacity formula is investigated to establish the relationship of the formula with the confining stress dependent failure criterion. Two kinds of analytical method were employed to consider the stress level dependent bearing capacity, in this study: modified slip line method and finite element method.

The modified slip line method employed is a kind of finite differential method for solving numerically the characteristics of stress field based on rigid plasticity; see Sokolovskii (1965). The nonlinear Mohr-Coulomb type failure criterion deduced from laboratory element test data on many sand samples by Maeda and Miura (1999), was combined to the slip line method; see Yamaguchi (1969).

Perkins and Madson (2000) presented a method that relies on Bolton's equations (Bolton, 1986) to describe the effects of nonlinear strength behavior and progressive failure on ultimate bearing capacity, while also accounting for footing geometry.

1.3 Objectives and scope

The main objective of the present thesis is to propose estimation method of bearing capacity regarding stress level effects. In practical design, the lack of proper consideration on the size effects brings the over-design and caused a swell of construction cost. The author aims to overcome the imperfections appearing current design method.

In accordance with this objective, the present thesis consists of three parts: a study to improve the accuracy of centrifuge model tests, the experimental study on failure mechanism of granular media and experimental and analytical study on the size effects of bearing capacity of shallow foundation. The constitution of the present thesis is presented with the flow chart in **Fig. 1.5**.

- The study to improve the accuracy of centrifuge model tests consists of the following subjects to be performed:

1. Development of a new measuring method of displacement field for sand by image processing.
 2. Influence of boundary conditions on centrifuge model tests, especially concentrated in friction between sand and container.
 3. Influence of initial conditions on centrifuge model tests.
- The experimental study on failure mechanism of granular media consists of the following subjects to be performed:
 1. Development of a new direct shear testing apparatus to observe shear banding in granular materials.
 2. Observation of development of shear bands in granular materials.
 3. Observation of shear band patterns in granular materials.
 4. The presentation of the reference data for the thickness of shear bands in various kinds of granular materials.
 - Experimental and analytical study on the size effects of bearing capacity of shallow foundation consists of the following subjects to be performed:
 1. A series of loading tests on several granular materials under gravitational and centrifugal acceleration fields.
 2. Reappraisal of size effect of bearing capacity from plastic solution.
 3. Reappraisal of size effect of bearing capacity from elast-plastic finite element analysis.
 - Proposal of prediction methods of ultimate bearing capacity regard to stress level effects consists of the following subjects to be performed:
 1. Proposal of determination method of shear strength used to obtain bearing capacity.
 2. Proposal of simplified estimation method of bearing capacity on sands.
 3. Verification of the proposed methods with various kinds of experimental results conducted the author and other researchers.

1.4 Composition of the present thesis

The present thesis, entitled “On the bearing capacity of surface foundation on granular media”, consists of this introduction and the following seven chapters.

Chapter 1 In Chapter 1 “Introduction”, firstly background of the present study, a brief review of previous studies, objectives and scope of the present study and composition of this thesis are presented.

Chapter 2 This chapter describes overview and physical properties of granular materials used or referred in the present thesis. In the present thesis, eleven granular materials are employed. They are classified into four categories: four natural sands, two graded sands, one scoria and four artificial materials.

The description of the materials in this chapter is concentrated on physical properties, such as specific density G_s , grain size analysis, void ratio extent $e_{\max} - e_{\min}$ and particle shape. Particle shape is expressed by Lees’s two dimensional angularity.

Chapter 3 Chapter 3 describes apparatus and measurement used for centrifuge model tests conducted in this study. Section 3.2 describes centrifuge apparatus used in this study. In Section 3.3 set-up for loading tests is introduced. A brief description of devices to measure load and displacement are also presented. In Section 3.4, the author proposes a new measuring method, named CCIP method, orienting to simplicity and generality, for displacement field in model sand with an aid of image processing.

The proposed method requires no target installed in the model to measure displacement field on sand. A newly developed correction method for distortion of image needs no longer information than the coordination of four datum points covering the analyzing zone. These features can reduce labor in the preparation of model sand and the analysis of displacement fields. Evaluation of accuracy and examples are presented in the chapter. The results show that the method can measure both macroscopic and microscopic deformations in sand within an accuracy of two pixels of the images employed.

Chapter 4 To reproduce ideal stress distribution in models is essential to establish accurate and reliable testing technique. Friction between model sand and soil container is one of

the significant factors that dominates stress distributions in model sand. In Chapter 4, the author examines frictions between metal plates and sand, and discusses the effectiveness of a lubrication technique in improvement of stress distribution in model sand. Also, stress history is another factor that affects on initial stress distributions. Changes in stresses in model sand due to repetitious centrifuge operation and their influence on bearing capacity are discussed in Chapter 4.

Chapter 5 In Chapter 5, shear deformation patterns and the thickness of shear bands are discussed. To investigate the formation of shear band in granular materials, the author developed a new type of apparatus, which is a kind of direct shear box equipped with a microscope. With this apparatus, the deformation and shear banding on the surface of granular materials of different seven types were directly observed. The development of shear band during shearing deformation was carefully observed and the thickness of shear band was measured on the photographs taken with the microscope.

The results showed that the thickness of shear bands was independent of the dimensions, relative density and the amount of the shearing specimen. The ratio of the shear band thickness to the mean grain size of granular materials ranged from about 10 to 25. Furthermore it was found the ratio of shear band thickness is dependent on the grain shape (angularity) and the void ratio extent ($e_{\max} - e_{\min}$).

Chapter 6 In Chapter 6, the author examine the phenomena of size effect on bearing capacity of granular materials by means of centrifuge and 1g model loading tests. In the model loading tests, Toyoura sand and glass beads were employed. Data reported by other researchers are also compiled in this chapter.

Chapter 7 In Chapter 7, two methods for estimating the ultimate bearing capacity of surface footings, in which size effects are taken into account, are described. One of the methods, called M-I hereafter, is for footings on cohesionless soil and the another method, called M-II hereafter, is for both cohesionless and cohesive soil. Clarification of size effects, which is indispensable for appropriate design of large foundations, may permit use of the results of small scaled loading tests and prevent increases in the construction costs of foundations.

The prediction method for cohesionless soils, M-I, is based on Shiraishi's empirical equation for ultimate bearing capacity. Shiraishi (1990) deduced an empirical equation from a series of

in-situ loading tests on piles for nonlinear relationship of bearing capacity coefficient N_γ . To clarify relationship between the nonlinearity of N_γ and stress dependency of shear strength, an extended slip line method is developed, in which dependency of the angle of internal friction on the confining stress is formulated. Based on results from a comprehensive series of calculations employing the extended slip line method, the authors reappraise size effects on bearing capacity, and investigate the relationship between strength parameters of sand and size effects on bearing capacity. A modified formula and several diagrams that provide a simple estimation method are proposed. Also physical meaning of Shiraishi's empirical equation is discussed.

The another method for both cohesionless and cohesive soil, M-II, is based on Terzaghi's bearing capacity formula for surface footing. The method M-II incorporates a method for determination of shear strength constants c and ϕ that should be applied to the bearing capacity formula. A nonlinear Finite Element Method which incorporates the confining stress dependent shear strength is used to calculate stress distribution in foundation ground beneath the footings. It is shown that the effective mean principal stress σ'_m tends to range from $2\gamma B$ to $10\gamma B$ for strip footings and from $1\gamma B$ to $15\gamma B$ for circular footings; within this range the parameters c and ϕ are determined from the straight fitting line.

In this chapter, above two methods M-I and M-II are verified with the results of series of model surface footing loading tests on cohesionless and cohesive ground presented in Chapter 6. This verification confirms the practicability of the proposed methods to strip, circular and square footings on cohesive and cohesionless foundation ground with sufficient accuracy for practical purposes.

Chapter 8 In Chapter 8, a series of conclusions shown in the above seven chapters are summarized.

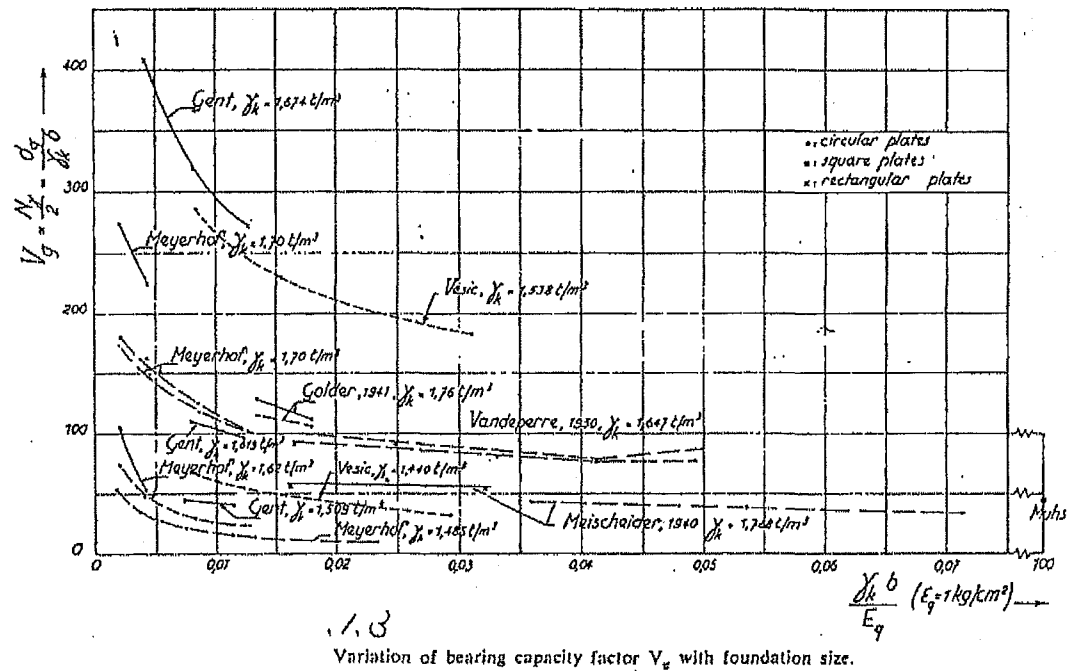
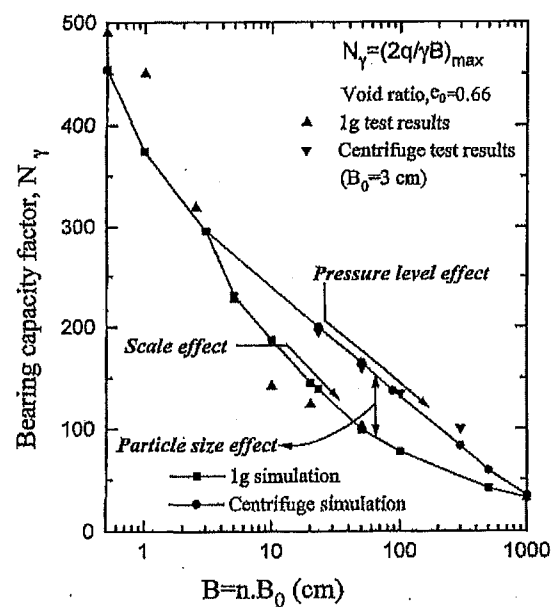


Figure 1.1: Variation of bearing capacity factor with foundation width after de Beer (1965)

Figure 1.2: Particle size effect (after Siddiquee *et al.*, 1999)

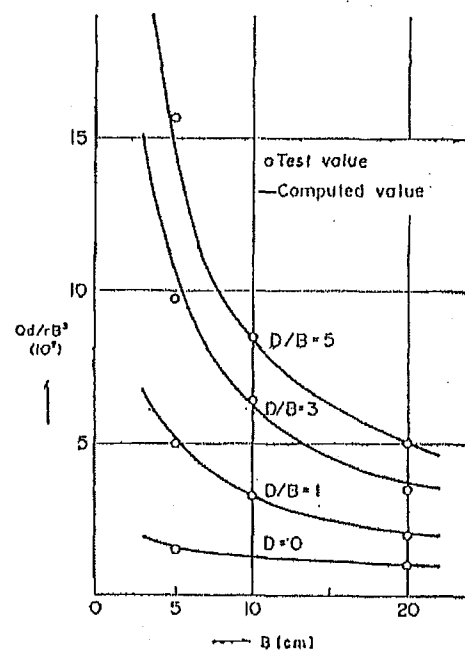
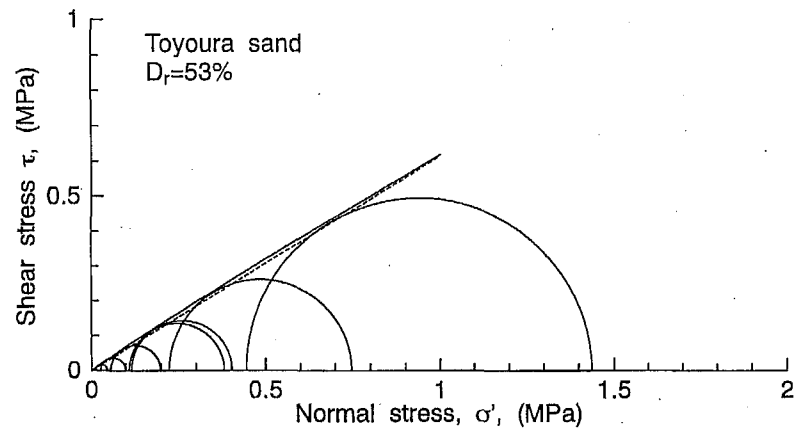
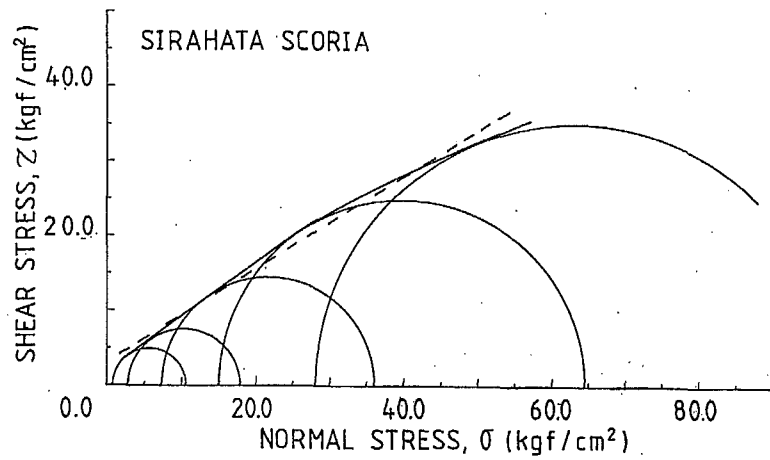
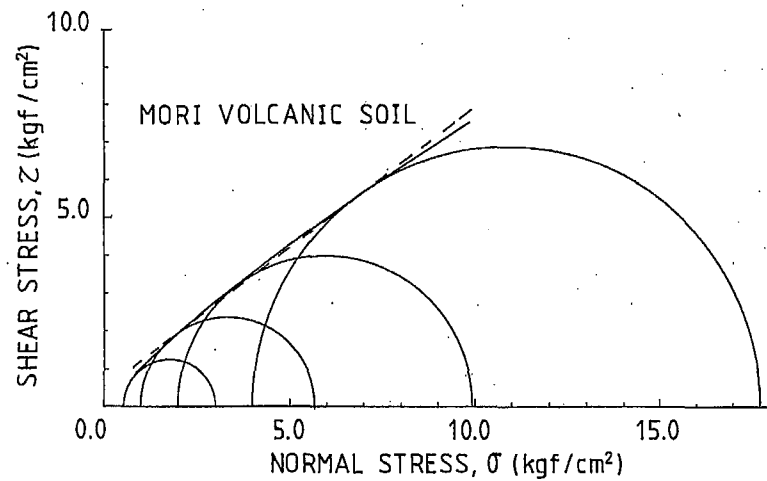
Fig. 4. Value of $Q_d/\gamma B^3$

Figure 1.3: Comparison of computed and measured value of $Q_d/\gamma B^3$ (after Shiraishi 1990).



(a) Toyoura sand after Miura (1981).

(b) Shirohata scoria after Kusakabe *et al.* (1992).

(c) Mori volcanic soil after Takeda (1992).

Figure 1.4: Mohr's stress circles at failure and failure envelopes obtained from conventional drained triaxial compression tests on.

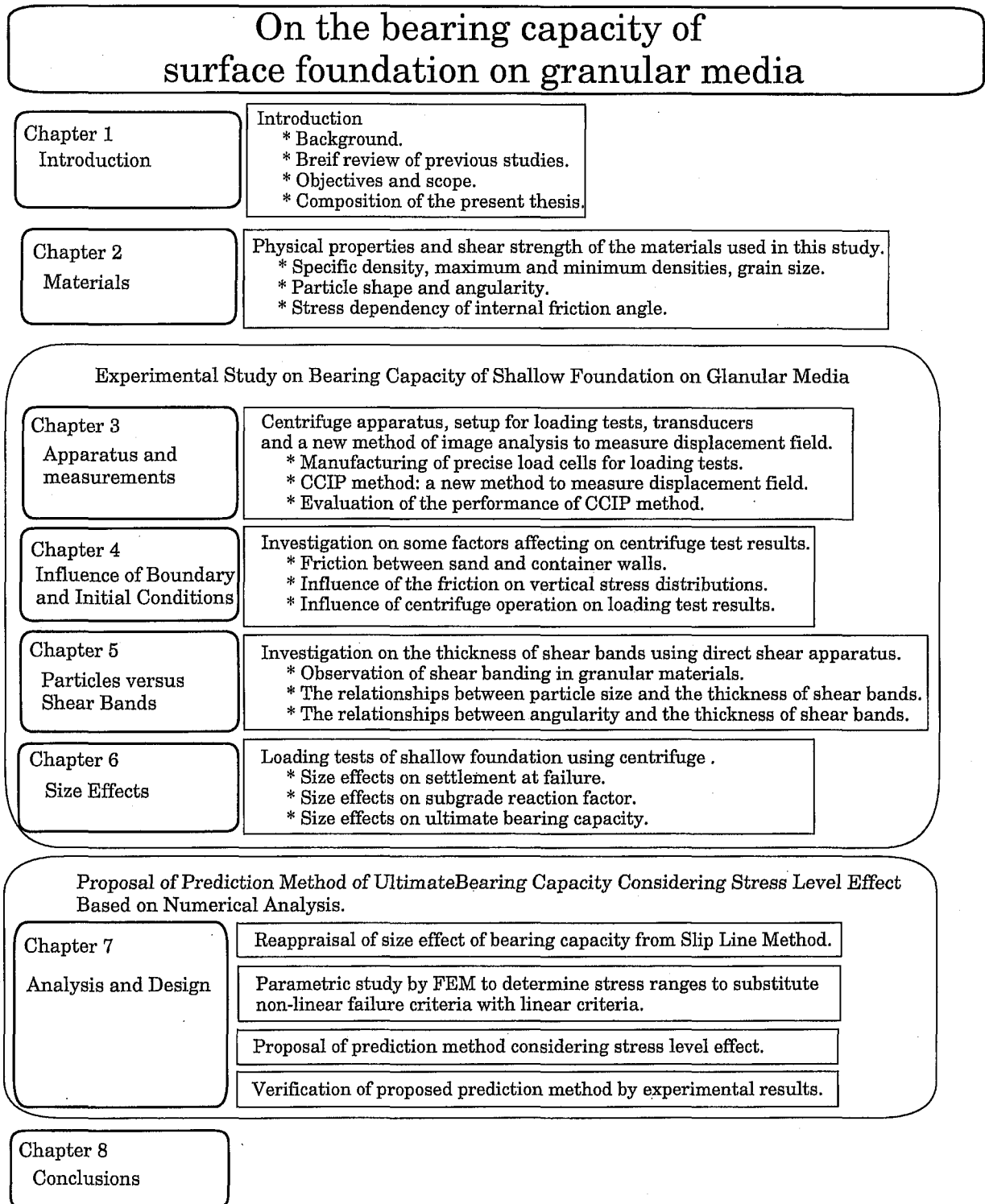


Figure 1.5: Composition of the present thesis

Chapter 2

Materials

2.1 General

This chapter describes overview and physical properties of granular materials used or referred in the present thesis. In the present thesis, eleven granular materials are employed. They are classified into four categories: four natural sands, two graded sands, one scoria and four artificial materials. The name of materials, category, objectives of the materials and citation are tabulated in **Table 2.1**.

The description of the materials in this chapter is concentrated on physical properties, such as specific density G_s , grain size analysis, void ratio extent $e_{\max} - e_{\min}$ and particle shape. Particle shape is expressed by two dimensional angularity, A_{2D} , firstly proposed by Lees (1964a). The definition and the method to determine the angularity will be introduced in Section 2.3.2.

2.2 Overview of materials

2.2.1 Natural sand

Aichi Noma Sand (AN): Aichi Noma sand is produced at Noma, Aichi Prefecture in Japan. This sand was used to investigate the relationship between particle size and the thickness of shear band in Chapter 5. Particle shape of Aichi Noma Sand is shown in **Photo 2.1(a)**.

Inagi Sand (IN) Inagi Sand (IN) is a sand having a coefficient of uniformity U_c of 2.71, produced at Inagi city, a western part of Tokyo Metropolis. This sand was used in a series of loading tests in Chapter 6. Physical properties and the grain size distribution of IN Sand are shown in Table 2.3 and Fig. 2.1, respectively.

Monterey 0/30 Sand (MO) Kutter *et al.* (1988) conducted a series of loading tests with Monterey 0/30 Sand in a centrifuge. Physical and shear strength properties of this sand were given by Kutter *et al.* (1988) with the results of the series of loading tests. The results of the loading tests will be cited in this study to evaluate the performance of the proposed method for ultimate bearing capacity in Chapter 6 and Chapter 7.

Shirahata Scoria (SS) Shirahata Scoria is a kind of uniform volcanic granular soil which comprises a diluvial deposit. Kusakabe *et al.* (1992) performed a series of in-situ loading tests on the deposit of Shirahata Scoria. The physical and mechanical properties of this material was cited from Kusakabe *et al.* (1991). In this study, the results of the series of loading tests conducted by Kusakabe *et al.* (1991) will be referred in Chapter 6 and Chapter 7 to verify practicability of the proposed prediction method for ultimate bearing capacity, as a typical material having a notable cohesion.

Toyoura Sand (TO) Toyoura Sand (TO sand) that is a uniform fine sand was used as Japanese standard sand in concrete testing. Many researchers of geotechnics in Japan used TO sand so that the author also uses this as a typical fine sand in Chapter 5, 4, 6 and 7. Figure 2.1 shows the grain size distribution of TO sand. Physical properties of TO sand are tabulated in Table 2.3. Particle shape of Toyoura Sand is shown in Photo 2.1(c).

2.2.2 Graded sand

Several kinds of sand were sieved out with a set of sieves standardized in Japanese Industrial Standards (JIS), and graded according to grain size distribution. All graded sands were only used to investigate the relationship between particle size and the thickness of shear band. Abbreviated notation of original sand followed by maximum and minimum grain size in micrometer is used for naming the graded sands. The grain size distribution and physical properties of graded sand are shown in Fig. 2.3 and Table 2.4, respectively.

Siliceous sand (CA): Siliceous sand, named CA, commercially available in Japanese, is used for an industrial material, such as for silicon products, or molding material for iron casting. The sand was sieved into eight classes with uniform grain size distributions as shown in **Fig. 2.3**. Well graded sample CA-850-75 was prepared by blending the uniform samples, which has a coefficient of uniformity of 2.22, and the grain size distribution was comparable to that of Aichi Noma Sand (AN) as shown in **Fig. 2.2**. Particle shape of Siliceous Sand is shown in **Photo 2.1(b)**. This sand will be used in Chapter 5.

Inagi Sand (INL, INS) Inagi Sand was sieved into two classes with uniform grain size distributions: INL and INS Sands. The grain size distribution and physical properties of INL and INS sands are shown in **Fig. 2.1** and **Table 2.3**, respectively. These sands were used in a series of loading tests in Chapter 6.

Toyoura Sand (TOG): Graded Toyoura Sand was prepared to investigate the relationship between particle size and the thickness of shear band. Natural Toyoura Sand whose grain size distribution is shown in **Fig. 2.2** was first milled in ceramic pot and then sieved into five uniform classes, as shown in **Fig. 2.3**. This sand will be used in Chapter 5.

2.2.3 Artificial materials

Artificial materials were mainly used to investigate the relationship between particle size and the thickness of shear band, except glass beads with a diameter of $100\mu\text{m}$. The grain size distribution and physical properties of artificial materials are shown in **Fig. 2.3** and **Table 2.5**, respectively. Also, photographs taking the shape of artificial materials are available in **Photo 2.1**.

Chopped grain (CG): Chopped grain made from a skinny polymer rod by chopping into about 2.8mm in length was named CG. Shape of CG is angular but its surface is relatively smooth, as shown in **Photo 2.1(d)**. This material will be used in Chapter 5.

Glass bead (GB): Glass bead (GB) is an artificial granular material made of glass and comprises of particles with a shape of sphere. Grain size of GB is industrially controlled by the manufacture. Three different sizes of glass bead were prepared: $100\mu\text{m}$, $400\mu\text{m}$ and

800 μ m. They will be used in Chapter 5. Only the glass beads having a diameter of 100 μ m will be used for loading tests in Chapter 6. The shape of GB, shown in **Photo 2.1(e)**, is almost spherical and its surface is smooth; therefore, their angularity A_{2D} is equal to zero. Due to the its spheric shape and negligible crushability of the glass beads particles, this material shows less confining stress dependency of shear strength compared with sands; see Maeda and Miura (1999).

Light Weight Aggregate (LWA): Light weight aggregate is made of sewage mud by burning and cementing in rotary oven. LWA will used in Chapter 5. Shown in **Photo 2.1(f)** is the shape of LWA particles. This material has crushable nature and spherical shape. The angularity A_{2D} of this material is rather small, even with rough surface. Four uniform grain sizes were prepared as listed in Table 2.5.

Rotund Grain (RG): Rotund Grain, shown in **Photo 2.1(g)**, is consist of ellipsoidal grains made of polymer. LWA will be used in Chapter 5. Grain size of RG is 2.71mm. Material RG has a small angularity A_{2D} comparable to LWA.

2.3 Physical properties

2.3.1 Soil indices

Physical properties of the materials used are obtained by conventional soil testing specified by Japanese Industrial Standards (JIS) or by the Japanese Geotechnical Society (JGS), as tabulated in **Table 2.2**.

The grain size distributions of materials used are shown in **Figs. 2.1-2.3**. Indices of the material used in the present thesis are tabulated in **Tables 2.3-2.5**.

2.3.2 Angularity

The index A_{2D} at the sixth column in **Tables 2.4-2.5** expresses two dimensional angularity of the grains. This angularity was defined by Lees (1964a, b) and used by Maeda (1994) and Miura *et al.* (1997) for characterizing the primary properties of sands.

The definition of *Angularity* for an angle of the particle observed in the projection is described with the following expression (**Fig. 2.4**).

$$Angularity = (180^\circ - a) \frac{X}{r} \quad (2.1)$$

where a is the corner angle; r , the radius of the inscribed circle of the particle projection; X , the distance measured from the centre of the inscribed circle to the corner. The two-dimensional angularity A_{2D} is obtained by summarizing the all corners in the projection, as

$$A_{2D} = \sum Angularity \quad (2.2)$$

The summation of three A_{2D} obtained from three projections on mutually orthogonal planes is called total degrees of angularity A . Lees (1964a, b) prepared the table that demonstrated total degree of angularities A for smashed rocks with their shape, and tried to derive the values of A from the table by means of visual inspection (see **Fig. 2.4**). Furthermore, he showed that the values derived from the illustration well agreed with the values determined from the definition. Based on Lees investigations, Maeda (1994) obtained the values of two dimensional angularity A_{2D} for the materials used in this study from comparison between a plane projection of 20 particles and the Lees's table. The values of angularity A_{2D} in **Table 2.4** and **Table 2.5** were thus obtained. The term of *angularity* that appears hereafter will mean two dimensional angularity A_{2D} in Eq. 2.2 instead of angularity A in Eq. 2.1.

The values of two-dimensional angularity A_{2D} in **Table 2.4** and **Table 2.5** were obtained from plane projection of every 20 particles. The value of A_{2D} is high for angular grains, while A_{2D} is zero for spheres.

According to Maeda (1994) and Miura *et al.* (1998), a clear positive correlation is found between void ratio extent ($e_{\max} - e_{\min}$) and angularity A_{2D} , and ($e_{\max} - e_{\min}$) is a function not only of grain shape but also of grain size distribution. From the examination of the physical and mechanical properties of some 200 granular materials, Maeda (1994) and Miura *et al.* (1998) showed that compressibility, shear stiffness and dilatancy of sands can be estimated based on the value of ($e_{\max} - e_{\min}$), because void ratio extent ($e_{\max} - e_{\min}$) implies the possible volume change or the degree of freedom of structure formed with constituent grains.

2.4 Stress dependency of shear strength

Figures 2.5(a-b) show the results of a series of conventional triaxial compression tests on

two different granular materials. While the Mohr-Coulomb failure criterion assumes straight failure envelopes as shown with dashed lines in these figures, actual failure envelopes for the materials are upper convex curves. Toyoura Sand (TO) in **Fig. 2.5(a)** is cohesionless, while Shirahata Scoria (SS) in **Figs. 2.5(b)** has notable cohesion in undisturbed state.

De Beer (1963) assumed that the secant friction angle ϕ is in proportion to the logarithm of effective mean stress σ'_m , and proposed the following expression

$$\phi' = \phi_0 - A \log \frac{\sigma'_m}{\sigma_a} \quad (2.3)$$

where A is a material constant that expresses the degree of stress dependency; σ_a , the referential stress equal to atmospheric pressure (98kPa); ϕ_0 , the value of ϕ' when $\sigma'_m/\sigma_a = 1$. Kimura *et al.* (1985) and Kusakabe *et al.* (1992) employs a similar expression; however, some difficulty may be appeared in De Beer's expression when we try to extend its applicability to cohesive soils.

Another expression can be used to express the stress dependency of shear strength. Maeda and Miura (1999) extensively investigated the effects of confining stress level on the mechanical properties of sands. Deformation-failure behavior of about 80 granular materials was observed in conventional consolidated-drained triaxial compression tests under various confining stresses. They showed that all the tested materials exhibited notable dependency of the internal friction angle ϕ on confining stresses. They also showed that linear relationships in the $\tan \phi^*$ vs. σ_m logarithmic plots can be modeled as shown in **Figs. 2.6(a, b)** and described as,

$$\tau_f = \sigma \tan \phi^* = \sigma \tan \phi_a^* \left(\frac{\sigma_m}{\sigma_a} \right)^{-\alpha} \quad (2.4)$$

where σ_a is reference atmospheric stress equal to 98kPa. Parameters α and ϕ_a^* are introduced to express the assumed linearity in $\log \tan \phi^*$ vs. $\log \sigma_m^*$ plots, shown in **Fig. 2.6(b)**. According to Maeda and Miura (1999) the parameter α would range from 0.02 to 0.12 and average 0.07 for the tested granular materials.

For cohesive soils, the failure criterion regards to stress dependency of shear strength can be extended by introducing a tensile strength σ_t . **Figures 2.7(a, b)** are conceptual drawings and **Eq. (2.5)** expresses the relationship,

$$\begin{aligned} \sigma_m^* &= \sigma_m + \sigma_t \\ \tau_f &= (\sigma + \sigma_t) \tan \phi^* \\ &= (\sigma + \sigma_t) \tan \phi_a^* \left(\frac{\sigma_m^*}{\sigma_a} \right)^{-\alpha} \end{aligned} \quad (2.5)$$

where σ_t is the tensile strength.

The stress dependency of shear strength of the materials used are presented in **Figs. 2.8(a, b)**, which were obtained from conventional drained triaxial compression tests conducted by many researchers. Strength parameters determined by using **Figs. 2.8(a, b)** and citations are tabulated in **Table 2.6**. The following paragraphs are supplemental descriptions of each material.

Toyoura Sand (TO) The strength characteristics of Toyoura Sand is cited from Miura (1981), where the confining stress dependency of shear strength was considered. **Figure 2.5(a)** shows Mohr's stress circles at failure obtained from a series of conventional triaxial compression tests conducted by Miura (1981). The relationships between friction angle and relative density are also examined by Miura (1981), as shown in **Fig. 2.9**. Assuming independence between confining stress dependency and density dependency, we determined the shear strength constants as a function of effective mean normal stress σ_m and relative density D_r

$$\left\{ \begin{array}{l} \sigma'_r = 196 \text{ kPa} \\ \tan \phi_a^* = 0.425 \times 10^{-2} D_r + 0.500 \\ \sigma_a = \frac{\sigma'_r}{1 - \sin \phi_a^*} \\ \tan \phi^* = \tan \phi_a^* \left(\frac{\sigma_a}{\sigma'_m} \right)^{0.050} \end{array} \right. \quad (2.6)$$

Inagi Sand (IN) The confining stress dependent shear strength of Inagi Sand (IN) was examined by the results of conventional triaxial compression tests reported by Kusakabe *et al.* (1991). The test results are shown in **Fig. 2.8(a)**, where a least square method was applied to find the best fit.

$$\tan \phi^* = 0.897 \left(\frac{\sigma_a}{\sigma'_m} \right)^{0.038} \quad (2.7)$$

Monterey 0/30 Sand (MO) Shear strength properties of Monterey 0/30 Sand were reported by Kutter *et al.* (1988). Failure criterion of this sand was determined with least square method as shown in **Fig. 2.8(a)**:

$$\tan \phi^* = 1.177 \left(\frac{\sigma_a}{\sigma'_m} \right)^{0.089} \quad (2.8)$$

Shirahata Scoria (SS) Kusakabe *et al.* (1992) conducted a series of large-scaled *in situ* loading tests with square footings with different breadth of $B = 0.3\text{m}$ to 1.3m on a natural deposit of Shirahata Scoria. **Figure 2.5(b)** shows Mohr's stress circles at failure obtained from a series of conventional triaxial compression tests. Because the Shirahata scoria is a diluvial deposit and cemented, it has notable tensile strength of $\sigma_t = 147\text{kPa}$:

$$\tan \phi^* = 0.816 \left(\frac{\sigma_a}{\sigma'_m + \sigma_t} \right)^{0.023} \quad (2.9)$$

Glass beads (GB) Due to its spheric shape and negligible crushability of the glass beads particles, this material shows less confining stress dependency of shear strength compared with sands; see Maeda and Miura (1999):

$$\begin{aligned} \tan \phi^* &= 0.578 \left(\frac{\sigma_a}{\sigma'_m} \right)^{0.019} \\ \tan \phi_a^* &= 0.587 \\ \alpha &= 0.019 \end{aligned} \quad (2.10)$$

Table 2.1: Granular materials used.

Material Name	Category ¹⁾	Chapter				Citation
		Chap. 4	Chap. 5	Chap. 6	Chap. 7	
AN	N	—	○	—	—	—
CA	G	—	○	—	—	—
CG	A	—	○	—	—	—
GB	A	—	○	○ ²⁾	○ ²⁾	—
IN	N	○	—	○	○	Kusakabe <i>et al.</i> (1991)
INL	G	—	—	○	—	—
INS	G	—	—	○	—	—
LWA	A	—	○	—	—	—
MO	N	—	—	—	○	Kutter <i>et al.</i> (1988)
RG	A	—	○	—	—	—
SS	N	—	—	—	○	Kusakabe <i>et al.</i> (1992)
TO	N	○	○	○	○	—
TOG	G	—	○	—	—	—

Note: ¹⁾ A is artificial materials; N, natural materials; G, graded sand.

²⁾ GB-125-105 only.

Table 2.2: Soil testing for indices.

Item	Code
Specific density	JIS A 1202
Grain analysis	JIS A 12004
Maximum and Minimum density of sand	JIS A 1224

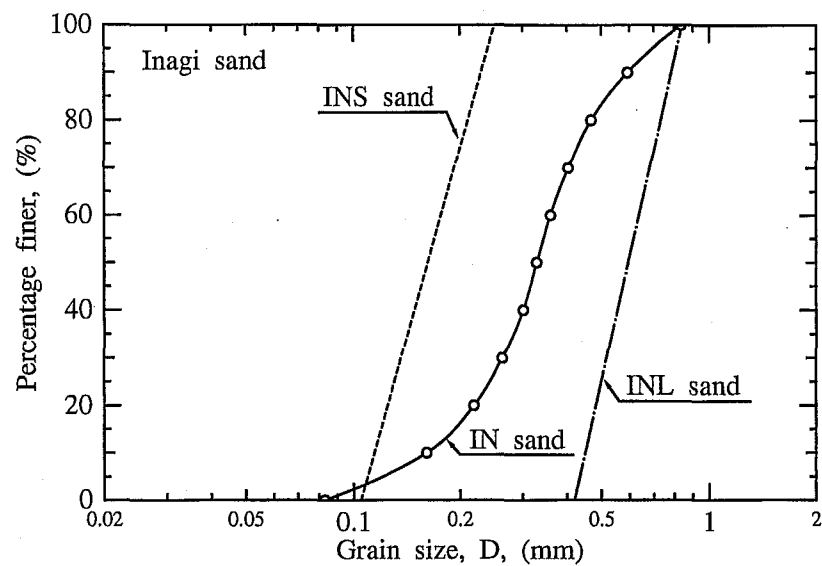


Figure 2.1: Grain size distribution curves of Inagi Sand (IN, INS and INL)

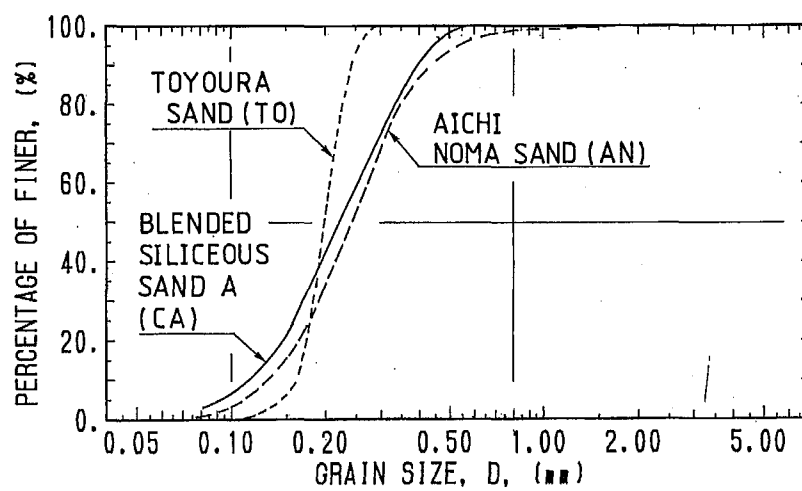


Figure 2.2: Grain size distribution curves of other natural sands.

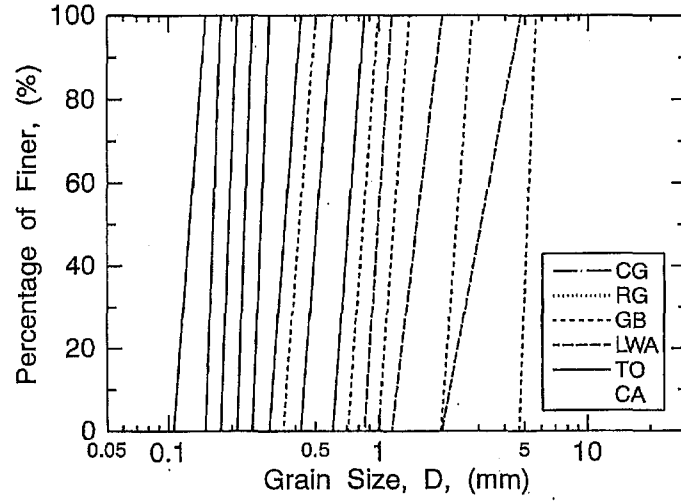


Figure 2.3: Grain size distribution curves of graded sands used in shear band tests in Chapter 5.

Table 2.3: Indices of materials used in loading tests (Chapter 4, 6) and direct shear tests for wall friction (Chapter 3).

Name	ρ_s (g/cm ³)	ρ_{dmax} (g/cm ³)	ρ_{dmin} (g/cm ³)	e_{max}	e_{min}	U_c
GB	2.489	1.588	1.411	0.764	0.567	1.09
IN	2.666	1.755	1.435	0.858	0.519	2.71
INL	2.651	1.678	1.424	0.862	0.580	1.414
INS	2.773	1.710	1.378	1.012	0.622	1.543
TO	2.660	1.660	1.340	0.985	0.602	1.34

Table 2.4: Physical properties of sand used in shear band tests in Chapter 5.

Material	ρ_s (g/cm ³)	D_{50} (μ m)	U_c	$(e_{\max} - e_{\min})$	A_{2D}
AN-1180-75 ¹⁾	2.650	231	2.12	0.445	460
CA-106-75	2.643	89	1.19	0.640	610
CA-150-106	2.660	126	1.19	0.587	620
CA-177-150	2.661	163	1.09	0.495	630
CA-212-177	2.647	194	1.09	0.488	640
CA-250-212	2.637	230	1.09	0.468	640
CA-300-250	2.634	274	1.10	0.456	650
CA-425-300	2.635	357	1.19	0.434	650
CA-600-425	2.629	505	1.19	0.398	670
CA-850-75 ²⁾	2.640	219	2.22	0.469	620
TOG-150-106	2.652	126	1.19	0.450	530
TOG-177-150	2.646	163	1.09	0.373	470
TOG-212-177	2.652	194	1.09	0.354	420
TOG-250-212	2.656	230	1.09	0.335	420
TOG-300-250	2.649	274	1.10	0.334	420
TOG-300-75 ¹⁾	2.650	200	1.30	0.367	420

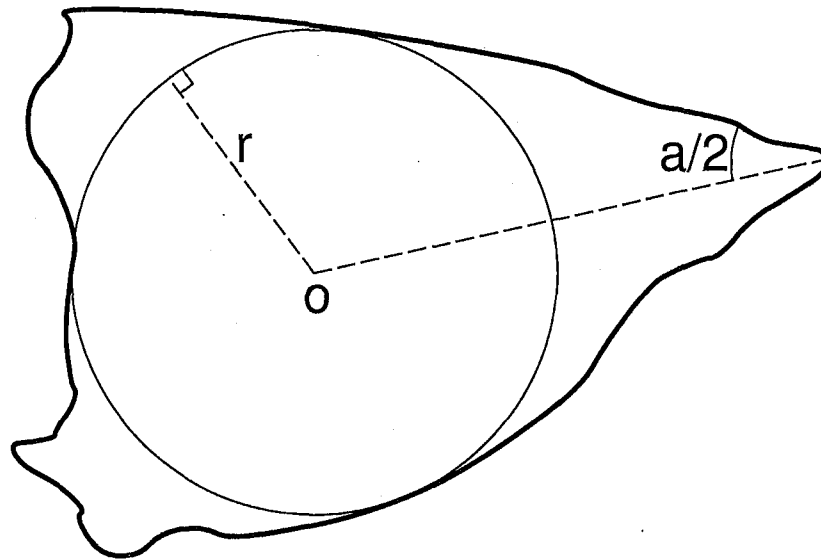
¹⁾ Natural sands²⁾ Blended sand

Table 2.5: Physical properties of artificial materials used in shear band tests in Chapter 5.

Material	ρ_s (g/cm ³)	D_{50} (μ m)	U_c	$(e_{\max} - e_{\min})$	A_{2D}
CG	—	2,760	1.00	0.247	450
GB-125-105	2.491	115	1.09	0.177	0
GB-500-350	2.491	418	1.29	0.158	0
GB-990-710	2.491	838	1.18	0.120	0
GB-1397-991	2.491	1,177	1.19	0.136	0
GB-2794-1981	2.491	2,353	1.19	0.099	0
GB-5613-4699	2.491	5,136	1.09	0.125	0
LWA-850-600	2.211	714	1.19	0.349	50
LWA-1150-850	2.141	1,001	1.18	0.287	50
LWA-2000-1180	2.148	1,536	1.30	0.279	50
LWA-4750-2000	2.240	3,082	1.54	0.261	50
RG	—	1,410	1.00	0.187	50

Table of angularity of some grains (Lees, 1964a, b).

0-99	100-199	200-299	300-399	400-499	500-599	600-699	700-799
800-899	900-999	1000-1099	1100-1199	1200-1299	1300-1399	1400-1499	1500-1599

Figure 2.4: Definition of *angularity* in plan view of a particle.

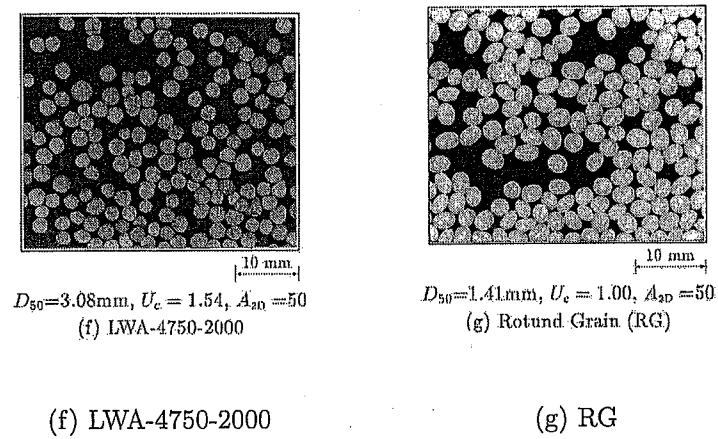
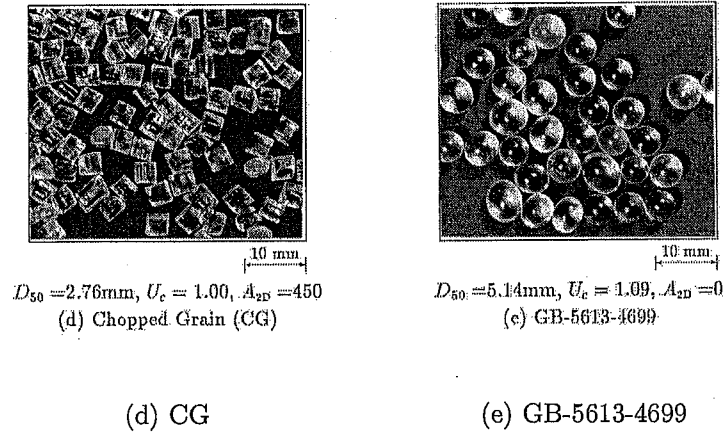
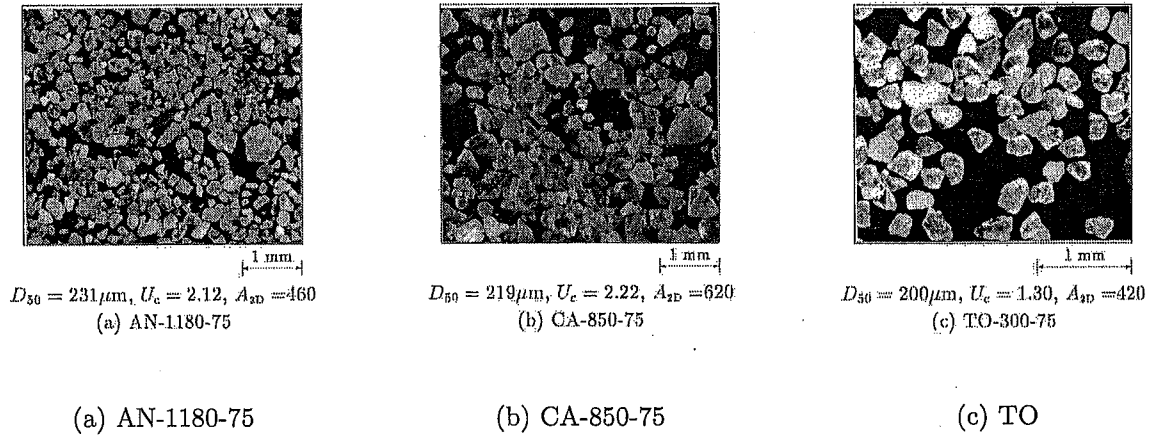
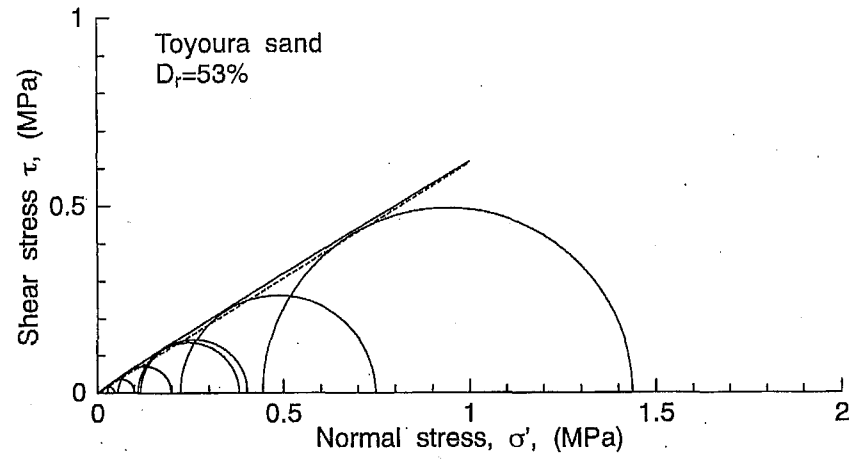


Photo 2.1: Particle shape of the materials used.



(a) Toyoura Sand after Miura (1981).

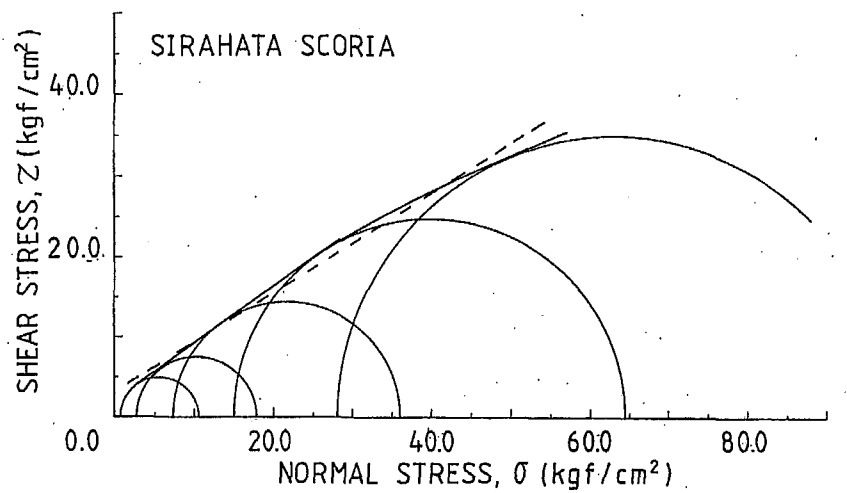
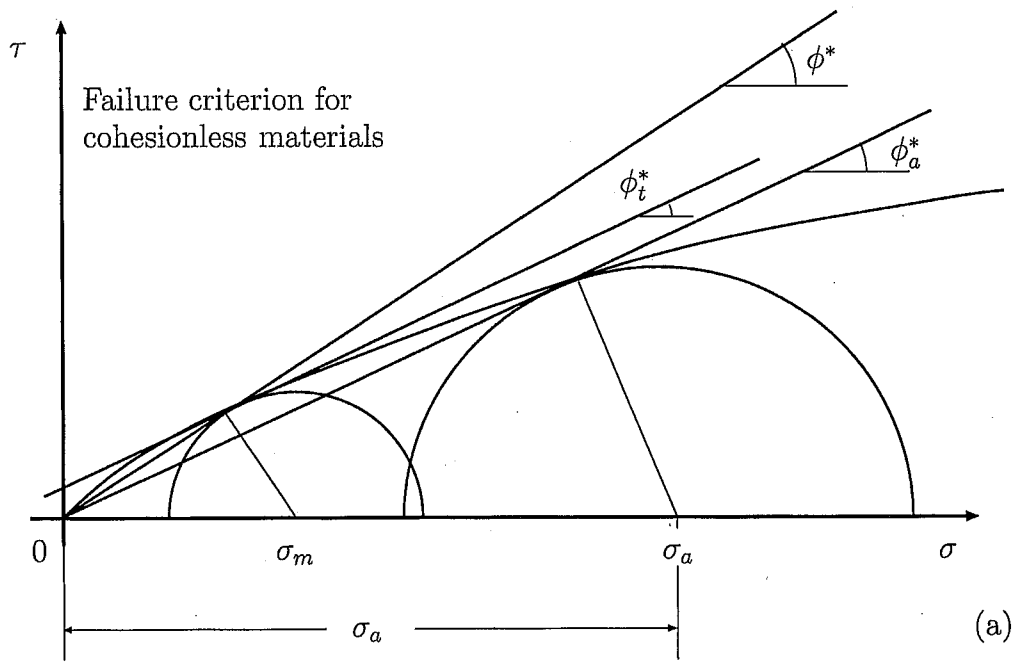
(b) Sirahata Scoria after Kusakabe *et al.* (1992).

Figure 2.5: Mohr's stress circles at failure



(a) Nonlinear failure envelope.

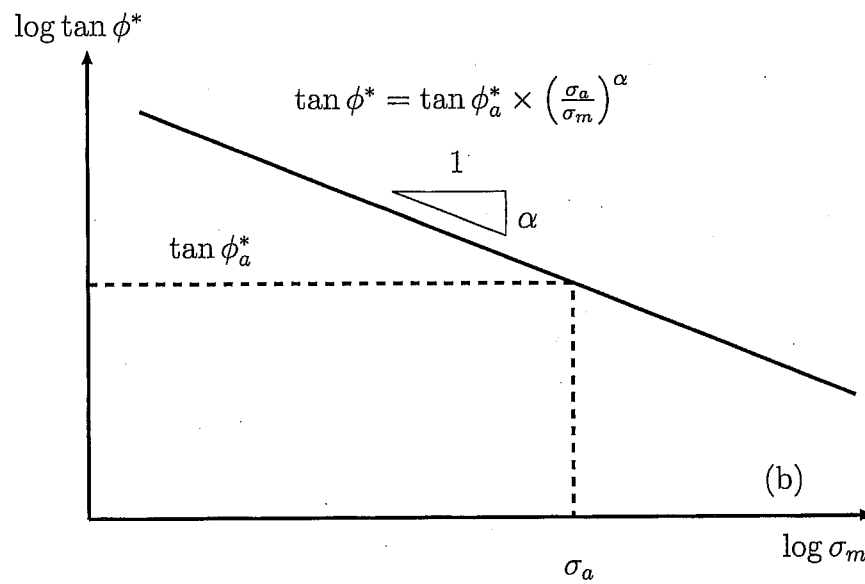
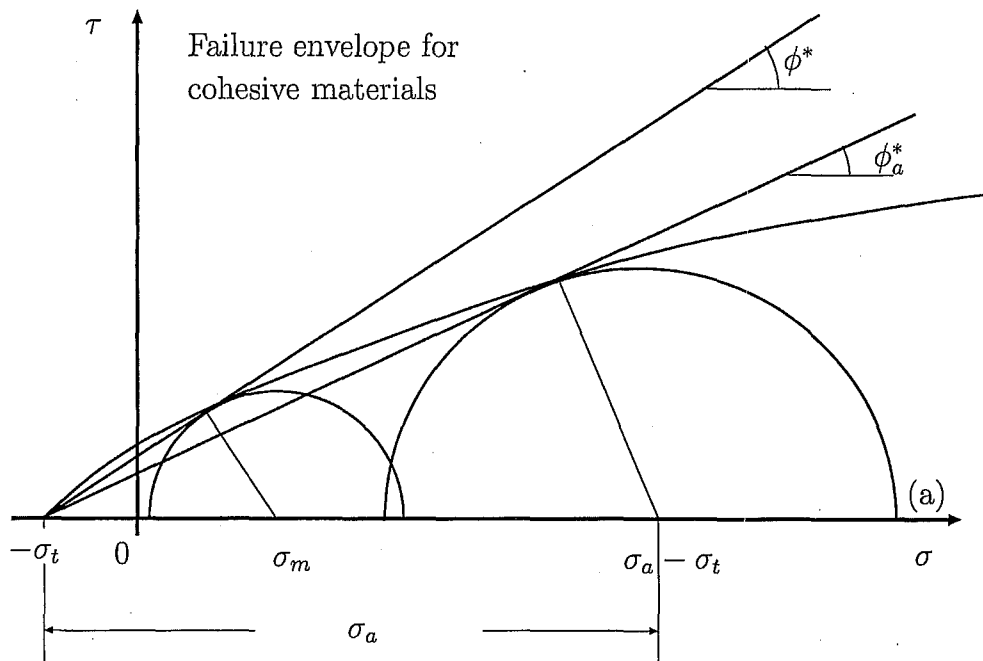
(b) Logarithmic linear relationship between $\tan \phi^*$ and mean normal stress σ_m^* .

Figure 2.6: Failure criterion for cohesionless soils with confining stress dependency of shear strength.



(a) Nonlinear failure envelope.

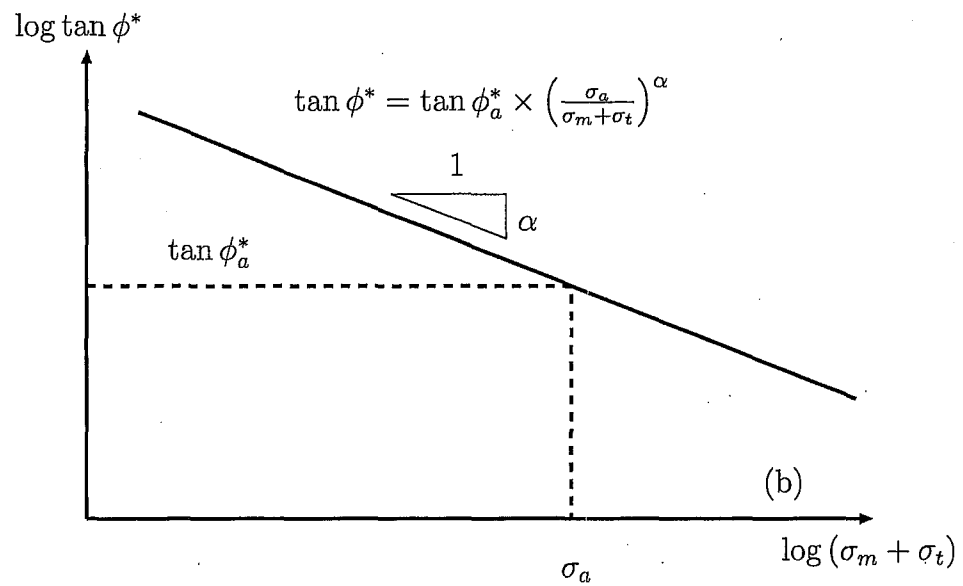
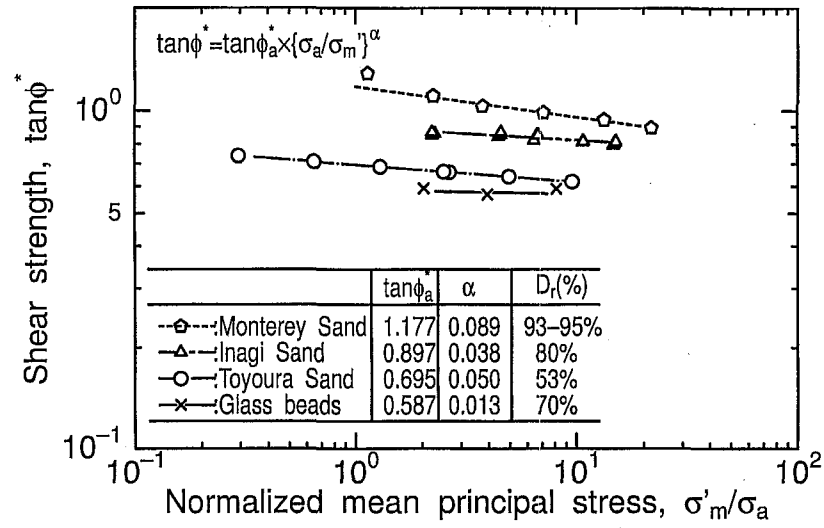
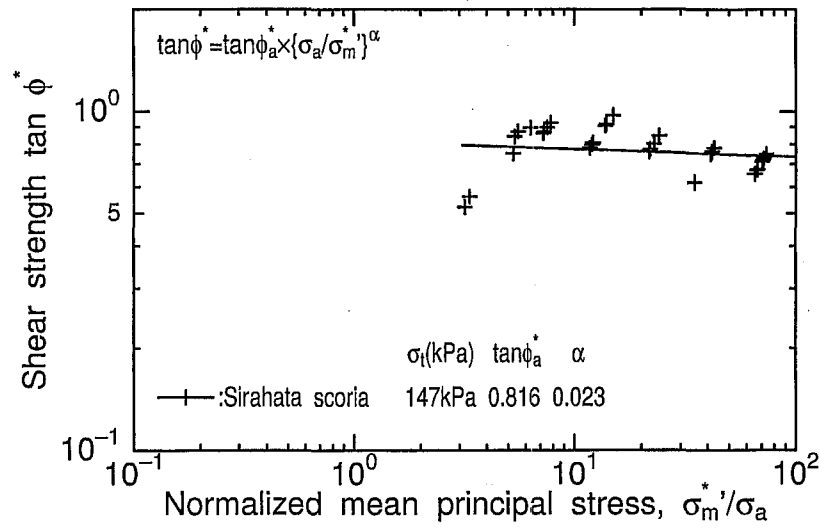
(b) Logarithmic linear relationship between $\tan \phi^*$ and mean normal stress σ_m^* .

Figure 2.7: Failure criterion for cohesive soils with confining stress dependency of shear strength.



(a) For cohesionless materials.



(b) For cohesive material (SS).

Figure 2.8: Determination of shear strength of granular materials from laboratory element tests with the aid of the proposed confining stress dependent failure criterion.

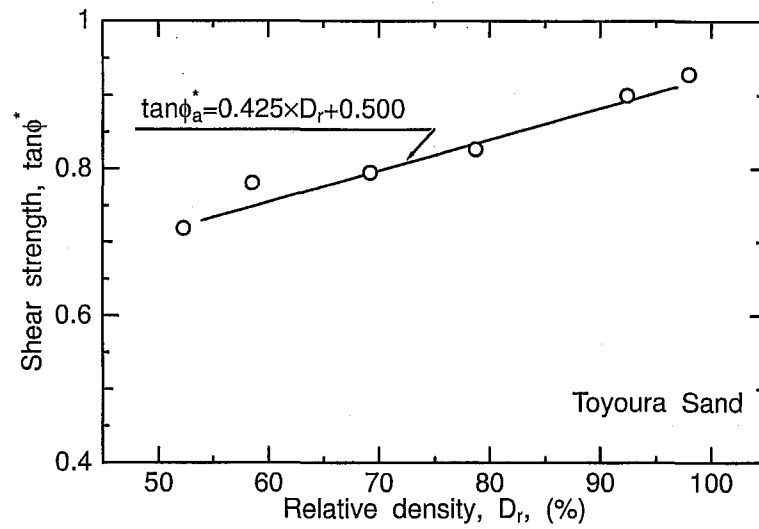


Figure 2.9: Dependency of shear strength of Toyoura Sand on relative density.

Table 2.6: Shear strength parameters for materials used in Chapter 7.

Material	$\tan \phi^*$	α	σ_t (kPa)	D_r (%)	Citation
TO	0.695	0.050	0	53	Miura (1981)
IN	0.987	0.038	0	80	Kusakabe <i>et al.</i> (1991)
MO	1.177	0.089	0	93-95	Kutter <i>et al.</i> (1988)
SS	0.816	0.023	147	—	Kusakabe <i>et al.</i> (1992)
GB	0.587	0.013	0	70	

Chapter 3

Apparatus and measurement

3.1 General

This chapter describes apparatus and measurement used for centrifuge model tests conducted in this study. Firstly, Section 3.2 describes centrifuge apparatus used in this study. Secondly, in Section 3.3 set-up for loading tests is introduced. Some descriptions of devices to measure load and displacement are also described. In Section 3.4, the author proposes a measuring method for displacement field in model sand with an aid of image processing. The aim of the method is simplicity and generality.

The author developed a simple and general displacement field measuring method, named CCIP method, by using a technique of image processing. The proposed method requires no target installed in the model to measure displacement field on sand. A newly developed correction method for distortion of image needs no longer information than the coordination of four datum points covering the analyzing zone. These features can reduce labor in the preparation of model sand and the analysis of displacement fields. Evaluation of accuracy and examples are presented here. The results show that the method can measure both macroscopic and microscopic deformations in sand within an accuracy of two pixels of the images employed.

3.2 Centrifuge apparatus

The centrifuge used is a balancing beam centrifuge located at Utsunomiya University. It was constructed in 1970 at Tokyo Institute of Technology and called Mark I centrifuge, which had been transferred to Chuo University in 1981 and then further transferred to Utsunomiya

University in 1987.

Shown in **Fig. 3.1** is the centrifuge facility used in this study. The facility consists of a balancing beam centrifuge and measuring and controlling equipment. **Photo 3.2** shows a loading jack equipped on the beam, slip ring stacks and the measuring and controlling equipment. **Figure 3.2** shows the balancing beam centrifuge; it has an effective radius of 1.18m and a maximum capacity of 15 ton-g, as tabulated in **Table 3.1**. The centrifuge is installed in a pit under the ground floor, having an inner diameter of 3,100mm and a depth of 1,082mm made of a cylindrical concrete wall and a red colored circular steel lid.

Thirty bridges of strain gauges and five transducers can be connected to a data logger via a multiplexer and a junction box on the beam of the centrifuge, respectively. All measurements can be acquired by the data logger, and send to a personal computer to be stored and processed. The data logger is for static data acquisition and has a scanning rate of 0.08sec per point, a measuring range of $\pm 4000 \times 10^{-6}$ strain and a resolution of 1×10^{-6} strain. To avoid EMI and to promote precise measuring, the transducers were connected to the data logger by using double shielded and twisted pair cables through a 20 pole slip ring stack. A guard electrode made of two copper plates isolates junctions and naked wires from electro-magnetic and electrostatic induction.

Power to drive and control models such as footings and others is of electric and pneumatic, which are supplied through a five-pole slip ring stack and a rotary joint. The five poles slip rings stack is attached on a top portion of the axis of the centrifuge, and connected to a switch array in the laboratory. Simple model manipulation, such as alternation of loading and unloading, is available with the switch array. The rotary joint is placed at the bottom end of the center axis of the centrifuge, and provides a one way pneumatic channel from a control panel to the model. The control panel consists of several valves, a regulator, a Bourdon gauge and an air filter. The magnitude of pneumatic pressure can be changed manually with the regulator. Negative pressure can be available by using an aspirator installed in the model, if required.

3.3 Setup for loading test

3.3.1 Soil container

Typical soil containers used in this study are rectangular strong boxes made of steel plates with a thickness of 9mm. Several steel bars are attached on the container walls to provide enough rigidity to the container. The inner dimensions of the container are 500mm in length, 320mm in depth and 150mm in width. Inner walls of the container are polished and lubricated to provide an ideal distribution of overburden pressure, by using a lubrication layer made from a 0.2~0.37mm thick latex membrane and a 0.017mm thick silicon grease layer, as shown in **Fig. 3.3**.

3.3.2 Loading system

Figure 3.4 shows setup for loading tests with circular footings in centrifuge, while **Fig. 3.5** for strip footings. The setup consisted of a strong rectangular soil container and an electric loading jack. The container was stiffen and smoothed, as described above. The loading jack, shown in **Photo 3.3**, powered by a 60W AC motor was controlled by the switch array placed in laboratory. The jack drove a model footing into sand with a constant loading rate of 1mm/min.

The footings used in the loading tests were of both circular and strip. For loading test with a circular foundation, the diameter of the footing used was 30mm and made of steel. The footing was rigidly connected to the loading jack through a load cell produced by a Japanese manufacture, Tokyo Sokki Kenkyujo Co., Ltd. For the tests with a strip footing, the footing used was divided into five 30mm square footings. Each of the five square footings was connected to five hand-made load cells gathered on a thick steel plate, and then connected to the loading jack, as shown in **Photo 3.3**.

3.3.3 Measurements and devices

Settlement of the sand surface was measured by a LVDT (DC differential transformer) to obtain a void ratio change of the sand under the acceleration at which the loading tests were conducted. **Figure 3.6** is a schematic drawing of the LVDT used and **Photo 3.4** shows the LVDT and its bracket made of aluminum alloy. This LVDT was contact type, however, the

weights of the core and the probe were small enough comparing with the bearing capacity of the sand used (approximately one thirty-th of the ultimate bearing capacity). Therefore, no influence of the self weight appeared in the measured settlement; see Nakatomi (1992). A calibration result of this LVDT is shown in **Fig. 3.7**. The result shows a good linearity between displacement and output within the range of $\pm 5\text{mm}$. When the LVDT was driven with a ten volts of regulated DC power supply, it provided a $1.9 \times 10^{-3}\text{mm}$ of resolution in displacement.

For the tests with circular footing, the settlement of the footing was measured by a linear voltage displacement transducer (LVDT) having a 10mm capacity and a 10^{-6}mm resolution. On the other hand for the tests with strip footing, a laser displacement transducer, shown in **Photo 3.5**, having a resolution of $10\mu\text{m}$ and a measuring range of $\pm 40\text{mm}$ was employed.

The load cells used in the tests with strip footing were hand-made. Finding no suitable load cell among the markets, the author designed and produced two brand new bi-axial load cells. Tani (1982) proposed the original design concept of this type of load cells. Sato *et al.* (1983) applied it to 1g loading tests, while Imamura *et al.* (1996) developed miniature load cells to measure the earth pressure acting on a miniature shield tunneling machine in a centrifuge. **Figure 3.8** and **Fig. 3.10** are the design diagrams, and **Table 3.2** shows the specification of the load cells. **Photo 3.6** and **Photo 3.7** show five BLC-01t and BLC-10k load cells with a divided strip footing made of steel. **Figure 3.9** and **Fig. 3.11** demonstrate that the linearity and isolation appearing in the calibration results are good enough to use the load cells in model loading tests under both centrifugal and gravitational acceleration fields.

In several cases of centrifuge model tests, vertical and horizontal earth pressure in sand was measured by using a pair of earth pressure cells, manufactured by Tokyo Sokki Kenkyujyo Co., Ltd. The earth pressure cells used had a capacity of 2.94MPa and dimensions of 30mm in diameter and 10mm in thickness, as shown in **Fig. 3.12**.

3.4 CCIP method: measuring method of displacement field for sand by image processing

3.4.1 Review on measurement of displacement in model tests

Measurement plays an important role in mechanical problems. In the modeling which replaces a physical problem by a mathematical problem, the identification of the governing mechanism of the phenomena is an essential. In model tests, observation of deformation is essential step to understand failure mechanism. Therefore, many researchers have tackled the development of the schemes.

Shimizu and Konaigai (1994) present a review of the technique of deformation visualization. Laser Speckle Method introduced by Tatsuoka *et al.* (1990) provide precise measurement of deformation, however the technique requires special equipments. On the other hand, there is a widely distributed simple method, called Target Method (Oda and Furuto, 1982, Shibuya *et al.*, 1997) In Target Method, displacements at each point are measured by tracing many nodes on a grid painted on a membrane or many targets embedded in the model. Although Target Method is a generic and simple method, reading of many nodes or targets requires terrible labor. Another disadvantage of this method is difficulty in measurement of local and/or microscopic deformation. Measurement is only available the point at which the targets are located, and resolution can be affected by the dimensions of the target used.

Recent development of computer and image processing equipment promotes application of image processing technology to geotechnical engineering. Allersma (1997), Horii *et al.* (1997) and Guler *et al.* (1999) introduced the technique of image processing in geotechnical measurement. Allersma (1997) showed various kinds of application of imaging technologies in experimental geotechnics. Horii *et al.* (1997) developed Matching Method and applied the method to compression tests on rock materials. Guler *et al.* (1999) traced the movement of sand particles subjected to direct shear.

In centrifuge tests, imaging technologies are useful tools for measuring displacement field in models, therefore many researches have been undertaken. Due to spatial limitation of centrifuges, to install numerous displacement transducers in the model is not practical. Allersma (1997) measured displacement due to embankment in centrifuge by using image processing. Davies and Jones (1998) tried to install a film camera in a centrifuge to acquire a clear image

for image processing. Taylor *et al.* (1998) developed a 3-D measuring system.

Unfortunately, the disadvantages of Target Method still remain even imaging technologies are employed, because most of the methods require a number of “targets”.

The author developed a displacement measuring method that uses no target to measure displacement in models. This feature can reduce labor in the preparation of the model and the analysis of displacement fields.

3.4.2 Development of a displacement field measuring technique

Figure 3.13 is a flow diagram showing how displacement fields are measured. The sequence of the method proposed consists of five parts. The followings are detailed descriptions of each part.

3.4.2.1 Photography of the sand model and input of pictorial image

A digital still camera and a conventional 35mm camera or a camera of large size are available for acquisition of digital images. Use of digital still camera reduces the cost and work for image data acquisition. If photo cameras are used, two methods for image acquisition are available; i) making a photo CD in a photograph shop, ii) use of a film scanner or an image scanner.

Because the resolution of the image data dominates accuracy of the measurements of displacement, finer image is preferable. In the case of a macroscopic measurement, it seems good to digitize a sand particle with an image with a dimension larger than 10×10 pixels. It is also good for pattern recognition to scatter dyed sand over the observation side.

Four nodal points of which coordinates are known are required to transform between image and real coordinate systems and correct distortion of images occurred during exposing and digitizing sequences.

3.4.2.2 CC method: pattern recognition

Figure 3.14 shows a flowchart of CC method which provides corresponding image coordinates between two frames of images: the origin coordinate in a certain frame of image (X_o, Y_o) , and the destination coordinate in the next frame of image (X_d, Y_d) . The method to obtain these coordinates is a kind of a technique of pattern recognition. In the method, a cross correlation is used to evaluate the similarity between the original and the destination images. The method

will be call CC method, hereafter.

Firstly a small image with a size of $(2n + 1) \times (2n + 1)$ pixels, called referential image hereafter, is extracted from an original frame at a region around an original coordinate (X_o, Y_o) . Also another small image with a size of $(2n + 1) \times (2n + 1)$ pixels, called test image hereafter, is extracted from the next frame at a tentative destination coordinate (X'_d, Y'_d) . (X_o, Y_o) and (X'_d, Y'_d) are the center points of referential image and test image, respectively. Referential image and test image are once transformed into one dimensional vector V_1 and V_2 , then coefficient of cross correlation between V_1 and V_2 , denoted by R_{12} , is obtained by **Eq. (3.1)**.

$$R_{12} = \frac{\sum_{i=1}^{(2n+1)^2} v_{1i} \cdot v_{2i}}{\sqrt{\sum_{i=1}^{(2n+1)^2} v_{1i}^2 \cdot \sum_{i=1}^{(2n+1)^2} v_{2i}^2}} \quad (3.1)$$

where v_{1i} and v_{2i} are components of vector V_1 and V_2 , respectively.

Changing tentative destination coordinate (X'_d, Y'_d) , the method seeks the maximum point of R_{12} . The maximum point can be recognized as the destination coordinate (X_d, Y_d) corresponding to original coordinate (X_o, Y_o) in the referential image. Repeating the above procedure with a number of different original coordinates (X_o, Y_o) , the method obtains whole combinations of (X_o, Y_o) and (X_d, Y_d) in analyzing area. When the images are of color, the average value of three coefficients of cross correlation obtained from three color components (red, green and blue) is used for pattern recognition.

3.4.2.3 IP method: distortion correction and coordinate transformation

To obtain displacement from the original and the destination coordinates (X_o, Y_o) and (X_d, Y_d) in the images, it is needed that correction of distortion in the images and transformation of coordinate system from images to a real object. The author considered an applicability of Affine transformation at first, however Affine transformation technique keeps parallelism of coordinates so that this technique cannot be applied to transformations between arbitrary quadrilateral. If an exposing plan in a camera used for image data acquisition is not parallel to an observing plan on an object, a square on the object will become a trapezoid in the obtained image. In particular in a centrifuge model test, to maintain parallelism in-flight seems to be difficult. Above disadvantage of Affine transformation is likely to lead a fatal error (Mochizuki and Mikasa 1984). The author noted that a technique used in finite element method (FEM) with isoparametric elements can be applied to transformation of coordinate

systems in order to exclude the disadvantage. The proposed transformation method will be called IP method, hereafter.

IP method can transform coordinates among arbitrary quadrilaterals, and the method can be used for distortion correction. When the distortion due to aberration of optical equipment is not negligible, you can divide the analyzing region into several small quadrilateral isoparametric elements and obtain more accurate values of coordinates. Because of a feature of isoparametric elements, the continuity at the boundaries, which is not ensured in Affine transformation, will be automatically maintained when the analyzing region is divided.

Another advantage of IP method is its simplicity. Other methods based on a technique of ray tracing require several parameters for the position and orientation of the camera used (Mochizuki and Mikasa 1984). On the other hand, IP method requires only the image and real coordinates of four nodal points in the observed plane that enclose analyzing regions. Therefore, IP method can treat pictures of old experiments, in which the position and orientation of the camera used are unknown. Additionally, an error optimization sequence required in other methods, such as Affine transformation and the method proposed in Mochizuki and Mikasa(1984), is no longer needed in IP method.

Figure 3.15 is a schematic diagram of IP method, an isoparametric element with four nodal points. In isoparametric element the coordinate system comprised of intermediate parameters (ξ, η) is called natural coordinate system. In FEM, displacement is associated with real coordinate. On the other hand in IP method, image coordinate system (X, Y) is associated with real coordinate system (x, y) instead of the displacement in FEM.

As we can see in FEM text books, natural coordinate (ξ, η) can be associated with image coordinate (X, Y) by using the equations for polynomial interpolation shown in **Eq. (3.2)**,

$$\begin{cases} X = \sum_i^n X_i N_i(\xi, \eta) \\ Y = \sum_i^n Y_i N_i(\xi, \eta) \end{cases} \quad (3.2)$$

where i is node number (1-4); (X_i, Y_i) , image coordinate of four fixed nodes; (X, Y) , arbitrary image coordinate corresponding with parameters (ξ, η) ; and $N_i(\xi, \eta)$, shape functions expressed in **Eq. (3.3)**.

$$\begin{cases} N_1(\xi, \eta) = \frac{1}{4}(1 - \xi)(1 - \eta) \\ N_2(\xi, \eta) = \frac{1}{4}(1 + \xi)(1 - \eta) \\ N_3(\xi, \eta) = \frac{1}{4}(1 + \xi)(1 + \eta) \\ N_4(\xi, \eta) = \frac{1}{4}(1 - \xi)(1 + \eta) \end{cases} \quad (3.3)$$

Inverting Eq. (3.2), we can obtain natural coordinate (ξ, η) associated with image coordinate (X, Y) . First, let X_i and Y_i into A_i and B_i , as follows,

$$\begin{cases} A_1 = (X_1 + X_2 + X_3 + X_4) \\ A_2 = (X_1 - X_2 + X_3 - X_4) \\ A_3 = (-X_1 + X_2 + X_3 - X_4) \\ A_4 = (-X_1 - X_2 + X_3 + X_4) \end{cases} \quad (3.4)$$

$$\begin{cases} B_1 = (Y_1 + Y_2 + Y_3 + Y_4) \\ B_2 = (Y_1 - Y_2 + Y_3 - Y_4) \\ B_3 = (-Y_1 + Y_2 + Y_3 - Y_4) \\ B_4 = (-Y_1 - Y_2 + Y_3 + Y_4) \end{cases} \quad (3.5)$$

Some reductions and rearrangements are applied,

$$\begin{cases} C_1 = B_2A_3 - A_2B_3 \\ C_2 = B_2(A_1 - 4X) + A_3B_4 - A_2(B_1 - 4Y) - B_3A_4 \\ C_3 = B_4(A_1 - 4X) - A_4(B_1 - 4Y) \end{cases} \quad (3.6)$$

Then we can obtain the inverted functions for ξ and η .

$$\xi = \begin{cases} -C_3/C_2 & C_1 = 0 \\ \frac{-C_2 \pm \sqrt{C_2^2 - 4C_1C_3}}{2C_1} & C_1 \neq 0 \text{ (singular)} \end{cases} \quad (3.7)$$

$$\eta = \frac{4Y - B_1 - B_3\xi}{B_2\xi + B_4} \quad (3.8)$$

Real coordinates of the four nodes (x_i, y_i) also can be associated with natural coordinate (ξ, η) . Substituting (ξ, η) obtained from Eq. (3.7) and Eq. (3.8) into Eq. (3.9), we can establish the relationship between the image coordinate system and the real coordinate system, i.e:

$$\begin{cases} x(\xi, \eta) = \sum_i^n x_i N_i \\ y(\xi, \eta) = \sum_i^n y_i N_i \end{cases} \quad (3.9)$$

3.4.3 Evaluation of accuracy of CC Method

A series of numerical experiments was conducted to examine accuracy and applicability of Cross Correlation Method (CC Method). CC Method is based on a technique of pattern recognition, therefore there are limitations in the magnitude of deformation. In the numerical

experiments, the relationships between amount of errors and magnitude of deformation will be examined with various sizes of referential image in order to determine the limitations of deformation.

Photo 3.8 shows the original image with a size of 500×500 pixels used in the numerical experiments. The material used was graded silica sand of which physical property is tabulated in **Table 3.3**. The sand was poured into a rectangular container made of glass with dimensions of 600mm in length, 160mm in width and 150mm in depth (see **Photo 3.9**), and dry model sand with a density of 1.5g/cm^3 was prepared. A photograph used to make the original color image was taken with a Bronica size camera (Mamiya 645proTL) placed in front of the soil container, and digitized with an image scanner. Red, blue, yellow, orange and black colored sands are mixed to provide clear contrast in the color image. A part of the image was clipped with a size of 500×500 pixels and the original image in **Photo 3.8** was obtained. The maximum grain size of the sand was $D_{\max} = 1.10\text{mm}$, of which size approximately equals 11×11 pixels of the original image.

In the numerical experiments, four types of deformation shown in **Fig. 3.16** were examined: isotropic compression, isotropic expansion, pure shear and counter clockwise rotation. The original image shown in **Photo 3.8** was distorted with an image processing software, and input images for the numerical tests were prepared.

Table 3.4 presents the test conditions of the numerical experiment. The author applied CC Method to obtain corresponding image coordinate (X, Y) at $n = 16$ locations in the original image (100 pixels each). The size of the referential images employed ranged from 5×5 pixels to 41×41 pixels. A comparison between the coordinate (X, Y) obtained by CC Method and the coordinate estimated from applied deformation (X_0, Y_0) was conducted. The standard deviation between (X, Y) and (X_0, Y_0) , defined in **Eq. (3.10)**, was employed to express the error of CC Method.

$$\left\{ \begin{array}{l} \sigma_x = \sqrt{\frac{\sum_{i=1}^{n=16} \{(X_i - X_{0i})^2\}}{n}} \\ \sigma_y = \sqrt{\frac{\sum_{i=1}^{n=16} \{(Y_i - Y_{0i})^2\}}{n}} \\ e = \frac{\sigma_x + \sigma_y}{2} \end{array} \right. \quad (3.10)$$

Figures 3.17(a-c) show the relationship between errors and the magnitude of deformation. The values of the error became extremely large when pattern recognition completely

failed. Shown in **Figs. 3.17(a-c)** are only the plots, in which the errors are smaller within two pixels. Errors within one pixel are attributed to the quantization error, so that it can be noted that the pattern recognition by CC Method was properly conducted and obtained coordinates (X, Y) are of correct.

Applicability of CC Method was confirmed when the magnitude of deformations is smaller than 15% by using referential images with a size larger than 11×11 pixels. **Table 3.5** summarizes the relationship between the size of referential image and the limitations of applicable deformations. The referential image with a size of 5×5 has less applicability except for isotropic expansion. The size is too small to retain sufficient information and to detect differences among images. On the other hand, in the cases with the referential images larger than 11×11 pixels no differences in accuracy appear when the range of deformation remains within 10%. Although some differences in accuracy become visible when the range of deformations are beyond 10%, the measurement provides sufficient accuracy up to 15% of deformation ($\tan(10\text{deg.}) = 0.176$).

If deformation becomes larger than 15%, the size of referential image that provides the smallest amount of error is 11×11 pixels. A reduction in errors with an increasing size of referential image was expected at first because of increase of image information, the results, however, show an opposite tendency on the size of referential images. It can be considered that the referential images with a large size cannot follow large uniform deformation so that error occurred during pattern recognition. This tendency implies that an excessive size of referential image is likely to lead not only a swell of calculation time but also deterioration of accuracy.

The following conclusions are drawn from the results of a series of numerical experiments.

1. The size of the referential image is required at least 11×11 pixels.
2. The accuracy becomes suddenly worse as the size of referential image becomes smaller than the size of sand particles. Therefore, the size of referential image should be grater than the size of sand particles.
3. Applicable range of deformation of CC Method between two images is 15%. Within this range of deformation no significant differences in accuracy appear among different sizes of referential image.

4. Considering above three points, the author can provide the following recommendation. Setting the size of referential image equal to the size of sand particles, exposing photographs of each 15% of deformation, and employing CC Method, we can measure displacement on model sand within an accuracy of two pixels.

3.4.4 Example of application

This section introduces an application of CCIP method in a loading test conducted under gravitational acceleration field. The sand used was graded silica sand of which physical properties are tabulated in **Table 3.3**. **Photo 3.10** is an example of the image used, and test condition is tabulated in **Table 3.6**.

A step by step calculation using five photographs taken at different five stages of settlement was employed to obtain displacement field on the model surface.

Figure 3.18 shows the distribution of accumulated maximum shear strain $\gamma_{max} = \varepsilon_1 - \varepsilon_3$ observed at the peak stage. **Figures 3.19(a, b)** show the distributions of maximum shear strain γ_{max} at just before the peak and just after the peak stages. Two clear slip lines passing through active and passive zones reach the surface of model at just after the peak stage, as shown in **Fig. 3.19(b)**. Because the width of the footing was 30mm, general shear failure could be occurred in this case. It can be said that CCIP method can be used for measurement of displacement field and is also an effective tool for model tests.

3.5 Summary

This chapter describes apparatus and measurement used for centrifuge model tests conducted in this study. Descriptions of a centrifuge, measuring and controlling equipments, and several models including transducers and measurements used in this study were presented. Additionally a newly developed displacement field measuring technique based on image analysis, named CCIP method, was introduced.

CCIP method consists of two parts. The first part, named CC method, is pattern recognition. A cross correlation of two images is used to evaluate the similarity between the original and the destination images in CC method. The second part, named IP method, is a newly developed coordinate transformation method. Using IP method, we can transform image coordinate system to real coordinate systems, and correct distortion of images occurred during

exposing and digitizing sequences. With an aid of IP method, CCIP method requires no target installed in the model to measure displacement field on sand. Only coordinates of four fixed points covering the analyzing zone are required to determine real coordinates. These features can reduce labor in the preparation of model sand and the analysis of displacement fields.

According to evaluation of accuracy based on a series of numerical experiments, the following conclusions are drawn from the results.

1. The size of the referential image is required at least 11×11 pixels.
2. The accuracy becomes suddenly worse as the size of referential image becomes smaller than the size of sand particles. Therefore, the size of referential image should be greater than the size of sand particles.
3. Applicable range of deformation of CC Method between two images is 15%. Within this range of deformation no significant differences in accuracy appear among different sizes of referential image.
4. Considering above three points, the author can provide the following recommendation. Setting the size of referential image equal to the size of sand particles, exposing photographs of each 15% of deformation, and employing CC Method, we can measure displacement on model sand within an accuracy of two pixels.

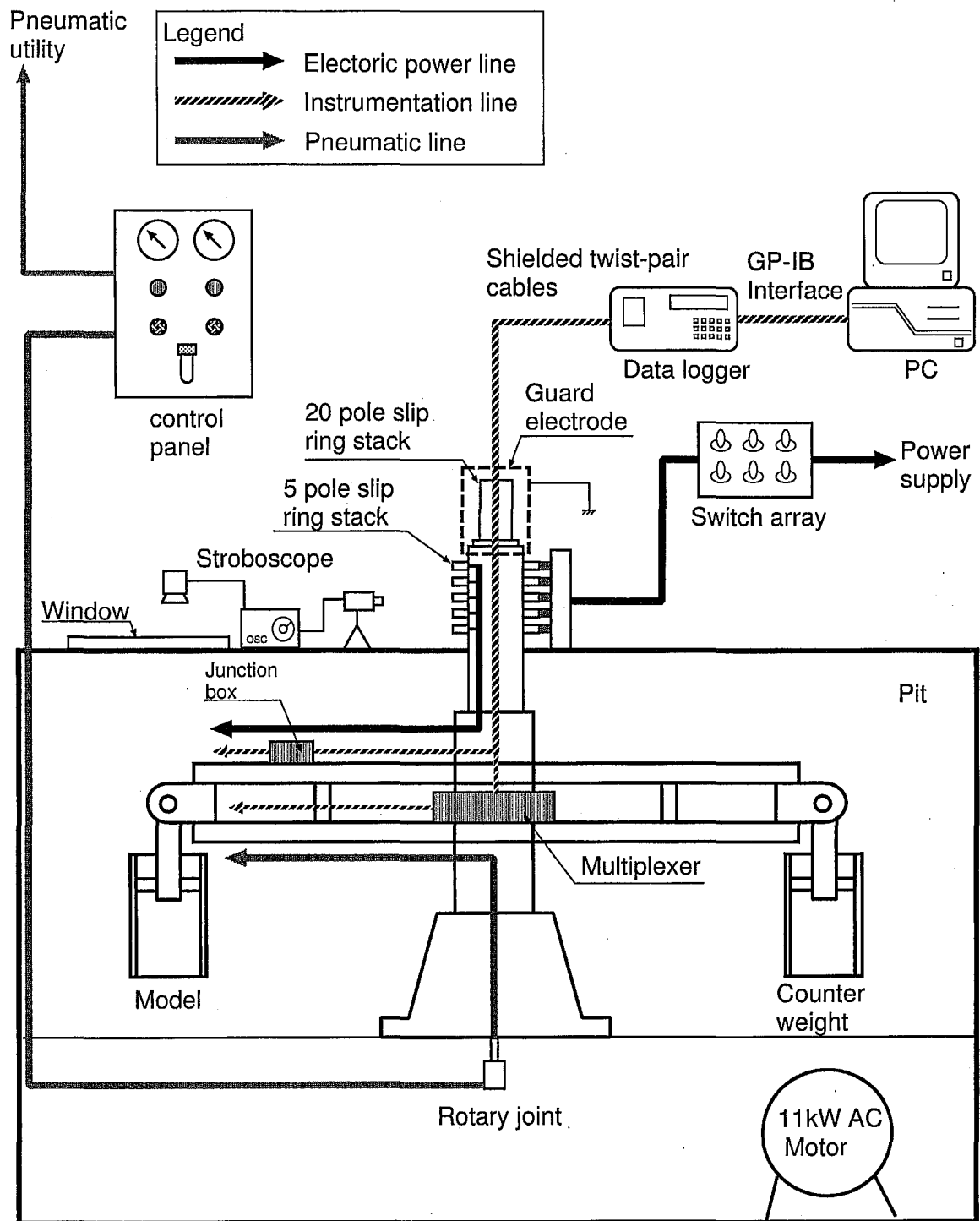


Figure 3.1: Centrifuge facility at Utsunomiya University.

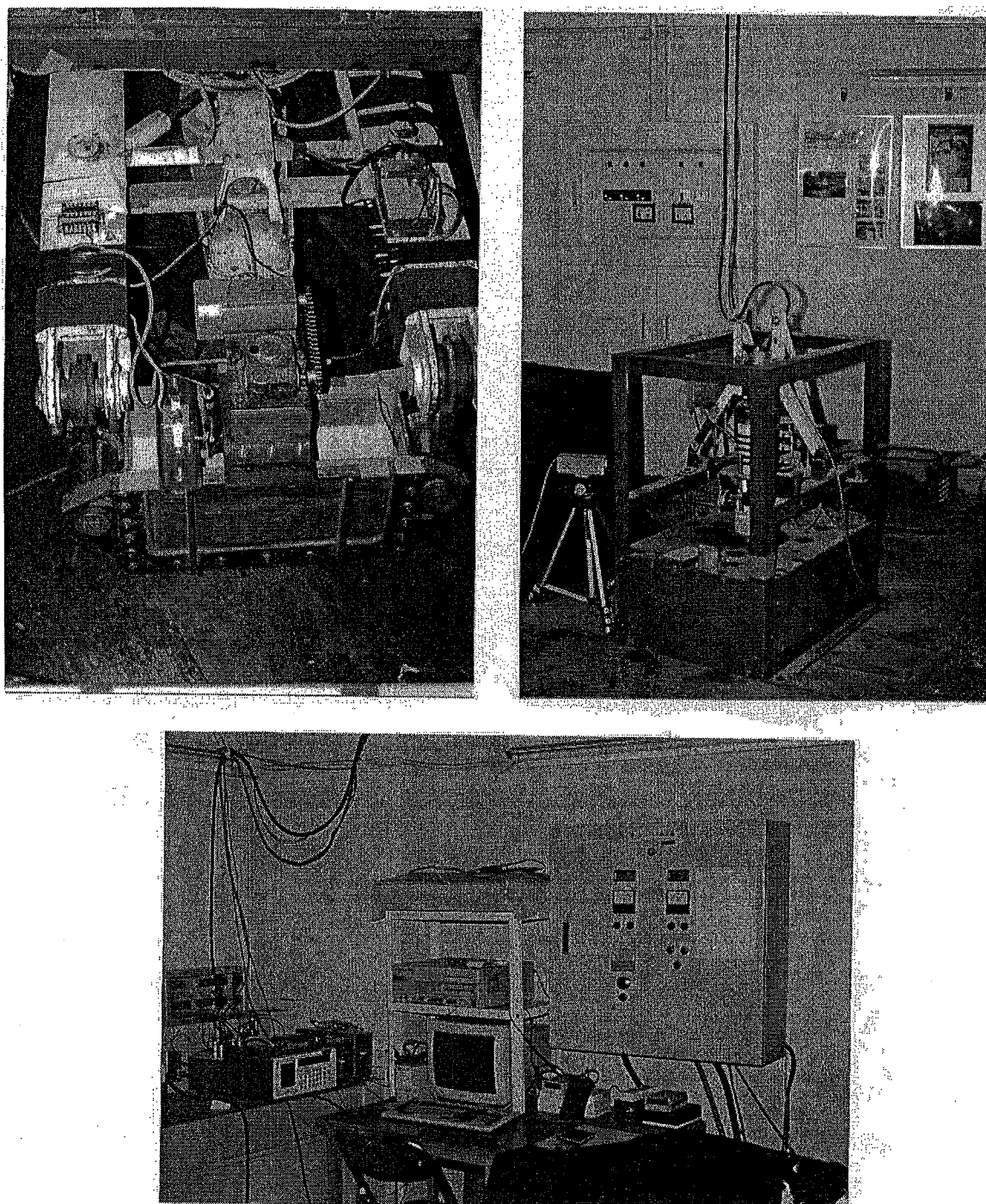


Photo 3.2: Centrifuge facility and a loading jack.

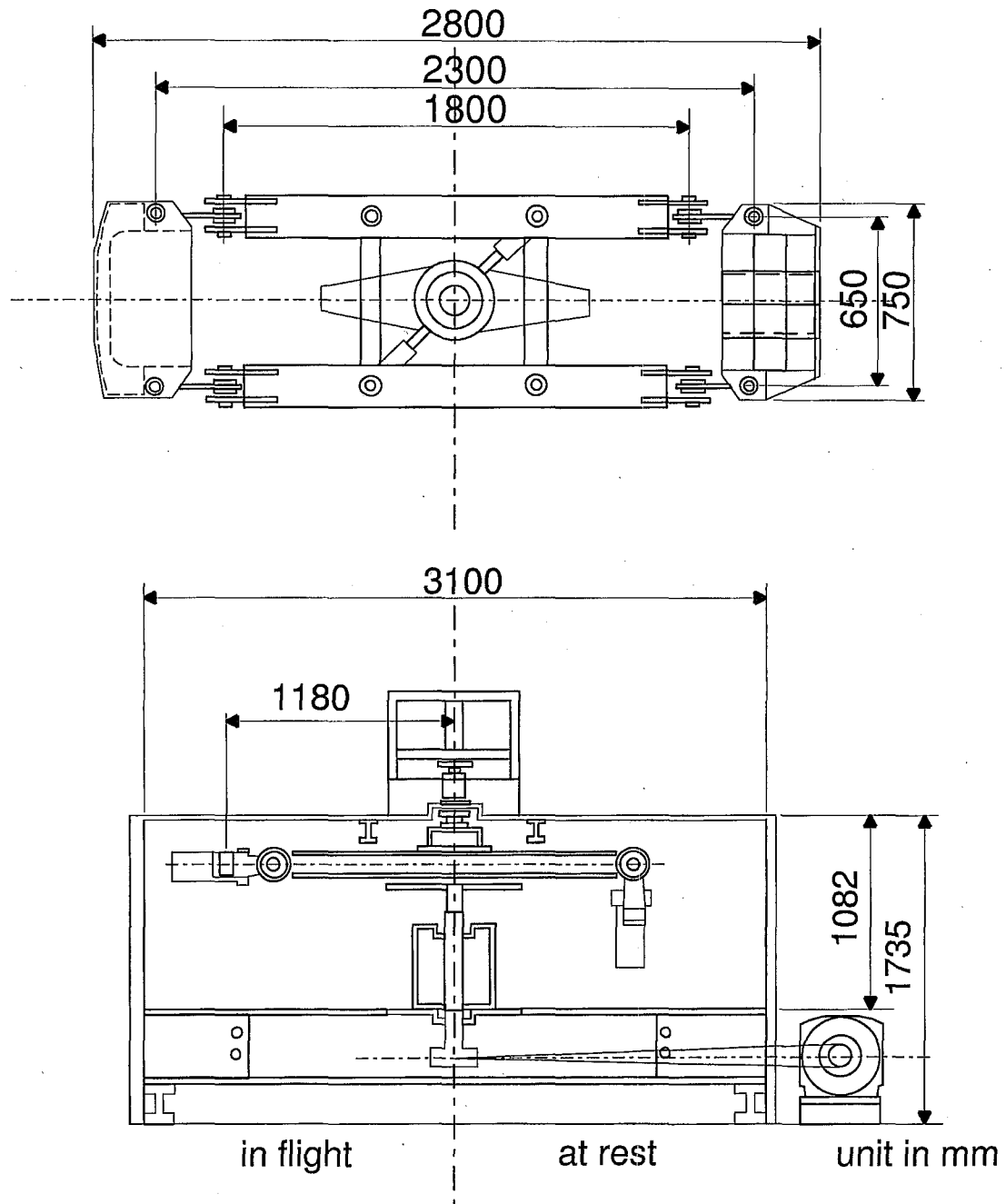


Figure 3.2: Balancing beam type centrifuge at Utsunomiya University.

Table 3.1: Specification of the centrifuge.

Effective radius	1.18	(m)
Maximum acceleration	120	(g)
Payload	150	(kg)
Capacity	18	(ton-g)
Drive power	11	(kW)

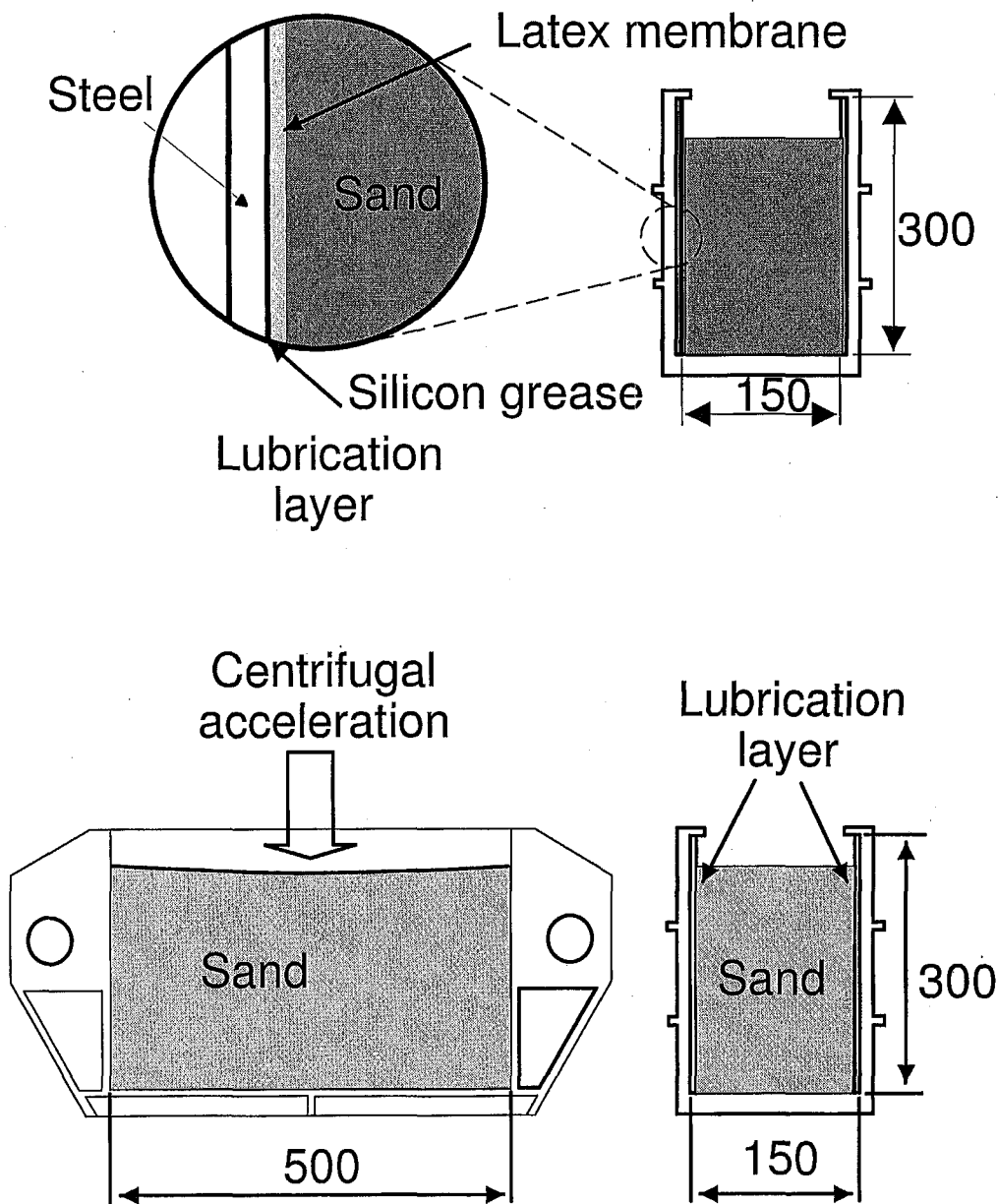


Figure 3.3: Typical container and lubrication of its inner walls.

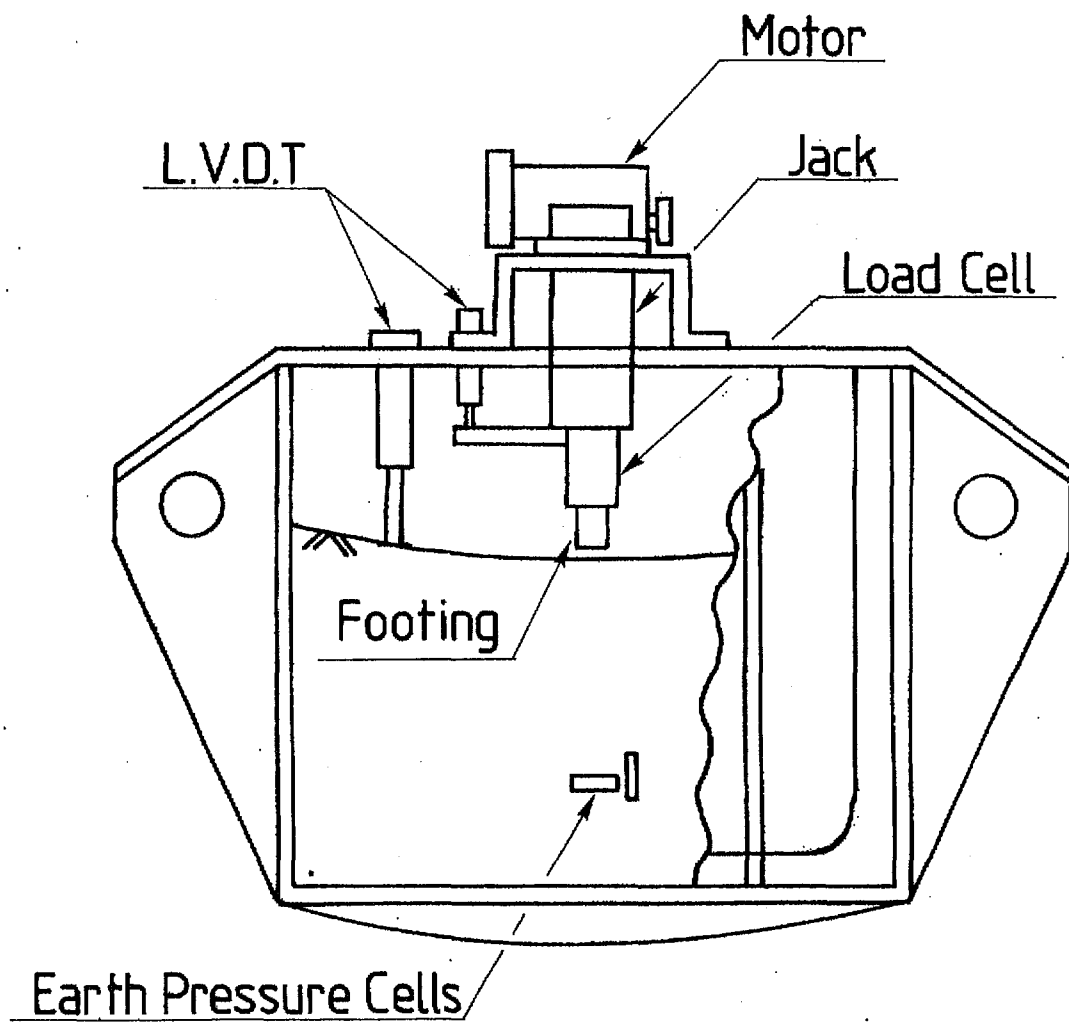
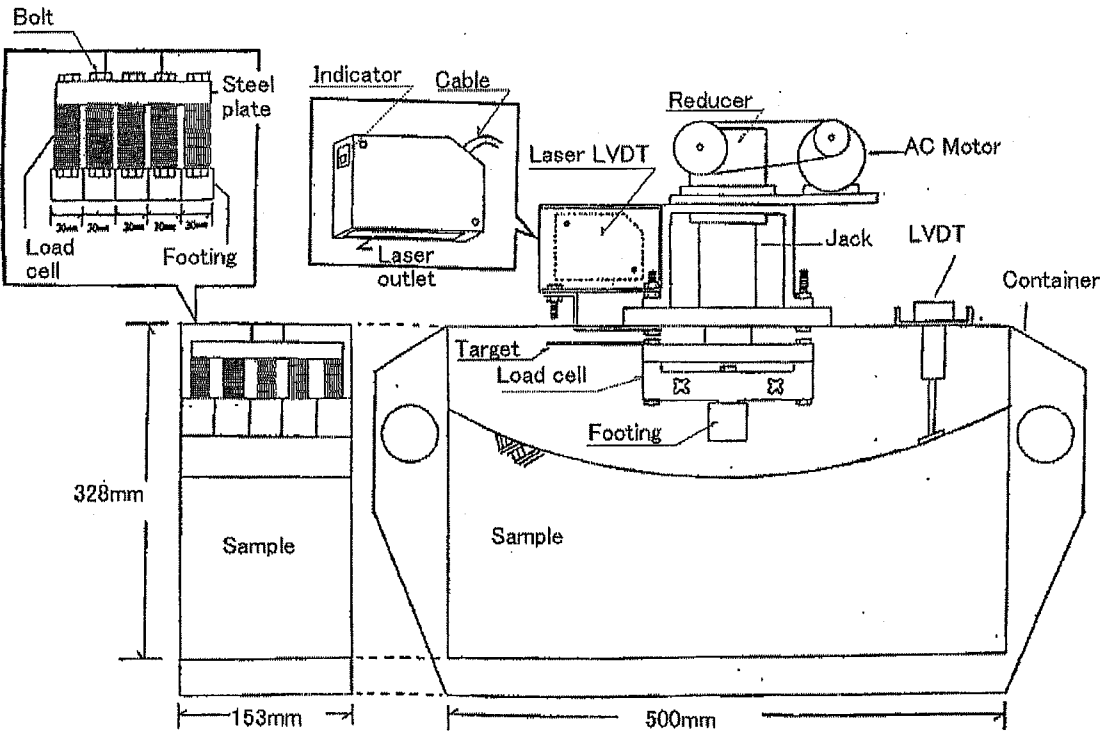
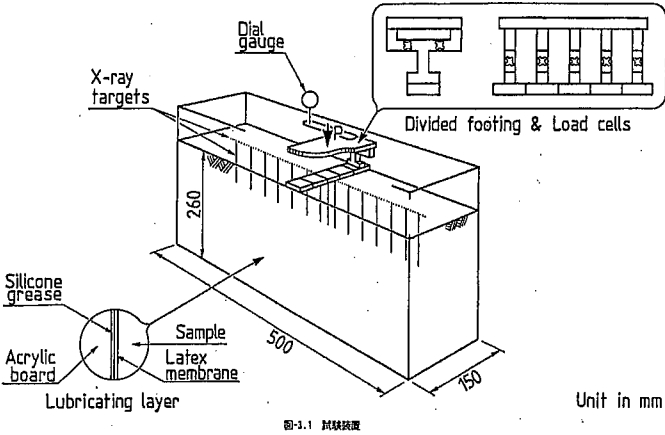


Figure 3.4: Setup for loading test with circular footing



(a) For centrifuge tests



(b) For centrifuge tests

Figure 3.5: Setup for loading test with strip footing

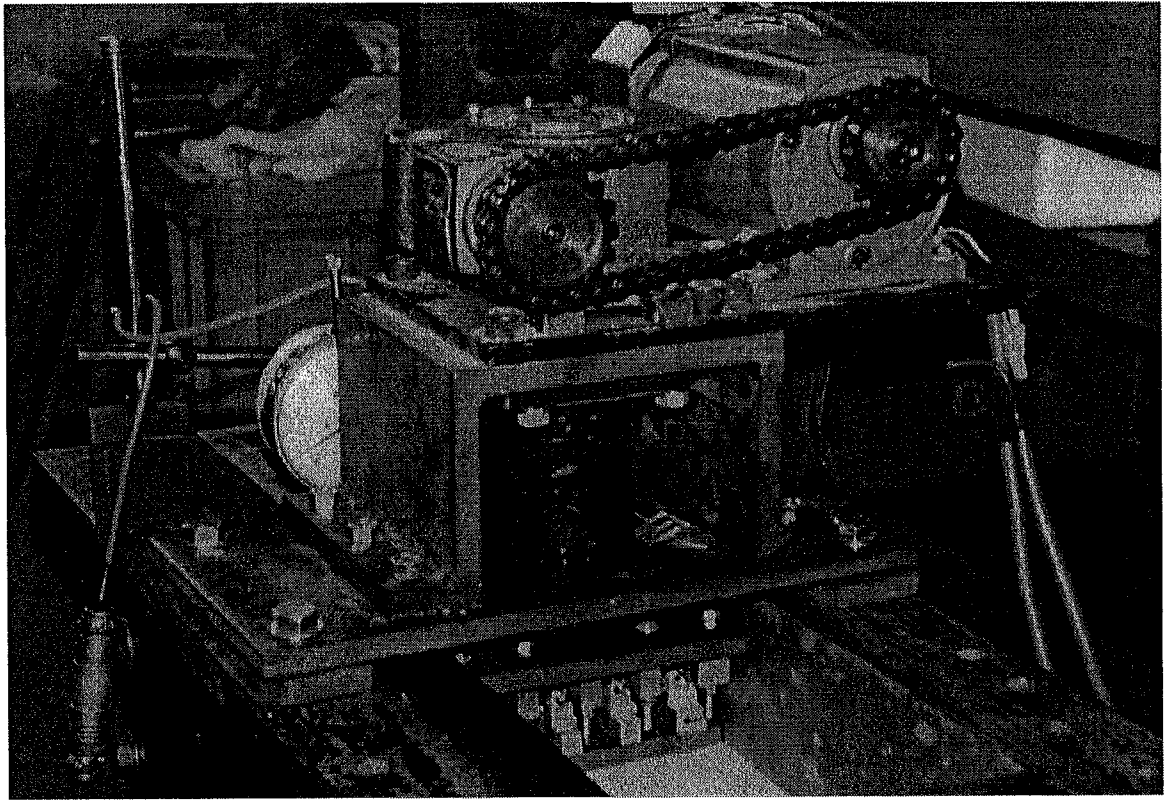


Photo 3.3: Loading jack with five divided strip footings for loading test under gravity acceleration field

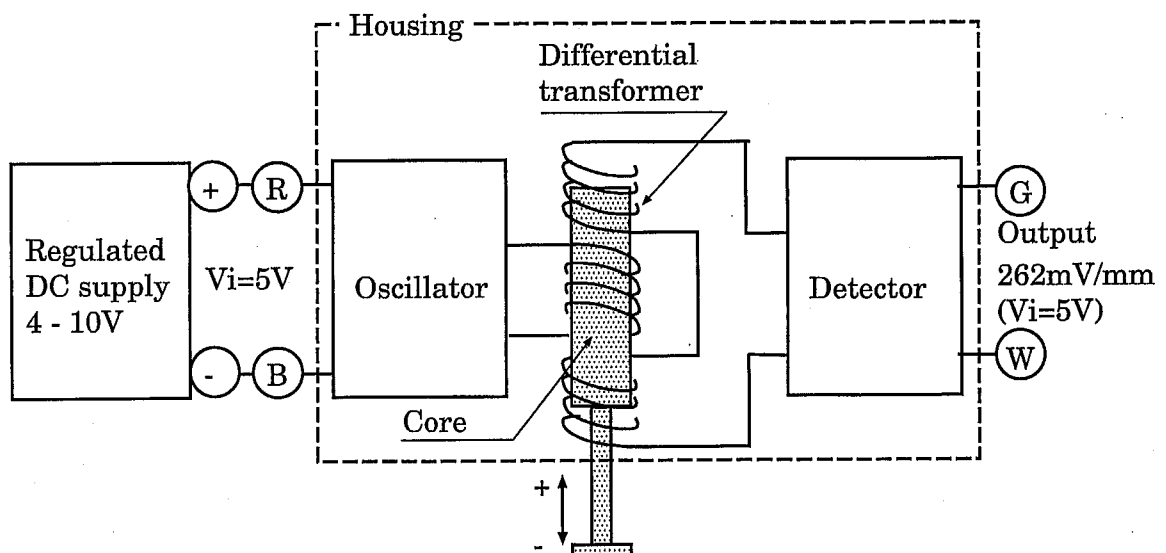


Figure 3.6: A LVDT measuring surface settlement.

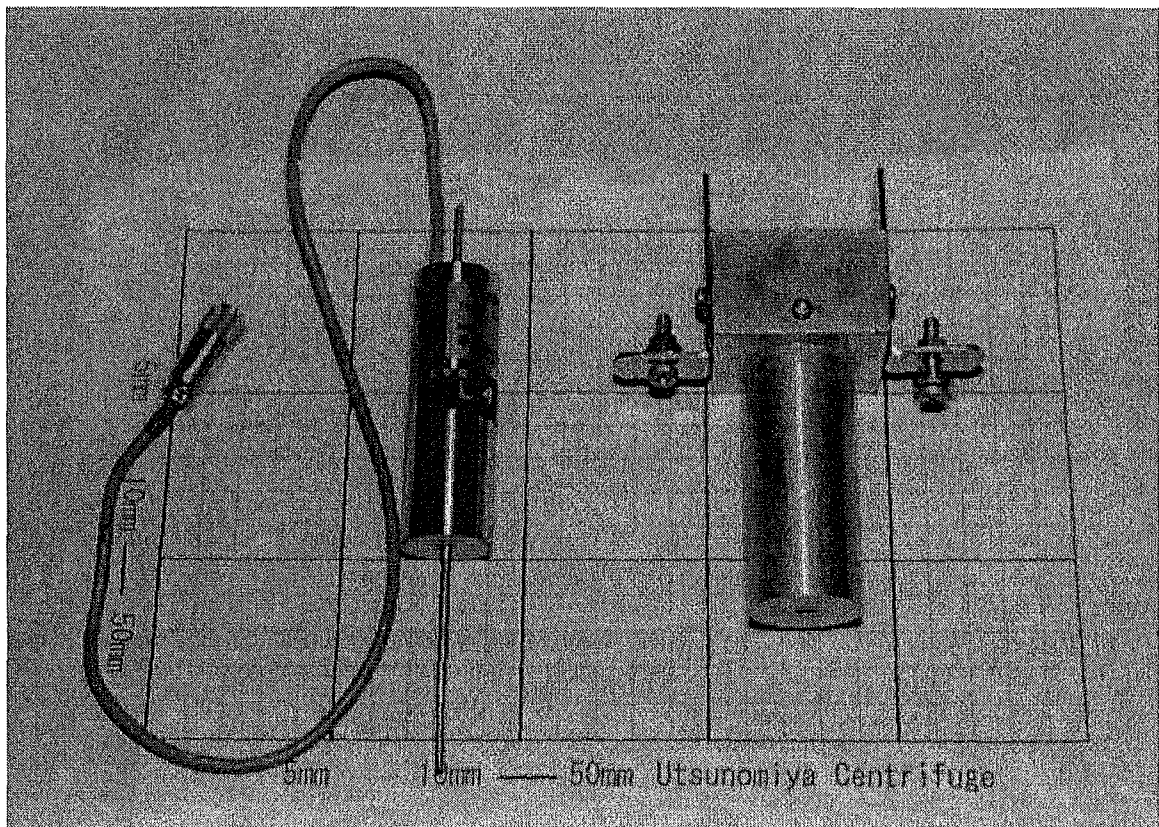


Photo 3.4: A LVDT and its bracket made of aluminum alloy.

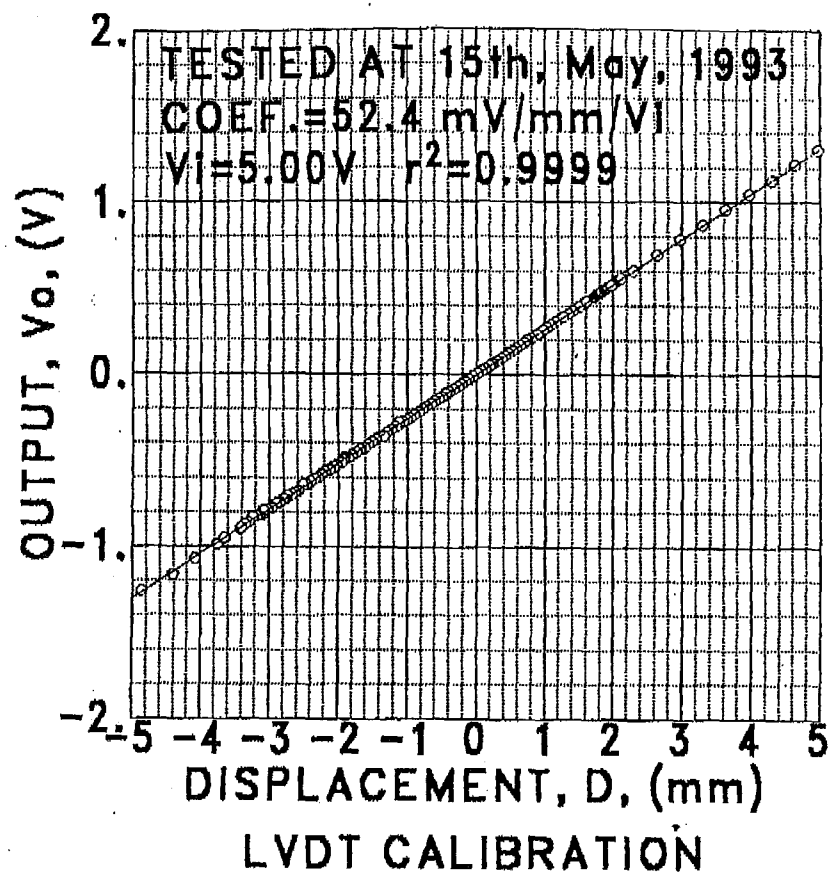


Figure 3.7: Calibration results of the LVDT used

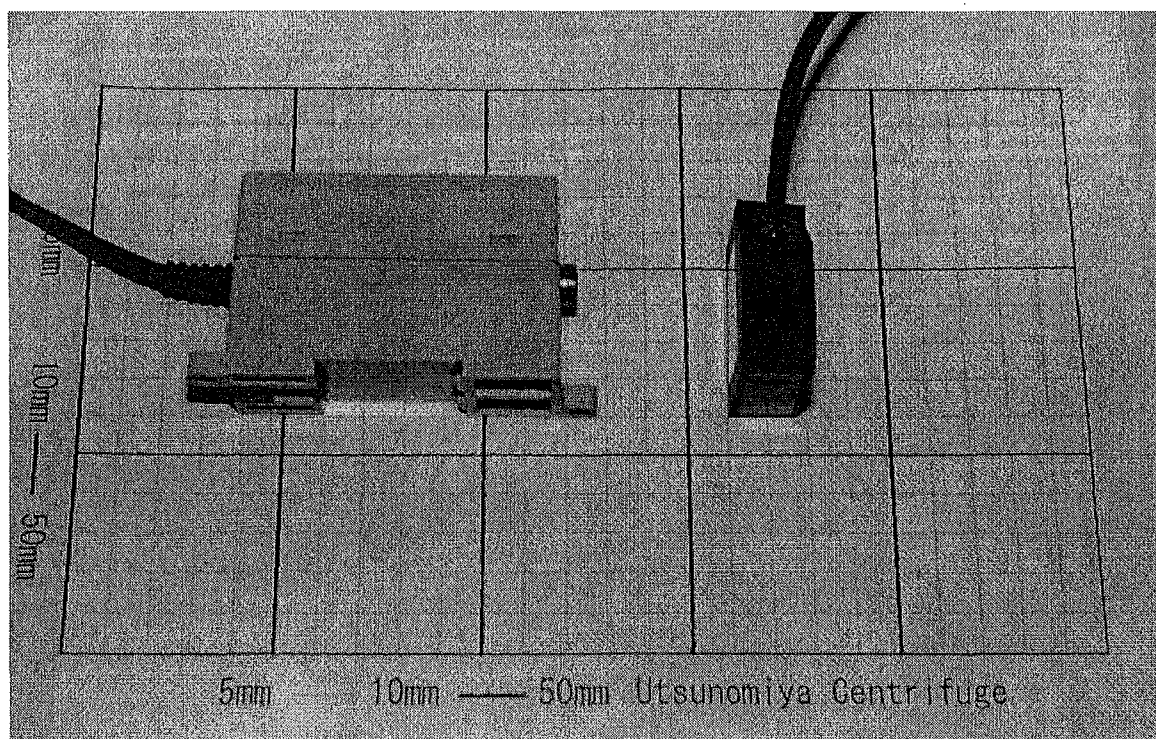


Photo 3.5: A laser displacement transducer.

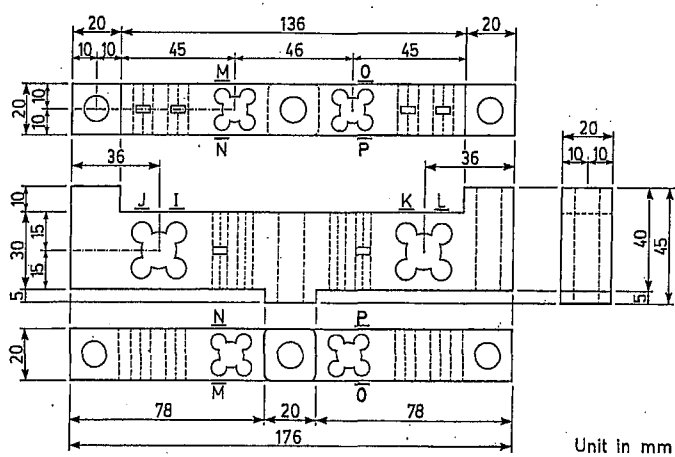


Figure 3.8: Design diagram of the load cell for loading tests with strip footing in centrifuge (capacity is 9.8kN)

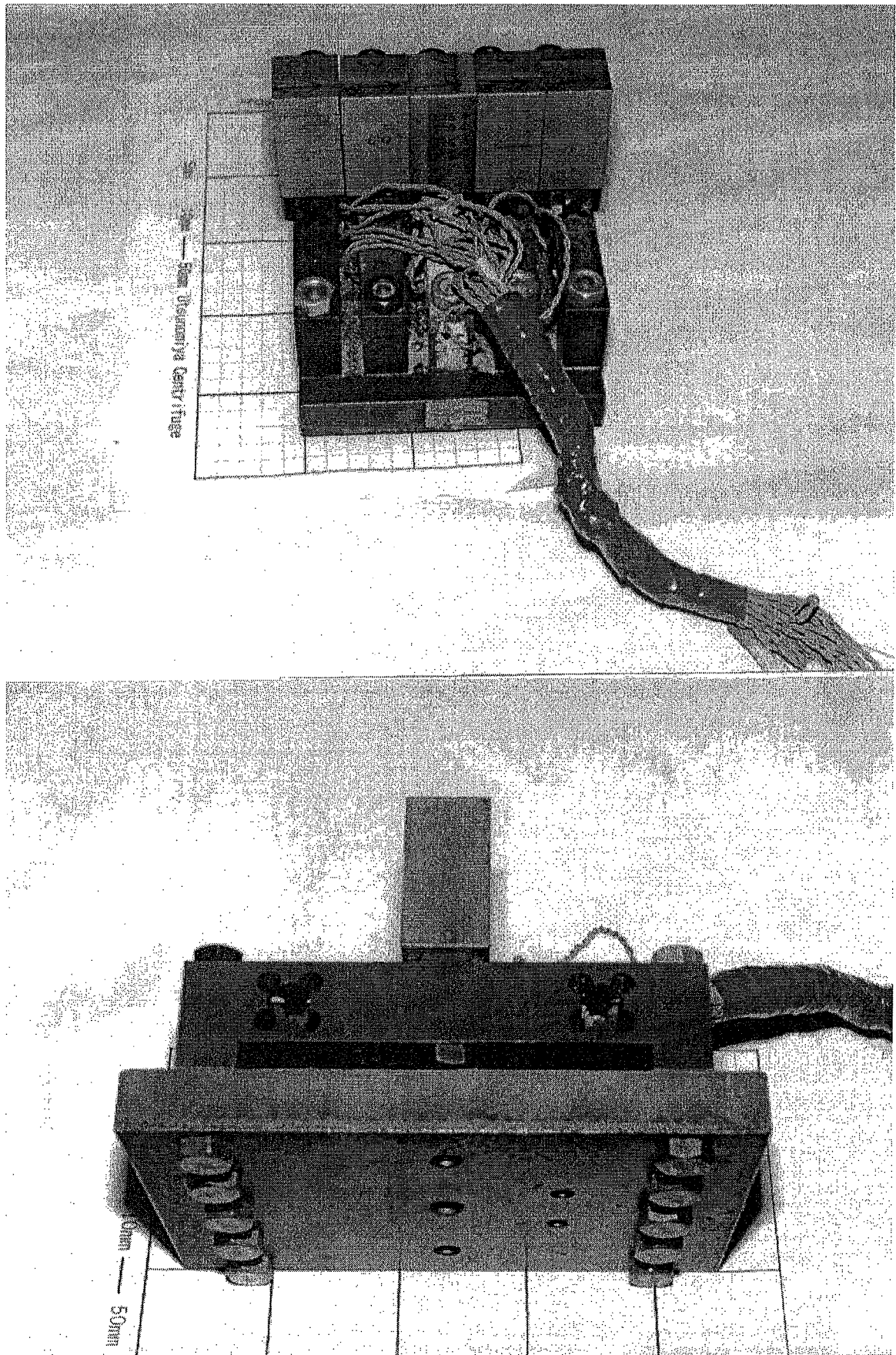


Photo 3.6: Divided footing and five BLC-01t load cells.

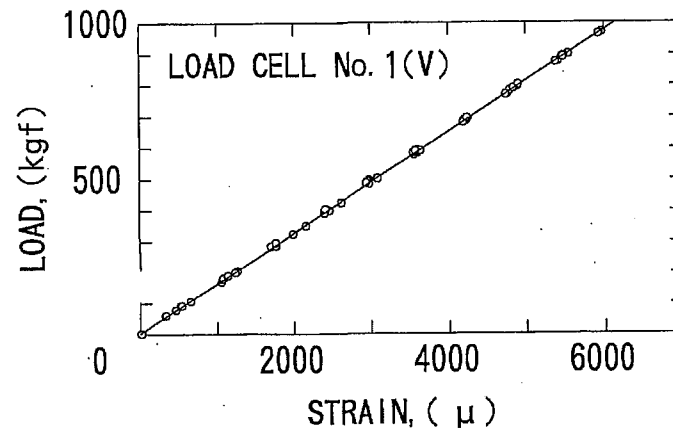


Figure 3.9: Calibration results of a type BLC-01t load cell

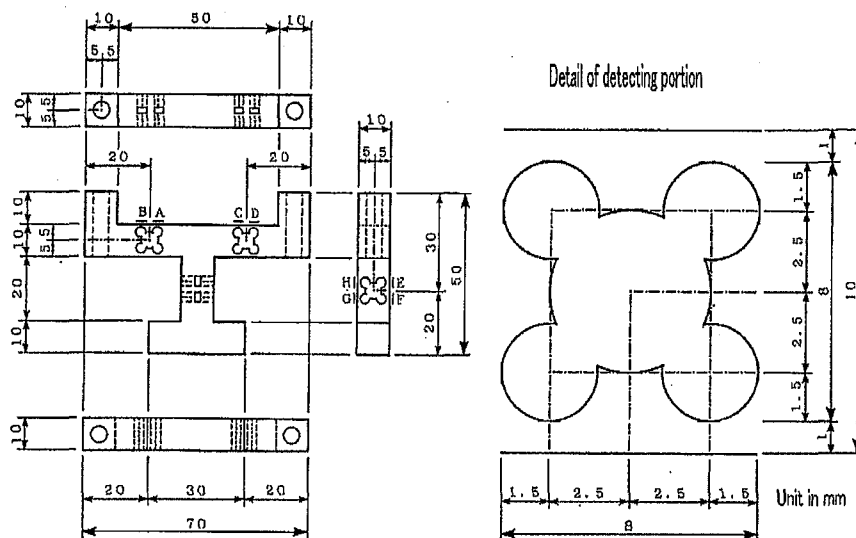


Figure 3.10: Design diagram of BLC-10k load cell for loading tests with strip footing under the gravity acceleration field (capacity is 98N)

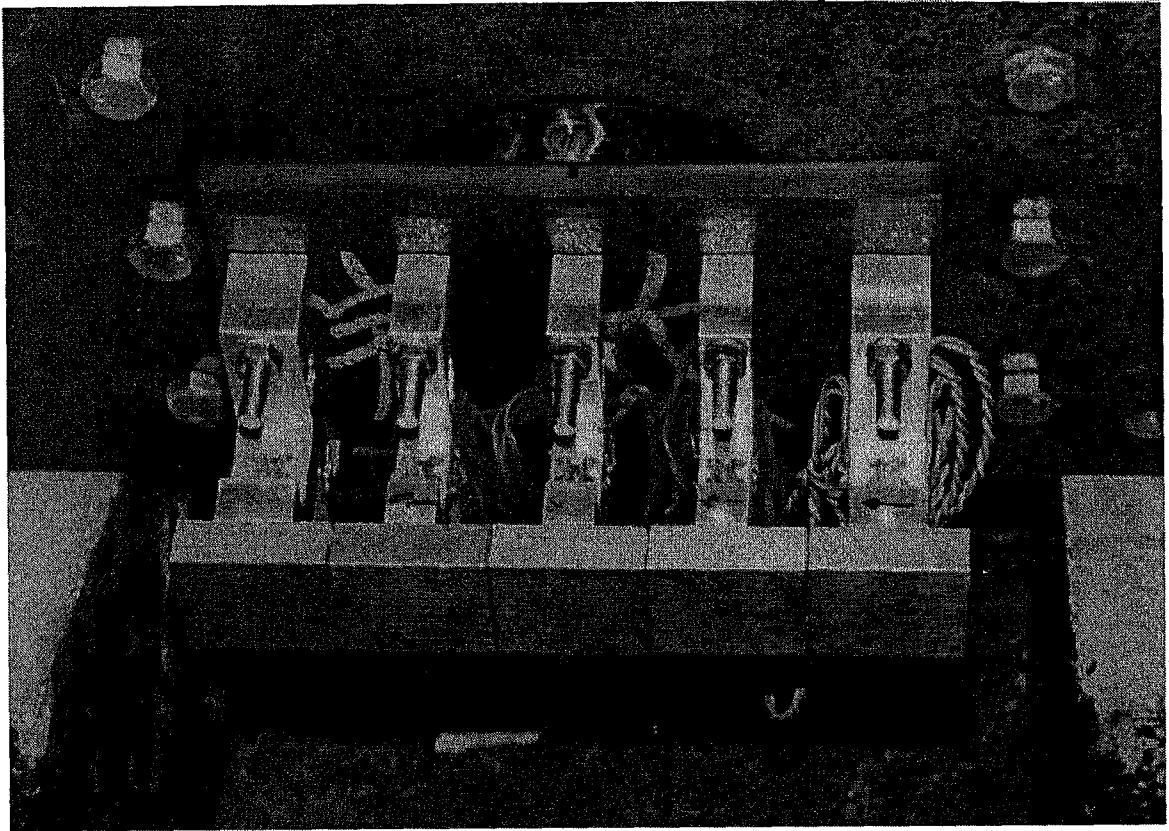


Photo 3.7: Divided strip footing with five BLC-10k load cells

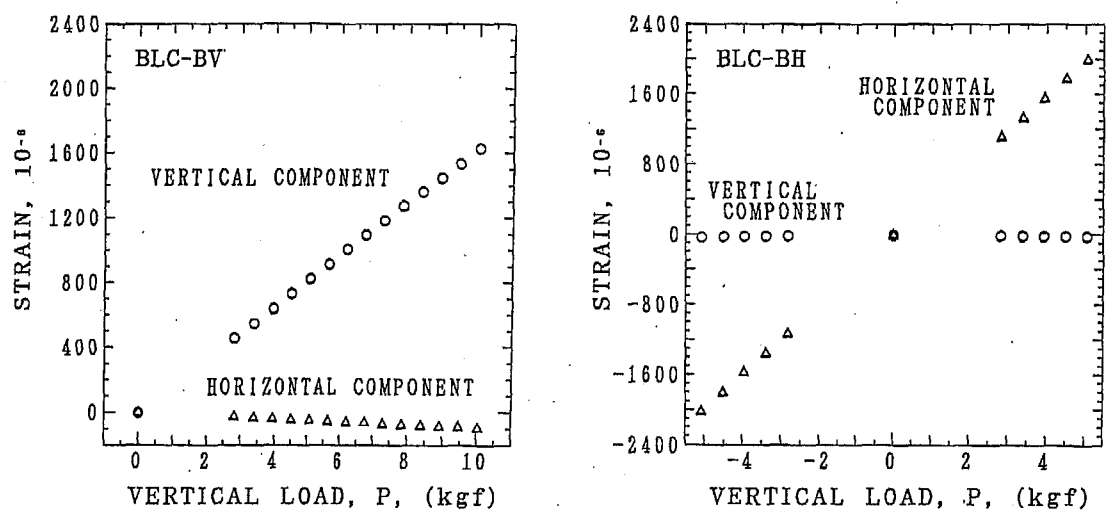


Figure 3.11: Calibration results of a type BLC-10k load cell

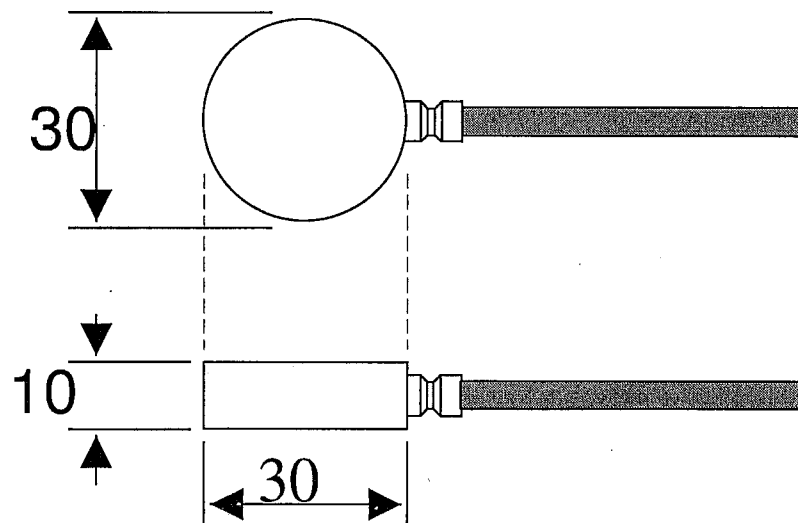


Figure 3.12: Earth pressure cell

Table 3.2: Design specification of the load cells

Name	BLC-01t	BLC-10k
Material	Steel	Aluminum alloy
Capacity	9.8kN	98N
Sensitivity (Typical)	$1.6(\text{N}/\mu)$	$6.1 \times 10^{-2}(\text{N}/\mu)$
Coef. of correlation R^2	> 0.9999	> 0.9999

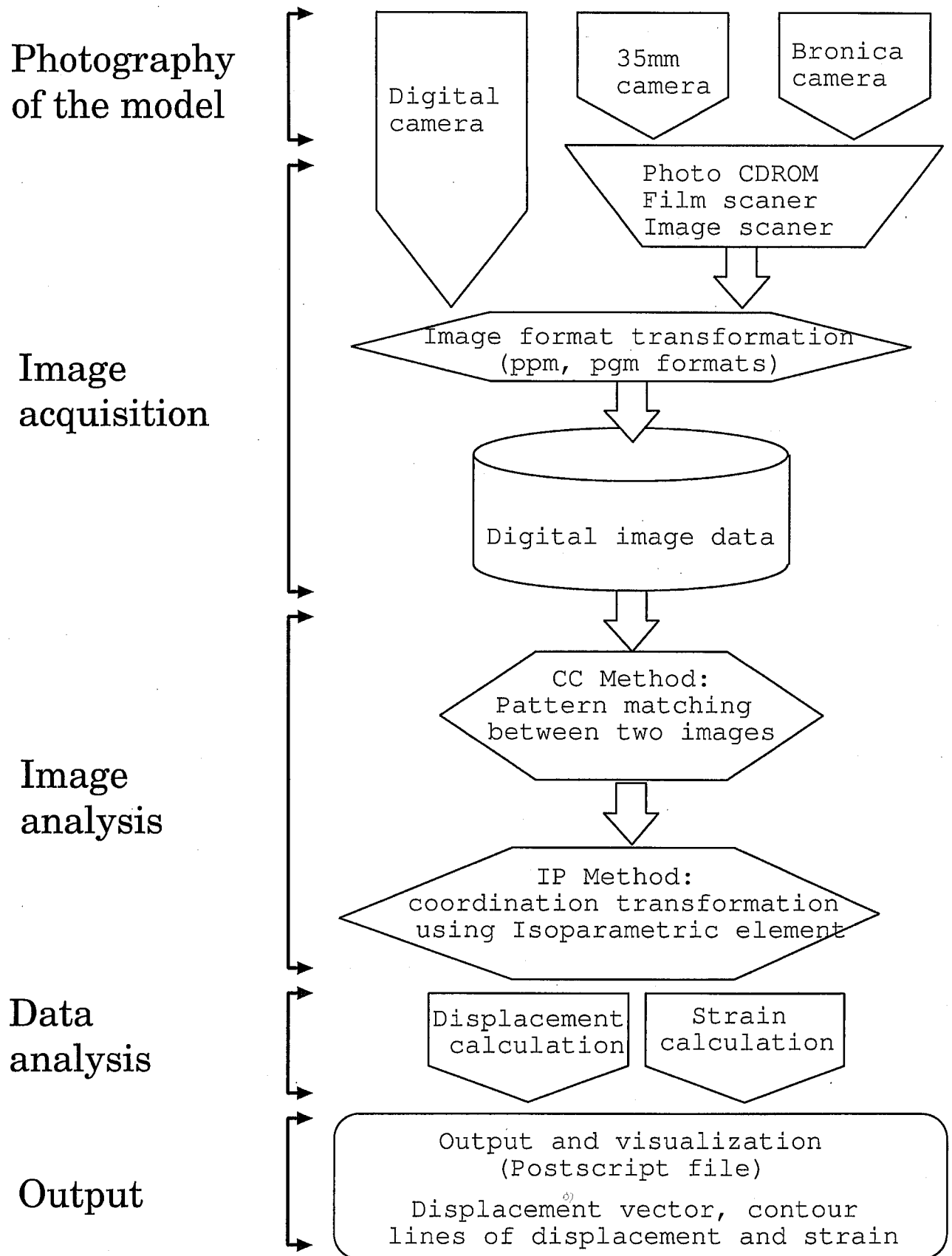


Figure 3.13: A flowchart of displacement field measurement process by image analysis

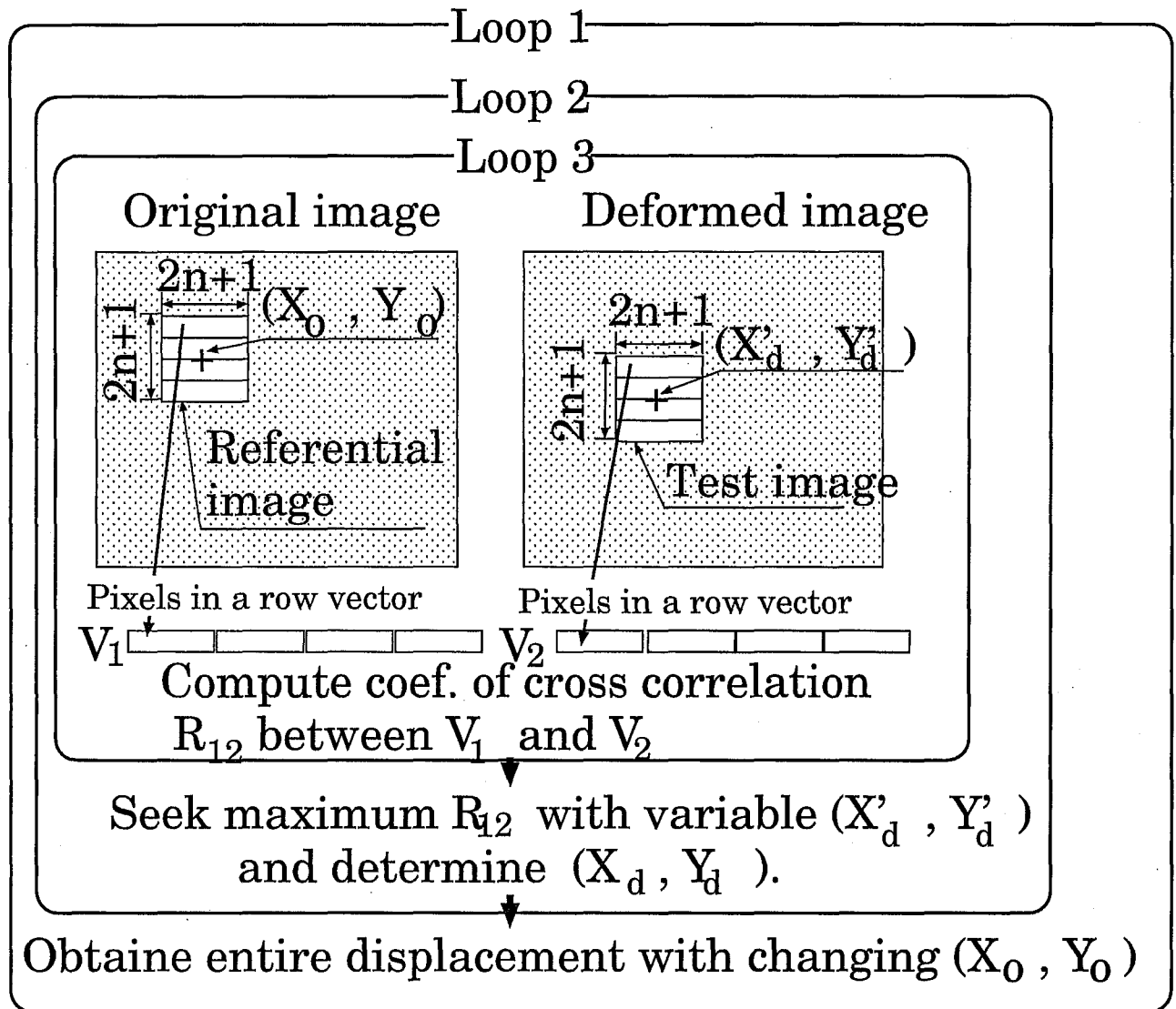


Figure 3.14: Flowchart of pattern recognition by CC method

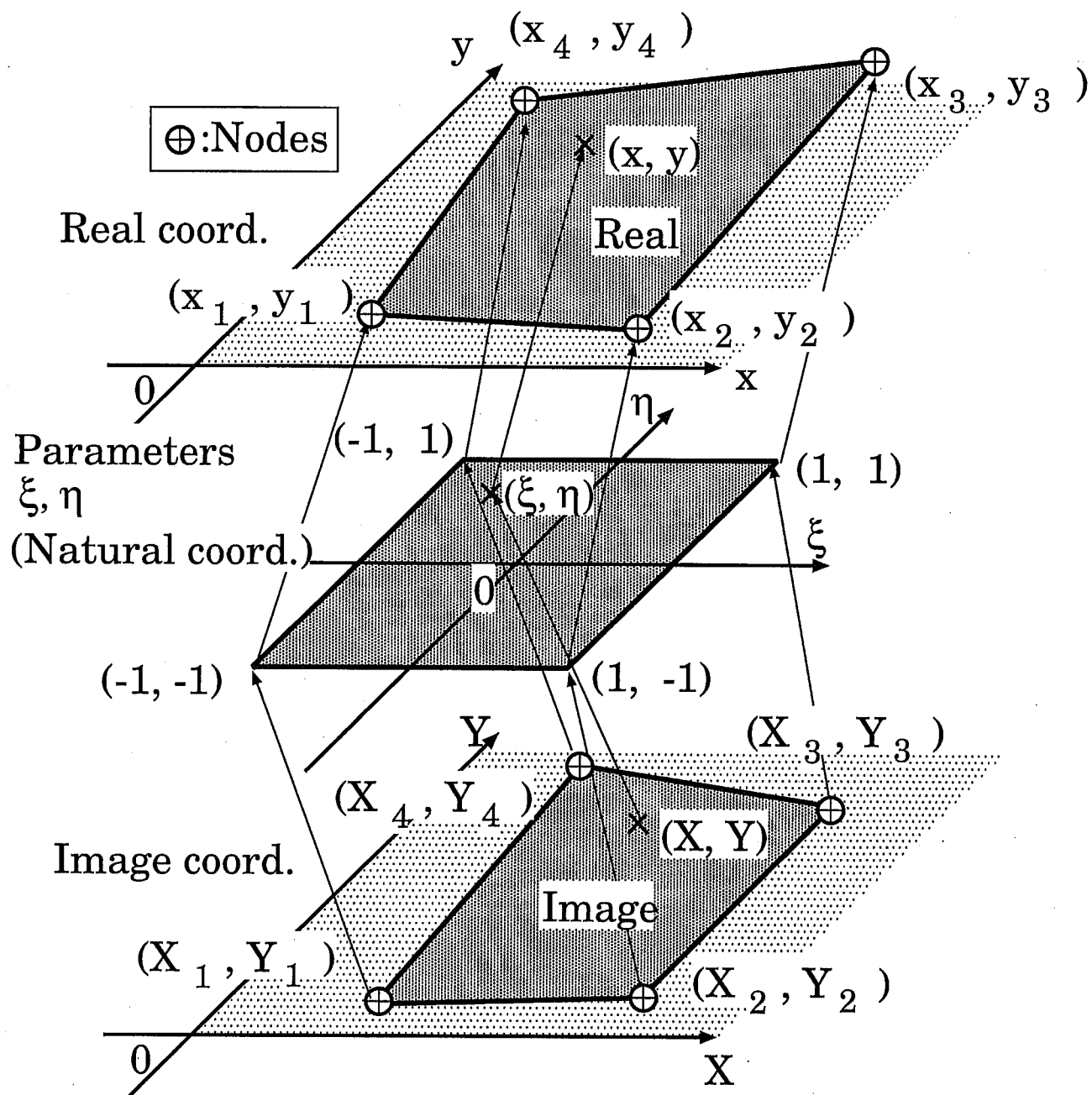


Figure 3.15: A schematic diagram of IP method

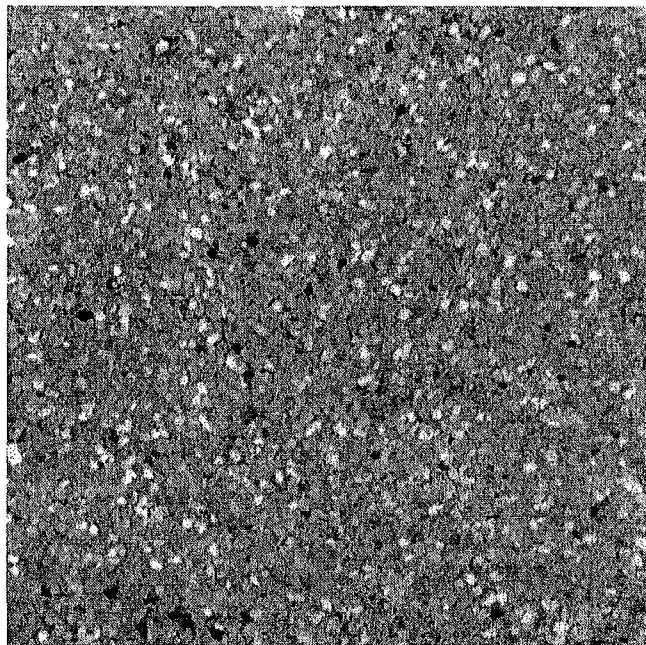


Photo 3.8: Original image used in numerical experiments(500×500pixels, 51mm×51mm, approximate resolution is 0.1mm/pixel)

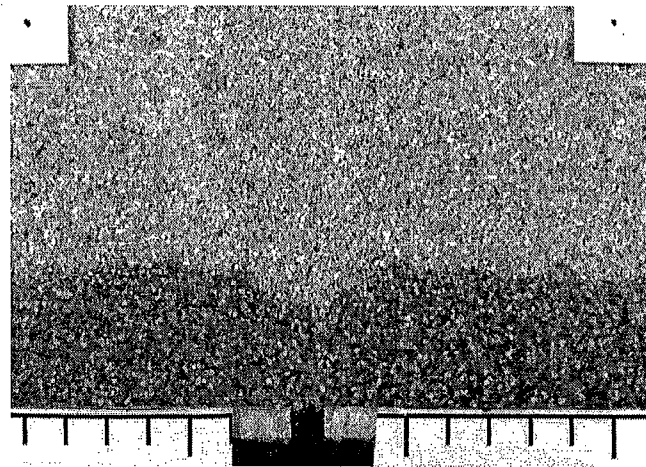


Photo 3.9: Sand with colored particles in soil container

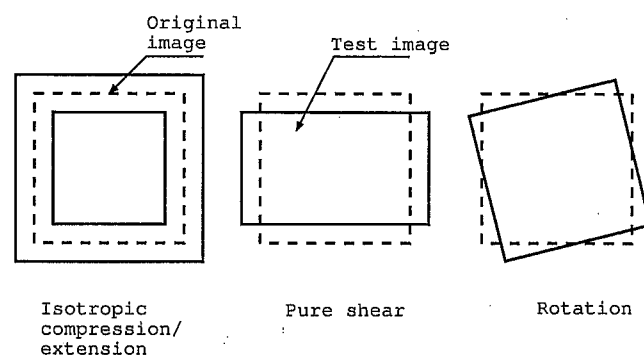


Figure 3.16: Deformation examined

Table 3.3: Physical property of graded silica sand

$D_{\max}$	1.10	(mm)	U_c	1.86
D_{60}	1.04	(mm)	U'_c	1.52
D_{50}	1.02	(mm)	ρ_d	2.622 (g/cm ³)
D_{10}	0.56	(mm)		

Table 3.4: Test conditions of the numerical experiment

Item	Range	Deviation
Isotropic compression and expansion	-30~30(%)	each 2.5(%)
Pure shear	0~30(%)	each 2.5(%)
Rotation (CCW)	0, 3, 5, 10, 15, 20(deg.)	
Size of referential image(pixels)	5×5, 11×11, 21×21, 41×41	

Table 3.5: Relationship between size of referential image and limitation of deformations (error within 2pixels)

Size of referential image (pixels)	Isotropic		pure shear (%)	rotation (deg.)
	compression (%)	expansion (%)		
5×5	0-7.6	-22.4-0	0-7.5	0-3
11×11	0-20.0	-30.0-0	0-23.0	0-15
21×21	0-15.2	-25.0-0	0-20.3	0-10
41×41	0-20.0	-22.0-0	0-20.3	0-10

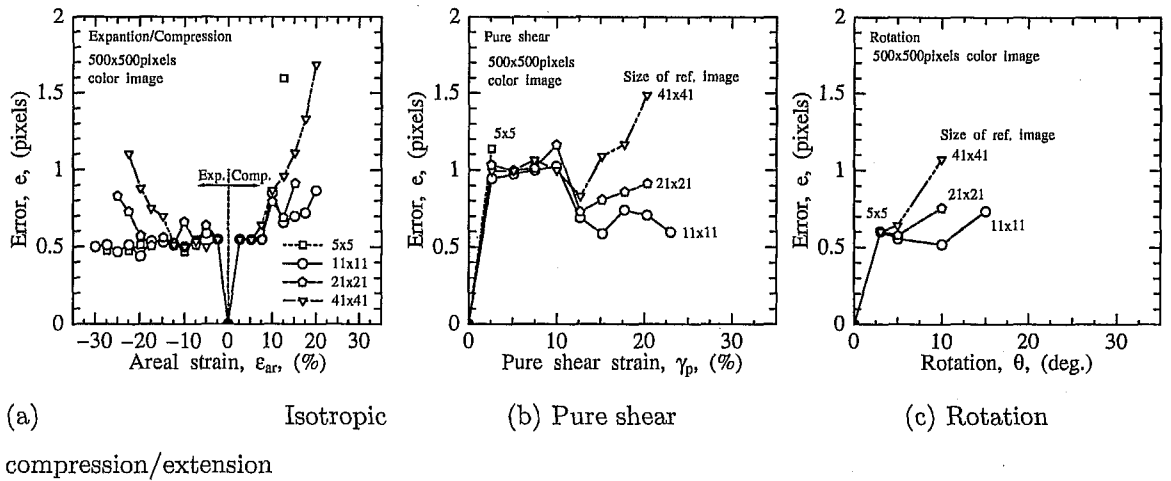


Figure 3.17: The results of numerical experiments

Table 3.6: Test condition

Type of experiment	Loading test
Footing width	30 (mm)
Material	graded dry silica sand
Dry density of sand	1.530 (g/cm ³)
Camera used	6×4 film camera
Method of image acquisition	from films with an image scanner
Resolution	5.0×10^{-2} (mm/pixel)
Size of referential image	41×41 (pixels)

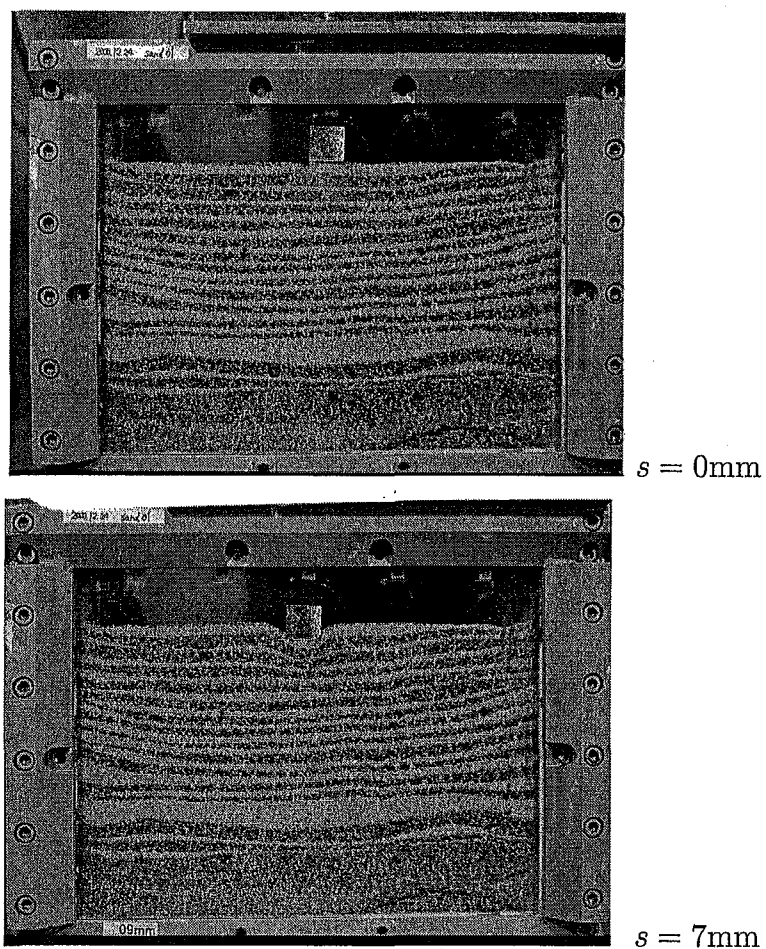


Photo 3.10: Loading tests

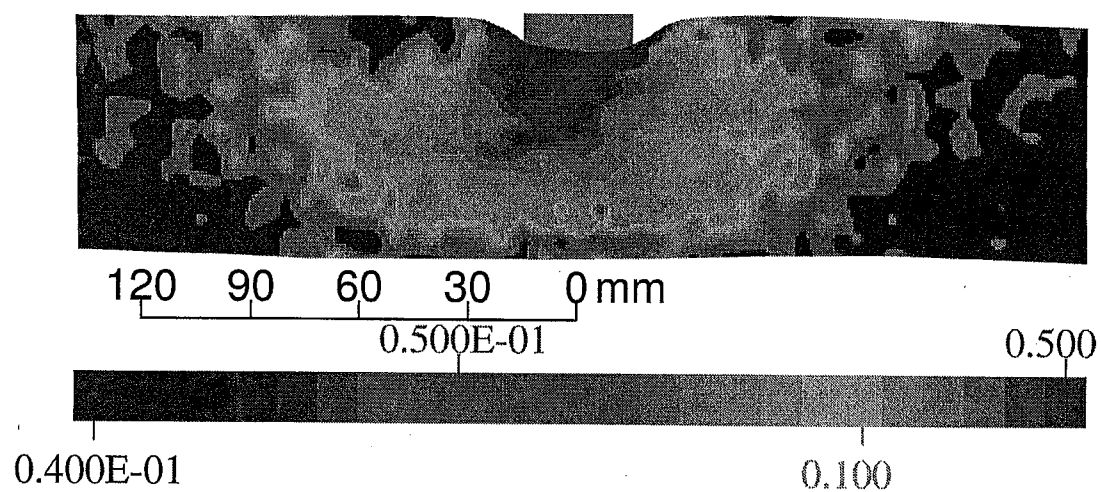


Figure 3.18: Distribution of accumulated maximum shear strain γ_{max} ($s = 0 - 6\text{mm}$).

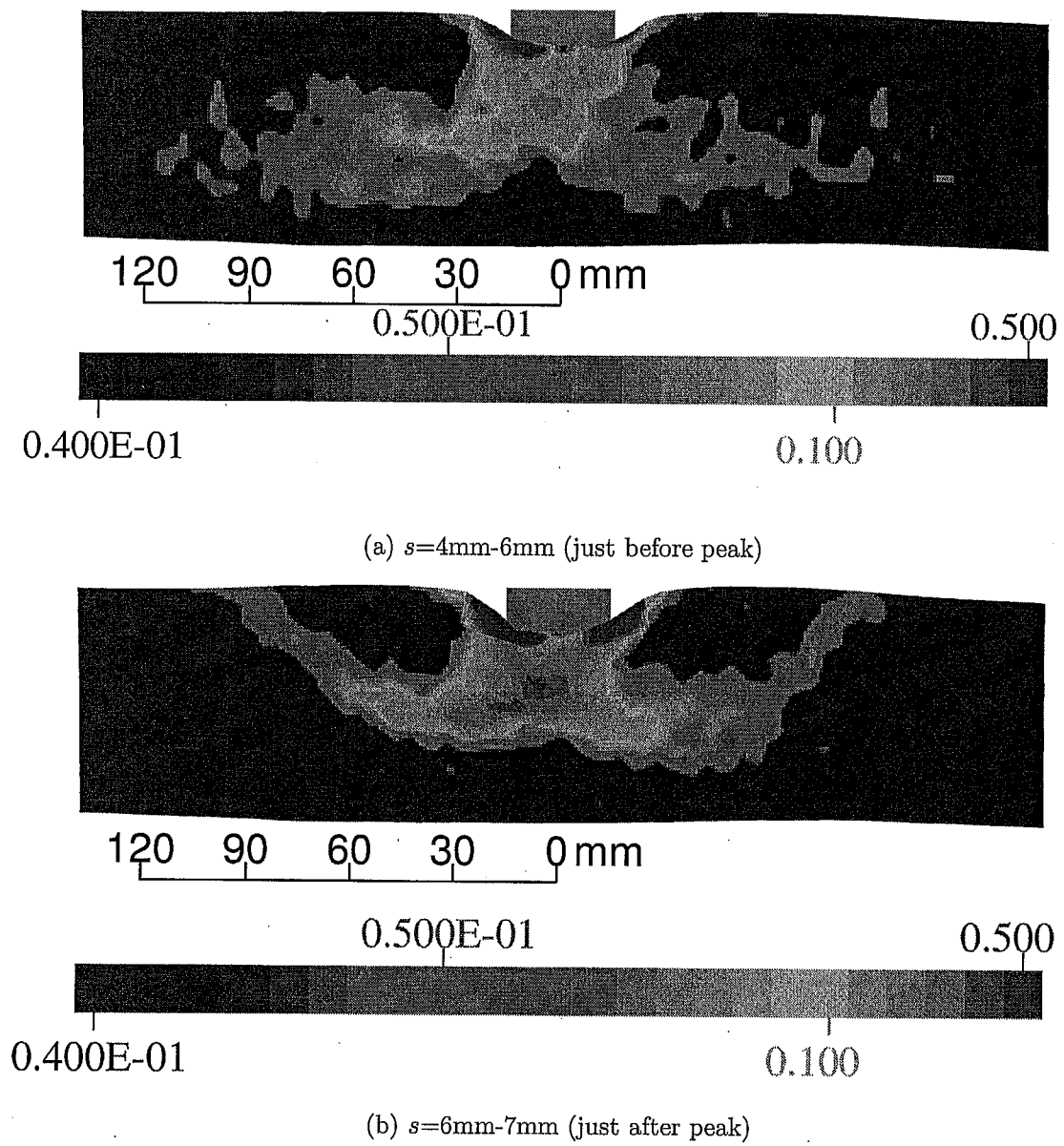


Figure 3.19: Distribution of maximum shear strain increment $\Delta\gamma_{max}$.

Chapter 4

Influence of boundary and initial conditions

4.1 General

To reproduce ideal stress distribution in models is essential to establish accurate and reliable testing technique. Friction between model sand and soil container is one of the significant factors that dominates stress distributions in model sand. In this chapter the author examines frictions between metal plates and sand, and discusses the effectiveness of a lubrication technique in improvement of stress distribution in model sand. Also, stress history is another factor that affects on initial stress distributions. Changes in stresses in model sand due to repetitious centrifuge operation and their influence on bearing capacity are investigated in the present thesis.

4.1.1 Influence of boundary friction

Lubrication on the inner walls of the container is important if realistic model tests are to be conducted because there are two types of significant influence of boundary friction that cause errors in the model tests (see **Fig. 4.2**). First, additional shear resistance arising from boundary friction might bring misunderstanding of test results. Second, the boundary friction between the soil container and the soil reduces vertical stress because of the silo effect (Terzaghi, 1942).

Boundary friction directly impedes movements of soil mass. For example, in loading test-

s with strip footings, boundary friction impedes movements of the soil mass and generally increases the values of ultimate bearing capacity. It results in overestimation of bearing capacities (Tatsuoka *et al.*, 1983, Tatsuoka and Haibara, 1984a, 1984b, 1984c, 1984d and Tatsuoka *et al.*, 1984)(see Fig. 4.1 after Nishida *et al.*, 1996). In stability problems, overestimated safety factor would be obtained if boundary friction exists. In earth pressure problems, boundary friction may cause excess passive earth pressure and deficient active earth pressure. Santamarina and Goodings (1989) observed boundary friction effects in reinforced sand retaining wall models in a centrifuge. They recommended that the appropriate model width should be about four times the model wall height or more. Khoo *et al.* (1994) reported evidence which shows that the influence of boundary friction impeded movement of a model concrete caisson in a centrifuge.

The silo effect is also likely to lead erroneous interpretation of test results. Feld *et al.* (1991) showed that vertical stress in a circular container with dimensions of 30cm in diameter and 30cm in depth became from 0.2 to 0.73 times smaller than the ideal values. In model tests for underground structures the reduction of the vertical stress due to boundary friction is likely to lead deterioration of shear resistance of soil. It might result in an underestimation in the load acting on underground structures, the tip resistance of piles, and so forth. Not only strength characteristics but also deformation characteristics change due to the silo effect. Assumption of Gibson's ground (Gibson, 1967), which is widely employed in interpretation of test results, is based on linear relationship between deformation modulus and stress. Therefore, reproduction of ideal stress distribution in a soil container is essential, especially when underground structures are modeled.

Note that boundary friction greatly inhibits satisfaction of similarity rule. The principle of centrifuge model tests is based on producing a model ground having a stress distribution expressed by

$$\sigma'_v(z_m) = \int_0^{z_m} \rho a dz \quad (4.1)$$

where z_m is the depth of model ground; ρ , the density of the soil; a , the centrifuge acceleration. From the similarity rule, the scale of the model ground can be set to a prototype scale. If the stress distribution $\sigma_z(z_m)$ in model ground is affected by the boundary friction and decreases, the similarity rule is no longer satisfied. Discrepancy between ideal stress distributions and that really produced in model ground results in errors due to boundary friction.

4.1.2 Influence of repetitious centrifuge operation

To conduct precise centrifuge model tests, the author investigated the influence of initial stress condition on the bearing capacity of shallow foundation. The bearing capacity of shallow foundations on soils has been studied and summarized by many researchers, such as De Beer (1965) and Vesić (1975). Little attention, however, has been paid to the influence of initial stress conditions on bearing characteristics, especially in experimental studies on sand (Chiken *et al.*, 1992).

4.2 Friction between sand and container

4.2.1 Direct shear tests

A series of direct shear tests were conducted to examine wall friction between sand and both lubricated and non-lubricated metal plates. Fig. 4.3 illustrates a direct shear device. The direct shear device consisted of a base plate and an upper shear box. The base plate had a rectangular mesa with 70mm wide, 80mm long and 20mm high. The upper shear box had a cylindrical container of which inner dimensions were 62mm in diameter and 32mm in depth. Cross sectional area of the sand was 28.27 cm^2 . To reduce friction between the specimen and the upper shear box, the bottom of the upper shear box was lubricated with a 0.37mm thick latex membrane and a 0.017mm thick grease layer. The grease used was KS63G produced by Shin-etsu Chemical Co. Ltd., of which effectiveness for lubrication was examined by Park *et al.* (1992). The latex membrane had a center hole coinciding with the soil container in the upper shear box.

A specimen was attached on the mesa of the top of the flat base plate of the direct shear device. After setting the specimen on the mesa, the upper shear box was placed on the specimen. Then sand was poured into the upper shear box with a funnel and compacted into prescribed relative density by means of tamping method. The upper shear box was fixed through a load cell and the base plate moved with a shearing rate of 1.0mm/min from right to left as shown in Fig. 4.3. The load cell used was BLC-10k, of which performance was introduced by Fig. 3.9 in chapter 3.

4.2.2 Materials

Specimens used were metal plates made of steel and aluminum. The author prepared two surface condition; rough and fine. Both surface were planed with a milling machine and left as is. Maximum roughness of rough surface was ranging from 200 to 400 (μm), while that for fine condition was ranging from 1.6 to 3.2, approximately. Both lubricated and non-lubricated specimens were examined.

Lubricated specimens equipped a lubrication layer made from a 0.37mm thick latex membrane and a 0.017mm thick grease layer, except series SSTT. In series SSTT, latex membranes having thicknesses of 0.37, 0.74 and 1.11mm were used to examine the influence of the thickness on friction. The grease used was KS63G, comprising silica filler, provided by Shin-etsu Co. Ltd. Tatsuoka *et al.* (1984) employed KS63G to lubrication layers in element tests and showed the effectiveness of the grease as a friction modifier under relatively low confining stress level.

Two kinds of sand sample were examined: air dry Toyoura Sand (TO) and Inagi Sand (IN). Properties of these sands are tabulated in Table 4.1. Detailed descriptions of the sands were presented in Section 2.2.

4.2.3 Results

Tables 4.2-4.6 show test cases and the peak values of frictional resistance f . In the following notation f_l denotes the friction between sand and lubricated interface, while notation f_n means that between sand and non-lubricated interface.

Thickness of latex membrane Series SSTT tabulated in Table 4.2 examined the influence of thickness of membrane t on frictional resistance f_l . Table 4.2 shows that the frictional resistance f_l tends to become greater when the membrane becomes thick. Because rigidity of membrane impedes the movement of sand particles along interface, resistance becomes larger as the membrane becomes thicker.

Relative density Series SSDr examined the influence of relative density D_r on frictional resistance f_l on lubricated plate. The results are tabulated in Table 4.3. The values of resistance are scattered, and no clear tendency appears in the range of these experiments.

Effects of lubrication A distinguishable lubrication effect can be expected by using a lubrication layer made from a latex membrane and silicon grease. Table 4.4 and Table 4.5 present comparisons of friction resistance between the lubricated and non-lubricated interfaces. The values of effectiveness of lubrication defined as f_l/f_n are ranging from 0.011 to 0.088. The ratio f_l/f_n tends to decrease with increasing normal pressure p . **Figure 4.4** and **Fig. 4.5** illustrate the comparisons of interface friction between a lubricated and a non-lubricated soil container. These figures indicate that an application of lubrication layers with grease and latex membrane efficiently reduces the wall friction.

Figures 4.6(a, b) are elongated figures of **Fig. 4.4** and **Fig. 4.5** to determine failure criteria for lubricated interface. The results reported in Haibara and Tatsuoka (1984) are also plotted with inverted hollow triangles as a reference; lubrication was made of a 0.2mm thick latex membrane and a 0.06mm thick KS63G grease layer. The plots agree well with the results from SSLT and ASLT Tests.

Although plots in these figures are rather scattered, Coulomb's criteria can be used to express the relationships between frictional resistance f_l and normal pressure p . Small amount of cohesion intercepts can be recognized in the tests results, as well as Haibara's results. It can be considered that the cohesion intercepts are attributed to the viscosity of grease in the lubrication layer. The strength parameters obtained from a series of direct shear tests are tabulated in **Table 4.7**, in which c_w and ϕ_w are cohesion intercept and wall friction angle, respectively.

4.3 Stress distribution in a container

A series of centrifuge model tests, named EPNI, EPLI and C Tests, were conducted in order to investigate initial stress distribution in model sand. In EPNI and EPLI Tests the influence of wall friction between the sand and the soil container was examined, while in C Tests the influence of cyclic accelerating operation was investigated.

4.3.1 Influence of friction

4.3.1.1 Procedure

The author examined vertical stress distributions of Inagi Sand in a soil container made of steel with dimensions of 150mm thick and 500mm wide. Test set-up and installation of lubrication layers are illustrated in **Fig. 4.7**.

Test conditions are tabulated in **Table 4.8**. To examine the effects of lubrication, both lubricated and non-lubricated conditions were examined. For the lubricated condition, a lubrication layer made from a thin latex membrane with 0.37mm thick and silicon grease was employed.

Three earth pressure cells, having a capacity of 2.94MPa, were placed at the same depth (195mm) but at different positions, as shown in **Fig. 4.8**: the center of the container ($D=0\text{mm}$), near the wall ($D=60\text{mm}$) and at intermediate position ($D=37.5\text{mm}$). The notation D in **Fig. 4.8** indicates the distances from the center of the container to the center of the earth pressure cells.

4.3.1.2 Results

Figure 4.9 and **Fig. 4.10** show changes in vertical stress σ_v with changing centrifuge acceleration obtained from the series EPNI and EPLI, respectively. Because residual stresses were generated, plots in deaccelerating sequences are eliminated from these figures. Solid lines in **Fig. 4.9** and **Fig. 4.10** show ideal relationships calculated from **Eq. (4.2)**:

$$\sigma_z = \int_{r_0}^{r_0+z} \rho_a r \omega^2 dr \quad (4.2)$$

The broken lines in these figures show predictions, which will be described latter.

The lubrication technique improves the accuracy and quality of centrifuge model tests. In the non-lubricated container, examined in series EPNI, the influence of wall friction appears in the test results. The values measured with the earth pressure cell embedded at the center of the container ($D = 0\text{mm}$) concentrate along the ideal relationship, however the plots measured at near the wall ($D=60\text{mm}$) show a 25% of reduction. Differences in the vertical stress distribution in the horizontal direction are evidence of an obstacle shear stress in model sand. Note that discrepancy between ideal and measured stress distributions result in errors in model tests and lead misunderstanding of the test results. In contrast in the lubricated container,

examined in series EPLI, measured values of σ_v concentrate along the ideal relationship, and the results show good evidence of the similarity rule. Additionally, the differences in the horizontal direction vanish in the lubricated container. Note that the reproduction of the ideal stress distribution is essential of model tests. Lubrication with latex membrane and silicon grease is an effective technique to reproduce the ideal stress distributions in the model sand.

4.3.1.3 Predicting method

Theory of silo effects can be used for prediction of the influence of wall friction. Terzaghi (1942) derived an expression to estimate the stress distribution in a silo along depth. The author applied Terzaghi's equation to vertical stress distribution in a soil container.

Considering equilibrium of vertical forces acting on a small soil element shown in **Fig. 4.11**, we obtain

$$b\gamma_d dz - bd\sigma_z - df = 0 \quad (4.3)$$

where df is frictional resistance between the container and the soil element. According to the results obtained from a series of direct shear tests, df can be expressed by

$$df = (c_w + K_0 \sigma_z \tan \phi_w) dz \quad (4.4)$$

Substituting **Eq. (4.4)** into **Eq. (4.3)** and solving under a boundary condition of $\sigma_z = 0$ at $z = 0$, we obtain

$$\sigma_z = \frac{b\gamma_d - c_w}{K_0 \tan \phi_w} \left\{ 1 - \exp \left(\frac{-K_0 \tan \phi_w}{b} z \right) \right\} \quad (4.5)$$

where σ_z is vertical stress; b , width of the soil container; γ_d , dry unit weight of the sand; c_w , cohesion term of wall friction; ϕ_w , angle of wall friction; K_0 , coefficient of earth pressure at rest; and z , the depth. When a surcharge of σ_0 exists, the solution becomes

$$\sigma_z = \frac{b\gamma_d - c_w}{K_0 \tan \phi_w} \left\{ 1 - \exp \left(\frac{-K_0 \tan \phi_w}{b} z \right) \right\} + \sigma_0 \exp \left(\frac{-K_0 \tan \phi_w}{b} z \right) \quad (4.6)$$

The author extends above Terzaghi's equation to centrifuge model tests. In centrifuge model tests the unit weight of soil γ varies with depth because centrifuge acceleration varies along the distance from the center axis of the centrifuge. Therefore, **Eq. (4.3)** becomes

$$b(r_0 + z)\omega^2 \rho_d dz - bd\sigma_z - df = 0 \quad (4.7)$$

where r_0 is the radius at soil surface; ω , the angular velocity; and ρ_d , the dry density of soil. Solving Eq. (4.7), we obtain the following equation.

$$\sigma_z = \frac{-b^2 \rho_d \omega^2}{K_0^2 \tan^2 \phi_w} + \frac{b \rho_d \omega^2 (r_0 + z) - c_w}{K_0 \tan \phi_w} + \exp\left(-\frac{K_0 \tan \phi_w}{b} z\right) \cdot C_i \quad (4.8)$$

where C_i is an integral constant. Applying a boundary condition of $\sigma_z = \sigma_0$ at $z = 0$, we derive C_i as

$$C_i = \sigma_0 + \frac{b^2 \rho_d \omega^2}{K_0^2 \tan^2 \phi_w} + \frac{c_w - b \rho_d \omega^2 r_0}{K_0 \tan \phi_w} \quad (4.9)$$

The broken lines in Fig. 4.9 and Fig. 4.10 show predicted values obtained from Eq. (4.8). In the non-lubricated container the predicted values of σ_z appears an upper convex curve and becomes about 15% less than the ideal distribution at the depth of $z_m = 195\text{mm}$. The measured values are scattered around the estimated distribution according to the position at which the pressure cells were installed. Because the prediction gives an average vertical stress at the depth, it can be said that Eq. (4.8) can be used to evaluate the influence of wall friction on vertical stress distribution in model sand. The wall friction reduces vertical stress and likely to lead errors in model test results. Otherwise in the lubricated container the estimated distribution of σ_z agrees well with both measured values and the ideal distribution shown by a solid line. Note that lubrication made from a thin latex membrane and silicon grease can be expected to provide the ideal vertical stress distribution in a soil container under centrifugal acceleration field.

4.3.1.4 Discussion

A series of parametric computation was conducted to predict the distributions of vertical stresses under various conditions. Figures 4.12-4.16 show the results. The values of wall friction angle ϕ_w ranging from 17.5(deg.) to 40.0(deg.) were employed for non-lubricated containers. A wall friction angle of 17.5(deg.) is associated with the results from SSLT Tests, which is the smallest value among the results for non-lubricated interfaces. For the lubricated cases the values of cohesion term c_w and wall friction angle ϕ_w obtained from direct shear tests were applied in the parametric computation.

Under the gravity acceleration field without surcharge It is quit difficult to replicate ideal stress distributions in a small container under the gravity acceleration field. Figures 4.12(a-g) show the results for non-lubricated containers with variable width of containers

b. The distribution of vertical stress in non-lubricated container depends on the ratio of the depth versus the width of container, $z/2b$, and the angle of wall friction. A 5(deg.) of the angle of wall friction, measured on PYLEX(TM) glass plate by Haibara and Tatsuoka (1984), provides a 10% of reduction at $z/2b = 4$. On the other hand, $z/2b = 1$ is a limitation for finely milled steel container ($\phi_w \simeq 17.5(\text{deg.})$), if the reduction in vertical stress should be kept within 10%.

Another difficulty still remains in lubricated containers for 1g tests. **Figures 4.13(a-f)** show the results for lubricated containers with variable width of containers b . The distributions of vertical stress under the gravity acceleration field are significantly sensitive to cohesion term c_w because of extremely low stress level. Although the lubrication in SSLT Tests improves distribution of vertical stress, those in other tests lead terrible results. In particular in SRLT Tests, computed values of vertical stress become negative when the width of the container $2b$ is 20cm or narrower. This result implies that model sand did not propagate any vertical stresses into the container, if any attentions were not paid to treatments of container walls. If the inner walls of container were not treated well before lubrication, the performance of lubrication became worse, even the container seems sufficiently wide. Note that cohesion term c_w eliminates the effect of self weight at the initial state in spite of lubrication improves the influence of friction on moving soil mass. Above discussion implies the difficulty of accurate 1g tests.

Under centrifugal acceleration field Lubrication made of a latex membrane and a silicon grease layer is an effective technique to replicate the stress distributions in centrifuge model tests. **Figures 4.14(a-e)** and **Figs. 4.15(a-e)** show comparisons between the prediction for non-lubricated and lubricated containers with a width of 15cm. In non-lubricated containers, the shape of distribution seems to be independent from the acceleration. On the other hand in lubricated containers, the influence of cohesion term c_w vanishes as the acceleration increases. No significant discrepancy between ideal and predicted distributions appears under an acceleration of 100g. The results from SRLT tests are the worst case among lubricated containers, however the prediction shows the effectiveness of lubrication, even under a 12.5g of acceleration. Note that the centrifuge testing with the lubrication technique can replicate the ideal stress distribution, without any difficulty. This aspect is one of the advantages of centrifuge model tests.

On sand subjected to a surcharge The lubrication technique can be also utilized in model tests on sand subjected to a surcharge. **Figures 4.16(a-d)** demonstrate the effectiveness of the lubrication technique. Because the amount of cohesion term c_w becomes rather small comparing with the intensity of surcharge, the reduction in vertical stress becomes negligible. On the other hand, vertical stresses in non-lubricated containers under the gravity field decrease as the depth increases, as shown in **Fig. 4.16(a)**. Lubrication is essential when model tests on underground structures, such as piles and tunnels.

4.3.2 Influence of centrifuge accelerating operation

4.3.2.1 Procedure

A series of centrifugal self weight compression tests, named C Tests, tabulated in **Table 4.9**, was conducted to examine the void ratio change and the initial stress conditions in sand subjected to a centrifuge acceleration history. The test set-up used is shown in **Fig. 4.17**. In these tests, the sand used was Toyoura Sand, of which initial void ratio was ranging from 0.674 to 0.844, prepared by means of air pluviation method. The model sand was subjected to a five cycles of centrifuge accelerating/deaccelerating operation up to 100g (980m/sec²) and down to 1g(9.8m/sec²), at a constant rate of 10g/min. Settlement and both vertical and horizontal stresses at the depths tabulated in tabulated in **Table 4.9** were measured during the operation, by using three LVDTs and three pairs of earth pressure cells. Surface settlement was also measured by using the fourth LVDT in **Fig. 4.17**.

4.3.2.2 Results

Initial void ratio **Figure 4.19** shows the relationship between absolute void ratio change obtained from surface settlement and the number of acceleration cycles. This figure indicates that increasing the number of cycles causes decreasing void ratio during accelerating ($\Delta e(load)$) ranging from 0.01 to 0.031 in virgin compression to a negligible value of around 0.004. The value is approximately equal to the void ratio change during the unloading sequence ($\Delta e(unload)$) after the fifth loading cycle. In the fifth cycle and later, the absolute values of void ratio change become small enough to neglect the influence on deformation due to the centrifuge operation. Note further, **Fig. 4.19** suggests that precise experiments in dry sand will require either (a) the correction of the initial void ratio estimated under the grav-

itational field if a measurement of surface settlement under the centrifugal field is available; or (b) at most five cycles of centrifuge operation, even for loose sand, prior to a loading test to safely neglect void ratio change induced by centrifuge operation if the measurement is not available.

Void ratios (e) of model sand which have been subjected to both one and five centrifuge cycle(s) up to 100g are plotted against each initial void ratio (e_0) measured prior to operation in **Fig. 4.20**. As shown in **Fig. 4.20**, for these particular tests, void ratios under an acceleration of 100g are approximately expressed by a liner equation, that is;

$$e = 0.92e_0 + 0.041 \quad (4.10)$$

which was obtained by means of least square method. Differences induced by the number of acceleration cycles are small, except in the case of $e_0 = 0.844$.

Changes in coefficient of earth pressure at rest **Figure 4.21(a)** shows changes in K_0 -value at a depth of $D_0 = 0.175\text{mm}$ during accelerating sequence obtained from C-1 test while **Fig. 4.21(b)** shows that during deaccelerating sequences. Since both vertical and horizontal stresses at 1g were so small that they probably contained a large amount of measurement errors, measurements under an acceleration smaller than 10g were eliminated from **Figs. 4.21(a, b)**.

Figure 4.21(a) indicates that the values of K_0 slightly change ranging from 0.35 to 0.32 during the virgin accelerating sequence. The values are close to the values estimated with Jáký's equation (Jáký 1944), i.e:

$$\begin{aligned} K_0 &= 1 - \sin \phi_d \\ &= 0.33 \end{aligned} \quad (4.11)$$

The values of K_0 under an acceleration of 100g, where the value of OCR is unity, are independent from the number of acceleration cycles as shown in **Figs. 4.21(a, b)**. The values of K_0 during re-accelerating sequences tend to scatter more than the values during deaccelerating sequences. K_0 under accelerating larger than 50g, where the corresponding OCR is less than two, is a little larger than the value under 100g. On the other hand, the value under accelerations less than 50g increases with decreasing acceleration and reaches up to twice larger than the value under 100g. Furthermore, K_0 under 10g (OCR=10) tended to increase with an increasing number of accelerating cycles during both accelerating and

deaccelerating sequences. Therefore it can be concluded that preparing reproductive over-compressed model sand requires conducting a model tests during the deaccelerating sequence, and that effective over-compression requires an OCR value at least larger than two.

4.4 Loading Test

4.4.1 Test procedure

A series of centrifugal loading tests, named LN, LC and LO tests, tabulated in **Table 4.10**, were conducted to investigate the influence of initial condition. Several factors that possibly affect on loading test results were examined in these loading tests: cyclic centrifuge operation, over-compression and wall friction of container.

LN tests are conventional centrifuge loading tests with a circular footing having a diameter of 3.0cm, of which results provide references for LC and LO test results. LC tests examined the influence of cyclic centrifuge operation at a constant rate of 10g/min. After a set of five cycles of centrifuge accelerating/deaccelerating history was imposed on the model sand, a series of loading tests were conducted under a 100g of centrifuge acceleration. LO tests were for over-compression. First model sand was accelerated up to a 100g of centrifuge acceleration, then deaccelerate down to 50, 25 and 10g and loading tests were conducted.

In LN, LC and LO tests, the sand used was Toyoura sand, of which initial dry density, ρ_d , was ranging from 1.585 to 1.605 (g/cm³), prepared by means of air pluviation method. Test set-up for these tests is shown in **Fig. 4.22**. Surface settlement and both vertical and horizontal stresses were measured during the operation, by using a LVDT placed at surface and a pair of earth pressure cells embedded at a depth of about 170mm.

4.4.2 Test results

4.4.2.1 Influence of cyclic centrifuge operation

Changes in void ratio **Figure 4.18** shows the measured settlement at surface with the number of accelerating/deaccelerating cycles. Noticeable settlements occurred during centrifuge acceleration up to 100g in LC-tests, while sharp rebounds appeared at the end of operating cycles during the following unloading sequence down to 1g. Although settlements measured increased with increasing number of operating cycles, most amount of settlement was

induced in the virgin loading sequence, and the increments of settlement gradually decreased with an increasing number of operating cycles.

Figures 4.24(a, b) show $e - \log_{10} \sigma_v$ relationships obtained from LC tests. The values of vertical stress were estimated from dry density, acceleration and the half depth of the model sand (140mm). The relationships are similar to those obtained from conventional oedometer tests (see Fig. 4.25). According to these figures, the sand used in LC-1-1 test seems to be lightly overconsolidated comparing with LC-1-2 tests. Therefore, surface settlement obtained from LC-1-1 test became smaller than that from LC-1-2 tests, even initial void ratio in LC-1-1 test was greater than that in LC-1-2 test. In LC-1-2 test most changes in void ratio occurs in the first accelerating sequence. Increments of void ratio at 100g decrease as the number of centrifuge accelerating/deaccelerating sequences increases. Considering the amount of errors in density of the model sand induced in sand preparation by air pluviation method, we can neglect the changes in void ratio due to centrifugal accelerating histories; to maintain the deviation of void ratios within 0.002 among a number of experiments is very difficult.

Bearing characteristics Figure 4.26 shows a comparison of normalized bearing capacity versus settlement behavior of the sand for one cycle (LN-tests) and five cycles (LC-tests) of the centrifuge operations. Curves for LC tests results coincide well each other, while in LN tests one result shows rather low bearing capacity. Although test results obtained from LN-tests somewhat show higher bearing capacities than those from LC-tests, no significant influence related to the number of centrifuge cycles appeared among the tests results in this case.

4.4.2.2 Influences of over-compression

Figures 4.27(a-c) show the comparison of the relationship between bearing capacity and bearing capacity over-compressed sand (LO-tests) and normally compressed sand (LN-tests). Because stress level effects on soil properties affect bearing characteristics of sand, LO-test results are compared with LN-test results under the identical centrifuge acceleration. The results of LO-tests are slightly higher bearing capacities than the results of LN-tests, as shown in Fig. 4.26(a). The OCR only slightly affects on bearing characteristics. Figs. 4.26(b) and (c), in which loading tests were conducted under a lower acceleration and higher OCR, also show smaller influence of OCR.

4.4.3 Discussion

4.4.3.1 Influence of over-compression on subgrade reaction factor

Figure 4.28 shows the relationship between subgrade reaction factor K_s and OCR. Over-compression leads to higher values of K_s , and this tendency becomes larger with increasing centrifuge acceleration.

Subgrade reaction factor K_s is defined as Eq. (4.12).

$$K_s = \frac{q_{50}}{s_{50}} \quad (4.12)$$

where q_{50} is a half value of ultimate bearing capacity q_u obtained from loading tests. s_{50} is the settlement when q_{50} is mobilized. The ultimate bearing capacity, q_u , determined by subtracting the contribution of surcharge on bearing capacity determined by fitting onto the linear portion appearing at the end of loading curves. The method is illustrated in Fig. 4.29.

4.4.3.2 Influence of K_0 on ultimate bearing capacity

Although most of normalized load intensity – settlement curves duplicated each other, some curves showed extremely low normalized load intensity. This fact implies that there is a missing parameter which influences bearing characteristics of dry sand.

Figure 4.30 shows normalized ultimate bearing capacities ($2q_u/\gamma B$) versus K_0 measured in loading tests. For a set of test results under 10g, values of $2q_u/\gamma B$ concentrate around 110; even the values of K_0 vary from 0.23 to 0.68. On the other hand, for three sets of test results under 25g, 50g, and 100g, the values of $2q_u/\gamma B$ increase with increasing K_0 . Higher stress level leads to a larger influence of K_0 . D'Appolonia and Lambe (1970) concluded that load-settlement behavior predicted by finite element analysis with an anisotropic model was strongly dependent upon the initial stress condition. Decreasing K_0 led to decreasing the first yield stress. Iizuka *et al.* (1987) applied finite element analysis to anisotropic consolidated clay. They demonstrated the influence of K_0 on bearing characteristics, and concluded that different K_0 caused different failure patterns in the analyzed mesh models. Note that not only on clay but also on sand, increasing K_0 leads to increasing normalized ultimate bearing capacity, and this tendency depends on stress level. The influence of K_0 can be understood as a resultant effect of both strength anisotropy and progressive failure, depending upon stress level.

4.5 Summary

4.5.1 On friction and lubrication

The author examined the friction acting on lubricated and non-lubricated metal by means of the direct shear test. The influence of the friction on vertical stress distributions was evaluated. The following comments were derived from the results.

1. Lubrication made from a thin latex membrane and silicon grease sufficiently reduced friction resistance of the interface between sand and metal. The friction resistance of lubricated interface tended to be smaller as latex membrane became thin. Thinner latex membrane is also preferable in consideration of the plane strain condition.
2. Roughness of the interface affected friction resistance even the interfaces were lubricated. It is preferable to polish the wall of the container to reduce friction. The Mohr-Coulomb criterion can be used to represent the friction resistance on the interfaces. On lubricated interfaces, a small amount of cohesion intercept, which was attributed to viscosity of silicon grease, was observed.
3. Vertical stress distribution in a container can be estimated by using Terzaghi's theory of silo effect. Estimated values of vertical stress were well agree with measured values.
4. Reproduction of ideal stress distribution under the gravity acceleration is very difficult, even container walls are lubricated. Cohesion intercepts on lubricated interfaces significantly affect stress distribution in the container, because stress levels in 1g model tests are, generally, extremely small comparing the cohesion intercepts.
5. If a surcharge was applied on the surface of model sand, or the model tests were conducted under 12.5g or higher accelerations, the lubrication technique using silicon grease and a latex membrane can be utilized because the stress level in the model sand becomes much higher than the cohesion intercepts. Combination of centrifuge testing technique and the lubrication technique can provide an ideal stress distribution in the model sand.

4.5.2 On initial condition

To investigate the influence of initial conditions affecting bearing characteristics of sand, a series of compression tests and loading tests were carried out. The following conclusions were

drawn from these test results.

1. Settlements due to centrifuge acceleration cycles were observed during virgin centrifugal accelerating operation. Precise experiments in dry sand will require either a measurement of deformation or a correction of void ratio.
2. Load-settlement curves were independent from over-compression and the number of centrifuge accelerating/deaccelerating cycles.
3. Increasing K_0 leads to increasing normalized ultimate bearing capacity under the acceleration larger than 50g. On the other hand under the acceleration of 10g, no influence of K_0 exists, even OCR equals ten. Neglecting the influence of K_0 on overconsolidated soil, as is done in current design methods, would result in a conservative estimation of bearing capacity.

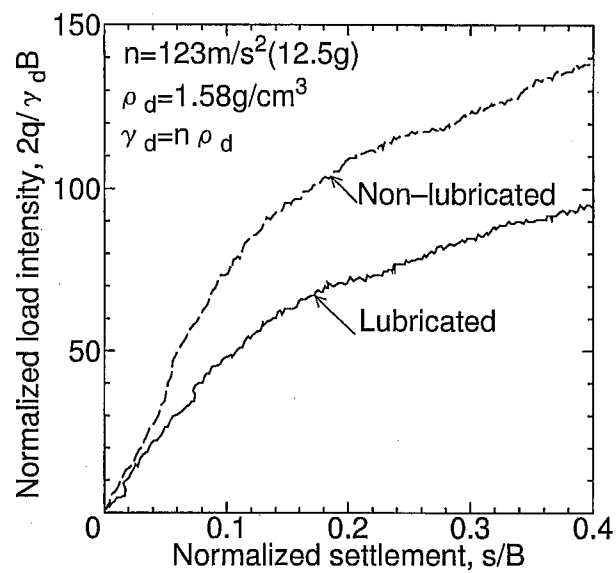


Figure 4.1: Influence of boundary friction on bearing characteristics (after Nishida *et al.* (1996)).

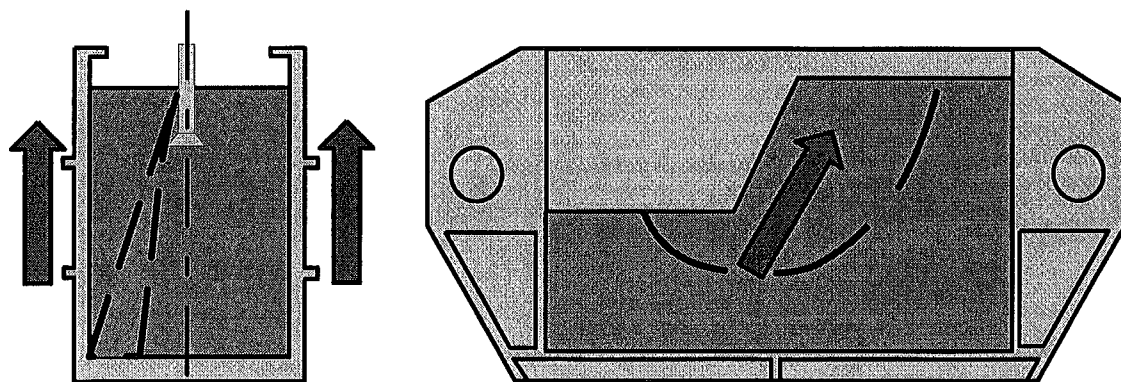
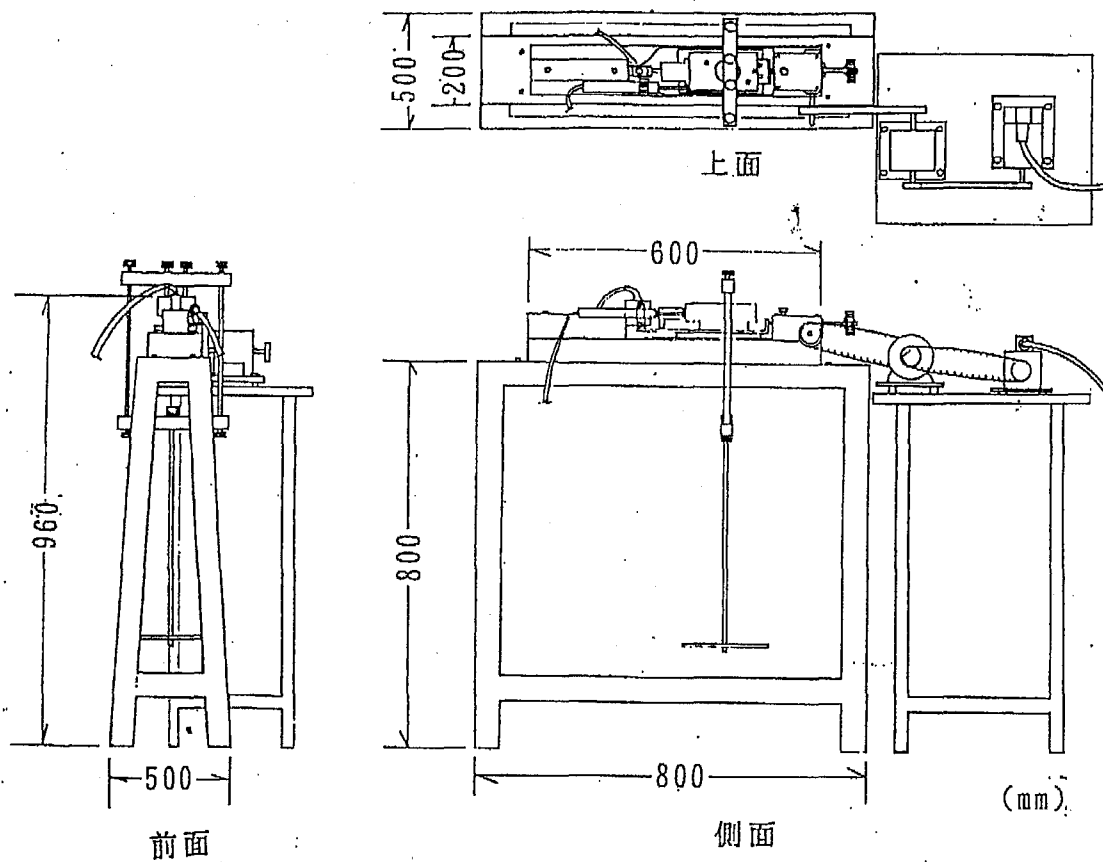
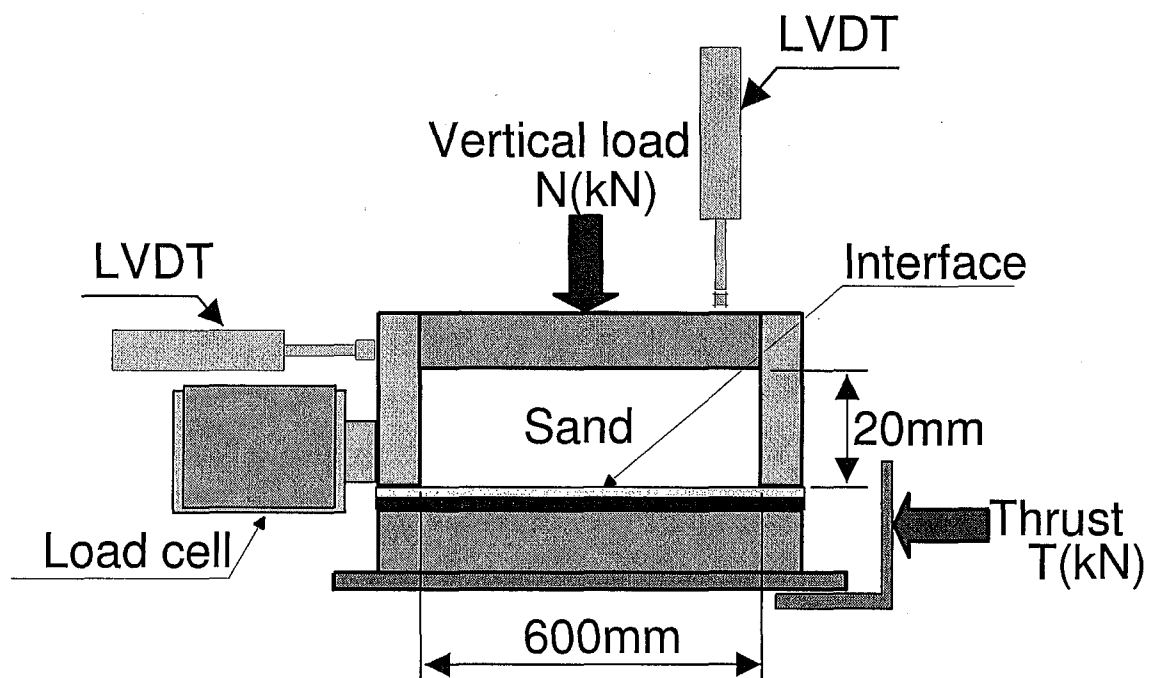


Figure 4.2: Influence of wall friction.



(a) Loading apparatus



(b) Shear box

Figure 4.3: Direct shear apparatus to measure friction between sand and interface.

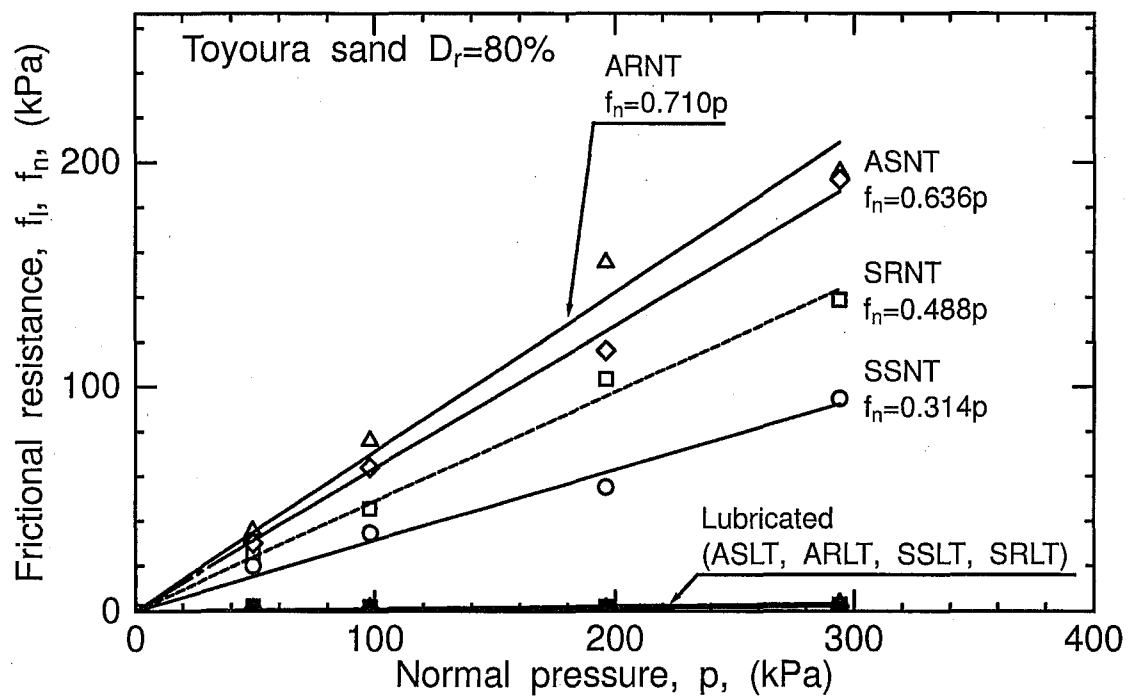


Figure 4.4: Comparison of friction resistance between lubricated and non-lubricated interfaces obtained for Toyoura Sand.

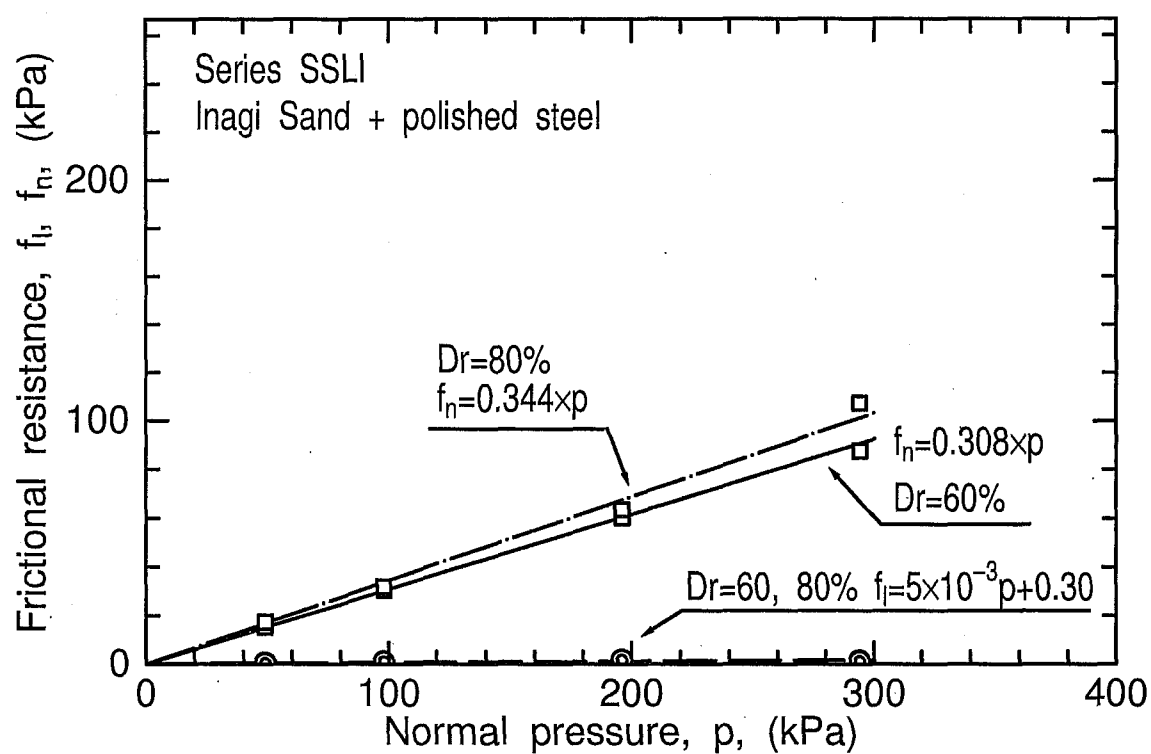
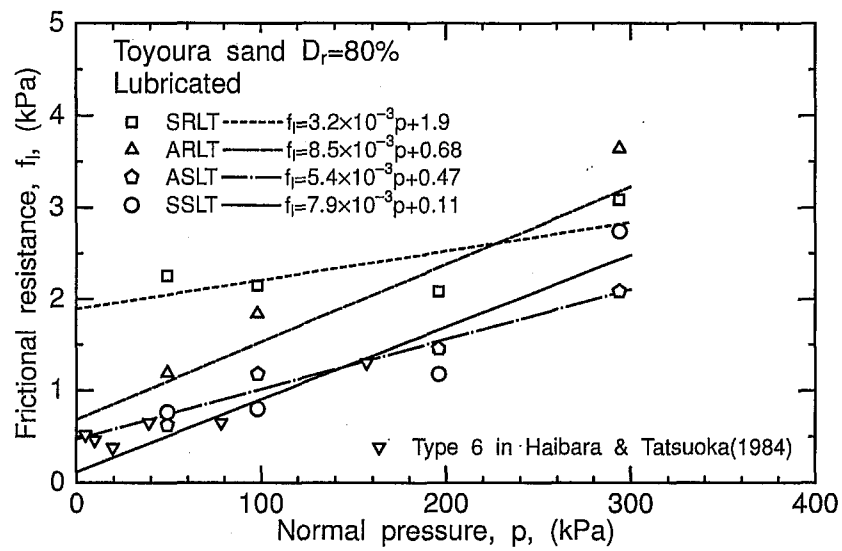
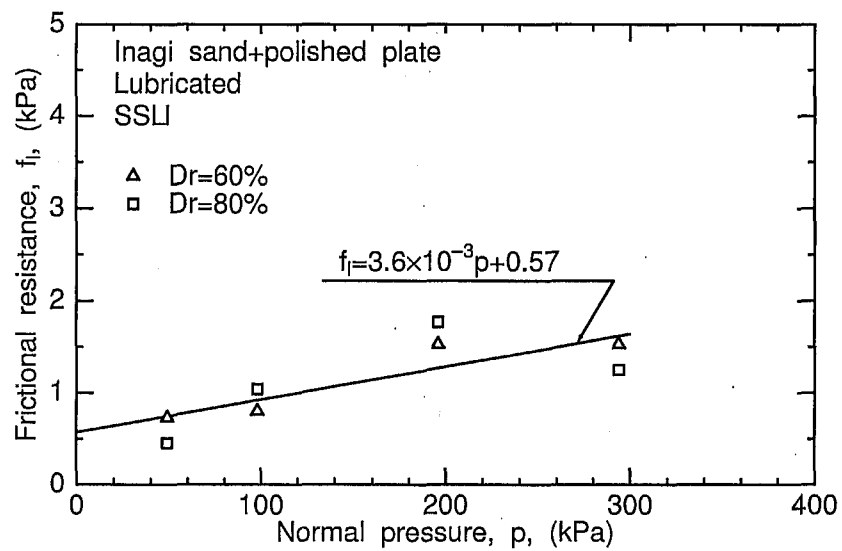


Figure 4.5: Comparison of friction resistance between lubricated and non-lubricated interfaces obtained for Toyoura Sand.



(a) Toyoura Sand (series ASLT, ARLT, SSLT and SRLT)



(b) Inagi Sand (series SSLI)

Figure 4.6: Frictional resistance on lubricated interface.

Table 4.1: Properties of sand.

	Toyoura Sand	Inagi Sand
Density, $\rho_s, (\text{g}/\text{cm}^3)$	2.660	2.666
Minimum density, $\rho_{\min}, (\text{g}/\text{cm}^3)$	1.340	1.435
Maximum density, $\rho_{\max}, (\text{g}/\text{cm}^3)$	1.660	1.755
Coefficient of uniformity, U_c	1.34	2.71
Internal friction angle, $\phi_d, (\text{deg.})$	42.0 ^a	36.5 ^a

^a $\sigma'_r=1.96\text{kPa}$, $Dr = 70\%$, CID triaxial test

Table 4.2: Direct shear tests using Toyoura Sand ($Dr = 80\%$) with changing thickness of latex membrane t .

Test Name	Thickness ^a t , (mm)	Materials	Normal pressure p , (kPa)	Resistance f , (kPa)
SSTT-1	0.37	Fine steel	196.0	1.1779
SSTT-2	0.74	Fine steel	196.0	1.6661
SSTT-3	1.11	Fine steel	196.0	1.4927

Note:^a thickness of latex membrane.

Table 4.3: Direct shear tests using Toyoura Sand with changing relative density D_r .

Test Name	Relative density D_r , (%)	Sand	Normal pressure p , (kPa)	Resistance f , (kPa)
SSDr-1	60	Toyouura Sand	196.0	1.249
SSDr-2	70	Toyouura Sand	196.0	1.493
SSDr-3	80	Toyouura Sand	196.0	1.178
SSDr-4	60	Inagi Sand	49.0	0.729
SSDr-5	80	Inagi Sand	49.0	0.453
SSDr-7	60	Inagi Sand	98.0	0.799
SSDr-7	80	Inagi Sand	98.0	1.040
SSDr-8	60	Inagi Sand	196.0	1.528
SSDr-9	80	Inagi Sand	196.0	1.769
SSDr-10	60	Inagi Sand	294.0	1.528
SSDr-11	80	Inagi Sand	294.0	1.249

Table 4.4: Direct shear tests using Toyoura Sand ($D_r = 80\%$) with various test conditions.

Test Name	Materials	Roughness	Normal pressure p , (kPa)	Resistance, (kPa)		Effectiveness f_l/f_n
				Lubricated, f_l	Non-lubricated, f_n	
SSLT-1	Steel	Fine	49.0	0.764	20.02	0.038
SSLT-2	Steel	Fine	98.0	0.799	34.49	0.023
SSLT-3	Steel	Fine	196.0	1.178	54.93	0.021
SSLT-4	Steel	Fine	294.0	2.741	94.59	0.029
SRLT-1	Steel	Rough	49.0	2.257	25.72	0.088
SRLT-2	Steel	Rough	98.0	2.151	45.42	0.047
SRLT-3	Steel	Rough	196.0	2.083	103.25	0.024
SRLT-4	Steel	Rough	294.0	3.088	138.77	0.022
ASLT-1	Aluminum	Fine	49.0	0.626	29.93	0.021
ASLT-2	Aluminum	Fine	98.0	1.181	63.85	0.018
ASLT-3	Aluminum	Fine	196.0	1.457	116.06	0.013
ASLT-4	Aluminum	Fine	294.0	2.083	192.29	0.011
ARLT-1	Aluminum	Rough	49.0	1.192	35.34	0.037
ARLT-2	Aluminum	Rough	98.0	1.839	75.91	0.024
ARLT-3	Aluminum	Rough	196.0	1.457	155.32	0.020
ARLT-4	Aluminum	Rough	294.0	3.643	195.68	0.019

Table 4.5: Direct shear tests using Inagi Sand ($D_r = 60\%$) with different normal pressure p .

Test Name	Materials	Roughness	Normal pressure p , (kPa)	Resistance, (kPa)		Effectiveness f_l/f_n
				Lubricated, f_l	Non-lubricated, f_n	
SSLI-1	Steel	Fine	49.0	0.729	15.32	0.048
SSLI-2	Steel	Fine	98.0	0.799	30.56	0.026
SSLI-3	Steel	Fine	196.0	1.528	60.24	0.025
SSLI-4	Steel	Fine	294.0	1.528	107.39	0.014

Table 4.6: Direct shear tests using Inagi Sand ($D_r = 80\%$) with different normal pressure p .

Test Name	Materials	Roughness	Normal pressure p , (kPa)	Resistance, (kPa)		Effectiveness f_l/f_n
				Lubricated, f_l	Non-lubricated, f_n	
SSLI-5	Steel	Fine	49.0	0.453	17.37	0.026
SSLI-6	Steel	Fine	98.0	1.040	31.87	0.033
SSLI-7	Steel	Fine	196.0	1.769	63.60	0.028
SSLI-8	Steel	Fine	294.0	1.249	87.58	0.014

Table 4.7: Friction parameters determined by direct shear tests.

Non-lubricated interface					
Test Name	Material	Sand	c_w (kPa)	ϕ_w (deg.)	$\tan \phi_w$
SSLT	Fine steel	Toyoura Sand	0	17.4	0.314
SRLT	Rough steel	Toyoura Sand	0	26.0	0.488
ASLT	Fine aluminum	Toyoura Sand	0	32.5	0.636
ARLT	Rough aluminum	Toyoura Sand	0	35.4	0.710
SSLI	Fine steel	Inagi Sand ($D_r=80\%$)	0	19.0	0.344
SSLI	Fine steel	Inagi Sand ($D_r=60\%$)	0	17.1	0.308
Lubricated interface					
Test Name	Material	Sand	c_w (kPa)	ϕ_w (deg.)	$\tan \phi_w$
SSLT	Fine steel	Toyoura Sand	0.11	0.45	7.9×10^{-3}
SRLT	Rough steel	Toyoura Sand	1.9	0.18	3.2×10^{-3}
ASLT	Fine aluminum	Toyoura Sand	0.47	0.31	5.4×10^{-3}
ARLT	Rough aluminum	Toyoura Sand	0.68	0.49	8.5×10^{-3}
SSLI	Fine steel	Inagi Sand	0.57	0.20	3.5×10^{-3}

Table 4.8: Test conditions

Test name	wall condition	Dry density ρ_d (g/cm ³)	Max. acceleration (g)
EPNI-1	Non-lubricated	1.561	12.5
EPNI-2	Non-lubricated	1.561	25.0
EPNI-3	Non-lubricated	1.561	50.0
EPLI-1	Lubricated	1.582	12.5
EPLI-2	Lubricated	1.551	25.0
EPLI-3	Lubricated	1.531	50.0

Three earth pressure cells were embedded at depth of 195mm.

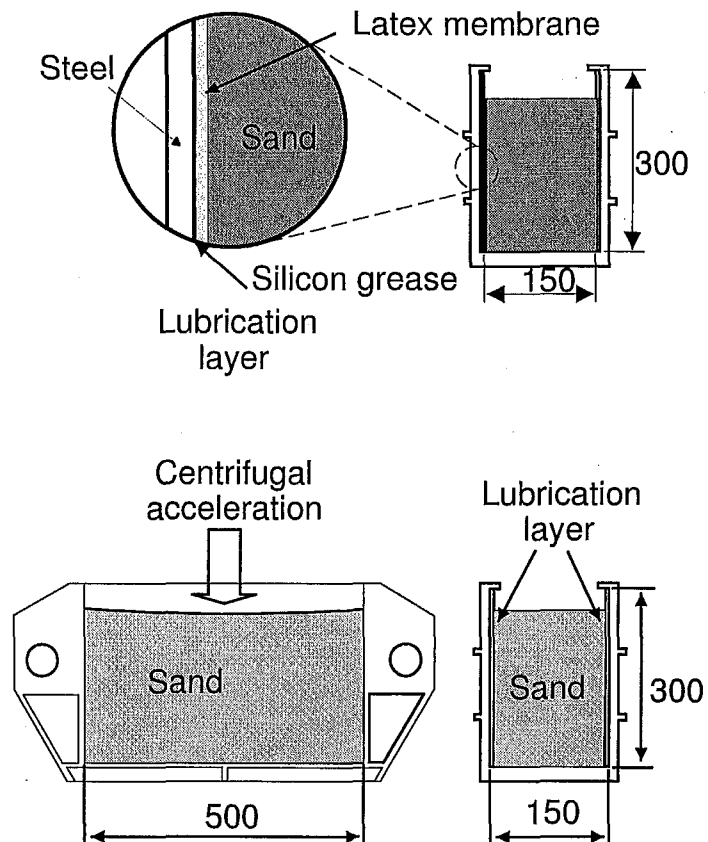


Figure 4.7: Set-up for EPNI and EPLI tests.

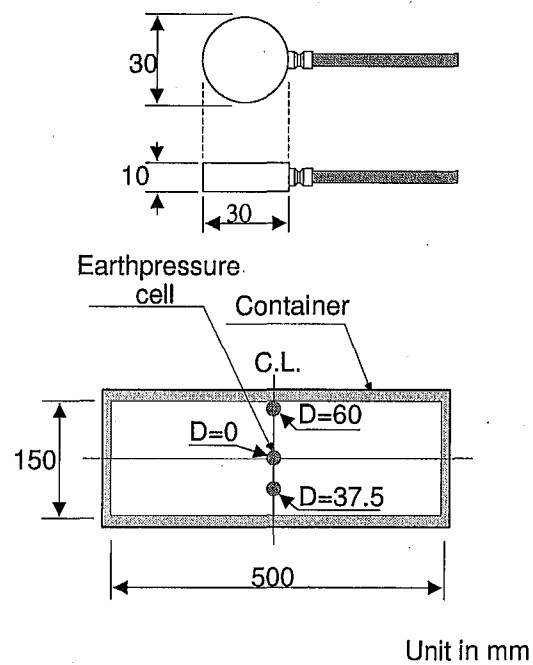


Figure 4.8: Earth pressure cells and their locations in a container.

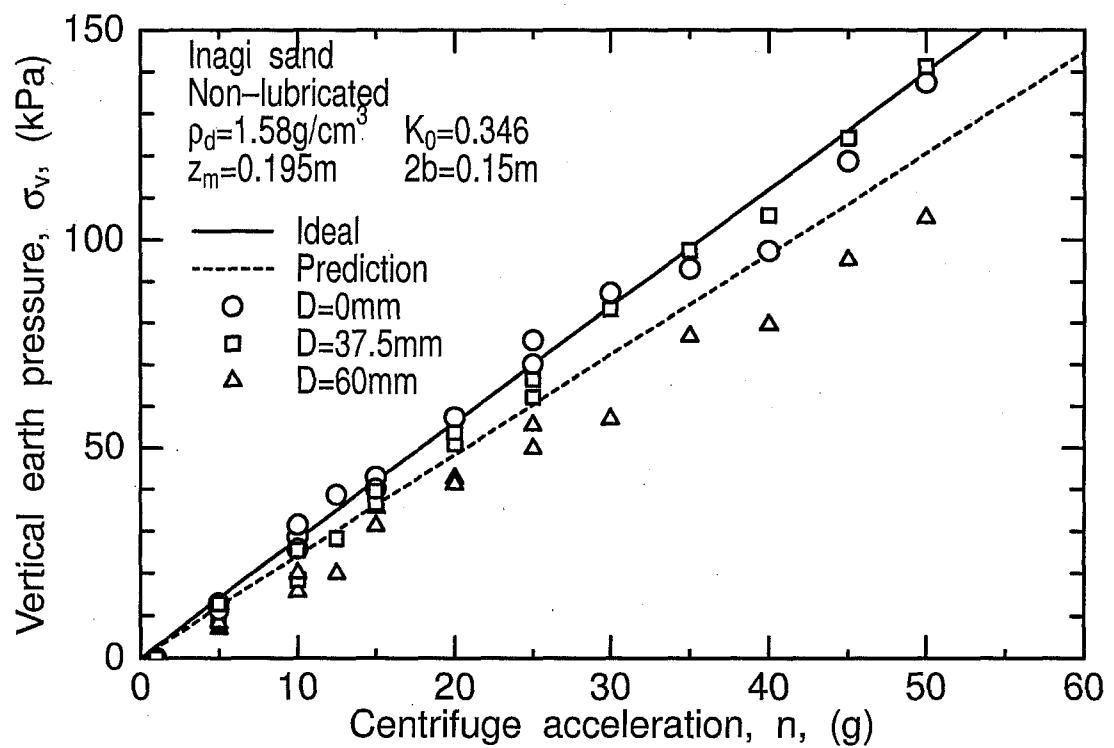


Figure 4.9: Comparison between predicted and measured vertical pressure σ_v in non-lubricated container ($2b=15\text{cm}$).

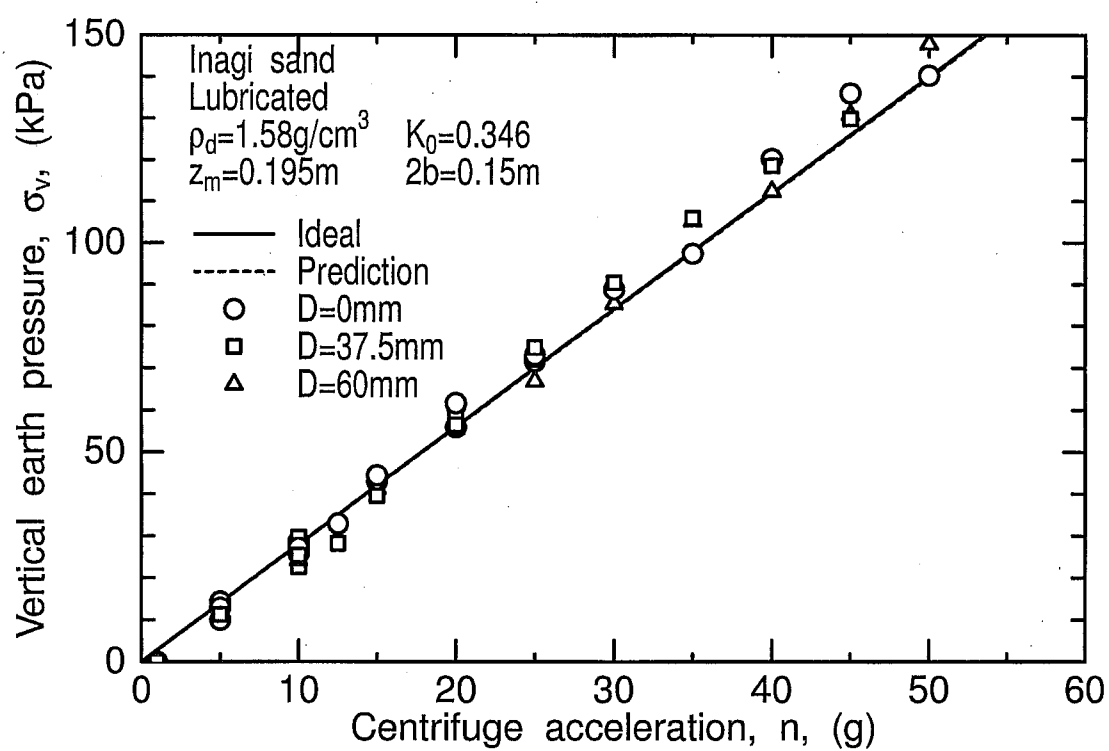


Figure 4.10: Comparison between predicted and measured vertical pressure σ_v in lubricated container ($2b=15\text{cm}$).

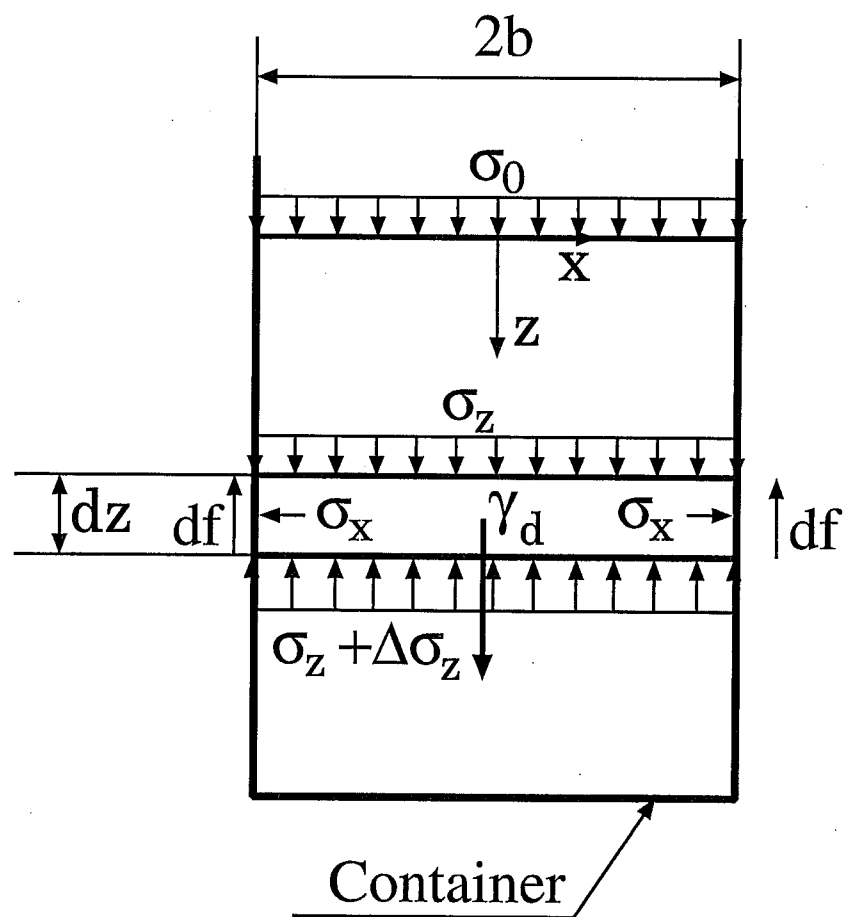


Figure 4.11: Force acting on a small soil element in a container.

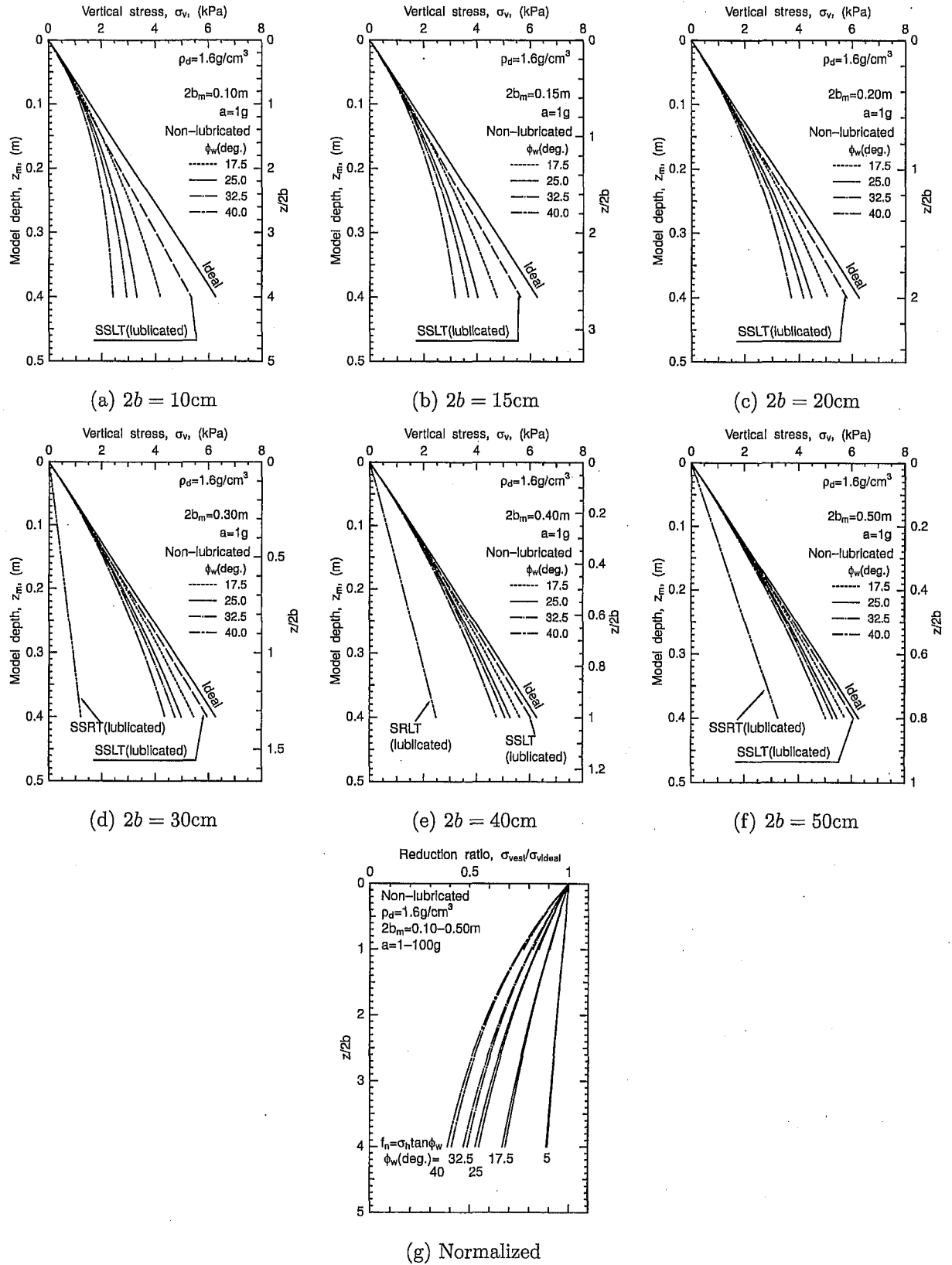


Figure 4.12: Predicted vertical stress distributions in non-lubricated container under the gravity acceleration field.

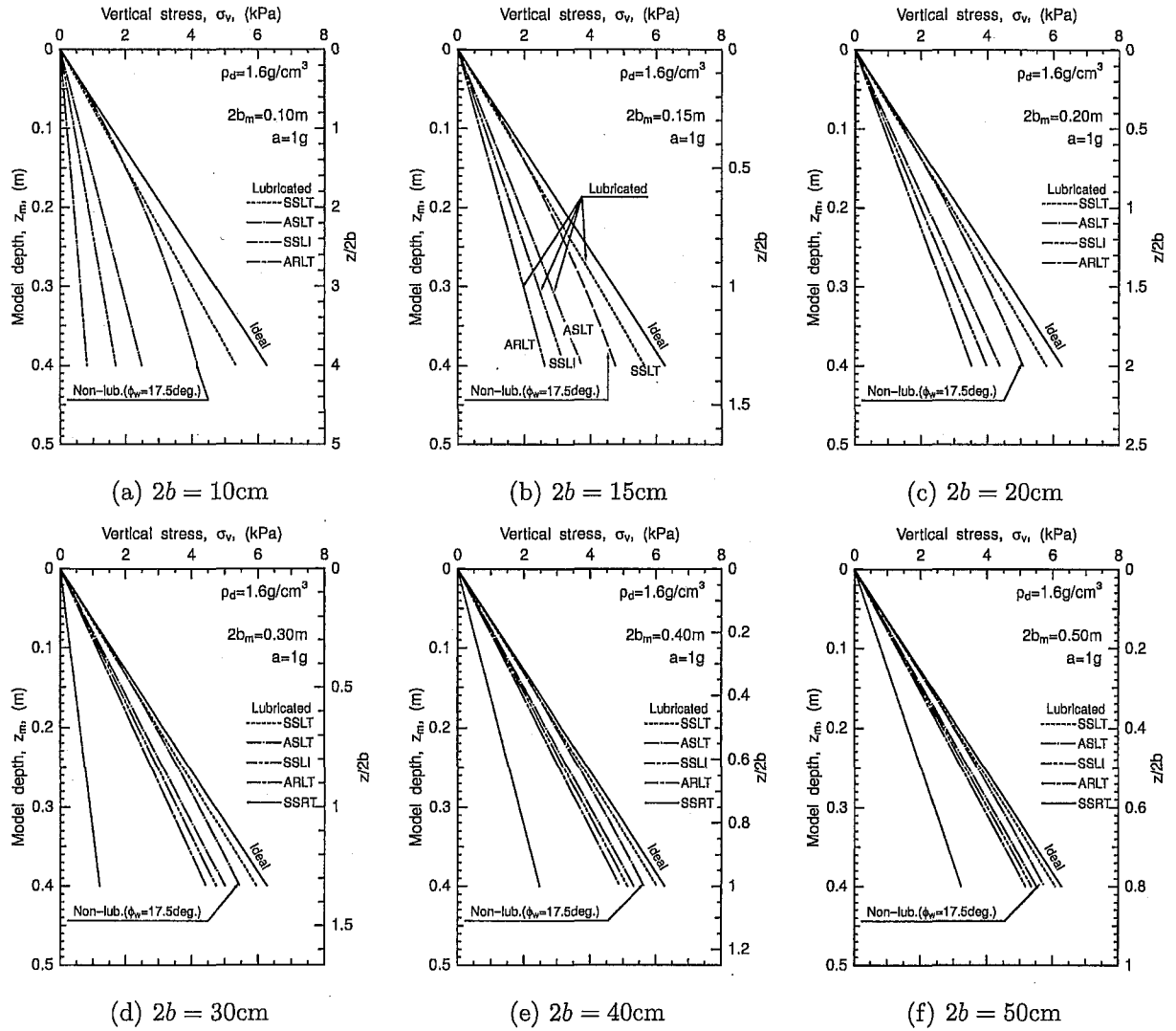


Figure 4.13: Predicted vertical stress distributions in non-lubricated container under the gravity acceleration field.

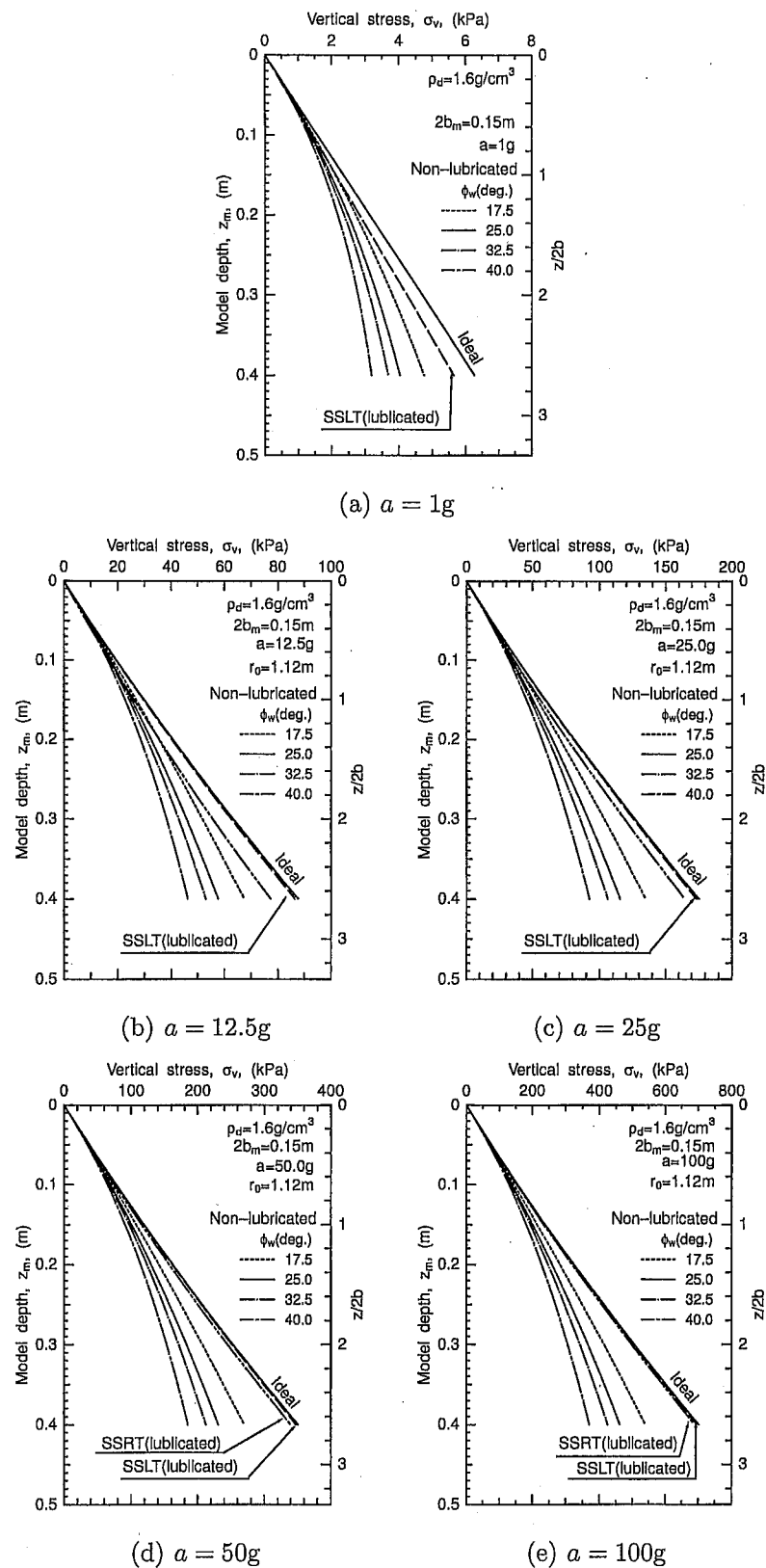


Figure 4.14: Predicted vertical stress distributions in non-lubricated container with a width of $2b = 15\text{cm}$ under various acceleration fields.

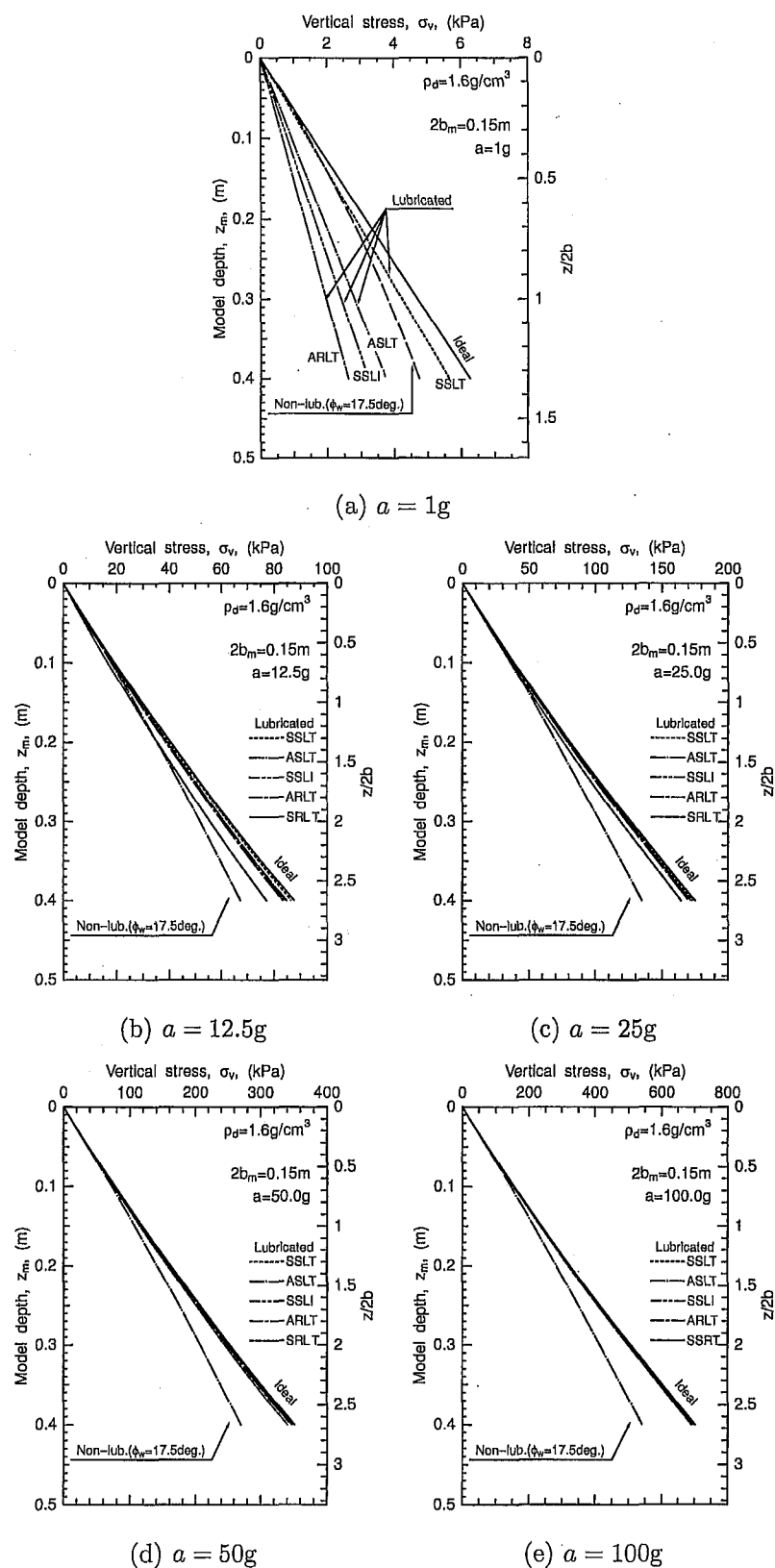


Figure 4.15: Predicted vertical stress distributions in lubricated container with a width of $2b = 15\text{cm}$ under various acceleration fields.

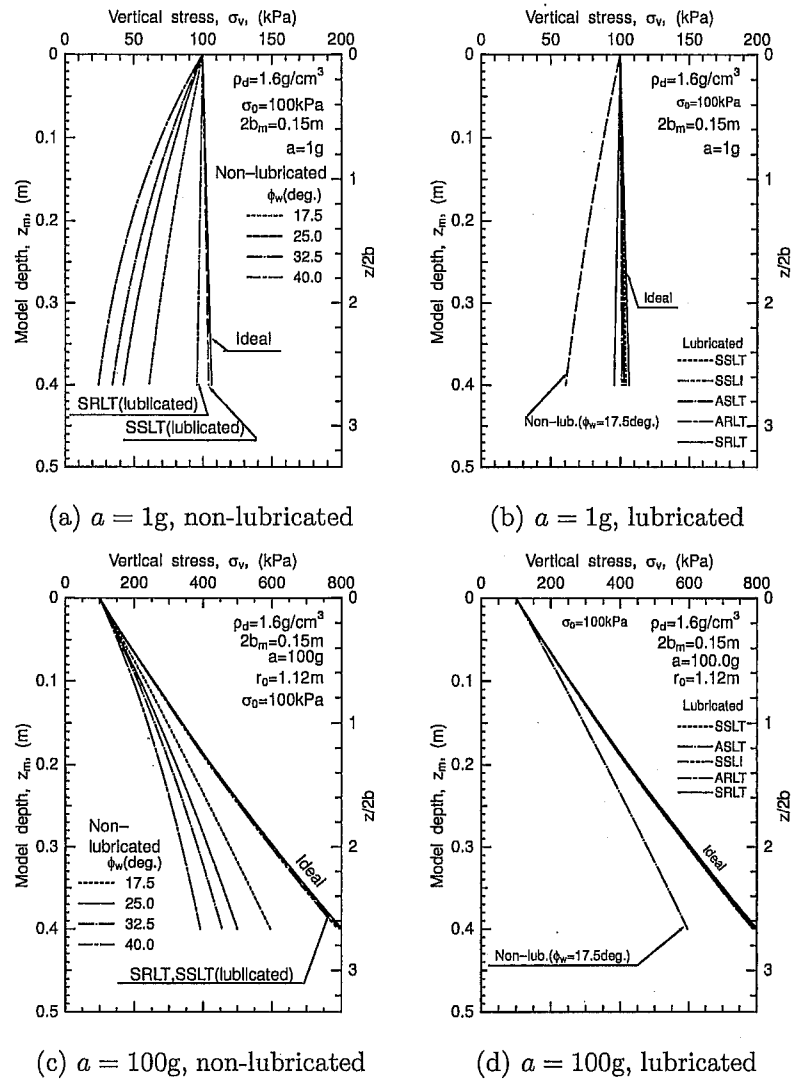


Figure 4.16: Predicted vertical stress distributions under a surcharge of 100kPa.

Table 4.9: Test conditions (C tests).

Test name	Void ratio, e_0	Relative density, $D_r(\%)$	Cycles operated, $N(\text{cycles})$	Measuring depth, $D_0(\text{m})$		
				No.1	No.2	No.3
C-1	0.674	81.3	10	0.059	0.125	0.175
C-2	0.709	72.0	10	0.045	0.102	0.166
C-3	0.716	70.2	10	0.048	0.108	0.174
C-4	0.753	60.5	10	0.054	0.105	0.169
C-5	0.774	55.1	10	0.054	0.097	0.160
C-6	0.844	36.9	10	0.045	0.105	0.164
C-7	0.676	80.8	7	0.039	0.105	0.164
C-8	0.705	73.1	1	0.046	0.112	0.174

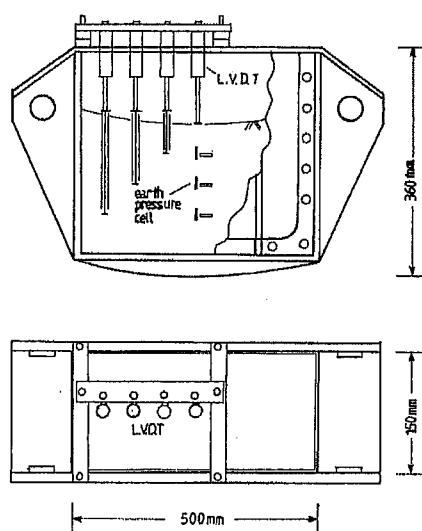


Figure 4.17: Set-up for C tests.

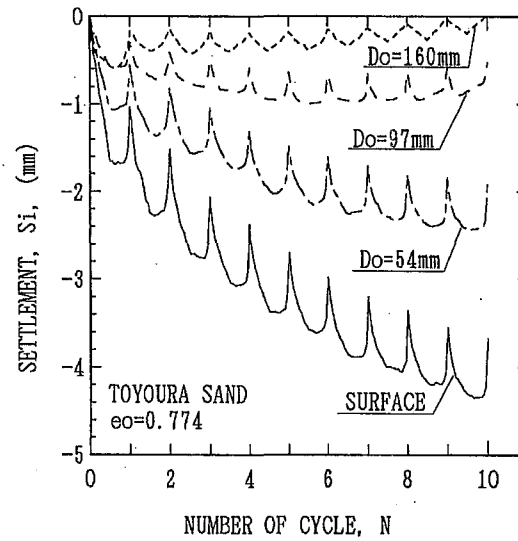


Figure 4.18: Settlement in C tests.

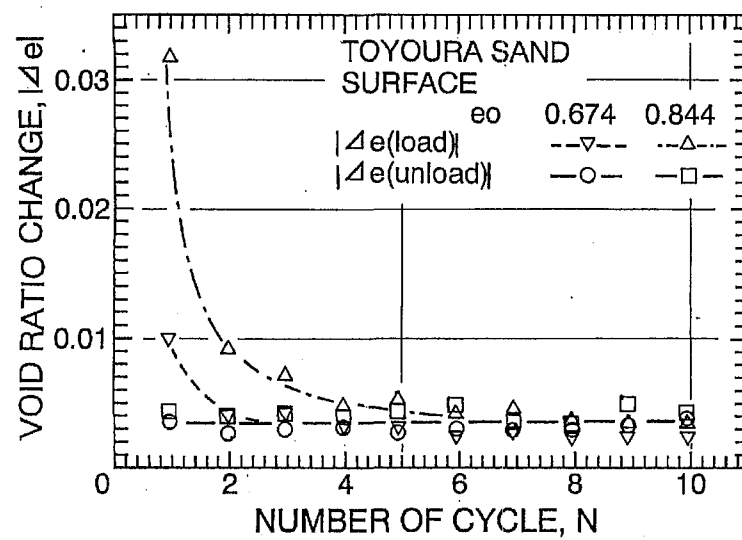


Figure 4.19: Void ratio change versus centrifuge operation cycle obtained from C tests.

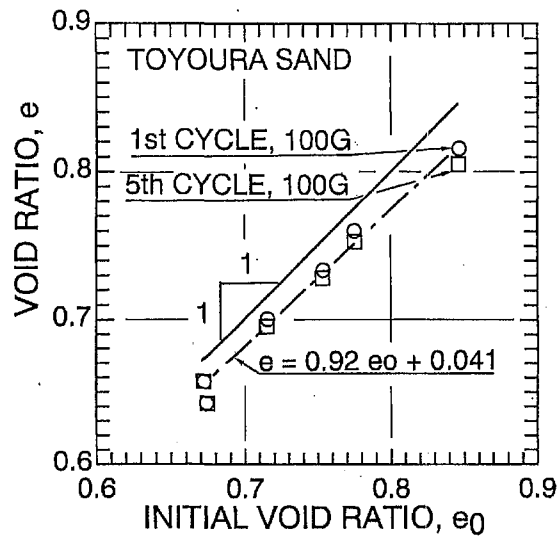
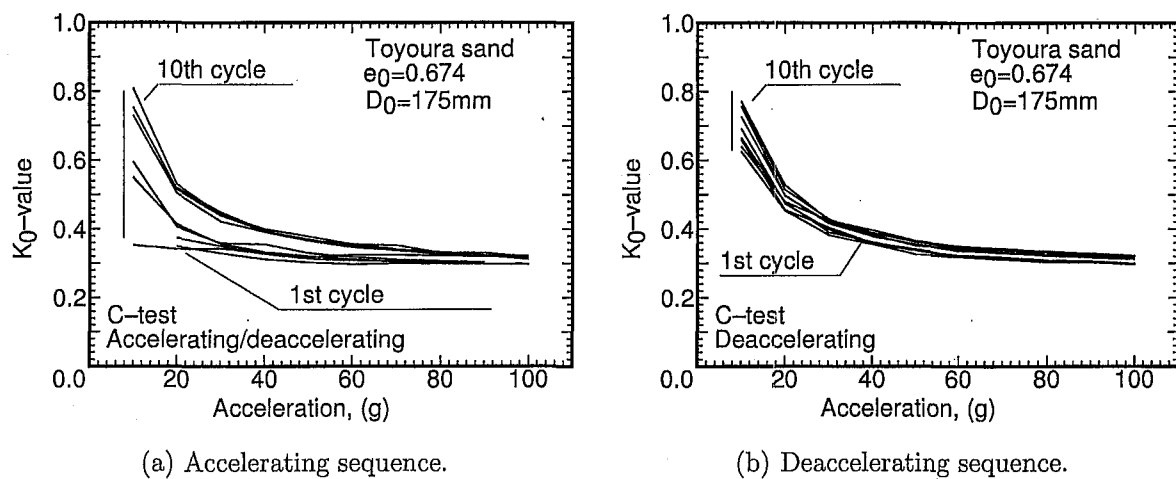


Figure 4.20: Void ratio under 100g.

Figure 4.21: K_0 obtained from C1-test: (a) accelerating sequence, (b) deaccelerating sequence.

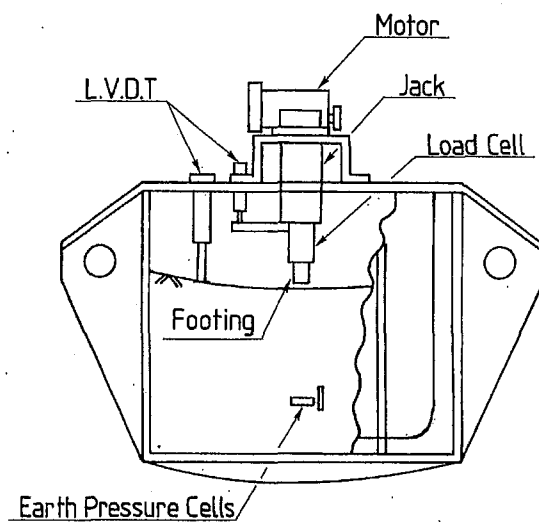


Figure 4.22: Test set-up for LN, LC and LO tests.

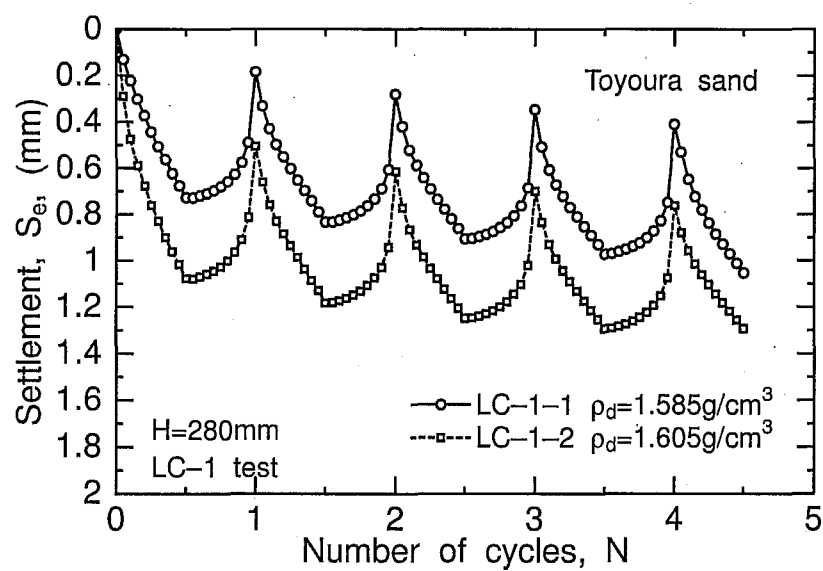
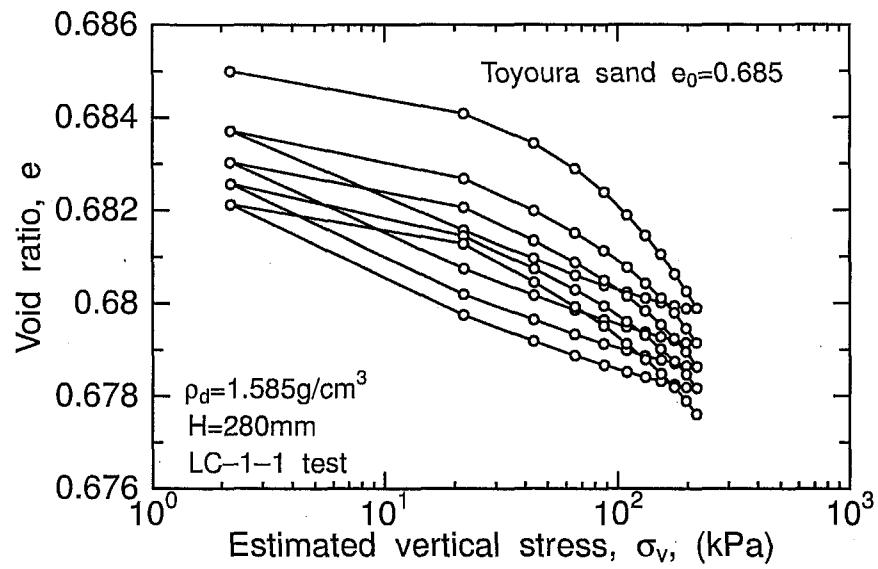


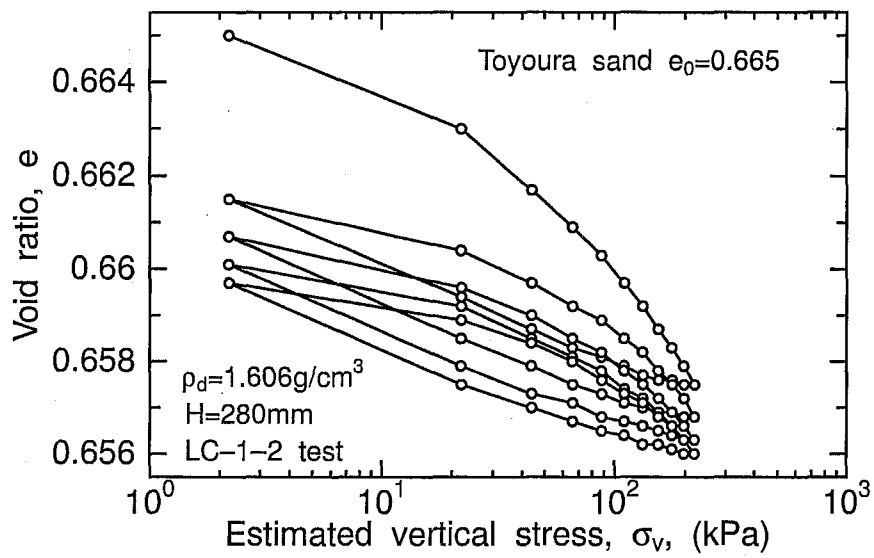
Figure 4.23: Sand surface settlement due to centrifuge operation.

Table 4.10: Loading test on Toyoura sand (LN, LC, LO tests).

Test name	Dry density ρ_d (g/cm ³)	OCR	Cycle(s) operated N	Accele- ration $n(g)$	Measuring depth, $D_0(mm)$	Value of K_0
LN-1-1	1.589	1	1	100	158	0.17
LN-1-2	1.594	1	1	100	172	0.25
LN-1-3	1.587	1	1	100	171	0.25
LC-1-1	1.585	1	5	100	171	0.28
LC-1-2	1.605	1	5	100	152	0.30
LN-2-1	1.582	1	1	50	179	0.23
LN-2-2	1.593	1	1	50	178	0.30
LN-2-3	1.596	1	1	50	175	0.33
LO-2-1	1.600	2	1	50	154	0.36
LO-2-1	1.597	2	1	50	185	0.45
LN-3-1	1.593	1	1	25	176	0.31
LN-3-2	1.599	1	1	25	176	0.37
LO-3-1	1.589	4	1	25	175	0.36
LO-3-2	1.595	4	1	25	179	0.46
LN-4-1	1.589	1	1	10	185	0.23
LN-4-2	1.596	1	1	10	164	0.55
LO-4-1	1.591	1	1	10	181	0.55
LO-4-2	1.594	1	1	10	177	0.53
LO-4-3	1.591	1	1	10	186	0.68



(a) LC-1-1.



(b) LC-1-2.

Figure 4.24: $e - \log_{10} \sigma_v$ relationships obtained from LC tests.

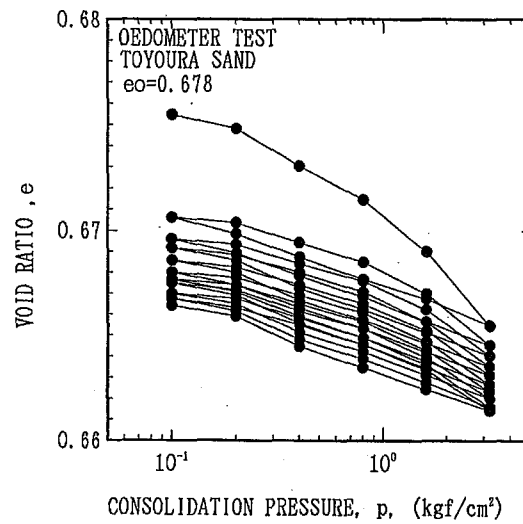


Figure 4.25: Oedometer test results.

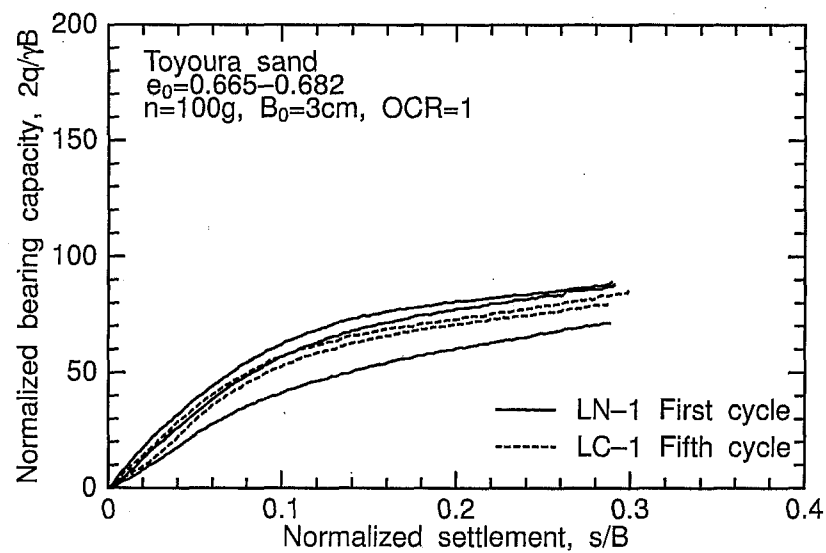
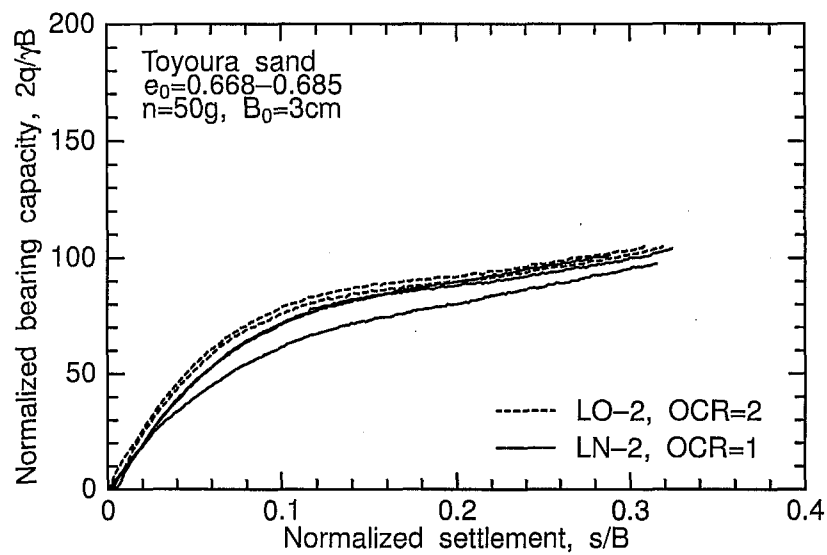
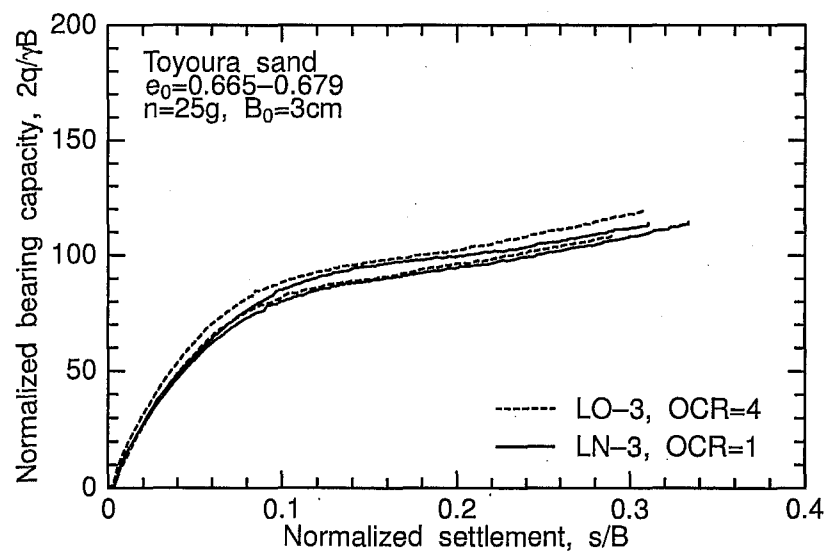


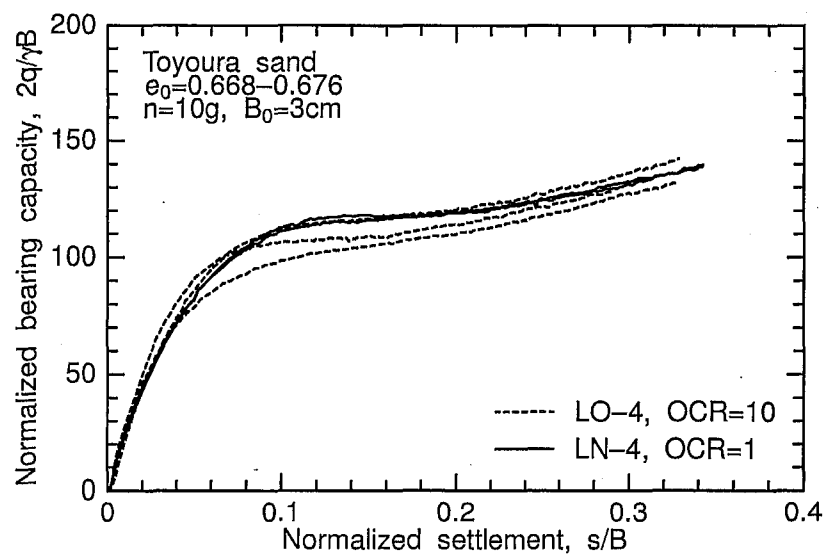
Figure 4.26: Comparison of LN-test results and LC-test results.



(a) 50g



(b) 25g



(c) 10g

Figure 4.27: Comparison of LN-test results and LO-test results.

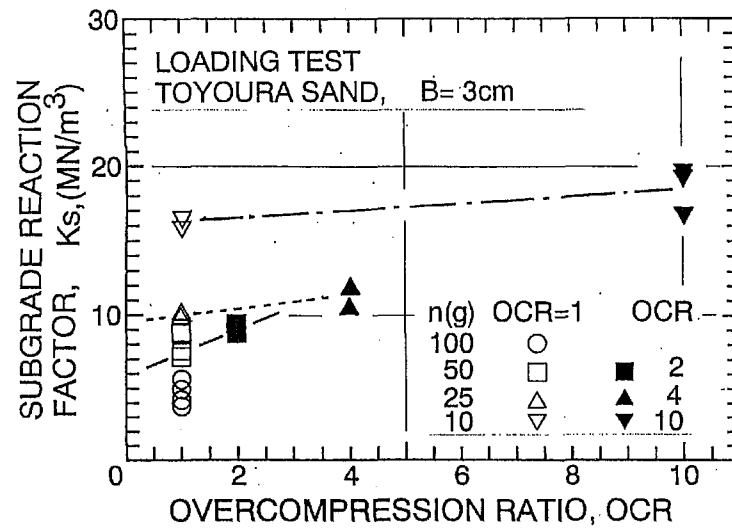


Figure 4.28: Effect of over-compression on subgrade reaction factor K_s .

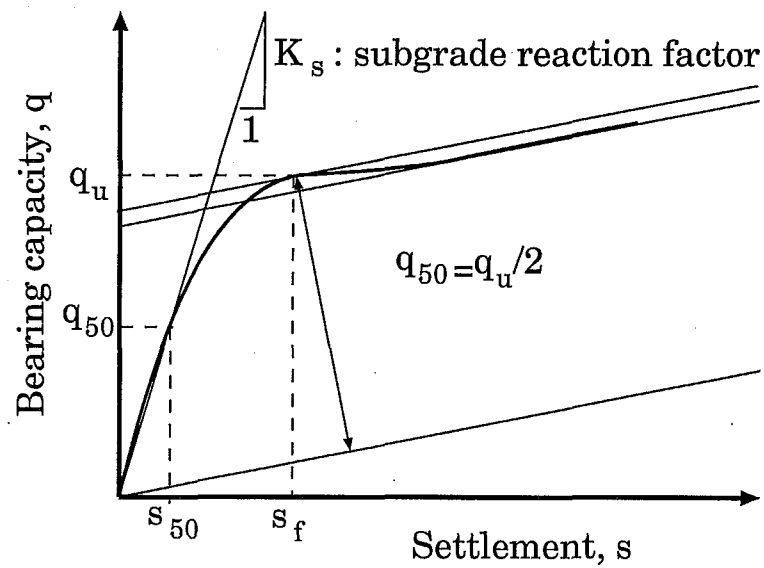


Figure 4.29: Determination of ultimate bearing capacity q_u and subgrade reaction factor K_s .

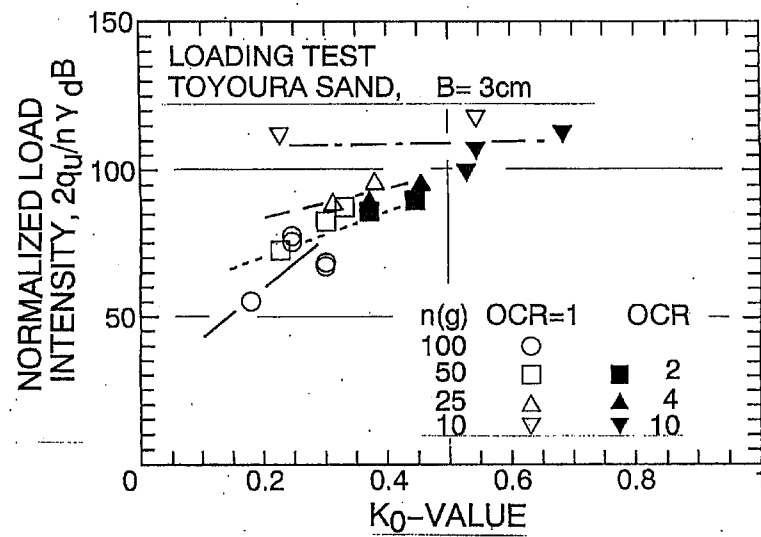


Figure 4.30: Influence of K_0 on normalized ultimate bearing capacity $2q_u/\gamma B$.

Chapter 5

Particle versus shear band

5.1 General

To investigate the formation of shear band in granular materials, the author developed a new type of apparatus, which is a kind of direct shear box equipped with a microscope. With this apparatus, the deformation and shear banding on the surface of granular materials of different seven types were directly observed. The development of shear band during shearing deformation was carefully observed and the thickness of shear band was measured on the photographs taken with the microscope.

The results showed that the thickness of shear bands was independent of the dimensions, relative density and the amount of the shearing specimen. The ratio of the shear band thickness to the mean grain size of granular materials ranged from about 10 to 25. Furthermore it was found the ratio of shear band thickness is dependent on the grain shape (angularity) and the void ratio extent ($e_{\max} - e_{\min}$).

5.2 Introduction

The formation and existence of shear bands in soil mass play important roles in many failure problems of various types in soil mechanics. In soil mass shear bands develop near the instance of shear failure. After the formation of shear band, deformation is concentrated or localized along the shear bands and, therefore, the strain that was defined macroscopically for the overall granular media has no mechanical meanings. The behavior associated with strain softening and/or progressive failure becomes evident (Oda and Furuto, 1982); this have been discussed

in several problems such as skin friction of friction piles (Uesugi *et al.*, 1988), stability of soil mass (Roscoe, 1970) and bearing capacities of foundation ground (Tatsuoka *et al.*, 1991 and 1992).

The thickness of shear band is an important information to find engineering solutions regarding strain softening and progressive failure phenomena. The Finite Element analysis on the deformation of soil mass with shear bands by Desai and Zaman (1984), Tatsuoka *et al.* (1991), Miura *et al.* (1991) and Yoshida *et al.* (1994) showed that the thickness of shear band affects significantly the failure behavior of soil masses. In the advanced analysis methods where the mechanical behavior of shear band is modeled with theoretical background, the numerical value of shear band thickness are required as a primary input parameter; see *e.g.* Mühlhaus and Vardoulakis (1987), Vardoulakis and Graf (1982 and 1985) and Pietruszczak and Niu (1993).

A direct observation on the movements of soil particles around shear bands is preferable in order to clarify the mechanism of shear bands. As the observation of the movement of actual soil particles is so difficult especially under confining stress, some physical model tests and numerical analysis methods where individual particles were numerically modeled, were utilized. Oda (1974) and Oda *et al.* (1985) used the aggregate of rod-like particles and Cundall and Strack (1979) introduced distinct element method. Also, the attempts for observing the shear bands in soils have been done by some researchers. Morgenstern and Tchalenko (1967) observed the shear bands in the clay sheared in direct shear boxes. Vardoulakis *et al.* (1985) and Yoshida *et al.* (1994) observed shear bands in sands sheared in bi-axial compression tests and plane strain compression tests, respectively, and measured the thickness of shear band.

The aim of this study is to clarify the formation and geometry of shear bands in granular materials regarding the effect of physical properties of materials. Shear bands in granular materials were observed in an open type direct shear boxes, which was equipped microscope for the accurate measurement of shear band thickness. The granular materials used consisted of sands with different grain size distributions and grain shapes, and some artificial materials such as glass beads and resin particles. The size of the shear zone and the thickness of shear band were examined and the effects of grain size and grain shape are discussed.

5.3 Material used

Seven kinds of granular materials, were prepared in this study: Aichi Noma Sand (AN), Siliceous sand (CA), Toyoura Sand (TO), Chopped grain (CG), Glass beads (GB), Light weight aggregate (LWA) and Rotund grain (RG). Detailed description of the material used was presented in Section 2.2.

5.4 Test apparatus

The author devised a new apparatus to measure shear bands formed in granular materials. The apparatus consisted of a split shear box, a stereo-graphic microscope and a close-up camera stand, as illustrated in **Photo 5.11(a, b)** and **Figs. 5.1, 5.2**. Three split shear boxes named Box I, Box II and Box III were prepared. The dimension of Box I was 240mm in width, 300mm in length and 100mm in depth. Box II was 120mm in width, 180mm in length and 50mm in depth. Box III had the dimensions of 300mm in length and 40mm in depth. The width of Box III could vary from 100mm to 200mm.

For the materials with large grain size such as CG, RG and LWA, Box I was mainly used. The surface of the split shear box was not covered in order to observe directly the shear bands in granular media without surface friction that impedes the movement of the granular particles. Therefore, no confining stress was applied to the materials; the effect of the confining stress on the formation of shear bands is out of the scope of this study. Although Yoshida *et al.* (1994 and 1994) observed the shear bands indirectly through the distortion of surface membrane in plane strain tests, the measurement of the geometry of shear band is questionable due to the concentrated tension of the membrane especially in the case of fine materials.

Box II was installed on a $x - y$ sliding table so that all the part of the material surface can be adjusted in the sight of the stereo-graphic microscope. The microscope, which was equipped with a 35mm camera and possesses an ability of variable elongation ratio from 10 to 67.5, provided clear and distortion free images of the movements of granular particles on films. The close-up camera stand consisted of a 35mm reflex camera and its stand, was used for overall observation of specimen surface. A ruler with a vernier scaling of 0.1mm indicated the amount of the shear box displacement, *i.e.* the amount of shearing.

Box III was used to observe deformation pattern on Toyoura Sand (TO). CCIP Method

introduced in Section 3.4 was applied to obtain distribution of maximum shear strain $\gamma_{\max} = \varepsilon_1 - \varepsilon_3$.

5.5 Test procedure

5.5.1 Specimen preparation

After air dried and weighed samples was poured into the shear boxes, the sample was compacted into prescribed relative density with a small handy vibrater having a 120 Hz frequency and 0.3mm stroke. During the compaction a flat plate with weight of 6.5 N was placed onto the specimen in order to make the surface leveled. A relative density of 70% was employed through the tests except a series for evaluating the effect of relative density. The depth of specimen d was measured at nine points on the specimen surface, and the dry density was calculated.

Seven black straight lines on the specimen were drawn with the particles died with blue black ink, and they were observed as marker lines. These died particles were identical to those comprising the specimen tested. Shear deformation of the specimens and the loci of the movement of granular particles were obtained with visual observation of the marker lines using either the close-up camera or the stereo-graphic microscope.

5.5.2 Shearing test specimen

The surface of the prepared sample was photographed by using the close-up camera. Microscopic photographs of the marker lines drawn on the specimens were also taken at this time. To make a reference of dimensions on microscopic photographs, a transparency with a fine grid was placed onto the specimen surface. The movable part of the split shear box was displaced until the prescribed amount of displacement. In every shearing stages, the deformation of the specimen was photographed using the close-up camera. Microscopic photographs of the marker lines were also taken when shear bands developed.

5.5.3 Test program

All the specimens were sheared and the deformation of specimen was observed. First the influence of the depth of specimen d was examined on the specimens of glass beads, GB-125-

105 and GB-990-710. The influence of the relative density D_r was examined on the specimens of natural Toyoura Sand (TO-300-75). Based on all the test results the influence of physical properties of materials, such as grain size and grain shape, on shear band thickness and shear zone width were discussed.

5.6 Deformation pattern and formation of shear band

Shown in **Photo 5.12(a-e)** is the observed deformation pattern on a specimen of CA-300-250 as a typical test result. The specimen with depth d of 40mm and relative density D_r of 70%, was sheared in Box II and was photographed at the amount of the shear box displacement s of 0, 5, 10, 20 and 30mm.

At the beginning of shearing, band-shaped discontinuity in which shear deformation concentrated, started to develop from both ends of the center line of the shear box, *i.e.* the edge of two parts of split shear box. This discontinuity was termed 'shear band'. At an early stage of shearing, the central part of the lens-shaped zone of specimen surface was sheared rather uniformly. This zone was termed 'shear zone'. Two shear bands grew across the specimen; this type of shear band is indicated in **Photo 5.12(b, c)** as shear band S_a . As the shearing proceeded, shear bands S_a stopped growing and another shear bands S_b started developing from both ends of the center line of the shear box, as shown in **Photo 5.12(d)**. Shear bands S_b grew longer than shear bands S_a and sometimes reached the opposite side of the shear box; the development of shear bands S_b was irregular, and more than two shear bands S_b appeared in certain cases. When the shear box displacement became large, another type of shear bands, which is designated as S_c , developed and they grew linearly across shear box and finally connected each other on the center line as shown in **Photo 5.12(e)**.

Figures 5.3-5.5 show transition of distributions of maximum shear strain $\gamma_{\max}$ obtained by CCIP Method. Three specimens made of Toyoura sand (TO, $e = 0.85$) with widths of 100mm, 150mm and 200mm, were sheared in Box III and photographed each 2mm of shear box displacements s . Photographs were digitized into image data of which resolution was 0.23mm/pixel. Displacements on a grid with a spacing of about 4.6mm (20pixels) were analyzed by CCIP Method and maximum shear strain $\gamma_{\max}$ was calculated.

Although spacing of the grid was too large to distinguish a shear band from others and the deformation was averaged within the 4.6mm square mesh, overall pattern of deformation can be

discussed by using **Figs. 5.3-5.5**. These transition of deformation patterns are similar to that observed from re-molded kaoline clay reported by Morgenstern and Tchalenko (1967). Also Kobayashi *et al.* (1999) showed similar results by FE simulation. According to **Figs. 5.3-5.5**, shear bands appear from both edges of the shear box, and gradually grow slant-wise toward the inside of specimen. Shear band S_a mentioned above appears at $s = 2.0mm$ (**Fig. 5.3(a)**, **Fig. 5.4(a)** and **Fig. 5.5(a)**). Shear band S_b starts to grow around $s = 6mm$ (**Fig. 5.3(c)**, **Fig. 5.4(c)** and **Fig. 5.5(c)**) and a region having a diamond shape appears. Then shear band S_c , clearly presented in **Figs. 5.5(f)-5.5(i)**, is generated around the center line of the shear box. On the other hand shear zone grows up to $s = 10mm$ then the thickness of shear zone remains fairly constant. In addition, the dimensions of shear zones were fairly identical among three specimens with different widths. Kinematic restriction of boundaries seems tolerant in the presented cases.

5.7 Definition of the thickness of shear band

The definition of the thickness of shear zone and share band is shown in **Figs. 5.3**; the width of shear zone is designated as D_d , and the thickness of shear band as D_b . The thickness of the shear zone D_d was measured in the direction perpendicular to the center line of shear box. The orientation of shear band was determined on micro-photographs taken at each stage of shearing. Then, the thickness of the shear band D_b was measured with vernier calipers in the direction perpendicular to the orientation of shear band. The micro-photographs clearly indicated the displacement of the particles that constituted the borders of shear bands.

The micro-photographs clearly indicated the displacement of the grains that constituted the borders of shear bands. Typical examples of micro-photographs are shown in **Photo 5.13(a, b)** and **Photo 5.14(a, b)**. Circles in **Photo 5.13(a)** and **Photo 5.14(a)** show the position where **Photo 5.13(b)** and **Photo 5.14(b)** were photographed. Micro-photographs **Photo 5.13(b)** and **Photo 5.14(b)** were analyzed by using CCIP method to show how correctly we can determine the thickness of shear band D_b on micro-photographs with vernier calipers. **Figure 5.6** shows displacement along the line denoted by a-a in **Photo 5.13(b)** obtained by CCIP method, together with the value of D_b obtained with vernier calipers. Although deformation in the shear band was too large to determine correct value of displacement within the shear band, similar results were obtained in both methods.

5.8 Test results and discussion

5.8.1 General remarks

Shown in **Figs. 5.7 and 5.8** are width of shear zone D_d and the thickness of shear band D_b observed on the specimens of glass beads. As shown in **Fig. 5.7** shear zone developed gradually at early stage of shearing. The development terminated at small shearing where the shear box displacement s is less than 20mm. At the same time the shear bands S_b stopped developing and the shear band S_c started developing. In **Fig. 5.8** the averaged thickness of the shear bands which were actively developing with shearing, is plotted against the shear box displacement s . The thickness of shear band is fairly constant during all shearing stages. Especially, it should be noted that the shear band thickness of S_c was constant even with large shear box displacement.

5.8.2 Influences of depth of specimen

In order to examine the influence of the depth of test specimen on the shear deformation of granular materials, the specimens of glass beads, GB-990-710 and GB-125-105, with variable depth d of from 20 to 90mm were sheared in Shear Box I.

Shown in **Figs. 5.9 and 5.11** are the plots of measured width of shear zone D_d and the thickness of shear band D_b against the relative depth of specimen with respect to mean grain size d/D_{50} . As shown in **Fig. 5.9**, the width of shear zone D_d is much dependent on the specimen depth d . The width of shear zone D_d increases with increasing specimen depth d . **Figure 5.10** shows all of normalized width of shear zone D_d/D_{50} against the relative depth of specimen with respect to mean grain size d/D_{50} . Plots except for GB series concentrates along a straight fitting line. On the other hand, the thickness of shear band D_b is independent of the specimen depth d as shown in **Fig. 5.11**. It can be said that the thickness of shear band is determined by the physical properties of granular materials, while the geometry of the shear zone is affected by the boundary condition such as the dimension of test specimen as well as physical properties of granular material.

5.8.3 Influence of relative density

In order to clarify the effect of the relative density D_r at initial condition before shearing on the width of shear zone D_d and the thickness of shear band D_b , the test specimens of natural Toyoura Sand (TO-300-75) with different relative density were sheared in Box II. The relative density of the specimen was varied from 35 to 88%; the depth of the specimen was kept constant of $d = 40\text{mm}$.

Shown in **Figs. 5.12** is the plot of the width of shear zone D_d against relative density D_r . This plot shows the clear tendency that the thickness of shear zone D_d increases with increasing relative density D_r . Shown in **Fig. 5.13** is the plot of the thickness of shear band D_b against the relative density D_r . As shown in this figure the thickness of shear band D_b is independent of relative density.

The change in soil structure and density concentrates in shear band due to localization of strain and dilatancy behavior. The behavior of shear banding is, therefore, expected to be influenced by the density. It is interesting that, nevertheless, the thickness of shear band D_b is independent of the initial condition such as relative density D_r , and only the location of shear banding is dependent on D_r .

5.8.4 Influence of mean grain size and grain shape

Shown in **Fig. 5.14** is the relationship of the shear zone width D_d with mean grain size D_{50} for all the granular materials used. All the test results on the specimens with depth d of 40mm and relative density D_r of 70% are plotted in this figure. There is a clear tendency that the width of shear zone is rather independent on grain size D_{50} .

Shown in **Fig. 5.15** is the relationship of shear bands thickness D_b with mean grain size D_{50} for all the granular materials used. All the test results on the specimens with various depth d and various relative density D_r are plotted in this figure. It should be remembered that the thickness of shear band is independent on the dimension and relative density of specimen. There is a tendency that shear band thickness D_b is strongly dependent on mean grain size D_{50} and is almost proportional to D_{50} even with some scatter. The ratio of D_b to D_{50} ranges from 12 to 28. In this figure the results on the graded materials AN-Sand and CA-850-75, whose uniformity coefficient U_c is large of 2.12 and 2.22. It will be allowed to evaluate the thickness of shear band D_b uniquely from its ratio to mean grain size (D_b/D_{50}) for granular

materials.

The ratio of shear band thickness to mean grain size (D_b/D_{50}) is plotted against angularity A_{2D} in **Fig. 5.16(a)**. As shown in this figure, there is clear negative correlation between (D_b/D_{50}) and A_{2D} . The ratio (D_b/D_{50}) becomes small for the materials composed with angular particles. On the other hand, shown in **Fig. 5.16(b)** is the plot of the ratio of shear band thickness to mean grain size (D_b/D_{50}) against void ratio extent ($e_{\max} - e_{\min}$). This figure clearly shows the negative correlation between (D_b/D_{50}) and ($e_{\max} - e_{\min}$); the shear band thickness is relatively small for the granular materials with large void ratio extent ($e_{\max} - e_{\min}$).

As calculated by Yoshida *et al.* (1994) the shear stress level reduces within shear band with an increase in shear strain. This mechanism in shear band is responsible for the various failure phenomena in soil mass associated with stress softening and progressive failure. The reduction of shear stress level must be attributed to the change in structure formed with constituent particles, which includes the decrease in density induced by dilatancy within shear band. Based on the positive correlation between angularity A_{2D} and void ratio extent ($e_{\max} - e_{\min}$) which imply the degree of possible change in the structure (Maeda; 1994 and Miura *et. al.*; 1998), the change in structure with in shear band will occur more in angular materials. Consequently it is considered that the reduction of shear stress level will occur inside the shear band of more angular materials in more concentrated manner. It can be said, therefore, that the shear band becomes thinner in more angular granular materials due to the more concentrated change in structure in shear band and the following significant reduction in shear stress level.

The correlation coefficient of (D_b/D_{50}) with void ratio extent ($e_{\max} - e_{\min}$) and angularity A_{2D} are comparable: -0.60 and -0.69, respectively. It can be said, however, that for the estimation of the shear band thickness D_b , void ratio extent ($e_{\max} - e_{\min}$) is more reliable as an index parameter, because $e_{\max}$ and $e_{\min}$ can be obtained in the laboratory test for determining the relative density D_r of samples.

5.9 Summary

The shear deformation and the shear banding in granular materials were observed in direct shear boxes. By means of the stereo-graphic microscope, the width of shear zone and the thickness of shear band were measured for the several granular materials used in this study. The findings about the influencing factors of shear zone and shear band can be summarized

as follows:

1. The width of shear zone is influenced by testing condition of direct shear test as well as by physical properties of granular materials. The size of shear zone on the specimen surface was determined by the dimension and relative density of the specimen. The size of shear zone is, however, rather independent of the size and shape of constituent particles.
2. The thickness of shear band is independent of the testing condition. The thickness in different three types of shear bands in lens-shape pattern, and it was constant independently of the amount of shearing specimen. The initial relative density of the specimen was not an influence factor on the thickness. The ratio of the thickness of shear band to mean grain size (D_b/D_{50}) ranges for 18 to 28; the ratio is determined as a function of angularity of constituent particles A_{2D} or void ratio extent ($e_{\max} - e_{\min}$); the ratio (D_b/D_{50}) decreases with an increase of A_{2D} and ($e_{\max} - e_{\min}$).

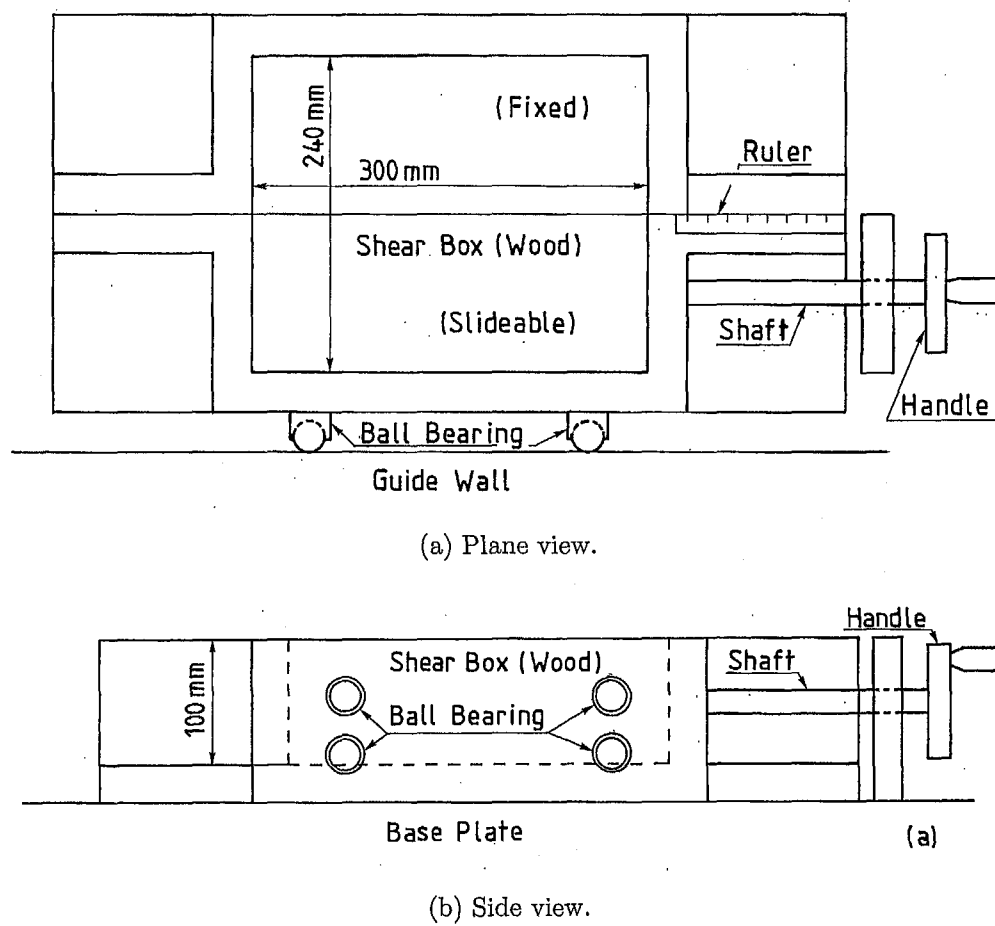
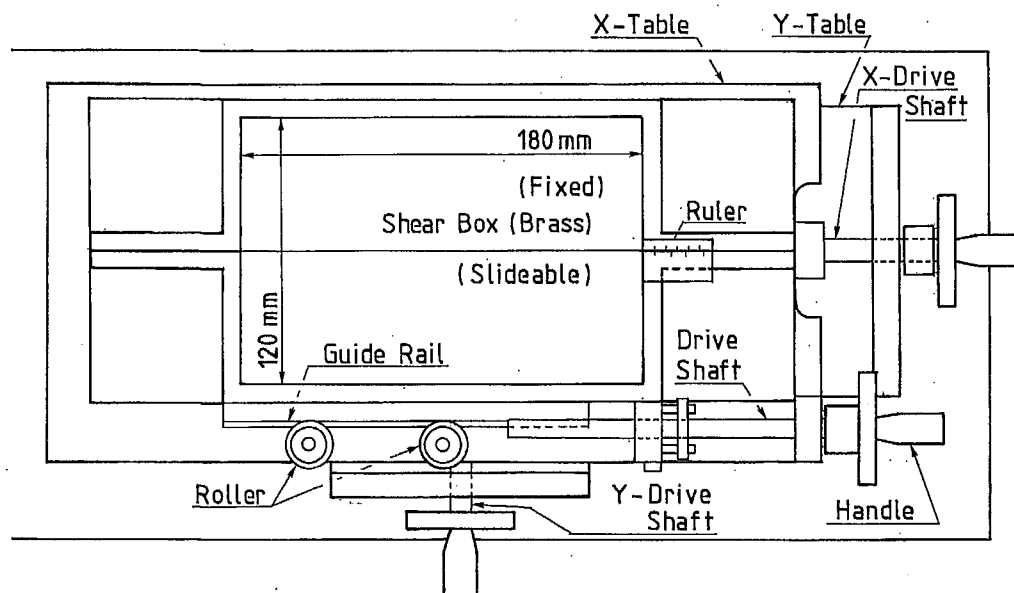
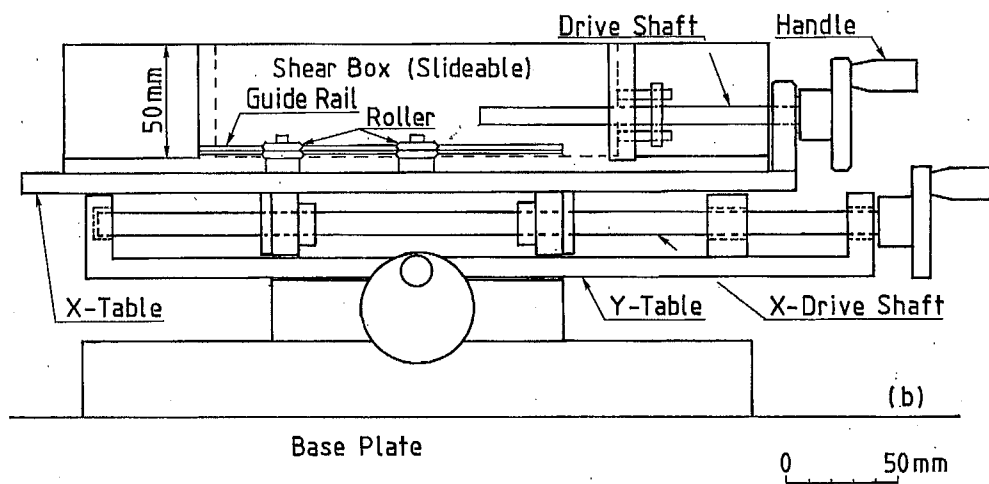


Figure 5.1: Test apparatus for observing shear bands with Box I.

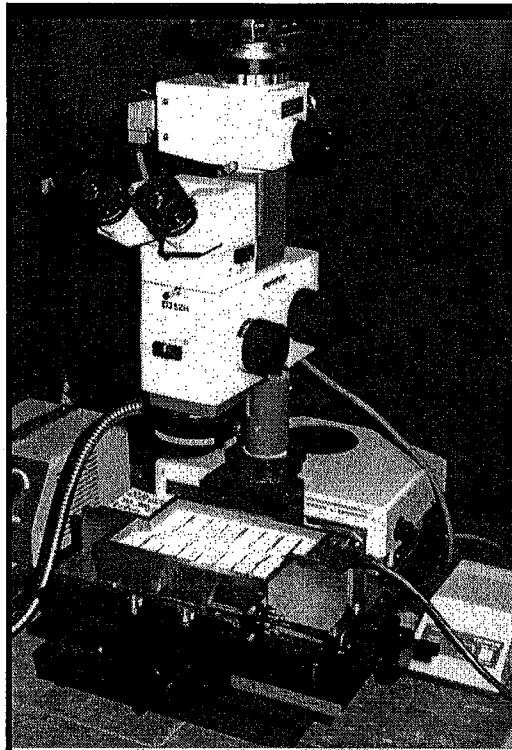


(a) Plane view.

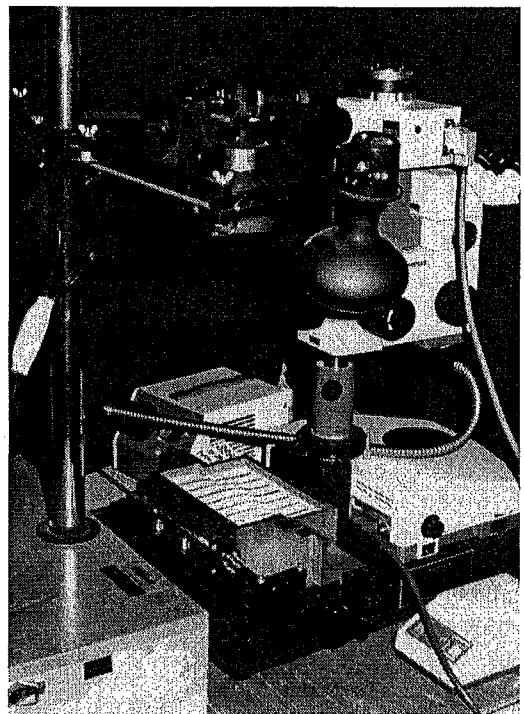


(b) Side view.

Figure 5.2: Test apparatus for observing shear bands with Box I.



(a) With a microscope.



(b) With a close-up camera.

Photo 5.11: Test apparatus for observing shear bands with Box II.

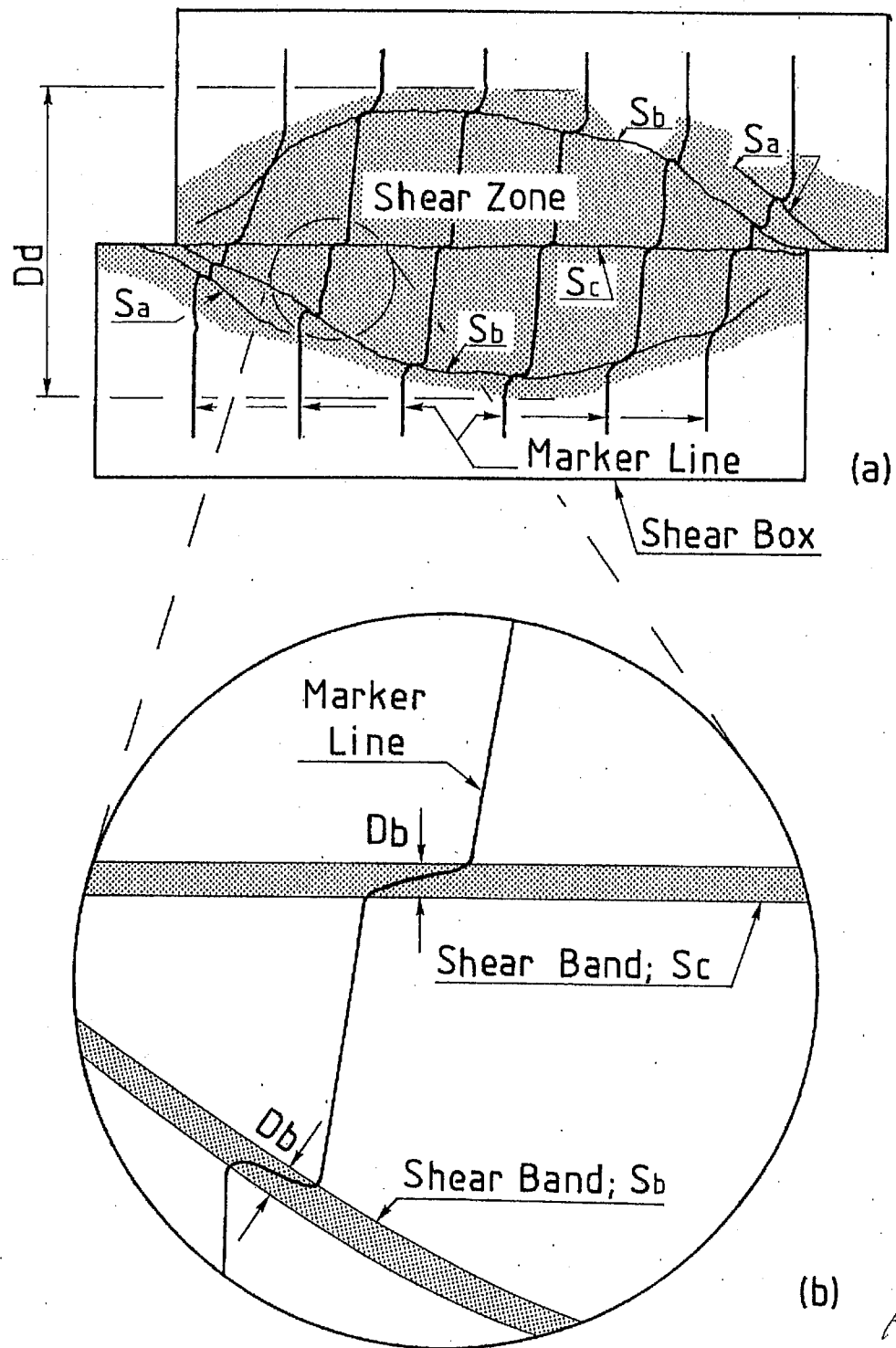
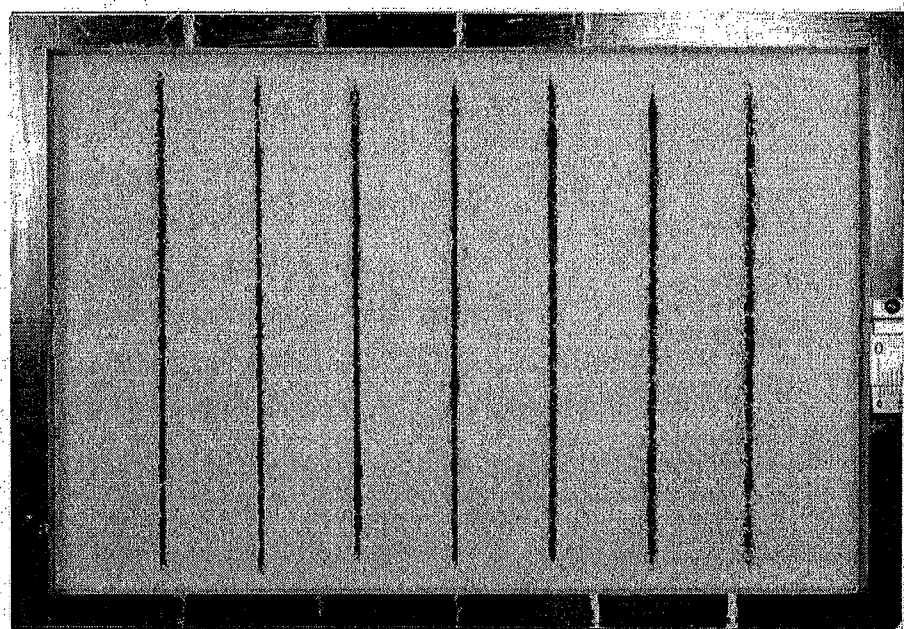
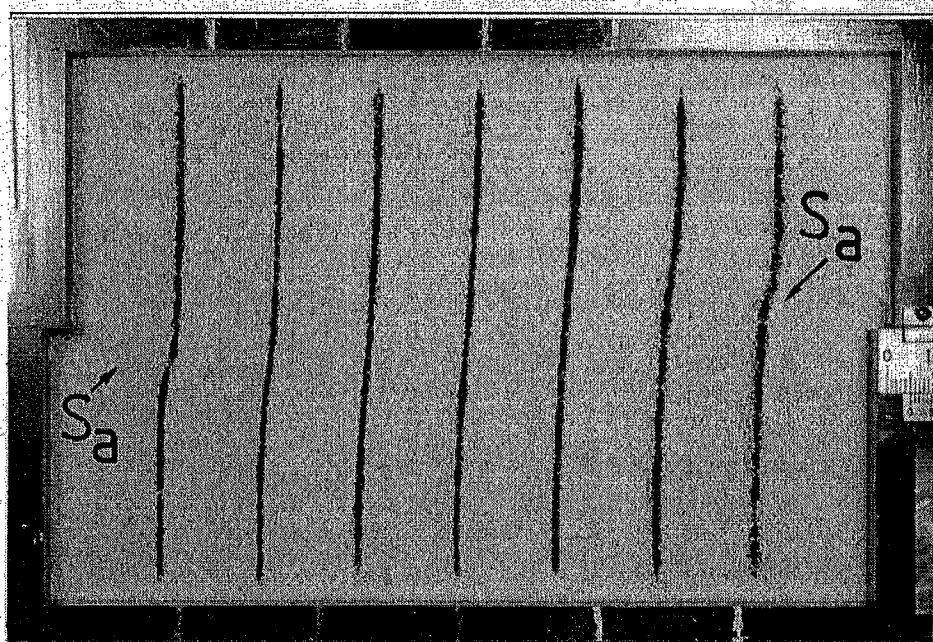


Figure 5.3: Illustration of shear deformation and definition of thickness of shear zone D_d and shear bands D_b .

 $s=0\text{mm}$

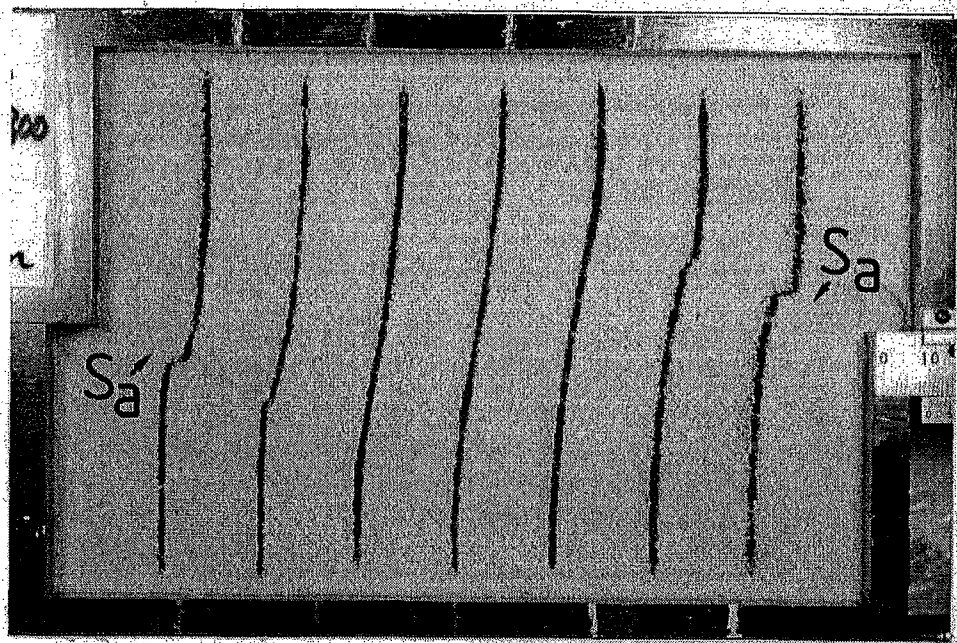
(a)

(a) $s = 0\text{mm}$  $s=5\text{mm}$

(b)

(b) $s = 5.0\text{mm}$

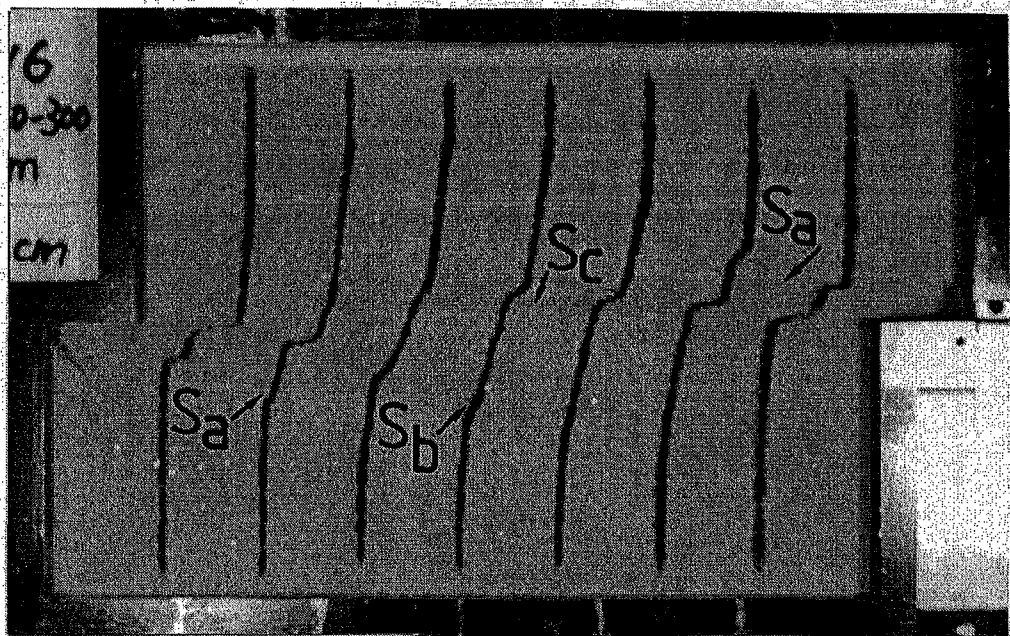
Photo 5.12: Observed shear deformation of the specimen of material CA-300-250 with depth of $d = 40\text{mm}$ and relative density of $D_r = 70\%$.



$s=10\text{mm}$

(c)

(c) $s = 10.0\text{mm}$

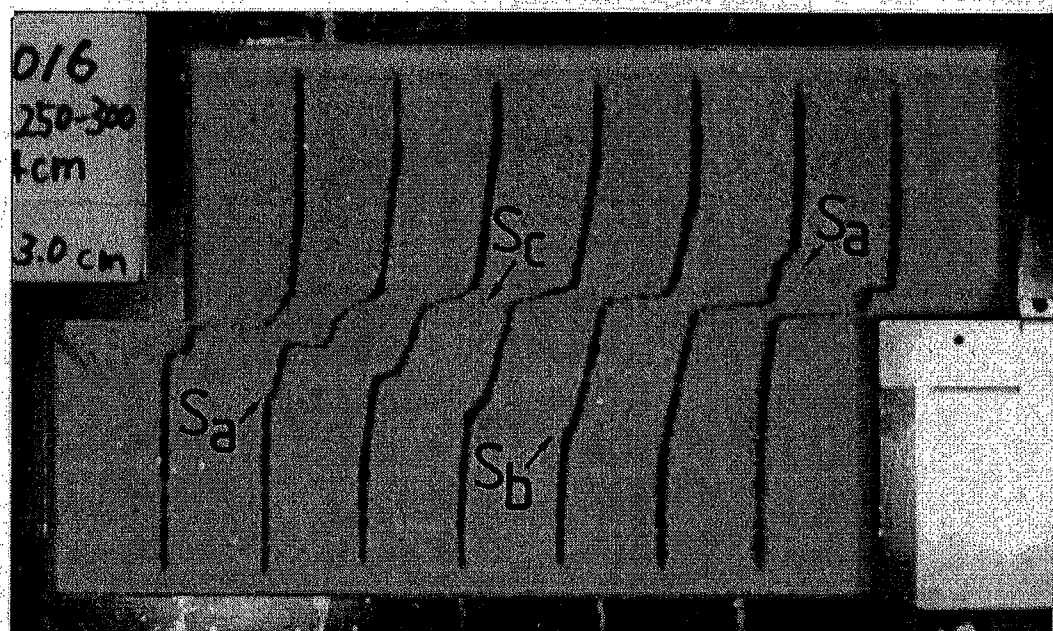


$s=20\text{mm}$

(d)

(d) $s = 20.0\text{mm}$

Photo 5.12: Observed shear deformation of the specimen of material CA-300-250 with depth of $d = 40\text{mm}$ and relative density of $D_r = 70\%$.



$s=30\text{mm}$

(e)

(e) $s = 30.0\text{mm}$

Photo 5.12: Observed shear deformation of the specimen of material CA-300-250 with depth of $d = 40\text{mm}$ and relative density of $D_r = 70\%$.

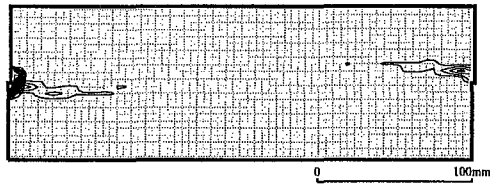
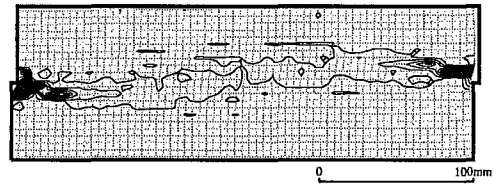
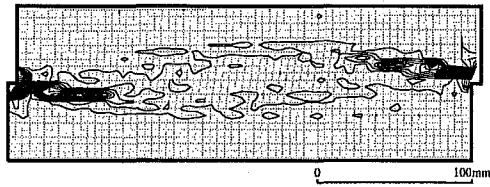
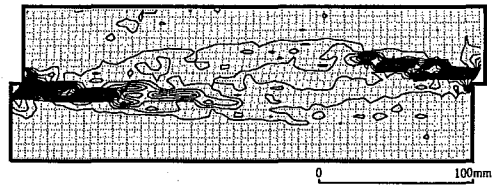
(a) $s=2\text{mm}$ (b) $s=4\text{mm}$ (c) $s=6\text{mm}$ (d) $s=8\text{mm}$

Figure 5.3: Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=100mm, 1div.=10%).

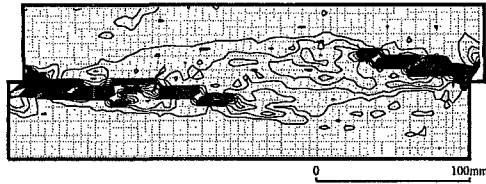
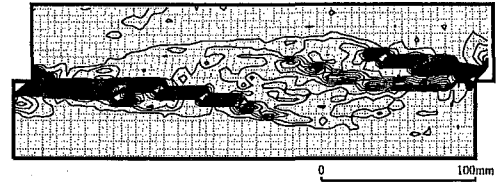
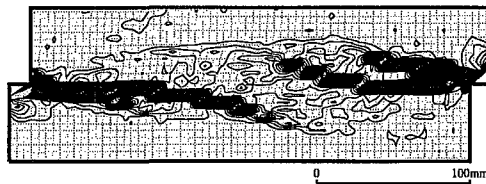
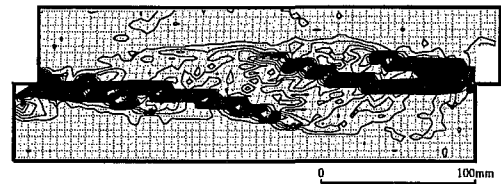
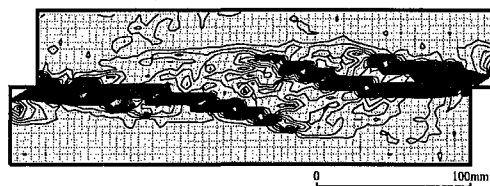
(e) $s=10\text{mm}$ (f) $s=12\text{mm}$ (g) $s=14\text{mm}$ (h) $s=16\text{mm}$ (i) $s=18\text{mm}$

Figure 5.3: Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=100mm, 1div.=10%).

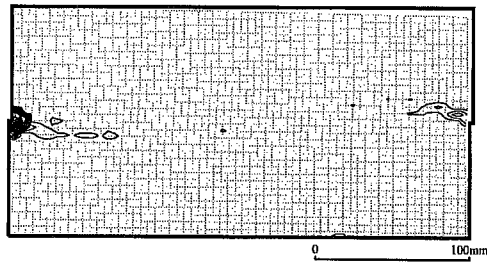
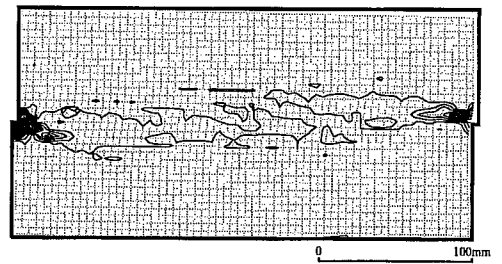
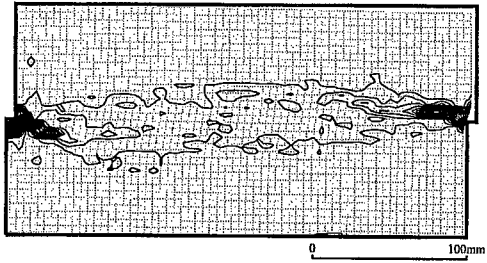
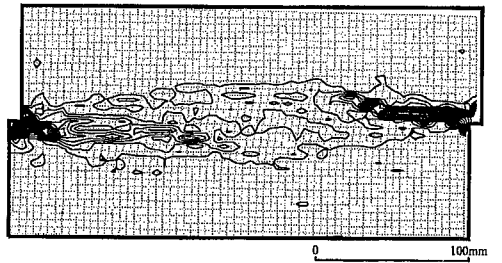
(a) $s=2\text{mm}$ (b) $s=4\text{mm}$ (c) $s=6\text{mm}$ (d) $s=8\text{mm}$

Figure 5.4: Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=150mm, 1div.=10%).

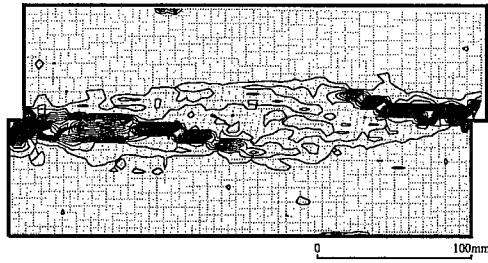
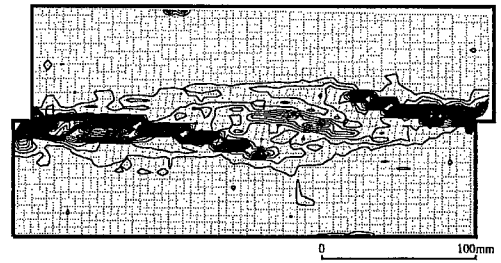
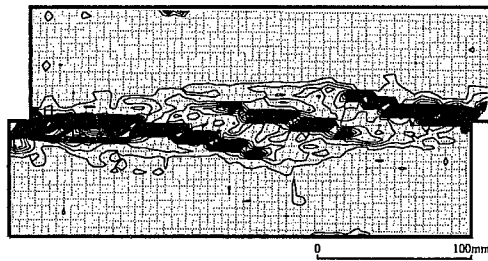
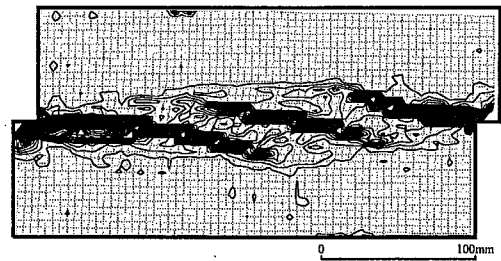
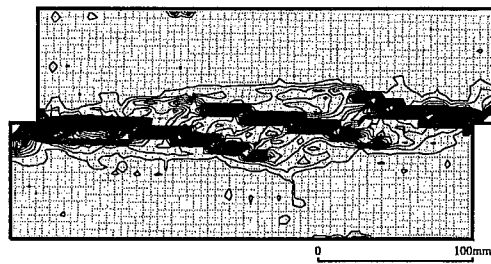
(e) $s=10\text{mm}$ (f) $s=12\text{mm}$ (g) $s=14\text{mm}$ (h) $s=16\text{mm}$ (i) $s=18\text{mm}$

Figure 5.4: Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=150mm, 1div.=10%).

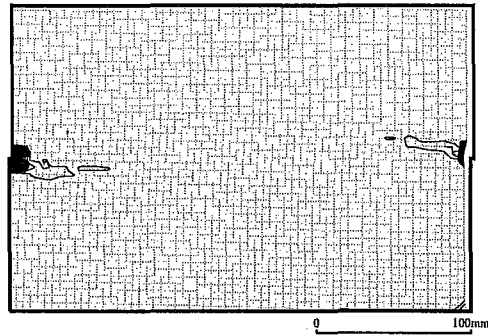
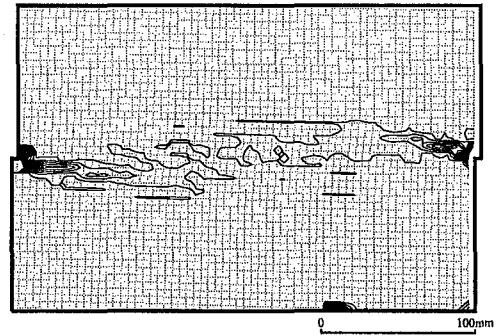
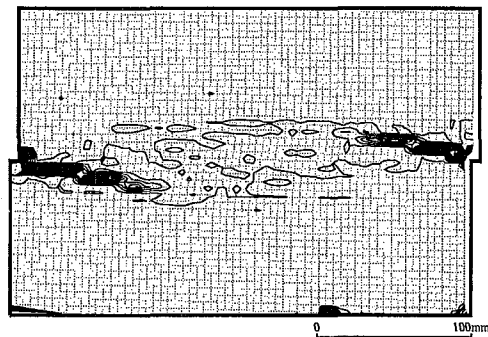
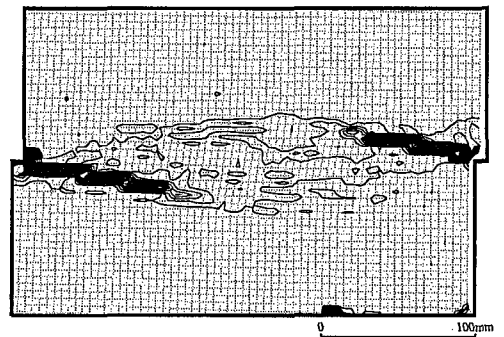
(a) $s=2\text{mm}$ (b) $s=4\text{mm}$ (c) $s=6\text{mm}$ (d) $s=8\text{mm}$

Figure 5.5: Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=200mm, 1div.=10%).

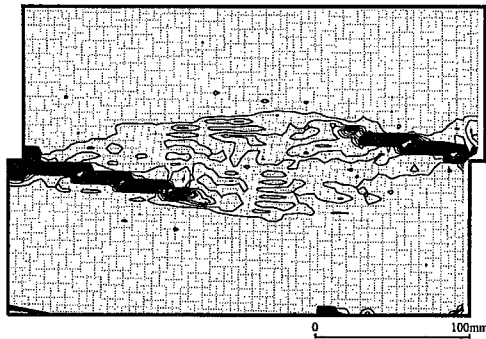
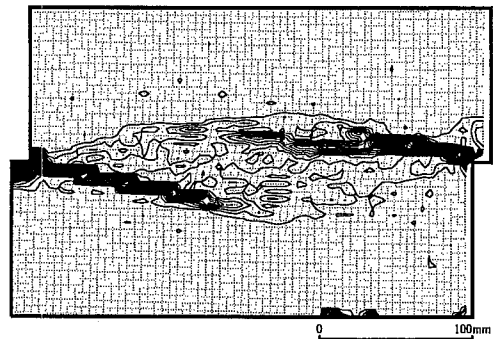
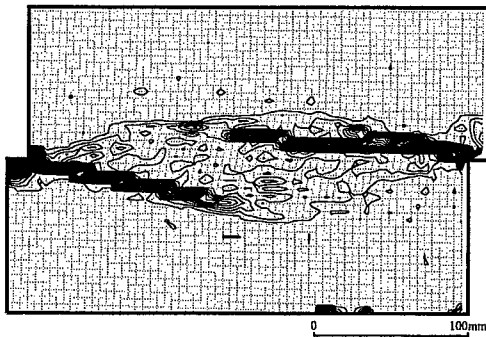
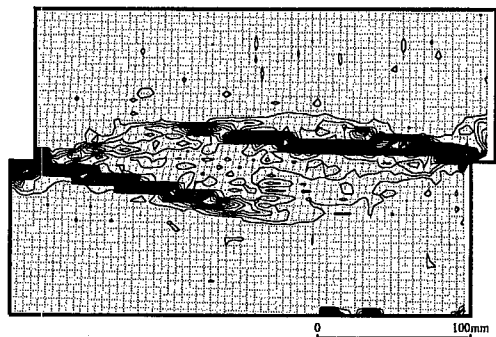
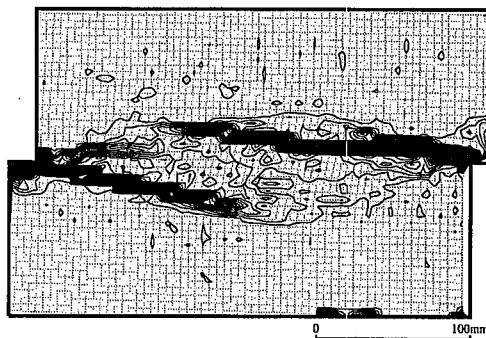
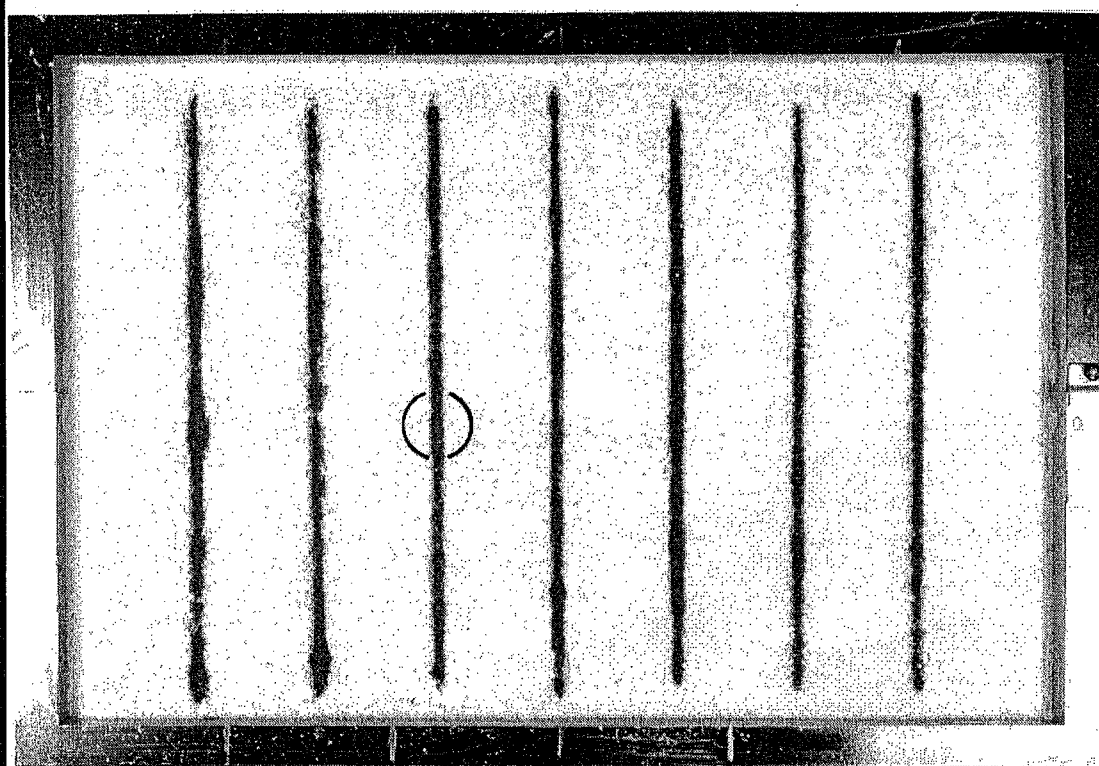
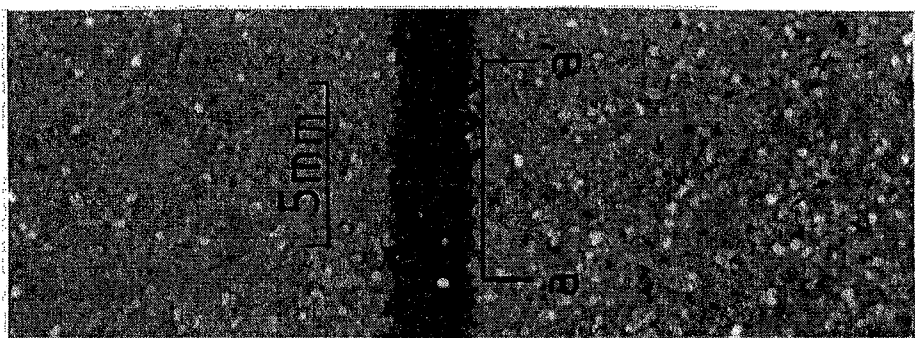
(e) $s=10\text{mm}$ (f) $s=12\text{mm}$ (g) $s=14\text{mm}$ (h) $s=16\text{mm}$ (i) $s=18\text{mm}$

Figure 5.5: Transition of distribution of shear strain γ_{max} in Box III shown by contour lines (width=200mm, 1div.=10%).



(a)



(b)

Photo 5.13: Example of photographs to measure the thickness of shear band D_b , taken for a specimen made of CA-177-150 with depth of $d=40\text{mm}$ and $s=0\text{mm}$; (a) close-up photograph, (b) microscopic photograph.

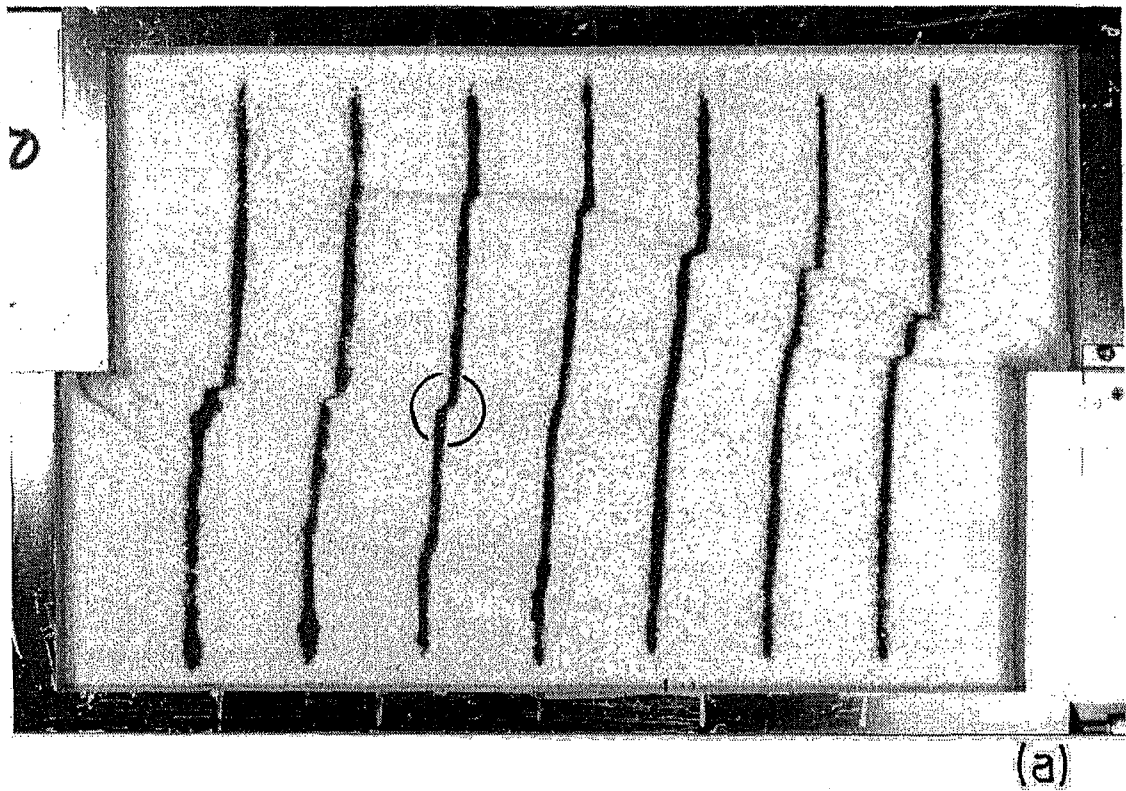


Photo 5.14: Example of photographs to measure the thickness of shear band D_b taken for a specimen made of CA-177-150 with depth of $d=40mm$ and $s = 10mm$; (a)close-up photograph, (b)microscopic photograph.

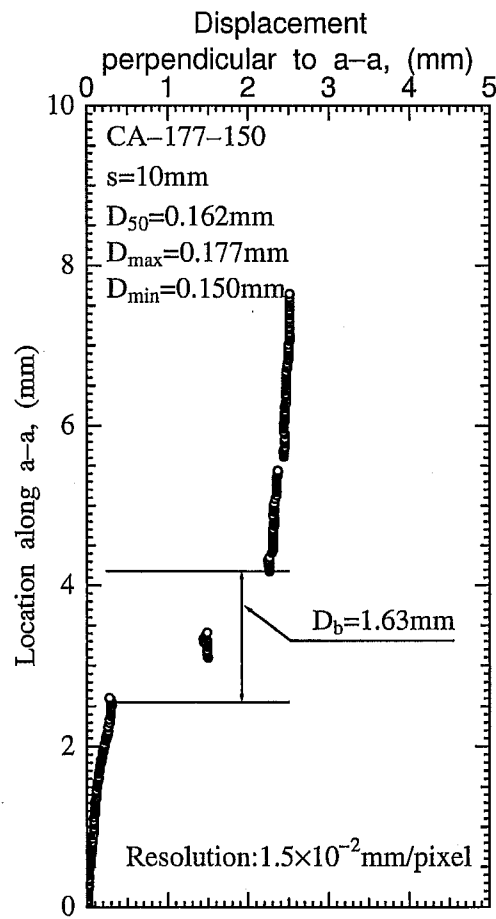


Figure 5.6: Displacement around a shear band obtained by CCIP method.

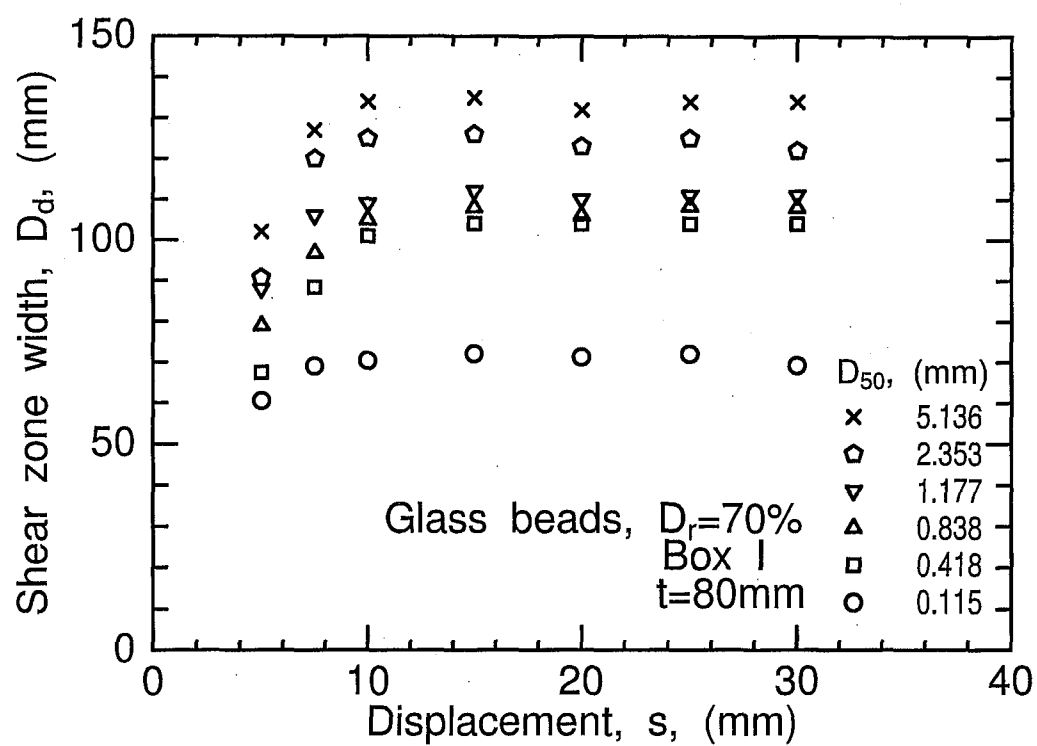


Figure 5.7: Width of shear zone D_d observed on the specimens of glass beads with depth $d = 80\text{mm}$ and relative density $D_r = 70\%$.

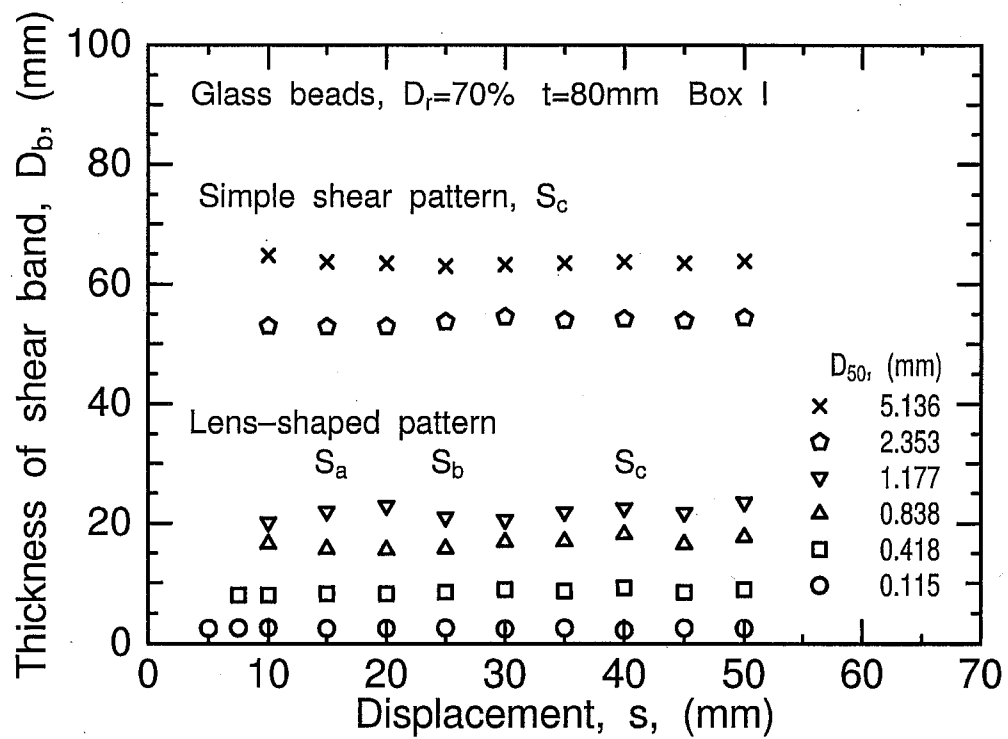


Figure 5.8: Thickness of shear band D_b observed on the specimens of glass beads with depth $d = 80\text{mm}$ and relative density $D_r = 70\%$.

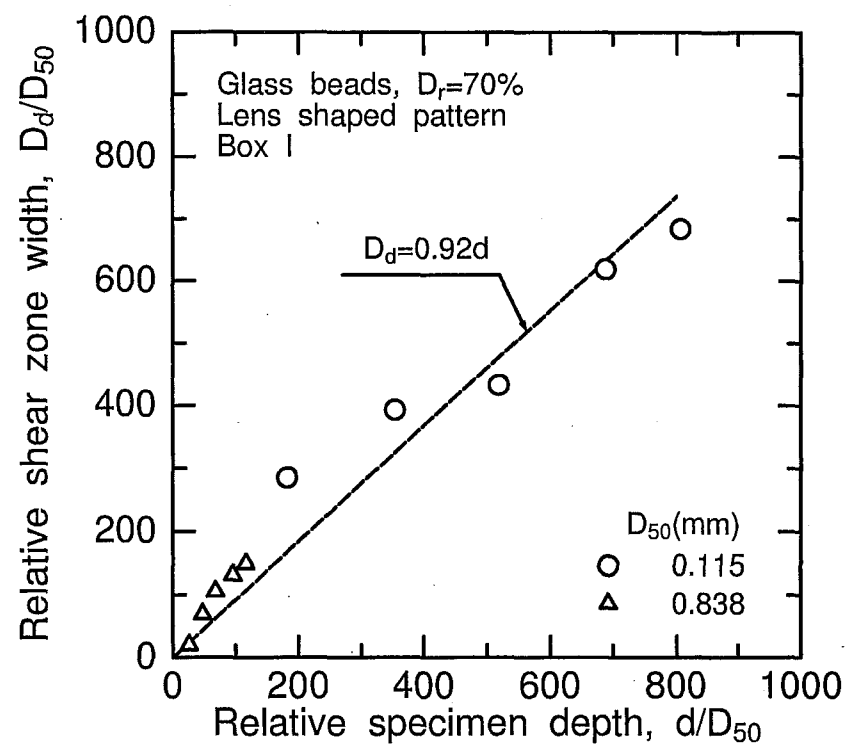


Figure 5.9: Influence of the depth of specimen d on the thickness shear zone D_d .

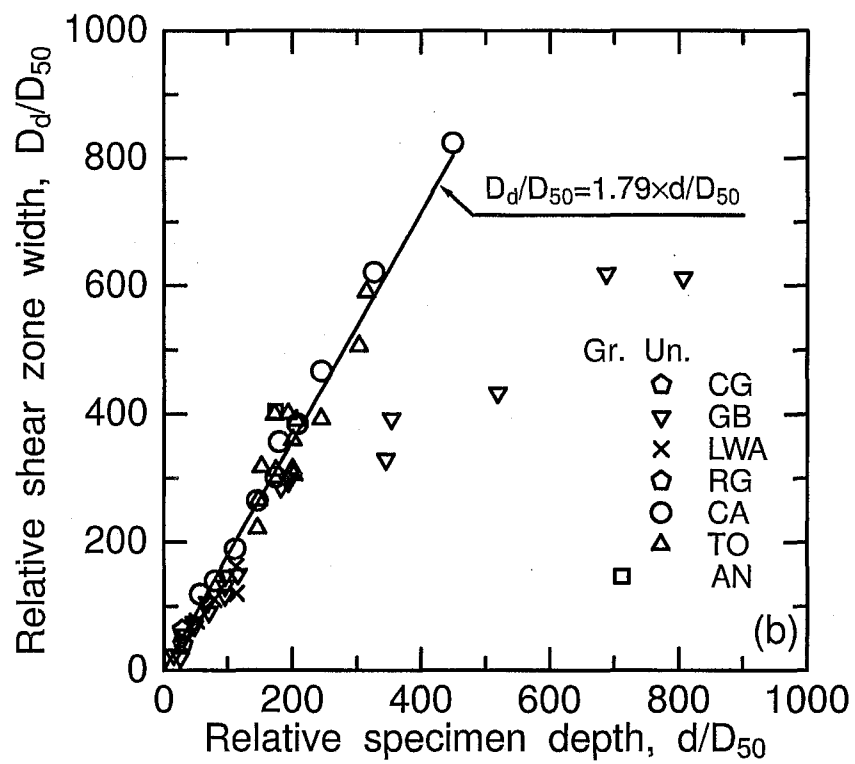


Figure 5.10: Relationship between relative depth of specimen d/D_{50} and thickness shear zone D_d .

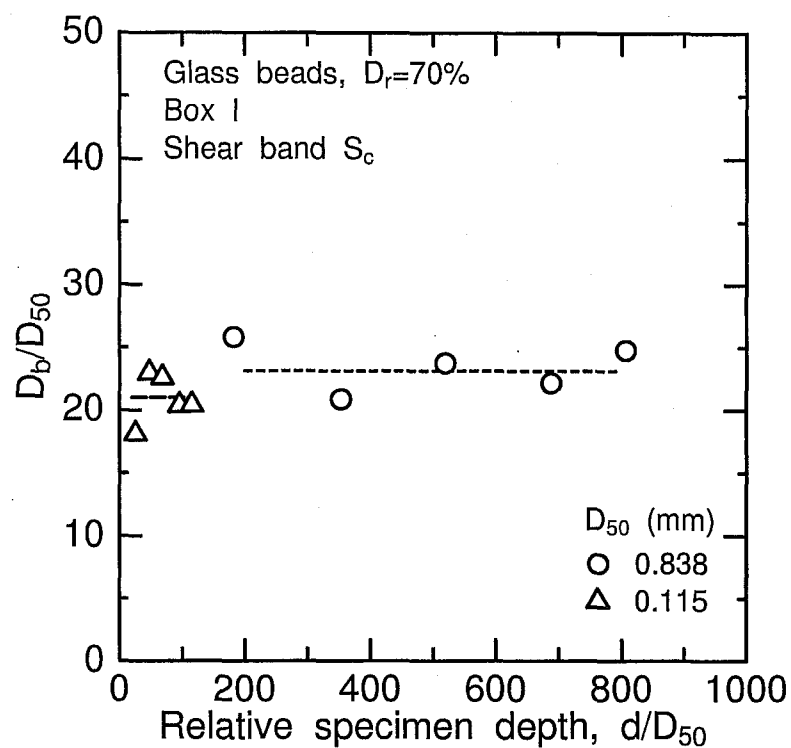


Figure 5.11: Influence of the depth of test specimen d on the thickness of shear band D_b .

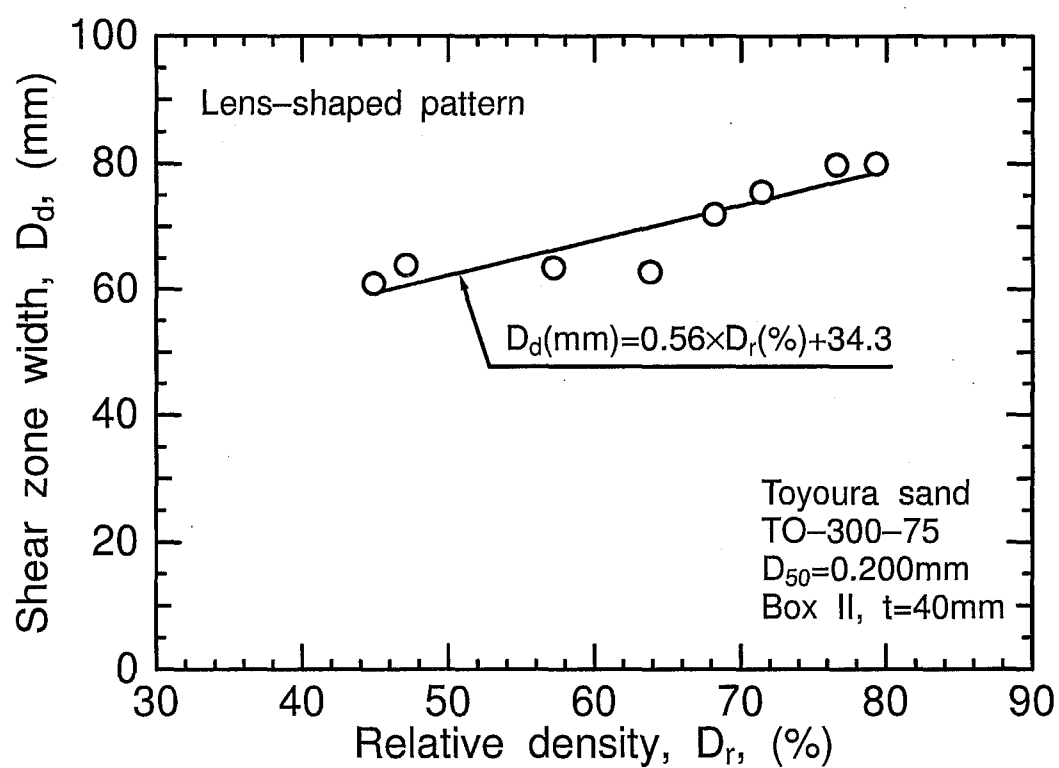


Figure 5.12: Influence of relative density D_r on the width of shear zone D_d .

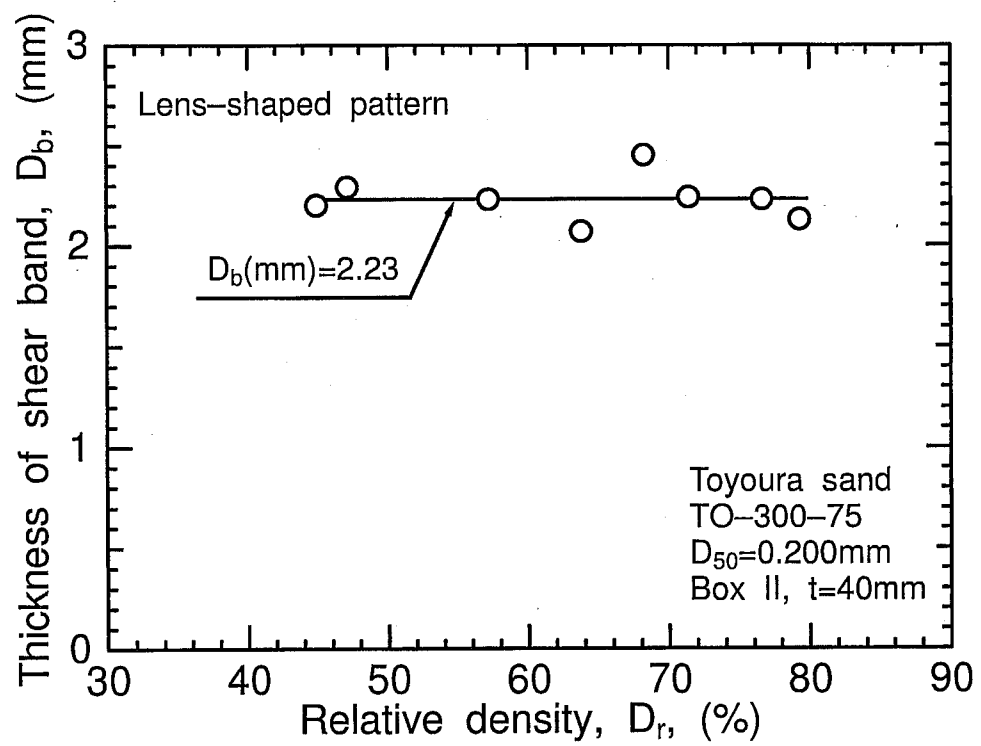


Figure 5.13: Influence of relative density D_r on the thickness of shear band D_b .

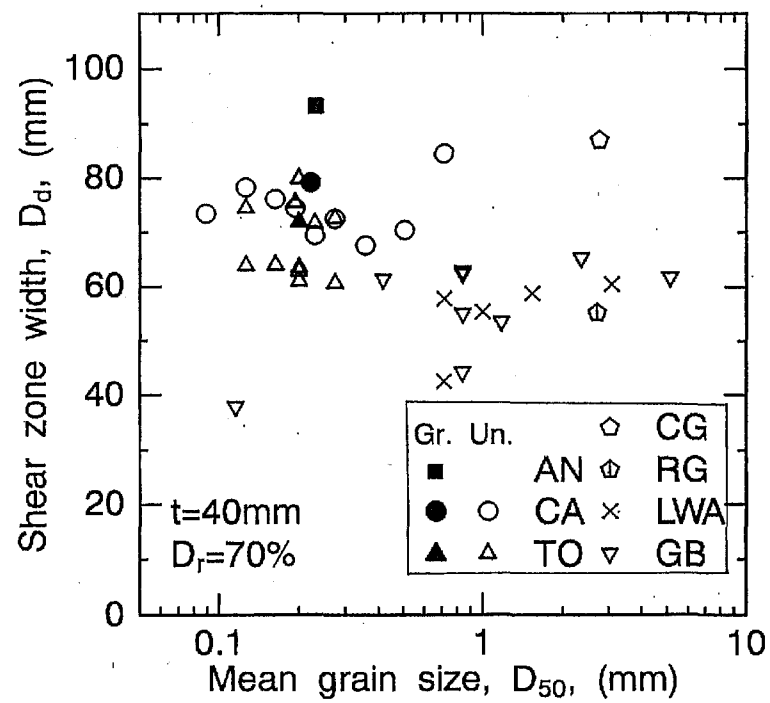


Figure 5.14: Relationship between width of shear zone D_d and mean grain size D_{50} ; specimen depth d of 40mm and relative density D_r of 70%.

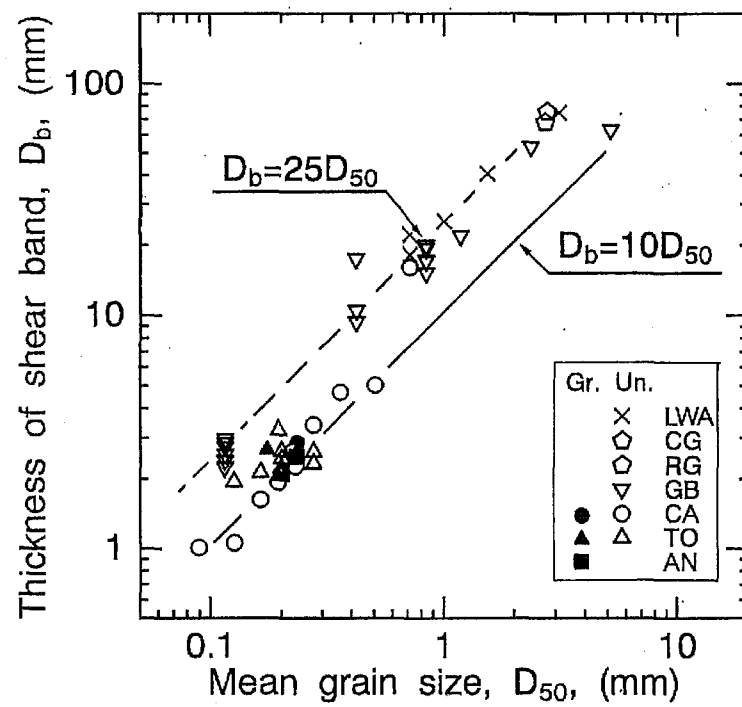


Figure 5.15: Positive correlation of the thickness of shear band D_b with mean grain size D_{50} .

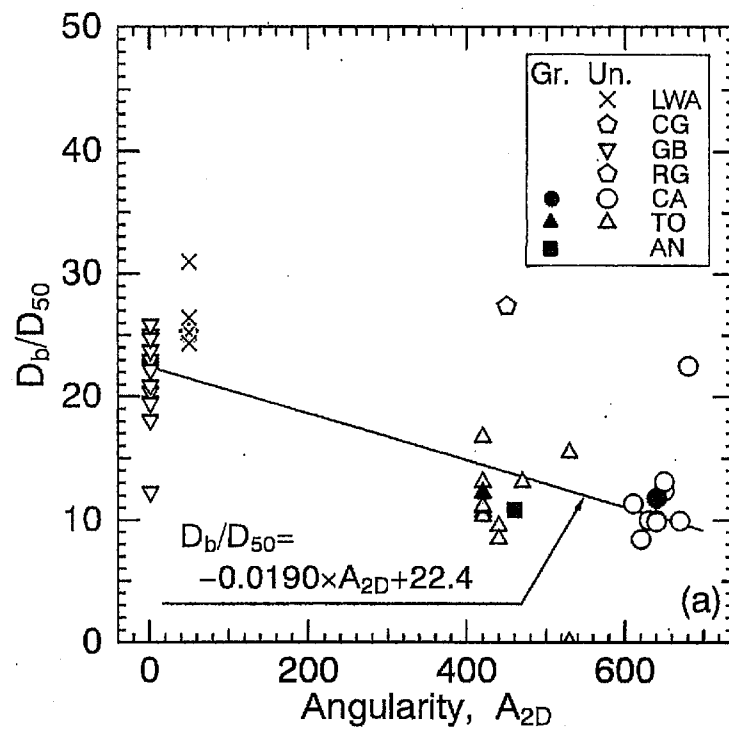
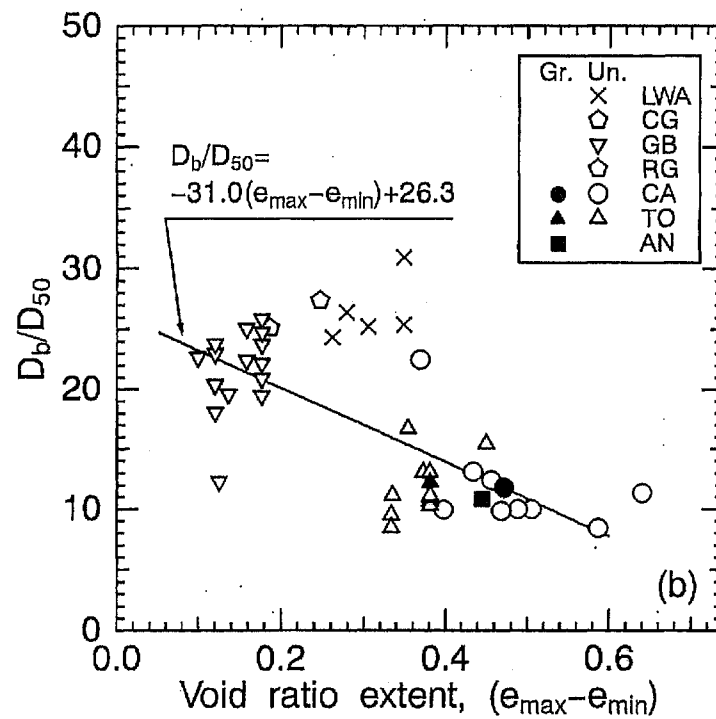
(a) Angularity A_{2D} (b) $(e_{\max} - e_{\min})$

Figure 5.16: Dependency of shear band thickness ratio (D_b/D_{50}) on; (a) angularity A_{2D} , (b) $(e_{\max} - e_{\min})$.

Chapter 6

Size effects on bearing capacity

6.1 General

Figure 6.1 shows the logarithmic relationships between normalized ultimate bearing capacity $2q_u/\gamma B$ and normalized footing breadth $\gamma B/\sigma_a$ of surface footings, which were obtained from test series conducted recently by the author and a number of other researchers. Test conditions and authorities of the test results quoted in Fig. 6.1 are given in Table 6.1.

According to Terzaghi's bearing capacity formula, for surface footings on cohesionless foundation ground, $2q_u/\gamma B$ should be equal to bearing capacity factor N_γ ; the test series shown in Fig. 6.1(a) are all for cohesionless materials. The normalization for the breadth of footing $\gamma B/\sigma_a$ is effective in demonstrating size effects on bearing capacity q_u , based on the results from the model tests including centrifuge tests, where γ increases with centrifugal acceleration. While N_γ is expected to be constant independently of the variation of B from the original meaning of Terzaghi's bearing capacity formula (Eq. (1.1)), the plots of $\log(2q_u/\gamma B)$ against $\log(\gamma B/\sigma_a)$ with negative correlation indicate a notable reduction in N_γ with increasing $\gamma B/\sigma_a$; this reduction is referred to as size effects (see e. g. De Beer; 1963, 1965).

The above mentioned confining stress dependency of the shear strength of ground material is one of the most important factors in controlling the size effect of surface footings; even for cohesionless materials, ϕ is not constant and reduces with an increase of confining stress. If a footing becomes larger, the location of the failure surface becomes deeper; the depth is approximately proportional to the breadth of footing B . Additionally, in an identical foundation ground made of non-cohesive soil, bearing capacity q_u could be approximately in

proportion to the value of γB , as long as the intensity of the confining stress dependency of shear strength is low. Therefore, it can be expected that the stress range to be interested in the foundation ground beneath the footing should be in proportion to the value of γB . Higher stress leads to a lower angle of internal friction ϕ , which yields a smaller bearing capacity factor N_γ . Consequently, N_γ does decrease with increasing γB ; this is a major mechanism of size effects related to stress level effect on shear strength.

The author conducted a series of centrifuge model loading tests to reappraise size effects in bearing capacity problems. The results of the loading tests will be presented in this chapter.

6.2 Procedure of loading test

A series of loading tests were conducted under both gravity and centrifugal acceleration field in order to reappraise size effects on bearing capacity of surface footings. **Table 6.2** shows the conditions of the loading test conducted by the author.

Series TC Series TC is quoted from the results of a series of centrifuge model loading tests, named Series LN, presented in Chapter 4. Details of this series of centrifuge model tests were described in Section 3.3 and Section 4.4. The series was conducted with a surface circular footing with a diameter of 3cm under the acceleration ranging 98m/sec² (10g) to 980/sec² (100g). Test conditions of Series TC are tabulated in **Table 6.3**.

Series GS Series GS was conducted on surface strip footing with rough base under 9.8m/sec² (1g) and 490m/sec² (50g) of centrifugal accelerations. A divided strip footing with width of 3cm was placed on the level ground consisting of uniform glass beads (GB) with $D_{50}=0.115\text{mm}$ and $U_c=1.091$. Due to the spheric shape and negligible crush-ability of the glass beads particles, this material shows less confining stress dependency of shear strength compared with sand (Maeda and Miura, 1999). Test conditions of Series GS are tabulated in **Table 6.4**. Test set-up for loading test were described in Section 3.3.

Series INSS and Series INLS Series INSS and Series INLS were conducted on surface strip footing with rough base under 123, 245, 368 and 490m/sec² (12.5, 25.0, 37.5 and 50.0g) of accelerations. The materials used were sieved Inagi sand: Series INSS employed IN-210-105, while Series INLS used IN-840-420. The origin of IN-210-105 and IN-840-420 was Inagi

sand, but particle sizes were different. The results obtained from loading tests performed on IN-210-105 and IN-840-420 will be compared to evaluate particle size effects on bearing capacity problems. Test conditions of Series INSS and INLS are tabulated in **Table 6.5** and **Table 6.6**. Test set-up for loading test were described in Section 3.3.

6.3 Results

Series TC **Figures 6.2** show the relationships between normalized load intensity $2q_u/\gamma B$ and normalized settlement S/B . Although the results obtained from TC-3-1 and TC-4-1 show somewhat small bearing capacities, other results show good repeatability. There are clear reducing tendency in normalized load intensity $2q_u/\gamma B$ as the centrifuge acceleration increases.

Ultimate bearing capacity q_u and settlement at failure S_f are tabulated in **Table 6.7** together with derivative normalized parameters $N_\gamma = 2q_u/\gamma B$, S_f/B , K_i and K_f , where parameters K_i and K_f are subgrade reaction factors determined at initial and failure points, respectively. Size effects on these parameters will be discussed in the succeeding section.

Series GS **Figures 6.3(a-d)** and **Figs. 6.4(a, b)** show the relationships between normalized load intensity $2q_u/\gamma B$ and normalized settlement S/B , observed under gravitational acceleration field. In these figures, load intensity curves obtained with three BLC-10k load cells are plotted against normalized settlement S/B .

Parameters q_u , S_f , $N_\gamma = 2q_u/\gamma B$, S_f/B , K_i and K_f were obtained from each load intensity - settlement curve. The values are tabulated in **Table 6.8**. Changes in the values of normalized ultimate bearing capacity $2q_u/\gamma B$ with increasing acceleration were small comparing to that observed in the results of Series TC.

Series INSS **Figures 6.5-6.8** show the relationships between normalized load intensity $2q_u/\gamma B$ and normalized settlement S/B , observed under centrifugal acceleration field from 123m/s^2 (12.5g) to 490m/s^2 (50.0g). In these figures, load intensity curves obtained with three BLC-01t load cells are plotted against normalized settlement S/B .

Parameters q_u , S_f , $N_\gamma = 2q_u/\gamma B$, S_f/B , K_i and K_f were obtained from each load intensity - settlement curve. The values are tabulated in **Table 6.9**. The values of normalized ultimate

bearing capacity $2q_u/\gamma B$ decrease with increasing centrifugal acceleration, as well as the values observed in the results of Series TC.

Series INLS Figures 6.9-6.12 show the relationships between normalized load intensity $2q_u/\gamma B$ and normalized settlement S/B , observed under centrifugal acceleration field from 123m/s^2 (12.5g) to 490m/s^2 (50.0g). In these figures, load intensity curves obtained with three BLC-01t load cells are plotted against normalized settlement S/B .

Parameters q_u , S_f , $N_\gamma = 2q_u/\gamma B$, S_f/B , K_i and K_f were obtained from each load intensity - settlement curve. The values are tabulated in Table 6.10. The values of normalized ultimate bearing capacity $2q_u/\gamma B$ decrease with increasing centrifugal acceleration, as well as the values observed in the results of Series TC and Series INSS.

The values of ultimate bearing capacity obtained from Series INLS tend to show smaller values than the values obtained from Series INSS although the particle size of IN-840-420 is approximately four times larger than that of IN-210-105. There is no clear tendency of particle size effects on the ultimate bearing capacity among the centrifuge test results.

6.4 Discussions

6.4.1 Size effects in load intensity - settlement curves

Figure 6.13 shows comparisons among normalized load intensity-settlement curves observed under four different centrifuge accelerations ranging from 10g (98m/sec^2) to 100g (980m/sec^2) to illustrate stress level effects in bearing characteristics. The normalized load intensity $2q/\gamma B$ becomes smaller as the centrifuge acceleration increases. On the other hand, normalized settlement at failure tends to be larger as centrifuge acceleration increases.

Figure 6.14 shows comparisons among normalized load intensity-settlement curves obtained from Series GS conducted under four different centrifuge accelerations ranging from 1g (9.8m/sec^2) and 50g (490m/sec^2) to illustrate stress level effects in bearing characteristics. No clear changes in the peak values of normalized load intensity $2q/\gamma B$ appear in the test results. On the other hand, normalized settlement at failure tends to be larger as centrifuge acceleration increases. This might be attributed mainly to a raise of the compressibility of the model ground caused by higher confining stress level under a 50g of centrifugal acceleration.

Figure 6.15 shows comparisons among normalized load intensity-settlement curves obtained from Series INSS conducted under four different centrifuge accelerations ranging from 12.5g (123m/sec²) to 50g (490m/sec²) to illustrate stress level effects in bearing characteristics. Although repeatability of loading tests becomes worse, load intensity tends to become smaller as centrifugal acceleration increases. Also normalized settlement at failure tends to be larger as centrifuge acceleration increases.

Figure 6.16 shows comparisons among normalized load intensity-settlement curves obtained from Series INLS conducted under four different centrifuge accelerations ranging from 12.5g (123m/sec²) to 50g (490m/sec²) to illustrate stress level effects in bearing characteristics. Although repeatability of loading tests becomes worse, load intensity tends to become smaller as centrifugal acceleration increases, as well as Series TC and Series INSS. Also normalized settlement at failure tends to be larger as centrifuge acceleration increases.

6.4.2 Size effects in settlement at failure

Figure 6.17 shows the relationships between normalized settlement at failure S_f/B and normalized footing breadth $\gamma B/\sigma_a$, where S_f is the settlement at failure; B , the footing breadth; and σ_a , the referential stress equal to 98kN/m³. The values of S_f/B tend to increase with increasing $\gamma B/\sigma_a$.

Comparing the settlement at failure obtained from Series INSS and Series INLS, we can see that settlement at failure in Series INLS tends to smaller than that in Series INSS. On the other hand, settlement at failure obtained from Series GS shows extremely smaller values in comparison with the values from other series.

6.4.3 Size effects in subgrade reaction factor

Figures 6.18(a-d) show the relationships between subgrade reaction factors K_i and K_f and normalized footing breadth $\gamma B/\sigma_a$, where K_i is determined at initial stage while K_f is determined at failure points. In all cases, both of subgrade reaction factors K_i and K_f decrease along logarithmic linear lines and they can be expressed by the following power functions, as shown in Figs. 6.18(a-d).

For Series TC:

$$\begin{cases} K_i = 3.513 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.530} & (\text{kN/m}^3) \\ K_f = 2.234 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.416} & (\text{kN/m}^3) \end{cases} \quad (6.1)$$

For Series GS:

$$\begin{cases} K_i = 2.817 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.294} & (\text{kN/m}^3) \\ K_f = 1.963 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.275} & (\text{kN/m}^3) \end{cases} \quad (6.2)$$

For Series INSS:

$$\begin{cases} K_i = 2.077 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.617} & (\text{kN/m}^3) \\ K_f = 1.673 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.507} & (\text{kN/m}^3) \end{cases} \quad (6.3)$$

For Series INLS:

$$\begin{cases} K_i = 3.144 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.496} & (\text{kN/m}^3) \\ K_f = 1.607 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.508} & (\text{kN/m}^3) \end{cases} \quad (6.4)$$

6.4.4 Size effects in ultimate bearing capacity

Figures 6.19(a-d) show the logarithmic relationships between normalized ultimate bearing capacity $N_\gamma = 2q_u/\gamma B$ and normalized footing breadth $\gamma B/\sigma_a$. Because all materials employed in this chapter are of cohesionless, the values of ultimate bearing capacity normalized by γB become identical to bearing capacity factor N_γ , according to the definition of N_γ . In all series, normalized bearing capacity N_γ decrease along logarithmic linear lines and they can be expressed by the following power function, as shown in Figs. 6.19(a-d).

$$\begin{aligned} N_\gamma &= N_{\gamma 0} \times \left(\frac{\gamma B}{\sigma_a}\right)^{-\beta} \\ \text{For Series TC: } N_\gamma &= 57.6 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.222} \\ \text{For Series GS: } N_\gamma &= 16.4 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.060} \\ \text{For Series INSS: } N_\gamma &= 65.5 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.274} \\ \text{For Series INLS: } N_\gamma &= 59.8 \times \left(\frac{\gamma B}{\sigma_a}\right)^{-0.257} \end{aligned} \quad (6.5)$$

In Eq. (6.5), $N_{\gamma 0}$ is a bearing capacity factor when $\gamma B/\sigma_a = 1$, while the index β expresses the magnitude of size effects on bearing capacity factor N_γ . A value of 0.060 of β for Series GS, which is rather small in comparison among other series, might be attributed to a weak stress dependency in shear strength (see Fig. 6.20), as Oda and Furuto (1982) mentioned.

As shown in Fig. 6.20(c) and Fig. 6.20(d), Series INSS provides somewhat higher values of ultimate bearing capacity. There is no clear differences about particle size effects.

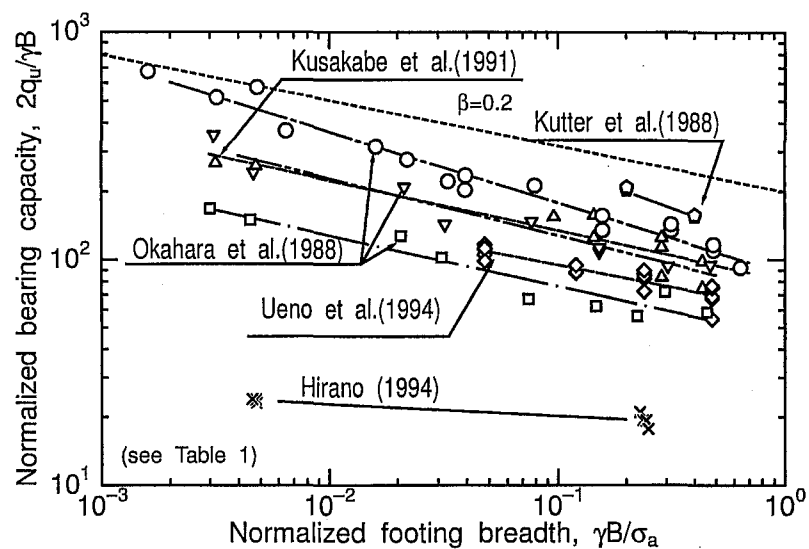
The values of $N_{\gamma 0}$ and β determined from the loading test results shown in **Fig. 6.1** and **Table 6.1** by means of the least square method are tabulated in **Table 6.11** together with the values obtained by the author. The values of β for cohesionless materials are ranging from 0.060 for Series GS to 0.402 for Series BS. Most of the values of β tends to concentrate within a range from 0.2 to 0.3, as Shiraishi (1990) recommended.

6.5 Summary

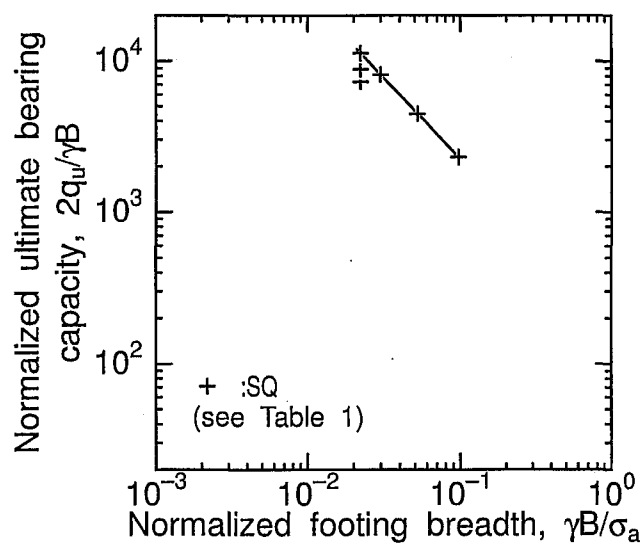
The author conducted a series of loading tests by using a centrifuge to examine size effects on bearing characteristics of granular materials. Size effects on load intensity-settlement curves, settlement at failure, subgrade reaction factors and ultimate bearing capacities were discussed in this chapter. The following conclusions were derived from the test results.

1. Clear size effects appeared in normalized load intensity - settlement curves obtained from Series INS, Series INL and Series TC.
2. Normalized ultimate bearing capacity normalized $2q_u/\gamma B$ tends to decrease with increasing stress level γB , except in Series GS.
3. In all series, normalized bearing capacity N_γ decrease along logarithmic linear lines and they can be expressed by the following power function; however a value of 0.060 of β for Series GS, which is rather small in comparison among other series, might be attributed to a weak stress dependency in shear strength (see **Fig. 6.20**), as Oda and Furuta (1982) mentioned.
4. The values of β for cohesionless materials are ranging from 0.060 for Series GS to 0.402 for Series BS. Most of the values of β tends to concentrate within a range from 0.2 to 0.3, as Shiraishi (1990) recommended.
5. The values of S_f/B tend to increase with increasing $\gamma B/\sigma_a$. Comparing the settlement at failure obtained from Series INSS and Series INLS, we can see that settlement at failure in Series INLS tends to be smaller than that in Series INSS. On the other hand, settlement at failure obtained from Series GS shows extremely smaller values in comparison with the values from other series.

6. In all cases, both of subgrade reaction factors K_i and K_f decrease along logarithmic linear lines and they can be expressed by power functions. The index expressing the reduction tendency for K_i ranged from 0.294 for Series GS to 0.617 for Series INSS. On the other hand for K_f , the index ranged from 0.275 for Series GS to 0.508 for Series INLS.
7. Series INSS provides somewhat higher values of ultimate bearing capacity than Series INLS. There is no clear differences about particle size effects according to the test results.



(a) Results obtained from centrifuge tests



(b) Results obtained from in-situ loading tests

Figure 6.1: Reduction of bearing capacity factor N_γ with increasing footing breadth (see Table 7.1).

Table 6.1: Testing conditions of loading tests of footings quoted in the present study.

Series	Footing	Method	Material ¹⁾	$D_r(\%)$	Citation
SQ	Square	in-situ	Shirahata scoria (SS)	—	Kusakabe <i>et al.</i> (1992)
IC	Circular	Centrifuge	Inagi sand (IN)	80%	Kusakabe <i>et al.</i> (1991)
TC	Circular	Centrifuge	Toyoura sand (TO)	70%	Ueno <i>et al.</i> (1994)
TS-1	Strip	Centrifuge	Toyoura sand (TO)	88%	Okahara <i>et al.</i> (1988)
TS-2				74%	
TS-3				58%	
GS	Strip		Glass beads (GB)	70%	Hirano (1994)
BS	Circular	Centrifuge	Monterey 0/30 sand (MO)	93–95%	Kutter <i>et al.</i> (1988)

Note: ¹⁾: See Chapter 2

Table 6.2: Conditions of loading tests conducted by the author.

Series	Footing	Acceleration	Material ¹⁾	$D_r(\%)$
TC	Circular	10g-100g	Toyoura sand (TO)	70%
GS	Strip	1g, 50g	Glass beads (GB)	70%
INSS	Strip	12.5-50g	Inagi sand (INS)	70%
INLS	Strip	12.5-50g	Inagi sand (INL)	70%

Note: ¹⁾: See Chapter 2

Table 6.3: Loading test on Toyoura Sand (Series TC).

Test name	Dry density ρ_d (g/cm ³)	Accele- ration $n(g)$	Value of K_0
TC-1-1	1.589	10	0.23
TC-1-2	1.596	10	0.55
TC-2-1	1.593	25	0.31
TC-2-2	1.599	25	0.37
TC-3-1	1.582	50	0.23
TC-3-2	1.593	50	0.30
TC-3-3	1.596	50	0.33
TC-4-1	1.589	100	0.17
TC-4-2	1.594	100	0.25
TC-4-3	1.587	100	0.25

Table 6.4: Loading test on glass beads (Series GS).

Test name	Dry density ρ_d (g/cm ³)	Accele- ration $n(g)$
GS-1-1	1.535	1
GS-1-2	1.533	1
GS-1-3	1.525	1
GS-1-4	1.488	1
GS-2-1	1.537	50
GS-2-2	1.535	50

Table 6.5: Loading test on Inagi Sand (IN-250-105, Series INSS).

Test name	Dry density ρ_d (g/cm ³)	Accele- ration $n(g)$
INSS-1-1	1.601	12.5
INSS-1-2	1.597	12.5
INSS-2-1	1.604	25.0
INSS-2-2	1.604	25.0
INSS-3-1	1.606	37.5
INSS-3-2	1.602	37.5
INSS-4-1	1.598	50.0
INSS-4-2	1.603	50.0

Table 6.6: Loading test on Inagi Sand (IN-840-420, Series INLS).

Test name	Dry density ρ_d (g/cm ³)	Accele- ration $n(g)$
INLS-1-1	1.600	12.5
INLS-1-2	1.607	12.5
INLS-1-3	1.606	12.5
INLS-2-1	1.629	25.0
INLS-2-2	1.596	25.0
INLS-3-1	1.602	37.5
INLS-3-2	1.609	37.5
INLS-4-1	1.604	50.0
INLS-4-2	1.600	50.0

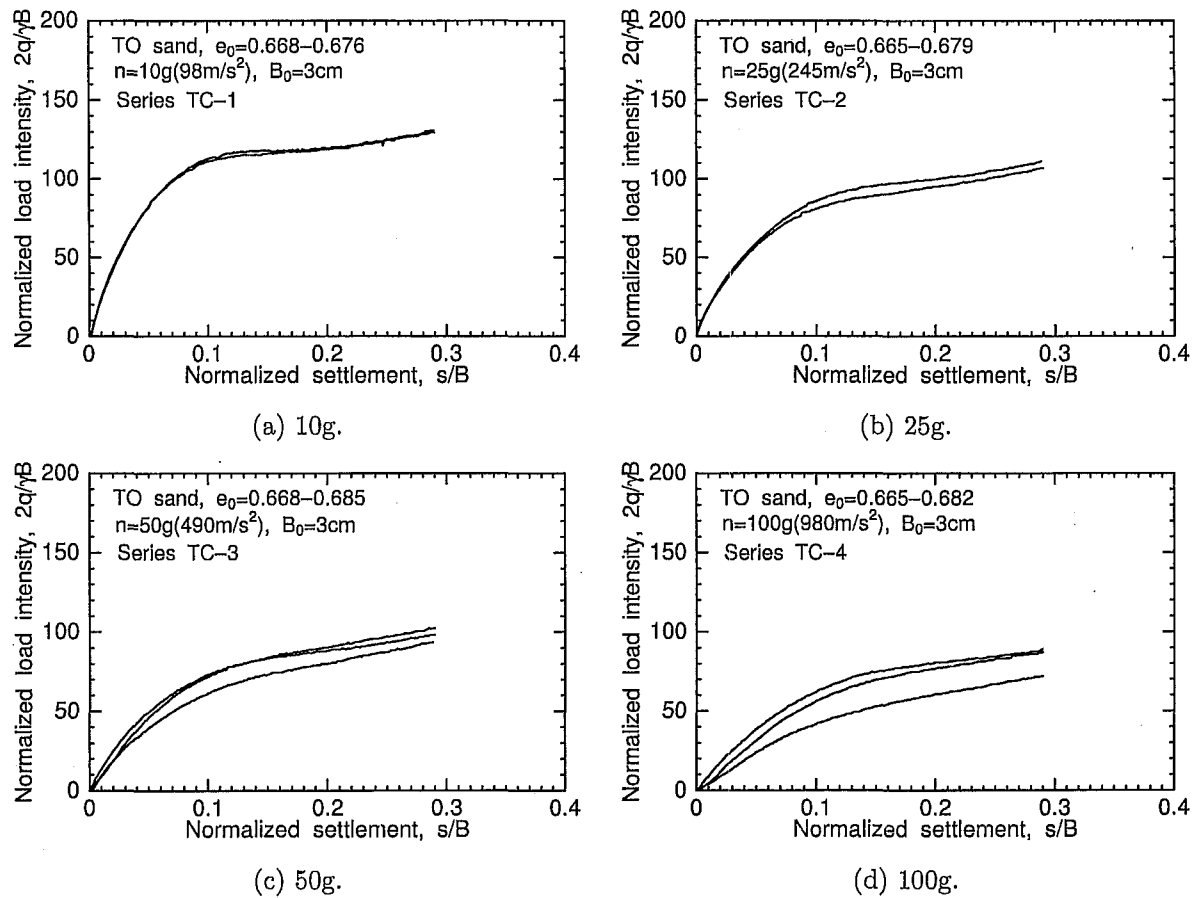
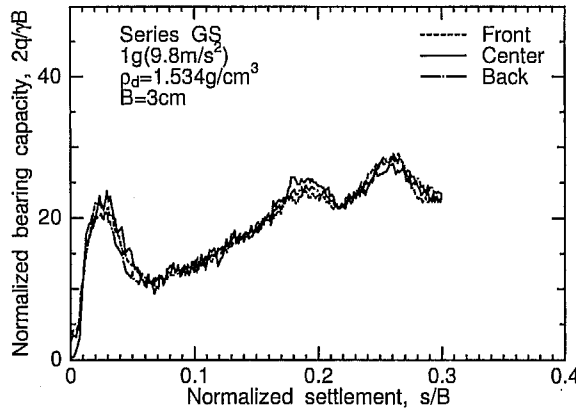


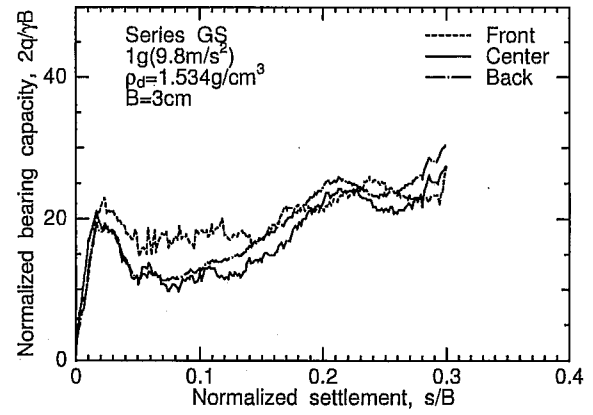
Figure 6.2: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B obtained from Series TC.

Table 6.7: Test results obtained from Series TC.

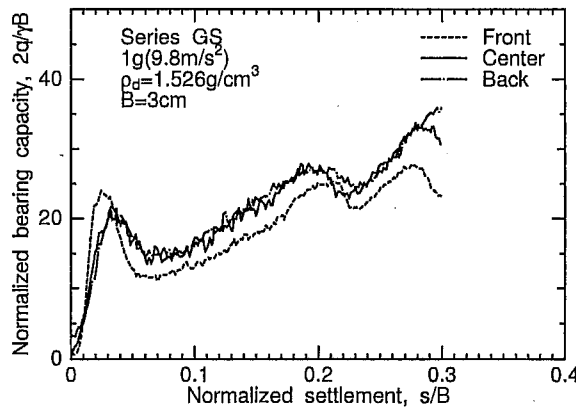
Name	ρ_d (g/cm ³)	Acc. (g)	γB (kPa)	q_u (kPa)	S_f (mm)	$N_\gamma =$ $2q_u/\gamma B$	S_f/B	K_i (kN/m ³)	K_f (kN/m ³)
TC-1-1	1.600	10.0	4.70	259.7	3.00	111.2	0.100	18030	8657
TC-1-2	1.600	10.0	4.70	273.4	3.51	116.7	0.117	15970	7796
TC-2-1	1.587	25.0	11.7	511.6	4.10	87.4	0.137	9722	4990
TC-2-2	1.587	25.0	11.7	555.7	4.25	94.5	0.142	10070	5229
TC-3-1	1.580	50.0	23.2	837.9	4.40	72.1	0.147	9428	3808
TC-3-2	1.593	50.0	23.4	962.4	4.55	82.2	0.152	7722	4235
TC-3-3	1.593	50.0	23.4	1019	5.10	86.5	0.170	10560	3997
TC-4-1	1.589	100	46.7	1274	4.96	54.5	0.165	3636	2568
TC-4-2	1.593	100	46.8	1754	5.48	74.9	0.183	4684	3201
TC-4-3	1.587	100	46.6	1784	4.87	76.5	0.162	5880	3663



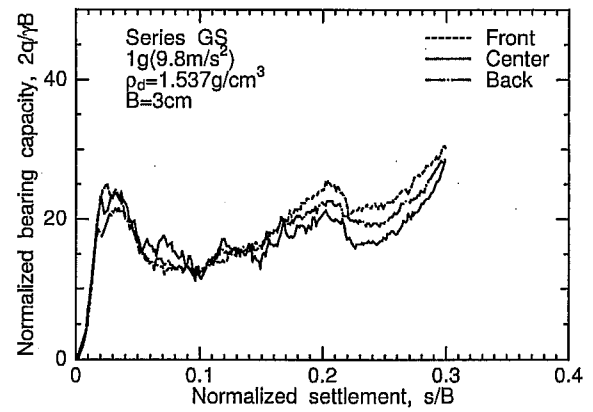
(a) GS-1-1



(b) GS-1-2

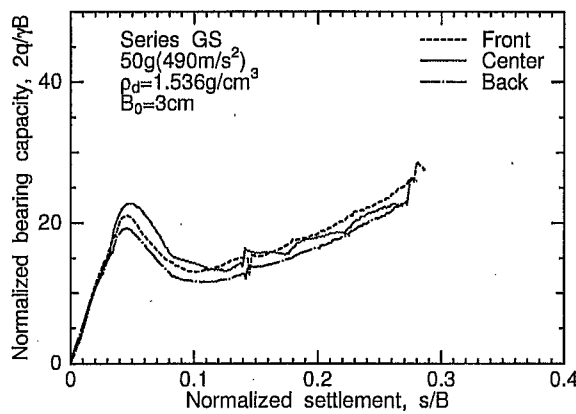


(c) GS-1-3

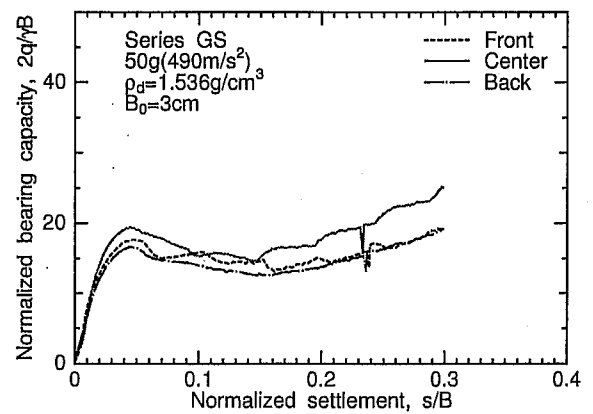


(d) GS-1-4

Figure 6.3: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B obtained from Series GS conducted under $1g$.



(a) GS-2-1

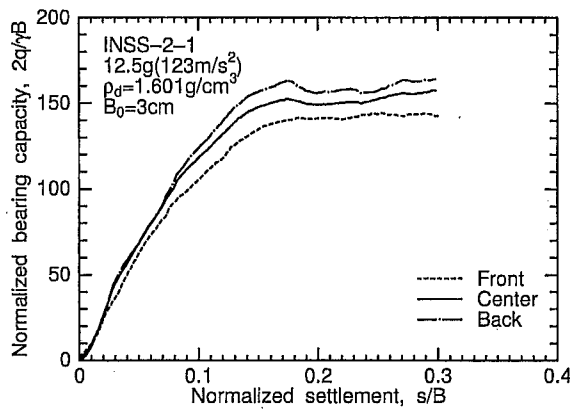


(b) GS-2-2

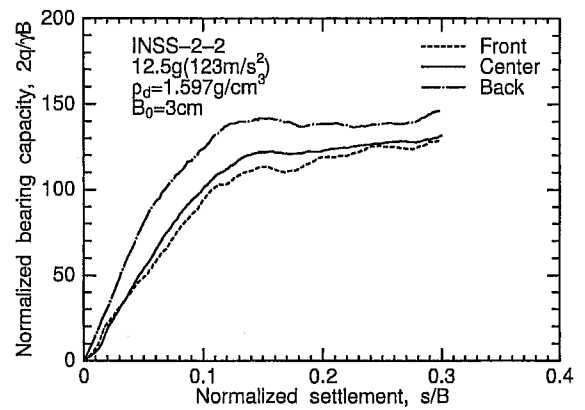
Figure 6.4: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B obtained from Series GS conducted under $50g$.

Table 6.8: Test results obtained from Series GS.

Name	Load cell position	Acc. (g)	γB (kPa)	q_u (kPa)	S_f (mm)	$N_\gamma = 2q_u/\gamma B$	S_f/B	K_i (kN/m ³)	K_f (kN/m ³)
GS-1-3 $\rho_d(\text{g/cm}^3)=$ 1.488	Front	1.0	0.437	3.26	1.29	14.9	0.043	8662	2523
	Center	1.0	0.437	2.72	1.43	12.4	0.048	3369	1896
	Back	1.0	0.437	3.18	1.71	14.5	0.057	2639	1852
GS-1-2 $\rho_d(\text{g/cm}^3)=$ 1.535	Front	1.0	0.451	5.66	0.52	25.1	0.017	15987	10874
	Center	1.0	0.451	5.23	0.63	23.2	0.021	17732	8272
	Back	1.0	0.451	4.87	0.79	21.6	0.026	10332	6166
GS-1-3 $\rho_d(\text{g/cm}^3)=$ 1.533	Front	1.0	0.451	4.98	0.64	22.1	0.021	11445	7735
	Center	1.0	0.451	5.40	0.64	24.0	0.021	17604	8396
	Back	1.0	0.451	4.67	0.57	20.7	0.019	15215	8171
GS-2-1 $\rho_d(\text{g/cm}^3)=$ 1.525	Front	1.0	0.448	4.84	0.57	21.6	0.019	13062	8474
	Center	1.0	0.448	4.62	0.50	20.6	0.017	8309	9221
	Back	1.0	0.448	5.37	0.79	23.9	0.026	6920	6815
GS-2-2 $\rho_d(\text{g/cm}^3)=$ 1.537	Front	50.0	22.59	237.	2 1.34	21.0	0.045	4578	3524
	Center	50.0	22.59	217.	6 1.40	19.3	0.047	3870	3102
	Back	50.0	22.59	256.	8 1.37	22.7	0.046	4397	3735
GS-3-1 $\rho_d(\text{g/cm}^3)=$ 1.535	Front	50.0	22.57	199.	9 1.40	17.7	0.047	6530	2844
	Center	50.0	22.57	186.	2 1.34	16.5	0.045	4860	2784
	Back	50.0	22.57	217.	6 1.26	19.3	0.042	7373	3461

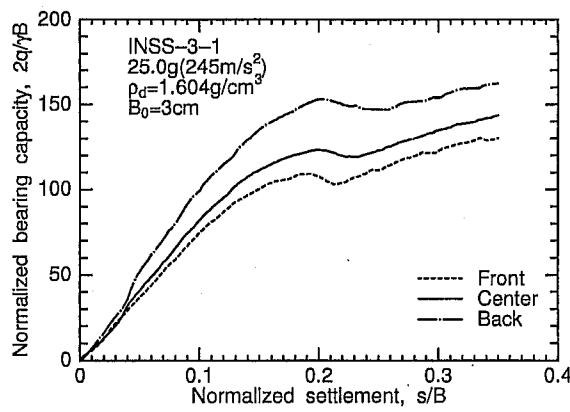


(a) INSS-1-1

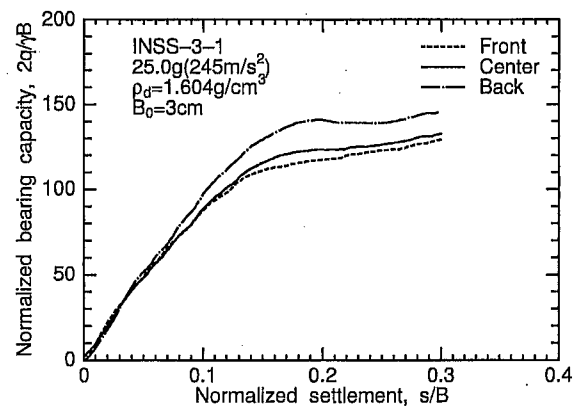


(b) INSS-1-2

Figure 6.5: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 12.5g obtained from Series INSS-1.

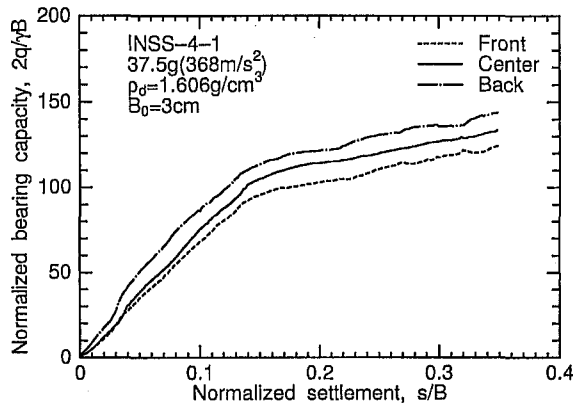


(a) INSS-2-1

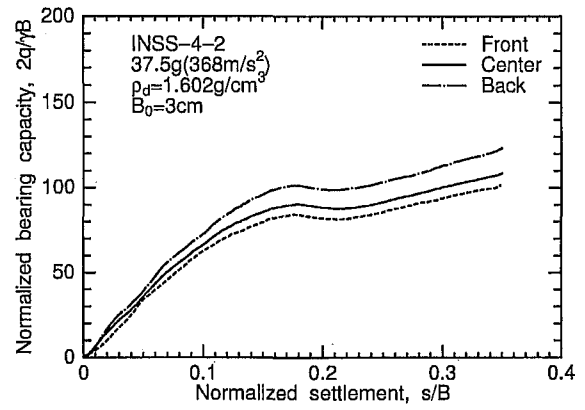


(b) INSS-2-2

Figure 6.6: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 25g obtained from Series INSS-2.

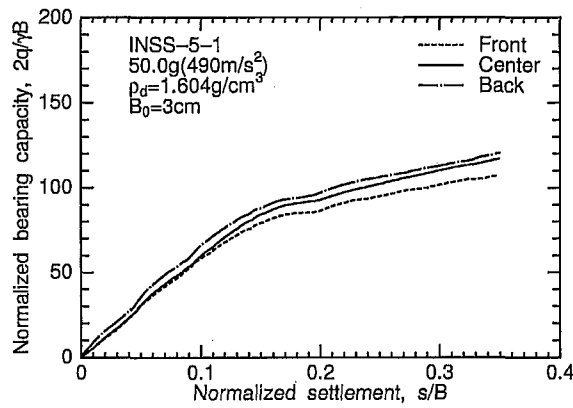


(a) INSS-3-1

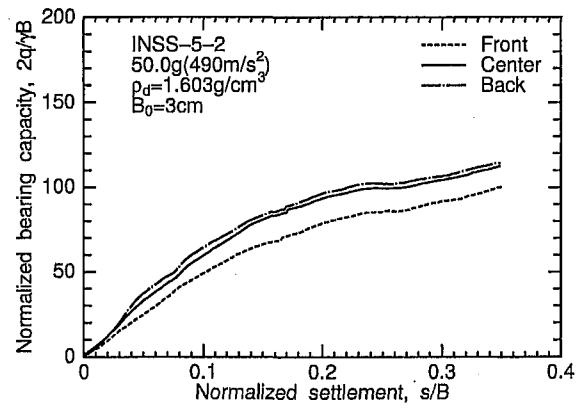


(b) INSS-3-2

Figure 6.7: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 37.5g obtained from Series INSS-3.



(a) INSS-4-1



(b) INSS-4-2

Figure 6.8: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 50g obtained from Series INSS-4.

Table 6.9: Test results obtained from Series INSS.

Name	Load cell position	Acc. (g)	γB (kPa)	q_u (kPa)	S_f (mm)	$N_\gamma = 2q_u/\gamma B$	S_f/B	K_i (kN/m ³)	K_f (kN/m ³)
INSS-1-1	Front	12.5	5.88	418.5	5.43	142.2	0.181	10449	6166
$\rho_d(\text{g/cm}^3)=$	Center	12.5	5.88	452.8	5.04	153.9	0.168	13368	7186
1.601	Back	12.5	5.88	481.2	5.00	163.6	0.167	14968	7704
INSS-1-2	Front	12.5	5.87	339.1	4.52	115.1	0.151	9750	5994
$\rho_d(\text{g/cm}^3)=$	Center	12.5	5.87	366.5	4.39	124.9	0.146	10001	6683
1.597	Back	12.5	5.87	430.2	4.52	146.3	0.151	13334	7606
INSS-2-1	Front	25.0	11.79	653.7	5.65	110.9	0.188	6555	4621
$\rho_d(\text{g/cm}^3)=$	Center	25.0	11.79	734.0	5.96	124.5	0.199	7388	4920
1.604	Back	25.0	11.79	922.2	6.04	156.6	0.201	7875	6115
INSS-2-2	Front	25.0	11.79	693.8	6.30	117.7	0.210	8143	4401
$\rho_d(\text{g/cm}^3)=$	Center	25.0	11.79	722.3	5.87	122.5	0.196	8143	4920
1.604	Back	25.0	11.79	831.0	5.87	141.0	0.196	9211	5659
INSS-3-1	Front	37.5	17.71	929.0	6.65	104.9	0.222	5666	3730
$\rho_d(\text{g/cm}^3)=$	Center	37.5	17.71	1026.1	6.57	115.9	0.219	6532	4171
1.606	Back	37.5	17.71	1100.5	6.57	124.4	0.219	6532	4470
INSS-3-2	Front	37.5	17.66	738.9	5.17	83.8	0.172	6091	3815
$\rho_d(\text{g/cm}^3)=$	Center	37.5	17.66	802.6	5.35	90.9	0.178	6139	4003
1.602	Back	37.5	17.66	893.8	5.35	101.3	0.178	7065	4459
INSS-4-1	Front	50.0	23.49	1015.3	5.96	86.4	0.199	4260	3414
$\rho_d(\text{g/cm}^3)=$	Center	50.0	23.49	1088.8	6.04	92.7	0.201	4369	3602
1.598	Back	50.0	23.49	1146.6	6.09	97.6	0.203	4792	3758
INSS-4-2	Front	50.0	23.56	1015.3	7.70	86.0	0.257	4037	2639
$\rho_d(\text{g/cm}^3)=$	Center	50.0	23.56	1169.1	7.35	99.2	0.245	5153	3173
1.603	Back	50.0	23.56	1215.2	7.26	103.0	0.242	5875	3346

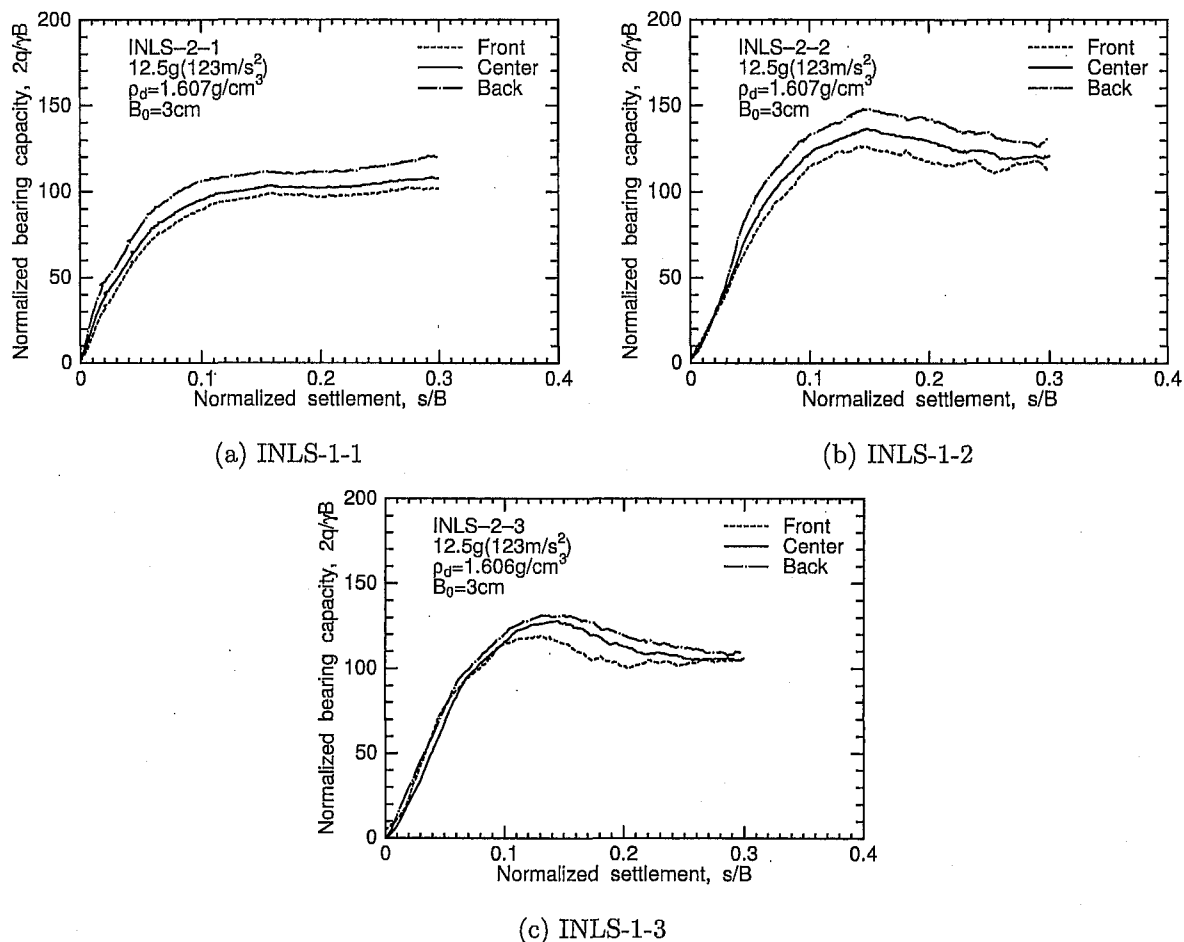


Figure 6.9: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 12.5g obtained from Series INLS-1.

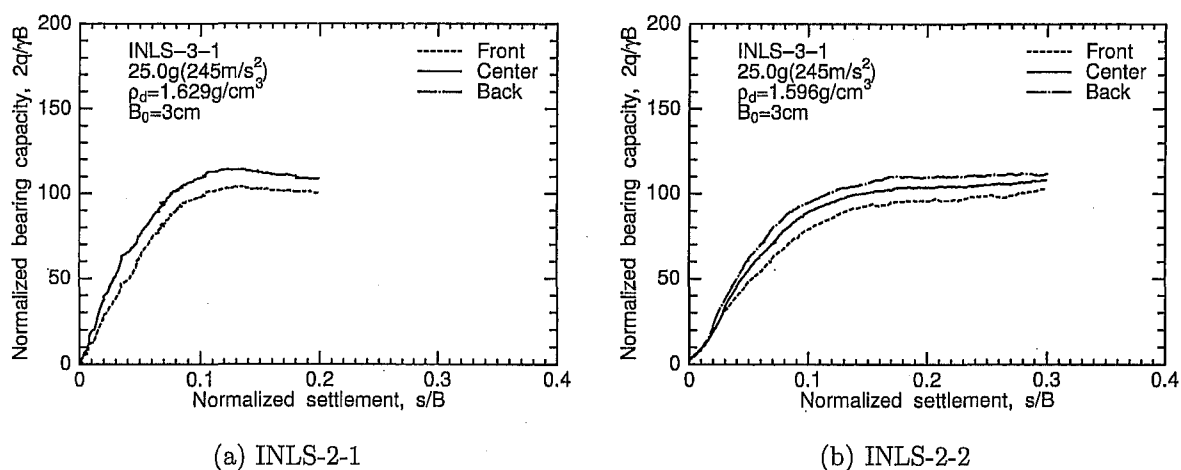
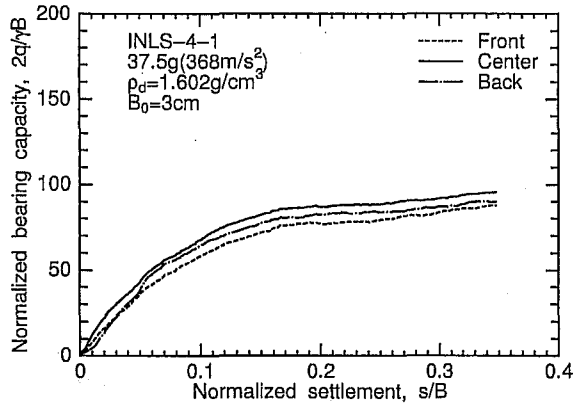
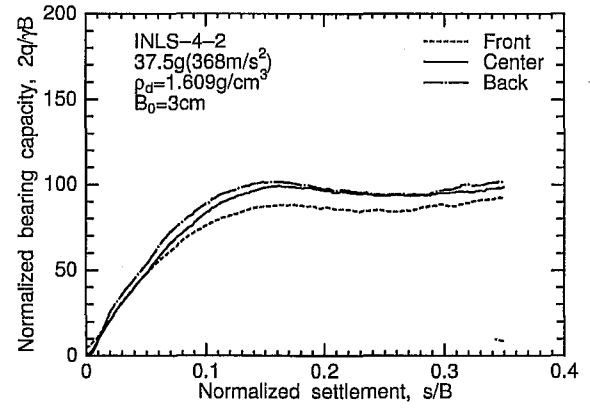


Figure 6.10: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 25g obtained from Series INLS-2.

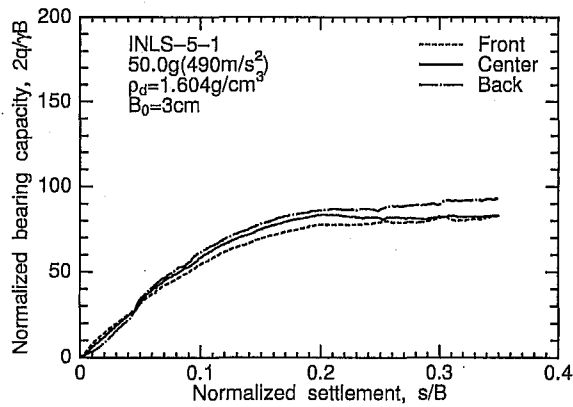


(a) INLS-3-1

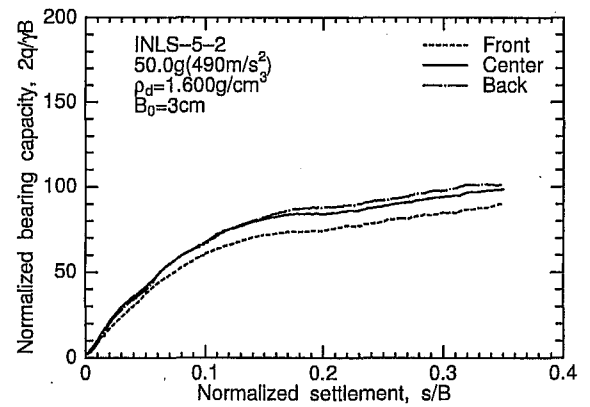


(b) INLS-3-2

Figure 6.11: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 37.5g obtained from Series INLS-3.



(a) INLS-4-1



(b) INLS-4-2

Figure 6.12: Normalized load intensity $2q/\gamma B$ vs. normalized settlement s/B under 50g obtained from Series INLS-4.

Table 6.10: Test results obtained from Series INLS.

Name	Load cell position	Acc. (g)	γB (kPa)	q_u (kPa)	S_f (mm)	$N_\gamma = 2q_u/\gamma B$	S_f/B	K_i (kN/m ³)	K_f (kN/m ³)
INLS-1-1	Front	12.5	5.88	285.2	4.87	96.9	0.162	11368	4688
$\rho_d(\text{g/cm}^3)=$	Center	12.5	5.88	299.9	4.57	101.9	0.152	12293	5253
1.600	Back	12.5	5.88	326.3	4.74	110.9	0.158	16276	5519
INLS-1-2	Front	12.5	5.91	368.5	4.30	124.7	0.143	12142	6851
$\rho_d(\text{g/cm}^3)=$	Center	12.5	5.91	399.8	4.30	135.3	0.143	13355	6725
1.607	Back	12.5	5.91	435.1	4.13	147.3	0.138	14583	8441
INLS-1-3	Front	12.5	5.90	353.8	3.78	119.8	0.126	13362	7492
$\rho_d(\text{g/cm}^3)=$	Center	12.5	5.90	378.3	3.96	128.1	0.132	12150	7649
1.606	Back	12.5	5.90	394.9	4.13	133.7	0.138	12150	7665
INLS-2-1	Front	25.0	11.97	617.4	4.17	103.1	0.139	9626	5923
$\rho_d(\text{g/cm}^3)=$	Center	25.0	11.97	680.1	4.09	113.5	0.136	13522	6657
1.629	Back	25.0	11.97	—	—	—	—	—	—
INLS-2-2	Front	25.0	11.73	558.6	5.87	95.2	0.196	8602	3801
$\rho_d(\text{g/cm}^3)=$	Center	25.0	11.73	607.6	5.78	103.5	0.193	9869	4207
1.596	Back	25.0	11.73	644.8	5.65	109.9	0.188	9400	4567
INLS-3-1	Front	37.5	17.66	680.1	6.96	77.0	0.232	6358	2606
$\rho_d(\text{g/cm}^3)=$	Center	37.5	17.66	776.2	7.13	87.8	0.238	8101	2904
1.602	Back	37.5	17.66	735.0	6.87	83.2	0.229	7033	2857
INLS-3-2	Front	37.5	17.74	781.1	5.60	88.0	0.187	6922	3721
$\rho_d(\text{g/cm}^3)=$	Center	37.5	17.74	880.0	4.87	99.2	0.162	7411	4825
1.609	Back	37.5	17.74	898.7	4.78	101.3	0.159	8893	5014
INLS-4-1	Front	50.0	23.58	918.3	6.22	77.8	0.207	4967	2955
$\rho_d(\text{g/cm}^3)=$	Center	50.0	23.58	980.0	6.30	83.1	0.210	4967	3112
1.604	Back	50.0	23.58	1035	6.87	87.7	0.229	5077	3018
INLS-4-2	Front	50.0	23.52	874.2	6.00	74.3	0.200	5394	2901
$\rho_d(\text{g/cm}^3)=$	Center	50.0	23.52	988.8	6.09	84.0	0.203	7558	3230
1.600	Back	50.0	23.52	1034.9	6.26	87.9	0.209	7009	3293

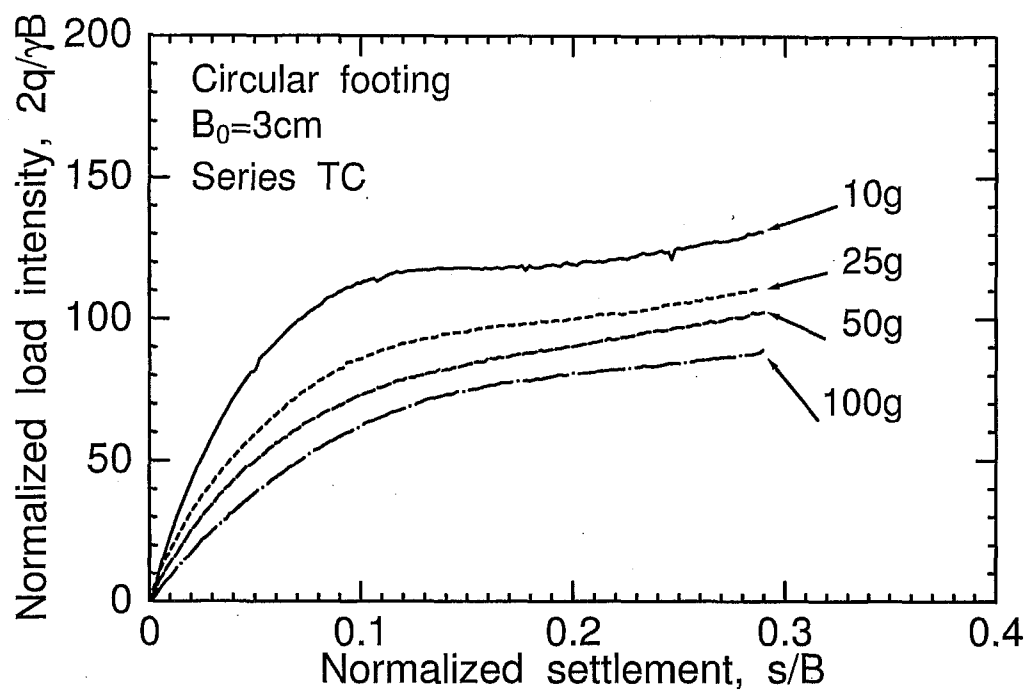


Figure 6.13: Size effects in load intensity vs. settlement curves (Series TC).

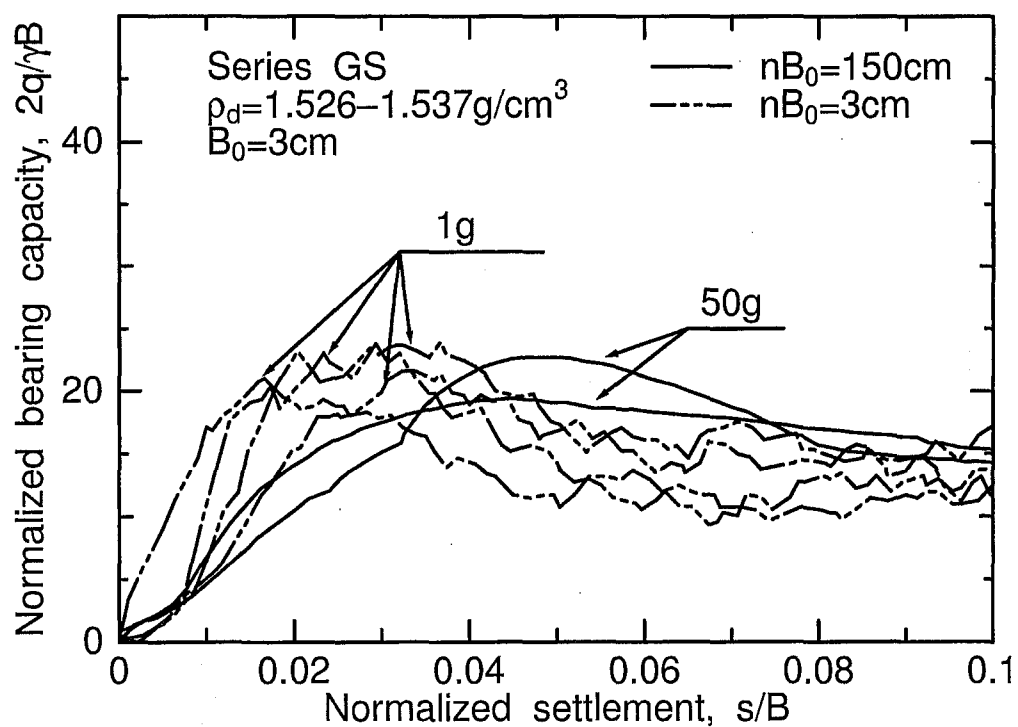


Figure 6.14: Comparison of $2q/\gamma B$ vs. s/B relationships obtained under 1g and 50g.

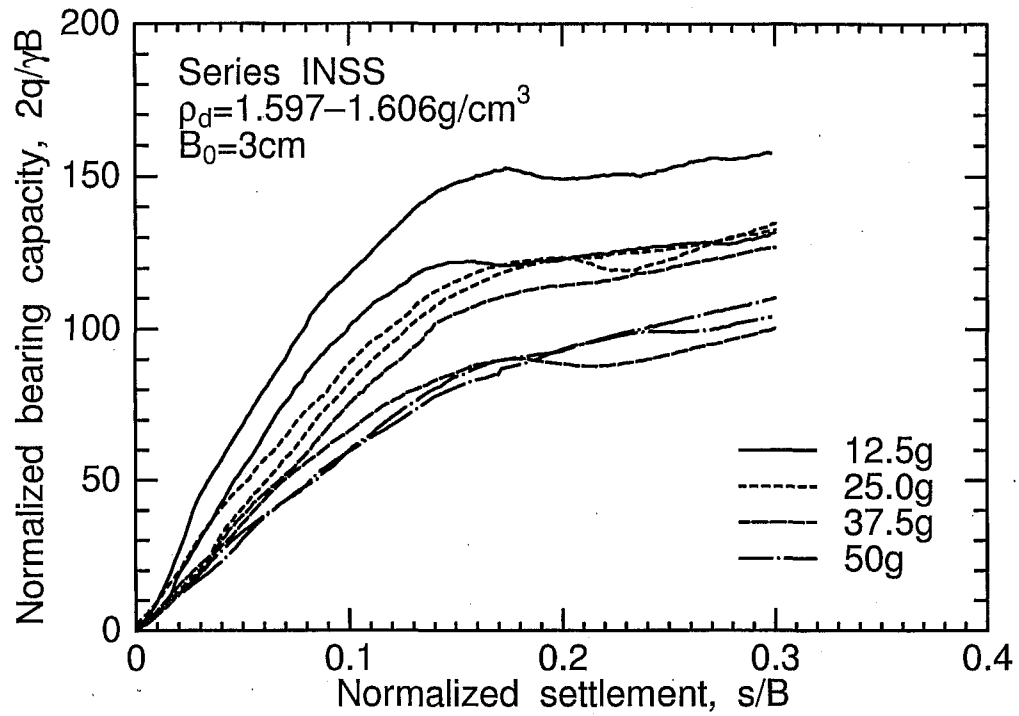


Figure 6.15: Size effects in load intensity vs. settlement curves (Series INSS).

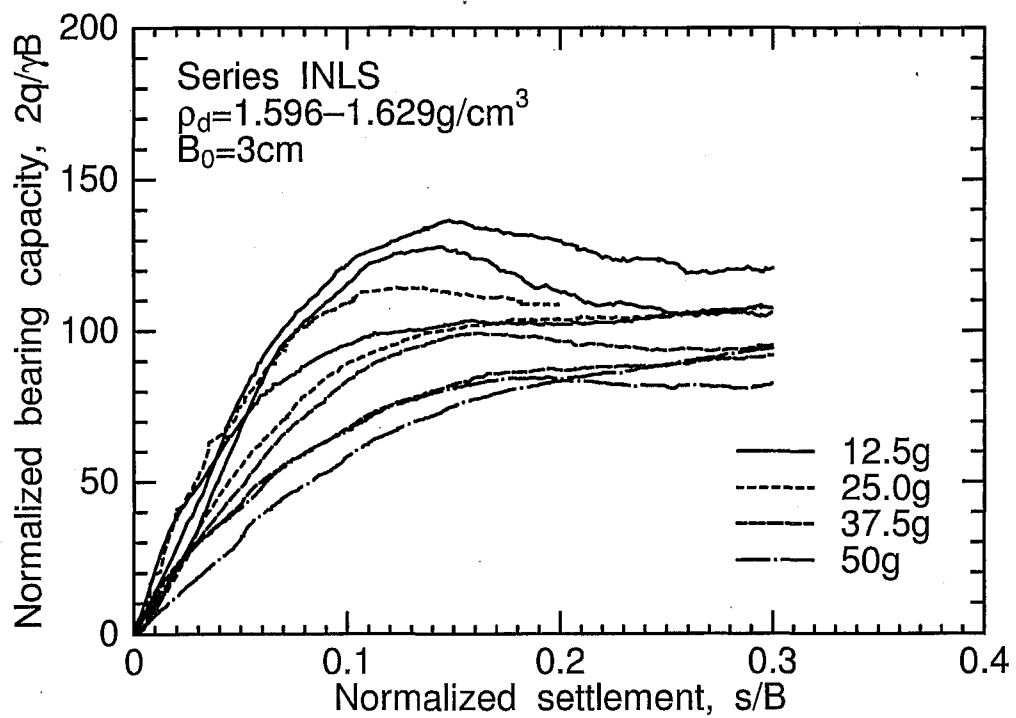


Figure 6.16: Size effects in load intensity vs. settlement curves (Series INLS).

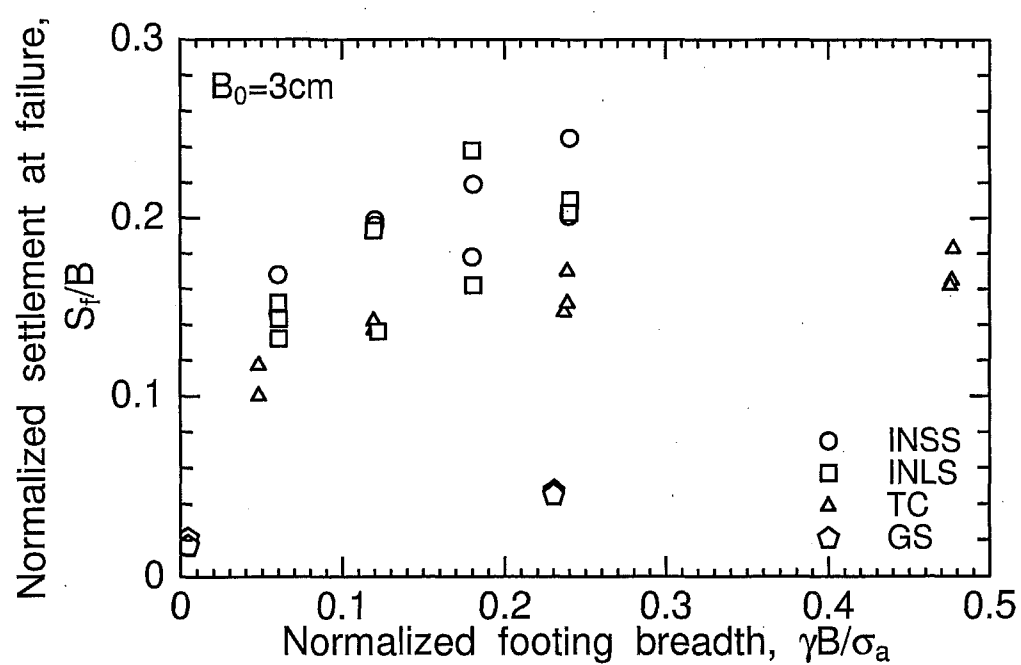
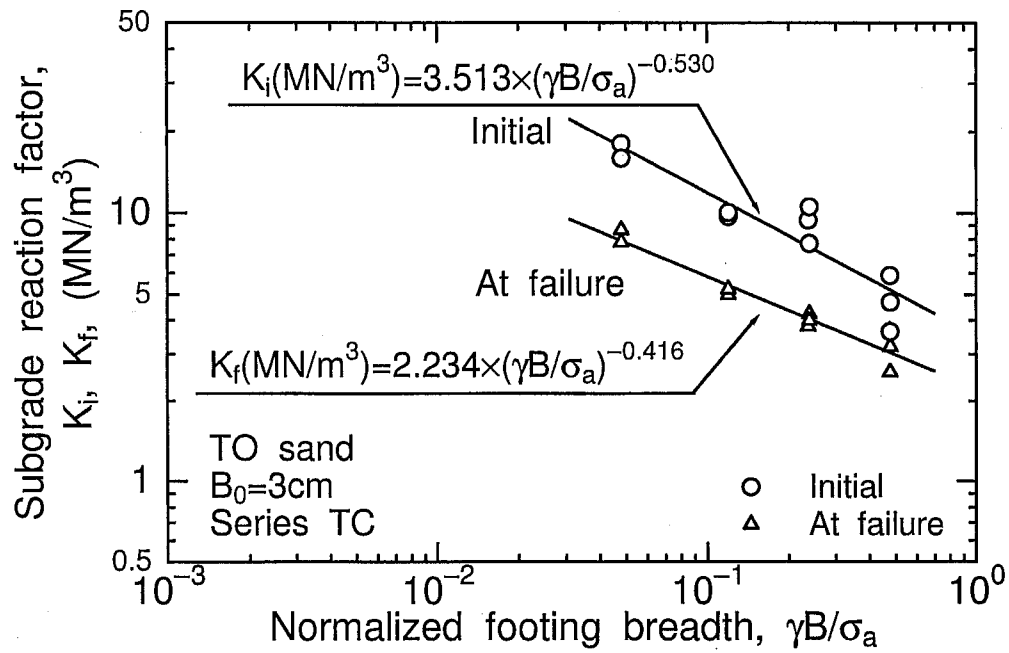
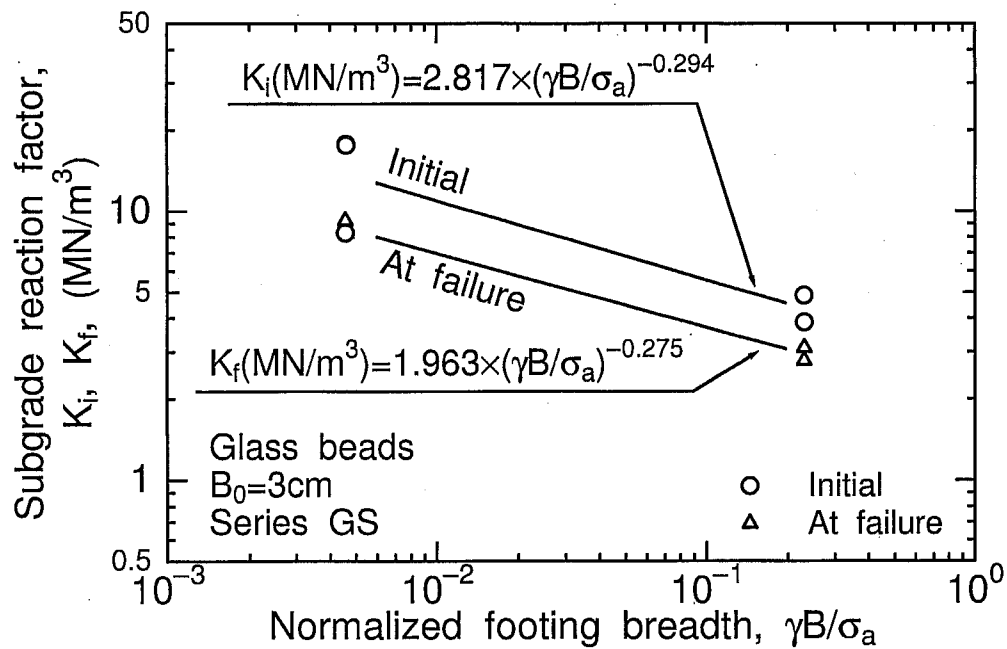


Figure 6.17: Changes in settlement at failure normalized by footing breadth S_f/B with increasing normalized footing breadth $\gamma B/\sigma_a$.

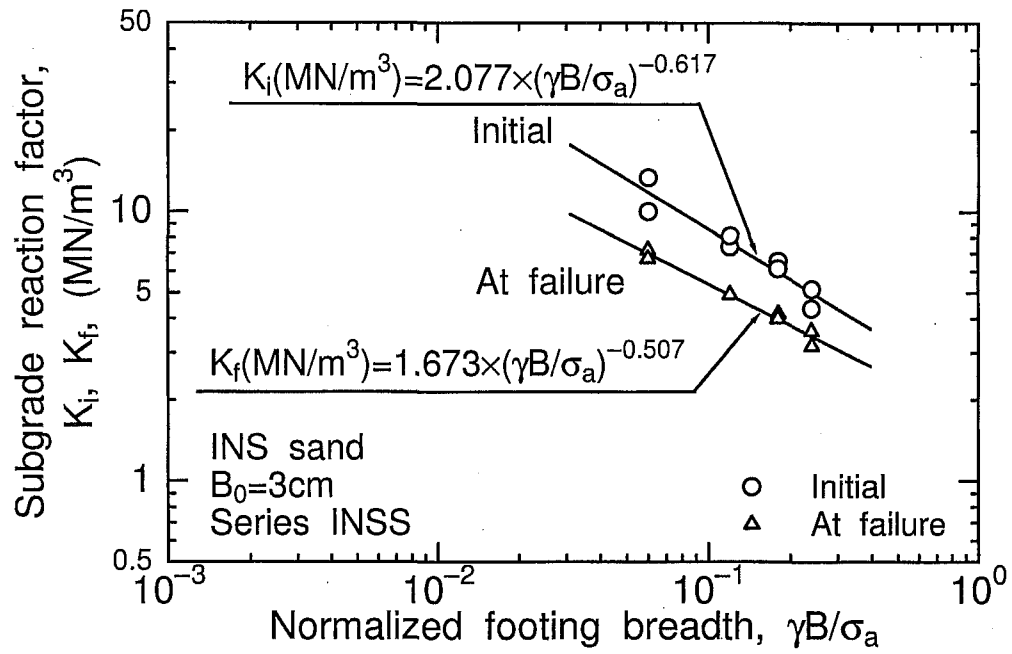


(a) Series TC

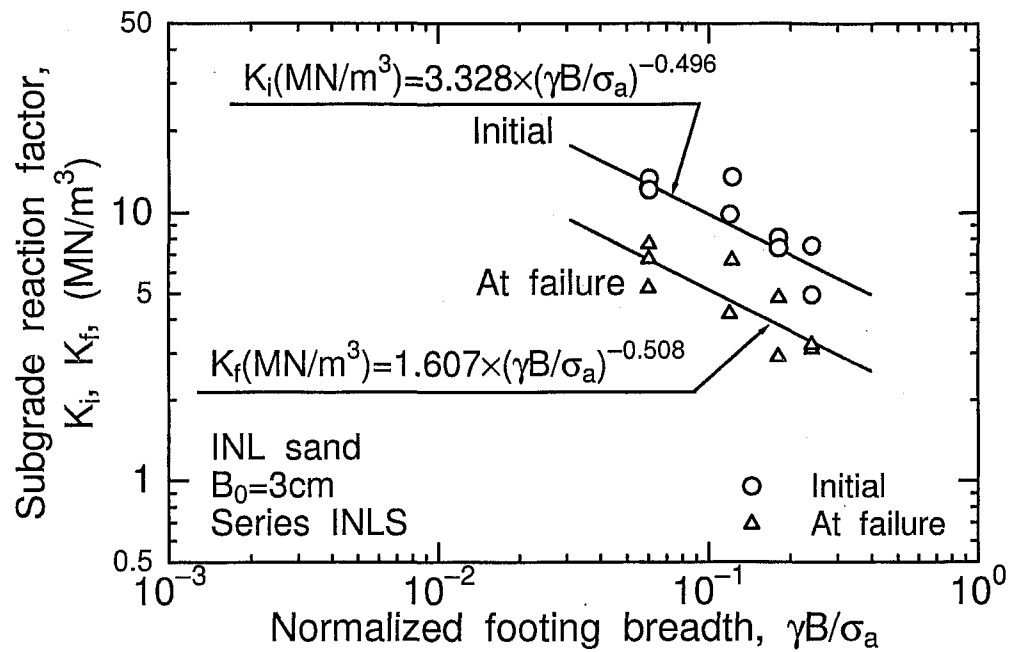


(b) Series GS

Figure 6.18: Changes in subgrade reaction factor K_i and K_f with increasing normalized footing breadth $\gamma B/\sigma_a$ (continued).

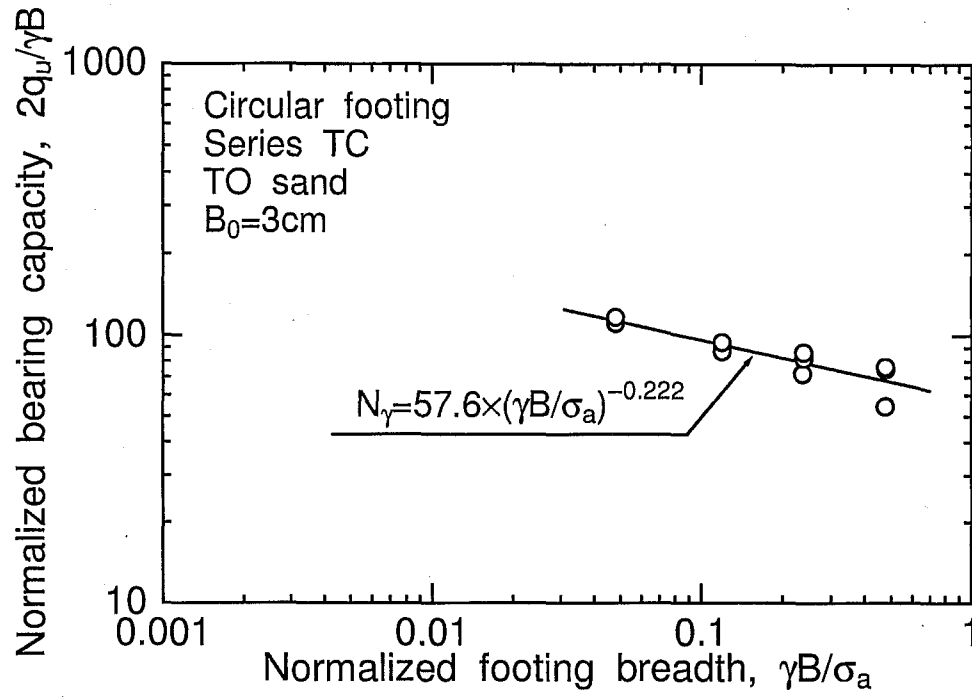


(c) Series INSS

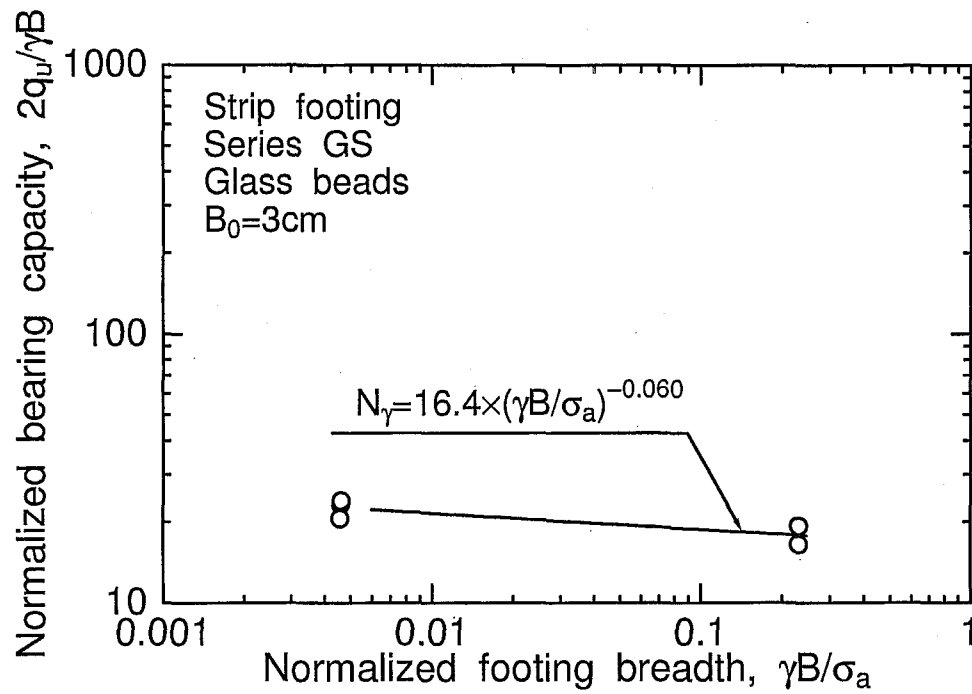


(d) Series INLS

Figure 6.18: Changes in subgrade reaction factor K_i and K_f with increasing normalized footing breadth $\gamma B / \sigma_a$.

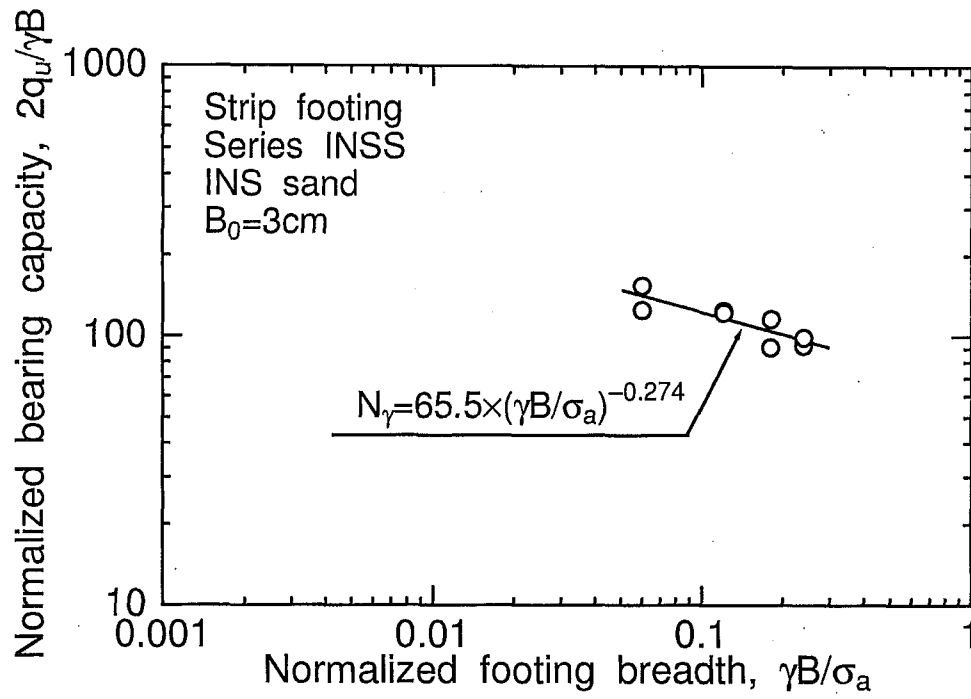


(a) Series TC

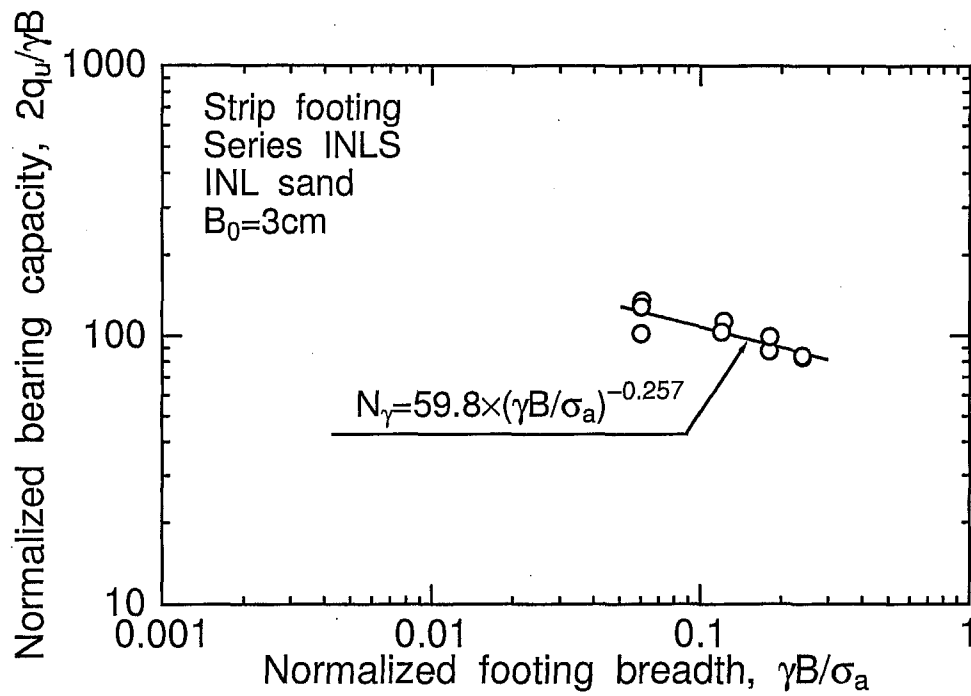


(b) Series GS

Figure 6.19: Changes in normalized bearing capacity $N_\gamma = 2q_u/\gamma B$ with increasing normalized footing breadth $\gamma B/\sigma_a$ (continued).



(c) Series INSS



(d) Series INLS

Figure 6.19: Changes in normalized bearing capacity $N_\gamma = 2q_u/\gamma B$ with increasing normalized footing breadth $\gamma B/\sigma_a$.

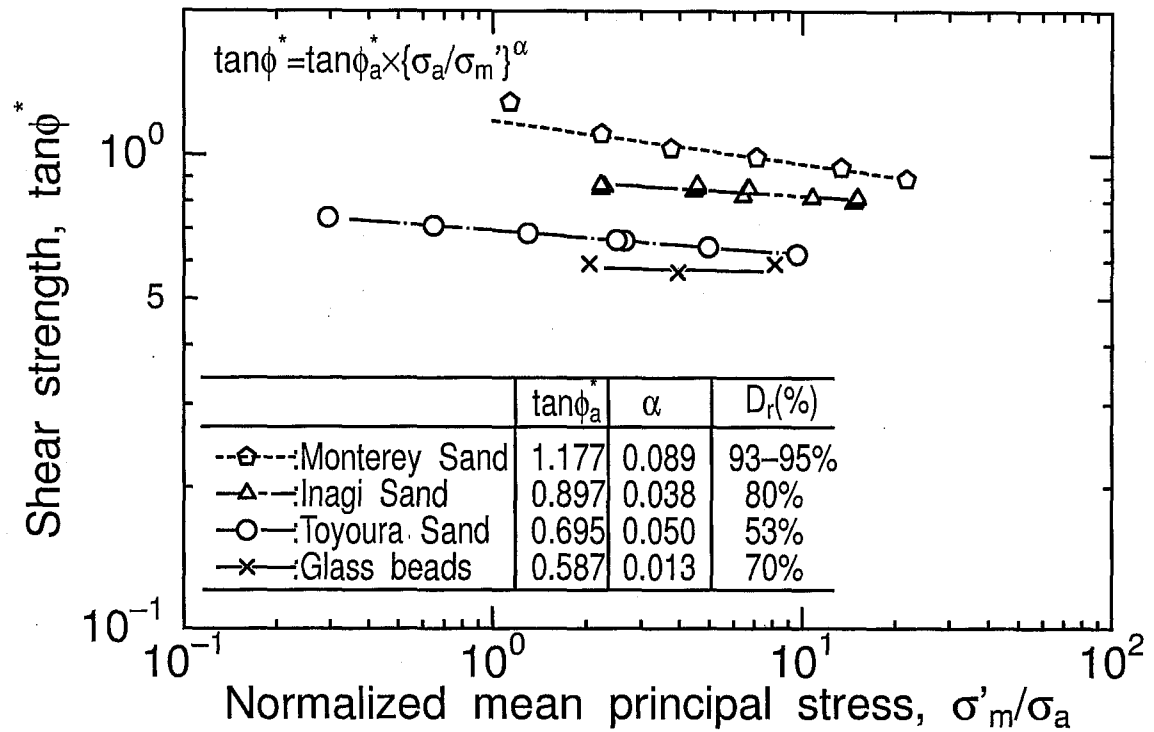


Figure 6.20: Determination of shear strength of ground materials from laboratory element tests with an aid of the proposed confining stress dependent failure criterion.

Table 6.11: Size effects of ultimate bearing capacity.

Series	Footing	Material	$D_r(\%)$	$N_{\gamma 0}$	β	Citation
SQ	Square	SS	—	284.	0.918	Kusakabe <i>et al.</i> (1992)
BS	Circular	MO	93–95%	108.	0.402	Kutter <i>et al.</i> (1988)
IC	Circular	IN	80%	81.0	0.220	Kusakabe <i>et al.</i> (1991)
TS-1	Strip	TO	88%	87.0	0.312	Okahara <i>et al.</i> (1988)
TS-2			74%	71.7	0.252	
TS-3			58%	45.4	0.225	
TC	Circular	TO	70%	57.6	0.222	
GS	Strip	GB	70%	16.4	0.060	
INSS	Strip	INS	70%	65.5	0.274	
INLS	Strip	INL	70%	59.8	0.257	

Chapter 7

Analysis and design

7.1 General

In this chapter, two methods for estimating the ultimate bearing capacity of surface footings, in which size effects are taken into account, are described. One of the methods, called M-I hereafter, is for footings on cohesionless soil and the another method, called M-II hereafter, is for both cohesionless and cohesive soil. Clarification of size effects, which is indispensable for appropriate design of large foundations, may permit use of the results of small scaled loading tests and prevent increases in the construction costs of foundations.

The prediction method for cohesionless soils, M-I, is based on Shiraishi's empirical equation for ultimate bearing capacity. Shiraishi (1990) deduced an empirical equation from a series of *in-situ* loading tests on piles for nonlinear relationship of bearing capacity coefficient N_γ . To clarify relationship between the nonlinearity of N_γ and stress dependency of shear strength, an extended slip line method is developed, in which dependency of the angle of internal friction on the confining stress is formulated. Based on results from a comprehensive series of calculations employing the extended slip line method, the authors reappraise size effects on bearing capacity, and investigate the relationship between strength parameters of sand and size effects on bearing capacity. A modified formula and several diagrams that provide a simple estimation method are proposed. Also physical meaning of Shiraishi's empirical equation is discussed.

The another method for both cohesionless and cohesive soil, M-II, is based on Terzaghi's bearing capacity formula for surface footing. The method M-II incorporates a method for

determination of shear strength constants c and ϕ that should be applied to the bearing capacity formula. A nonlinear Finite Element Method which incorporates the confining stress dependent shear strength is used to calculate stress distribution in foundation ground beneath the footings. It is shown that the effective mean principal stress σ'_m tends to range from $2\gamma B$ to $10\gamma B$ for strip footings and from $1\gamma B$ to $15\gamma B$ for circular footings; within this range the parameters c and ϕ are determined from the straight fitting line.

In this chapter, above two methods M-I and M-II are verified with the results of series of model surface footing loading tests on cohesionless and cohesive ground presented in Chapter 6. This verification confirms the practicability of the proposed methods to strip, circular and square footings on cohesive and cohesionless foundation ground with sufficient accuracy for practical purposes.

7.2 Introduction

Terzaghi's bearing capacity formula is widely used in current design codes (Terzaghi and Peck, 1967):

$$q_u = S_c c N_c + \frac{1}{2} S_\gamma \gamma B N_\gamma + S_q \gamma D_f N_q \quad (7.1)$$

where q_u is ultimate bearing capacity and c is cohesion; B and D_f are breadth and embedded depth of the footing, respectively. Parameters S_c , S_γ and S_q are shape factors, and N_c , N_γ and N_q are bearing capacity factors; $S_c = 1.3$, $S_\gamma = 0.6$ and $S_q = 1.0$ for circular footings, and $S_c = 1.3$, $S_\gamma = 0.8$ and $S_q = 1.0$ for square footings. According to Terzaghi and Peck (1967), Prandtl and Reissner derived the exact mathematical expressions for bearing capacity factors N_q and N_c :

$$N_q = \exp(\pi \tan \phi) \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \quad (7.2)$$

$$N_c = \cot \phi \left\{ \exp(\pi \tan \phi) \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right) - 1 \right\} \quad (7.3)$$

For N_γ , several formulae have been proposed by a number of researchers, as cited by Zadoroga (1994). Vesić (1973) recommended using the bearing capacity factor proposed by Caquot and Kérisel (1953) which can be approximated by

$$N_\gamma = 2(N_q + 1) \tan(\phi) \quad (7.4)$$

In this study Eq. (7.2) and Eq. (7.4) are employed for the calculation of ultimate bearing capacity of surface footings.

Bearing capacity factors N_c , N_γ and N_q are functions of angle of internal friction ϕ and are independent of footing breadth B in the formulae as shown by Eqs. (7.2-7.4); however, the bearing capacity factor N_γ calculated from the loading tests of footings on sand reduces with increasing footing size (Golder, 1941; De Beer, 1963, 1965; Okahara *et al.*, 1988; Kusakabe *et al.*, 1991, 1992; Ueno *et al.*, 1994). This phenomenon is referred to as the size effects of footing breadth on the bearing capacity factors. To compensate for the reduction of N_γ , exceedingly large safety factors are usually used. While Terzaghi's bearing capacity formula has contributed remarkably to the practical design of foundations, the lack of consideration of scale effects has prevented an appropriate design and led to inflated construction costs. It is, therefore, important to clarify the mechanism of size effects and develop rational prediction methods for the bearing capacity of footings.

Shiraishi (1990) developed an empirical equation for bearing capacity based on a series of *in-situ* loading tests on piles, where the bearing capacity factor N_γ is not constant, but is directly related to the footing width B as

$$q_u = S_\gamma \frac{\gamma B}{2} N_\gamma^* \left(\frac{B}{B^*} \right)^{-\beta} \quad (7.5)$$

where S_γ is the shape factor of the footing; γ , the unit weight of soil; N_γ^* , the reference bearing capacity factor; B^* , the reference footing width; and β , a factor for the dependency on the confining stress level. In this equation, the bearing capacity factor N_γ in Terzaghi's original form (Terzaghi and Peck, 1967) is modified as $N_\gamma^* (B/B^*)^{-\beta}$.

Shiraishi (1990) found β values from a limited number of *in-situ* loading tests. When B^* was set to be 1.40m, the β value was 0.2. Available centrifugal model test data of surface footings on sand, tabulated in Table 7.3 in Chapter 6, generally support the empirical bearing capacity equation Eq. (7.5), as shown in Fig. 6.1. While Eq. (7.5) has a practical value for its simplicity, the physical meaning of parameters β and B^* are not yet clear.

In this study, the physical meaning of Eq. (7.5) is explored in greater detail using the extended slip line method. A comprehensive series of parametric calculations are conducted to examine the relationships between the confining stress dependent angle of internal friction and the stress dependent parameters β and B^* . Then the author proposed a prediction method named M-I is developed.

Tatsuoka *et al.* (1991) investigated the size effects of the bearing capacity of spread footing experimentally and analytically; they found that size effects are caused by confining stress level

effects on shear strength, progressive failure of foundation ground and scale effects of footing breadth with respect to the particle size of ground materials. It should be noted that the scale effect cannot be detected loading tests in centrifuge, and Ovesen (1979) commented that the scale effect is emphasized only in small scaled footings whose footing breadth B is less than 30 times the grain size of foundation ground material. Tatsuoka *et al.* (1991) succeeded in simulating the size dependent behaviors of spread footings by means of an advanced FEM. However, simplified methods which incorporate the conventional bearing capacity formula are needed for practical design purposes.

There are simplified methods taking the stress level effects into account, in which the stress-level effect is regarded as a main cause of the size effects. Meyerhof (1950) and De Beer (1970) proposed that the angle of internal friction ϕ to be applied to the bearing capacity formula, must be selected in accordance with the stress level beneath the footing. Meyerhof (1950) suggested that the mean normal stress on the failure planes σ'_0 is about a tenth of ultimate bearing capacity q_u , and De Beer (1970) presented the formula for averaged σ'_0 as

$$\sigma'_0 = \frac{q_u + 3\gamma D}{4}(1 - \sin \phi) \quad (7.6)$$

Hettler and Gudehus (1988) supported Meyerhof's suggestion, and Perkins (1995) applied it to a highly frictional material. Though this type of methods seems to be promising for practical design, it is applicable only to cohesionless soils and not to materials with cohesion, such as cemented sand and gravel, because internal friction is determined from secant line to curved failure envelope, and the term for cohesion c in the bearing capacity formula is neglected in their methods. Also an iterative process is needed for determining the angle of internal friction ϕ corresponding to the stress range of interest, because the stress level is given as a function of ultimate bearing capacity q_u . Kutter *et al.* (1988) introduced a method for determining shear strength of the Mohr-Coulomb type with regard to apparent cohesion c even for cohesionless soils, and used the same constants for footings with different sizes.

In this chapter, two methods, M-I and M-II, for estimating the ultimate bearing capacity of surface footings, in which size effects are taken into account, are described. One of the methods called M-I is for footings on cohesionless soil and the another method called M-II is for both cohesionless and cohesive soil.

The prediction method for cohesionless soils, M-I, is based on Shiraishi's empirical equation for ultimate bearing capacity. To clarify relationship between the nonlinearity of N_γ and stress

dependency of shear strength, an extended slip line method is developed, in which dependency of the angle of internal friction on the confining stress is formulated. Based on results from a comprehensive series of calculations employing the extended slip line method, the authors reappraise size effects on bearing capacity, and investigate the relationship between strength parameters of sand and size effects on bearing capacity. A modified formula and several diagrams that provide a simple estimation method are proposed. Also physical meaning of Shiraishi's empirical equation is discussed.

The author proposes another method, M-II, for both cohesionless and cohesive to evaluate size effects on the bearing capacity of surface foundations, in which the stress level effect is considered as suggested by Meyerhof (1950) and others. To find the stress level beneath the footing, a nonlinear FEM devised by the author, which takes the confining stress dependency of the shear strength of ground materials into account, as applied in plane strain condition for strip footings and axial symmetrical condition for circular footings. In the proposed method M-II, the stress range concerned is given as a function of footing breadth B and unit weight of ground soil γ , but not of ultimate bearing capacity q_u . The strength constants c and ϕ that are given from a straight fitting line of curved failure envelope are applied to the bearing capacity formula (Eq. (7.1)).

7.3 Development of prediction method M-I

7.3.1 Extended slip line method incorporating stress level effects

To derive a prediction method for ultimate bearing capacity, an extended slip line method was developed. To reappraise size effects and to clarify relationships between stress dependent a shear failure criterion presented in Eq. (2.4), a comprehensive parametric computation was carried out. The extended slip line method can take account of the confining stress dependency of shear strength specified by Eq. (2.4).

7.3.1.1 Governing Equations for Equilibrium Condition

The stress condition considered is illustrated and stress parameters are defined in Fig. 7.1. The compression side is designated as positive. Partial differential equations governing equilibrium under the plane condition are given by,

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0 \\ \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \gamma \end{cases} \quad (7.7)$$

Linear relationships in the $\tan \phi^*$ vs. σ_m logarithmic plots can be modeled as shown in Fig. 2.6 and described as

$$\tau_f = \sigma \tan \phi^* = \sigma \tan \phi_a^* \left(\frac{\sigma_m}{\sigma_a} \right)^{-\alpha} \quad (7.8)$$

$$\sqrt{\frac{(\sigma_x - \sigma_z)^2}{4} + \tau_{zx}^2} = \sigma_m \sin \phi^* \quad (7.9)$$

$$\begin{cases} \sigma_x = \sigma_m (1 - \sin \phi^* \cos 2\theta) \\ \sigma_z = \sigma_m (1 + \sin \phi^* \cos 2\theta) \\ \tau_{zx} = \\ \tau_{xz} = \sigma_m \sin \phi^* \sin 2\theta \end{cases} \quad (7.10)$$

where ϕ^* is the secant internal friction angle, σ_a is the reference atmospheric stress equal to 98kPa, θ is the direction of maximum principals stress axis, and σ_m is the mean normal stress ($= (\sigma_x + \sigma_z)/2$). Parameters ϕ_a^* and α were introduced to express the assumed linearity in $\tan \phi^*$ vs. σ_m logarithmic plots, as shown in Fig. 2.6(b).

7.3.1.2 Derivation of extended slip line method

From Eqs. (7.7, 7.9, 7.10) the following simultaneous partial differential equations are derived

$$\begin{cases} \left\{ 1 - \frac{d(\sigma_m \sin \phi^*)}{d\sigma_m} \cos 2\theta \right\} \frac{\partial \sigma_m}{\partial x} + 2\sigma_m \sin \phi^* \sin 2\theta \frac{\partial \theta}{\partial x} + \\ \frac{d(\sigma_m \sin \phi^*)}{d\sigma_m} \sin 2\theta \frac{\partial \sigma_m}{\partial z} + 2\sigma_m \sin \phi^* \cos 2\theta \frac{\partial \theta}{\partial z} = 0 \\ \left\{ 1 + \frac{d(\sigma_m \sin \phi^*)}{d\sigma_m} \cos 2\theta \right\} \frac{\partial \sigma_m}{\partial z} - 2\sigma_m \sin \phi^* \sin 2\theta \frac{\partial \theta}{\partial z} + \\ \frac{d(\sigma_m \sin \phi^*)}{d\sigma_m} \sin 2\theta \frac{\partial \sigma_m}{\partial x} + 2\sigma_m \sin \phi^* \cos 2\theta \frac{\partial \theta}{\partial x} = \gamma \end{cases} \quad (7.11)$$

Since the secant internal friction angle ϕ^* is a function of σ_m as shown in Eq. (7.8), the derivative expression $d(\sigma_m \sin \phi^*)/d\sigma_m$ in Eq. (7.11) can be derived as follows;

$$\sin \phi^* + \sigma_m \cos \phi^* \frac{d\phi^*}{d\sigma_m} = \sin \phi^* (1 - \alpha \cos^2 \phi^*) = \sin \phi_t^*$$

because

$$\begin{cases} \frac{d \tan \phi^*}{d\sigma_m} = \frac{1}{\cos^2 \phi^*} \frac{d\phi^*}{d\sigma_m} = -\frac{\alpha}{\sigma_m} \tan \phi^*, \text{ and} \\ \frac{d\phi^*}{d\sigma_m} = -\frac{\alpha \sin \phi^* \cos \phi^*}{\sigma_m}. \end{cases} \quad (7.12)$$

The geometrical meaning of tangential internal friction angle ϕ_t^* is illustrated in **Figs. 7.1(c)** and **2.6(a)**. The simultaneous differential equations **Eq. (7.11)** can be simplified as

$$\begin{cases} (1 - \sin \phi_t^* \cos 2\theta) \frac{\partial \sigma_m}{\partial x} + 2\sigma_m \sin \phi^* \sin 2\theta \frac{\partial \theta}{\partial x} \\ \quad + \sin \phi_t^* \sin 2\theta \frac{\partial \sigma_m}{\partial z} + 2\sigma_m \sin \phi^* \cos 2\theta \frac{\partial \theta}{\partial z} = 0 \\ (1 + \sin \phi_t^* \cos 2\theta) \frac{\partial \sigma_m}{\partial z} - 2\sigma_m \sin \phi^* \sin 2\theta \frac{\partial \theta}{\partial z} \\ \quad + \sin \phi_t^* \sin 2\theta \frac{\partial \sigma_m}{\partial x} + 2\sigma_m \sin \phi^* \cos 2\theta \frac{\partial \theta}{\partial x} = \gamma \end{cases} \quad (7.13)$$

The directions of a set of slip lines S_1 and S_2 are defined as shown in **Figs. 7.1(b, c)**, and the distance along the slip lines is obtained by

$$\begin{cases} dS_1 = \frac{dx}{\sin(\theta+\mu)} = \frac{dz}{\cos(\theta+\mu)}; \text{ slip line } S_1, \\ dS_2 = \frac{dx}{\sin(\theta-\mu)} = \frac{dz}{\cos(\theta-\mu)}; \text{ slip line } S_2, \end{cases} \quad (7.14)$$

where μ is equal to $(\pi/4 - \phi_t^*/2)$. According to the Kotter's expressions the substitution of **Eqs. (7.14)** into **Eqs. (7.13)** yields

$$\begin{cases} \frac{d\sigma_m}{dS_1} + 2\sigma_m \tan \phi^* \frac{d\theta}{dS_1} = -\gamma \frac{\sin(\theta-\mu)}{\cos \phi^*} \\ \frac{d\sigma_m}{dS_2} - 2\sigma_m \tan \phi^* \frac{d\theta}{dS_2} = \gamma \frac{\sin(\theta+\mu)}{\cos \phi^*} \end{cases} \quad (7.15)$$

7.3.2 Numerical Calculation of Bearing Capacity

Numerical calculation of bearing capacity was performed within the framework of the finite difference method as explained by Sokolovskii (1965), Yamaguchi (1969) and Ko and Scott (1973).

A trapped rigid wedge with straight sides beneath the footing was assumed in the present calculation to consider the effect of the roughness of footing base on bearing capacity, where the weight of the trapped wedge is taken into account for the calculation of the base pressure, q_u , similar to the method adopted by Graham and Stuart (1971). Although they fixed the base angle of the wedge ψ to ϕ , here the author changed the base angle to find the minimum value of q_u (Kimura *et al.*, 1982). Meyerhof (1961) investigated the relationship $\psi = a\phi$ with a varying from 1 to 1.2 and found for $\psi = 30^\circ$ a minimum N_γ of 16.0. Vogel and Baracos (1973) established $a = 1.1$ for $\phi = 30^\circ$, at which N_γ is a minimum. Further they showed that a may vary from 0 to 1.22 with various ϕ . The results obtained here show that the angles of trapped rigid wedge, at which the minimum $2q_u/\gamma B$, ranged from 0.75 to 1.25 of the values of ϕ_a^* . **Figure 7.2** shows the relationships between calculated normalized bearing capacity $2q_u/\gamma B$ and the angle of trapped rigid wedge ψ .

Typical slip line meshes constructed by the numerical calculations are shown in **Figs. 7.3(a, b)**, in which the effects of confining the stress dependency parameter α are demonstrated. When parameter α becomes larger, the slip line mesh of the strip footing becomes smaller than that for $\alpha = 0.0$ in the case of footing width $B=10\text{m}$, because the secant internal friction angle ϕ^* or shear strength reduces with an increase in α . Thus, it is confirmed that the calculated base pressure q is very much sensitive to the value of α .

To obtain q_u for the surface footings, the surcharge q_s was monotonically reduced, approaching zero. Convergence of the calculated q_u to a certain value is demonstrated in **Fig. 7.4**. A ratio of $q_s/\gamma B$ of 10^{-2} was found to be small enough for eliminating practically all the influence of surcharge q_s .

7.3.3 Discussion on calculation results and derivation of simplified prediction method M-I

The ultimate bearing capacity q_u was calculated for a strip footing with variable breadth B . Shear strength parameter ϕ_a^* was changed parametrically from 20deg. to 50deg. and α was changed from zero to 0.15, respectively. Calculated normalized bearing capacity $2q_u/\gamma B$ and normalized footing breadth $\gamma B/\sigma_a$ relationships are presented in a logarithmic scale in **Fig. 7.5**. It was found that the relationship is almost linear, independently of ϕ_a^* and α . In this study, the linearity in the $(2q_u/\gamma B)$ vs. $(\gamma B/\sigma_a)$ relationships in a logarithmic scale was employed for practical evaluation of ultimate bearing capacity.

It is interesting to see that the straight lines for $\log(2q_u/\gamma B)$ vs. $\log(\gamma B/\sigma_a)$ relationships with a certain value of ϕ_a^* , intersect one another at the same point, independently of α as shown in **Fig. 7.5**. The ultimate bearing capacity corresponding to the intersection is free from the confining stress dependency and $N_\gamma = 2q_u/\gamma B$ becomes a function of only ϕ_a^* . The confining pressure associated with the intersections was regarded as a reference base pressure and denoted as p^* . Also, a reference bearing capacity factor N_γ^* , was newly introduced, corresponding to the reference pressure $\gamma B^* = p^*$. With both the reference base pressure p^* and reference bearing capacity factor N_γ^* , the empirical formula **Eq. (7.5)** proposed by Shiraishi (1990) was modified in this study as

$$q_u = S_\gamma \frac{\gamma B}{2} N_\gamma = S_\gamma \frac{\gamma B}{2} N_\gamma^* \left(\frac{\gamma B}{p^*} \right)^{-\beta} = S_\gamma \frac{\gamma B}{2} N_\gamma^* \left(\frac{B}{B^*} \right)^{-\beta} \quad (7.16)$$

In this formula, the bearing capacity factor N_γ in Terzaghi's original form (Terzaghi and Peck, 1967) is modified as $N_\gamma = N_\gamma^*(\gamma B/p^*)^{-\beta} = N_\gamma^*(B/B^*)^{-\beta}$, where S_γ is the shape factor; B^* , the reference footing breadth; and β , a parameter representing the degree of stress level effects.

Figure 7.6 shows the relationship between the calculated reference bearing capacity factor N_γ^* and the reference secant internal friction angle ϕ_a^* . In **Fig. 7.6** the values for strip footing by Caquot and K  risel (1953) are also shown by a broken line. The agreement of N_γ^* for strip footing with N_γ by Caquot and K  risel (1953) suggests that the optimization of the trapped rigid wedge adopted in the extended slip line method is appropriate.

Reference base pressure p^* is plotted against the reference secant internal friction angle ϕ_a^* in **Fig. 7.7**. Although the reference footing breadth B^* was assumed to be constant in the original formula **Eq. (7.5)** proposed by Shiraishi (1990), reference base pressure p^* would be a function of the reference secant internal friction angle ϕ_a^* . Through the reference base pressure p^* , the relationship between B^* and the soil parameters γ , ϕ_a^* and α was found out.

The stress level effect parameter β , which is the slope of $\log(2q_u/\gamma B)$ vs. $\log(\gamma B/\sigma_a)$ relationships shown in **Fig. 7.5**, is plotted against the reference secant internal friction angle ϕ_a^* in **Fig. 7.8**. The parameter β is much dependent on the confining stress effect parameter α on the shear strength, and the value of β increases with α . If the parameter α is equal to zero, which corresponds to the ordinary Mohr-Coulomb's failure criterion, β becomes zero, and the normalized ultimate bearing capacity $2q_u/\gamma B$ becomes constant as indicated in Terzaghi's original formula.

According to Maeda and Miura (1999) the parameter α would range from 0.02 to 0.12 and average 0.07 for the tested sands; therefore, it can be said that the values of β would range from 0.2 to 0.3. Thus, if information is not available about the confining stress dependency of shear strength of the ground materials, a value of β ranging between 0.2 to 0.3 may be used for estimation of ultimate bearing capacity. The 0.2 value of β , which was obtained by Shiraishi (1990) from a limited number of loading tests on piles with variable sizes, is in accordance with the above consideration.

7.4 Development of prediction method M-II

7.4.1 Finite element method

To derive a prediction method for ultimate bearing capacity, the author analyzed idealized numerical model ground by means of a Finite Element Method (FEM) developed by the author. The FEM can take account of the confining stress dependency of shear strength specified by Eqs. (2.4, 2.5).

7.4.1.1 FEM analysis

The constitutive model used was a bi-linear model incorporating the Mohr-Coulomb criteria. The model reduces maximum shear stress τ_m and shear modulus G to the failure value τ_{mf} and a ten-thousandth of the initial value of G if the maximum shear stress τ_m in an element exceeds the failure value obtained from Eq. (2.5), as shown in Figs. 7.9(a, b). To dissipate the excess shear stress over the failure criterion, the dynamic time historical calculation schema proposed by Zienkiewicz and Cormeau (1974) was incorporated. In the schema, the inertial force and quasi viscous stress by Rayleigh's method are combined to stabilize calculation processes. The time step used was equal to 0.02 sec, and the iterative calculation step was continued until the excess shear stress becomes less than a hundredth of the critical failure value in all the finite elements. Further information about this dynamic nonlinear FEM is provided in Ueno (1991).

7.4.2 Numerical Calculation of Bearing Capacity

Figure 7.10 shows the shape of a numerical model foundation ground, whose dimensions are $4.5B$ in width and $4B$ in depth; the proportion was kept identical through a series of analysis. Both plane strain condition for strip footings and axial symmetric condition for circular footings were accounted for through the series of analysis.

The properties of footings and model ground are tabulated in Table 7.1. The numerical values of the moduli of elasticity including Young's modulus E , Poisson's ratio ν and shear modulus G , were employed as typical values, because it was certified in the preliminary investigation by Ueno (1991) that the selection of these values does influence the load-displacement curves, but not the ultimate bearing capacity and ultimate stress distribution in failure zone.

Shown in **Figs. 7.11(a, b)** are typical calculation results for strip and circular footings by the FEM employed in this study: $B = 1.0\text{m}$ and 2.0m , $\phi_a^* = 40^\circ$ and $\sigma_t = 17\text{kPa}$. These figures show that the ultimate bearing capacity can be determined easily, due to the efficiency of the above mentioned algorithms for the dissipation of stress fraction exceeding failure condition and the stabilization of the calculation after the failure of the foundation ground.

Figures 7.12(a, b) show bearing capacity factors N_c and N_γ of the numerical model ground obtained by FEM analysis for the case of no stress dependency of shear strength ($\alpha = 0$). The factors N_c and N_γ were determined from the linear relationship between q_u and c shown in **Fig. 7.13**, where the slope and the intercept correspond to N_c and N_γ , respectively. This linear relationship assumed in **Eq. (7.1)** was also recognized in the parametric study with the FEM and other numerical methods; see Ueno (1991) and Pope (1975).

Bearing capacity factors given by **Eqs. (7.2, 7.4)** are also plotted in **Figs. 7.12(a, b)**. The tendency is recognized in these figures for strip footing N_c and N_γ by FEM to be smaller than those by **Eq. (7.2)** and **Eq. (7.4)** in the region of the large angle of internal friction ϕ_a .

7.4.3 Stress range in the ground beneath footing

The distributions of stress in the failure zone of the foundation ground are shown in **Figs. 7.14(a-h)**; the area of elements judged to have failed with $\tau_m > 0.98\tau_{mf}$ was arranged for normalized effective mean principal stress σ_m/σ_a in these histograms. While only limited histograms are presented in this chapter, the following useful aspects were found:

- The stress distributions are represented almost uniquely with normalized effective mean principal stress $\sigma_m^*/\gamma B$, despite the difference in footing breadth B and the strength parameters ϕ_a^* , α and σ_t as shown in **Figs. 7.14(a-h)**.
- Though the distribution pattern is different for strip and circular footings, it is clear that the stress distribution of the failure zone can be uniquely presented by $\sigma_m^*/\gamma B$ for both types of footings.
- The range of $\sigma_m^*/\gamma B$ of interest is derived from all the calculation results by the FEM including those not shown in **Figs. 7.14**; $\sigma_m^*/\gamma B$ is assumed to range from 2 to 10 for strip footing and from 1 to 15 for circular footing.

In this process the mean normal stress σ_m^* normalized by γB was selected as the best parameter for the unique representation of the stress range independently of the footing breadth B (Figs. 7.14(a, b)), the stress dependency of shear strength α (Figs. 7.14(c, d)), secant internal friction angle ϕ^* (Figs. 7.14(e, f)) and existence of cohesion $c = \tan \phi \sigma_t$ (Figs. 7.14(g, h)). If the normal stress on the failure plane σ_0 is used as suggested by Meyerhof (1950) and others, the stress distribution cannot be explained uniquely and is largely affected by cohesion c . Minor and major principal stresses, σ_3 and σ_1 , could also not be used for the same reason.

7.4.4 Proposal of a prediction method M-II for cohesive and cohesionless soils

First appropriate strength constants c and ϕ for particular foundations which should be applied to Terzaghi's bearing capacity formulae (Eq. (7.1)), are determined. As shown by the calculation results from the FEM, the stress range of interest for determining c and ϕ can be represented as

$$\sigma_m^* = [a\gamma B, b\gamma B] \quad (7.17)$$

where parameters a and b for strip and circular footings are listed in Table 7.2. A conceptual illustration for the relationship between footing breadth and confining stress range is shown in Fig. 7.15; soil elements beneath foundations are subjected to the confining stress whose magnitude depends on the breadth or diameter of footings. In this figure the breadth of Foundation-2 B_2 is three times larger than that of Foundation-1 B_1 ($=B_2/3$), and the stress range for Foundation-2 would become higher and wider than that for Foundation-1. Consequently, ϕ_2 is smaller than ϕ_1 , while c_2 is larger than c_1 .

The procedure proposed for the prediction of ultimate bearing capacity is as follows:

- Prepare the test results of shear strength obtained by laboratory element tests such as triaxial compression tests for the stress range from $a\gamma B$ to $b\gamma B$. If proper data is not available, the shear strength for the range can be estimated by interpolation or extrapolation with the aid of a confining stress dependent failure criterion by Fig. 2.6, Fig. 2.7 and Eq. (2.5).
- Obtain the appropriate shear strength parameters η and ψ on plane as shown in Fig. 7.16(a). If two failure shear stresses τ_a and τ_b corresponding to $a\gamma B$ and $b\gamma B$ are determined, c

and ϕ can be given by

$$\begin{aligned}
 \sin \phi &= \tan \psi \\
 &= \frac{\tau_a - \tau_b}{a\gamma B - b\gamma B} \\
 c &= \frac{\eta}{\cos \phi} \\
 &= \frac{\tau_a - a\gamma B \sin \phi}{\cos \phi}
 \end{aligned} \tag{7.18}$$

- Calculate ultimate bearing capacity q_u by substituting c and ϕ obtained above into the bearing capacity formulae Eqs. (7.1-7.4).

7.4.5 Verification with FEM results

By means of the FEM introduced in Section 7.4.1, the behaviors of strip and circular footings were simulated. The bearing capacity of the footings on the ground with different confining stress dependent shear strength were calculated. The conditions of the calculations are listed in Table 7.1.

Figures 7.17(a-d) and Figures 7.18(a-d) show comparisons between the normalized ultimate bearing capacity $2q_u/\gamma B$ predicted by the proposed method and that calculated by the FEM analysis. The sensitivity of the calculated values of ultimate bearing capacity were examined with respect to the parameter and for strip footings in Figs. 7.17(a-d). These comparisons presented in Figs. 7.18(a-d) show that the predicted and calculated values agree with sufficient accuracy for engineering practice under various situations for strip and circular footings. The proposed method can predict the trends of the reduction in normalized bearing capacity $2q_u/\gamma B$ with increasing footing breadth for non-cohesive soils with confining stress dependent shear strength as shown in Figs. 7.18(a-d), and it seems also to be applicable to cohesive soils with tensile strength as shown in Figs. 7.18(e, f). While only limited cases are presented in Figs. 7.18(a-f), the good agreement between the predicted and calculated values can be recognized for circular footings as well as strip footings.

7.5 Verification of the proposed prediction methods

The proposed estimation method for the bearing capacity of surface footing is verified in this section; experimental results presented in Fig. 6.1 and Table 7.3 are used for this verification. The characteristics of shear strength of the materials used are presented in Figs. 2.8(a, b) and Fig. 2.9, which were obtained by conventional drained triaxial compression tests. Even

with some scatters included, the test results were fitted to the confining stress dependent failure criteria **Eq. (7.8)**, and the parameters $\tan \phi_a^*$ and α were determined by means of the least square method.

The surcharge term in the bearing capacity formula vanishes, because all the footings are on the surface. For method M-I, bearing capacity parameters $N_{\gamma}^*, p^*(= \gamma B^*)$ and β were determined using **Figs. 7.6, 7.7 and 7.8**. For method M-II, strength parameters c and ϕ for the stress range of interest from $\sigma_m = a\gamma B$ to $b\gamma B$ were determined and substituted into the bearing capacity formula **Eq. (7.1)**. The values of bearing capacity factors N_c and N_{γ} were evaluated by **Eq. (7.3)** and **Eq. (7.4)**, except for Series BS. For Series BS, Hansen's formula of N_{γ} presented in **Eq. (7.19)**, was used instead of **Eq. (7.4)** to make a comparison with the prediction by Kutter *et al.* (1988).

$$N_{\gamma} = 1.5(N_q - 1) \tan \phi \quad (7.19)$$

Values of these parameters for bearing capacity calculation are tabulated in **Table 7.4** and **Table 7.5**. The following five paragraphs are brief descriptions of these verifications.

[Series TS] Series TS is a series of centrifuge model loading tests on surface strip footings by Okahara *et al.* (1988). The frictional condition of the footing base without any lubrication can be assumed to be rough. The sand used in Series TS was air-dried Toyoura Sand, and three different relative densities D_r of 58%, 74% and 88% were employed.

The strength characteristics of Toyoura Sand is cited from Miura (1981), here the confining stress dependency of shear strength was considered. Assuming independence between confining stress dependency and density dependency, we determined the shear strength constants as a function of effective mean normal stress σ_m and relative density D_r

$$\left\{ \begin{array}{l} \sigma_r' = 196 \text{ kPa} \\ \tan \phi_a^* = 0.425 \times 10^{-2} D_r + 0.500 \\ \sigma_a = \frac{\sigma_r'}{1 - \sin \phi_a^*} \\ \tan \phi^* = \tan \phi_a^* \left(\frac{\sigma_a}{\sigma_m} \right)^{0.050} \end{array} \right. \quad (7.20)$$

The comparison between estimated and measured $2q_u/\gamma B$ is shown in **Fig. 7.19** and **Table 7.4**. Measured values lie between the two types of estimation. The method M-I provides a lower bound, while the method M-II gives an upper.

Good agreements can be seen between the measured values and the values for $D_r=58\%$ calculated by method M-I. In the cases of $D_r=74\%$ and $D_r=88\%$, the calculated values tend to be underestimated slightly; however, the degree of stress level dependency of ultimate bearing capacities is almost the same both in the measured and calculated bearing capacities.

[Series TC] Series TC was quoted from the results of centrifuge model loading tests conducted by the author (Ueno *et al.* (1994)) on a surface circular footing. The sand used was Toyoura Sand, so that the expression of shear strength used is identical to that for Series TS. Therefore, it can be expected that these test results are suitable to verify the applicability of the proposed methods under various conditions for identical ground material.

Comparisons between estimated and measured values are shown in **Fig. 7.20**. Prediction by method M-I, assuming the shape factor of $S_\gamma = 0.6$, fits a lower bound of the measured values, on the other hand prediction by M-II gives an upper bound as well as in the case of Series TS. The tendencies of decrease in $2q_u/\gamma B$ with an increase in B by both methods of prediction and the experiments are in good agreement.

For practical estimation of the bearing capacity of circular footings, estimation of strip footing with a shape factor $S_\gamma = 0.6$ is acceptable. From the comparisons in Series TS and Series TC, it can be said that the methods M-I and M-II accurately estimate the ultimate bearing capacity of circular and strip footings with regard to stress level effects.

[Series IC] Series IC, conducted by Kusakabe *et al.* (1991), is a series of centrifuge loading tests on surface circular footings on air dried Inagi Sand (IN) having no cohesion. The confining stress dependent shear strength of Inagi Sand was examined from the results of conventional triaxial compression tests reported by Kusakabe *et al.* (1991).

The normalized bearing capacities, $2q_u/\gamma B$, obtained from the estimation method and the loading tests are compared in **Fig. 7.21**. In this series the estimation, with a shape factor $S_\gamma = 0.6$, tends to slightly underestimate q_u . There are few discrepancies between the estimation obtained by two methods. The tendency of the reduction in $2q_u/\gamma B$ with increasing footing breadth B is in good agreement with both estimated and measured values.

[Series BS] For Series BS, centrifuge model tests were conducted by Kutter *et al.* (1988) on surface circular footing on dense Monterey 0/30 Sand. Shear strength properties of Monterey 0/30 Sand were also reported.

Kutter *et al.* (1988) demonstrated the accuracy of their prediction method compared with the methods proposed by Meyerhof (1950) and De Beer (1963). **Table 7.6** shows comparisons between the author's estimation and that by Kutter *et al.* (1988). The estimation for the circular base, assuming a shape factor of $S_\gamma=0.6$, shows good agreement with the measured value as well as Kutter's estimation.

[**Series GS**] Series GS was a series of loading tests, using surface strip footing with a rough base, conducted under 1g and 50g of accelerations, by Hirano (1994) under the supervision of Ueno. The width of footing was 30mm, and the model ground was made of uniform glass beads with $D_{50}=0.115\text{mm}$ and $U_c=1.091$. Due to their spherical shape and negligible crushability of the glass beads, this material shows less confining stress dependency of shear strength compared with sand (see Maeda and Miura, 1999).

A significant agreement is evident between the calculated and measured values. Comparisons between estimated and measured ultimate bearing capacities are presented in **Fig. 7.22**. Ultimate bearing capacity shows a low degree of stress level dependency compared with other series on the footings on sand. This tendency is related to the less confining stress dependency of shear stress of glass beads. As shown in this figure, applicability of the proposed estimation method to soils with a wide range of stress level dependency of bearing capacity is recognized.

The evaluation method proposed by Ueno *et al.* (1998) was also applied to Series GS; the method overestimates to a certain degree although both of the methods are practicable.

[**Series SQ**] Series SQ was conducted as part of a series of large-scaled in situ loading tests by Kusakabe *et al.* (1992); square footings with different breadths of $B = 0.3\text{m}$ to 1.3m on a natural deposit were loaded in a pneumatic caisson. The submerged stratum on which the loading tests were conducted consists of Shirahata scoria, which is a kind of uniform volcanic granular soil. Because the Shirahata scoria is a diluvial deposit and cemented, it has notable tensile strength of $\sigma_t = 147\text{kPa}$:

$$\tan \phi^* = 0.816 \left(\frac{\sigma_a}{\sigma'_m + \sigma_t} \right)^{0.023} \quad (7.21)$$

The comparison between measured ultimate bearing capacities and their predictions by method M-II is presented in **Fig. 7.23**. Series SQ is the only case where the footings were placed on the foundation ground with cohesion; as shown in this figure applicability of the

proposed prediction method to cohesive ground as well as non-cohesive foundation ground can be confirmed.

7.6 Summary

Two prediction methods for the ultimate bearing capacities of surface footings were derived. In these methods, the stress level effects were taken into account. The concept of the predication methods proposed is that the stress level dependency of the ultimate bearing capacity is attributed to the confining stress dependency of shear strength of ground material.

This relationship was investigated through a comprehensive parametric examination by means of the extended slip line method. The following modified bearing capacity formula was derived from various independent loading tests:

$$q_u = S_\gamma \frac{\gamma B}{2} N_\gamma^* \left(\frac{B}{B^*} \right)^{-\beta} \quad (7.22)$$

The parameters N_γ^* , p^* , B^* and β were associated with the parameters in nonlinear failure criterion with confining stress dependency, ϕ_a^* and α . Thus, the parameters N_γ^* , p^* , B^* and β can be determined from a series of laboratory element tests for shear strength under variable confining stress using **Figs. 7.6, 7.7 and 7.8**. From the test results on shear strength of various sands, it can be said that the parameter β in the formula would be from 0.2 to 0.3 for ordinary natural sands. Also, results obtained from a series of in situ loading tests on footings with variable size can be used to determine the parameters in **Eq. (7.22)**.

The applicability of the proposed method was verified with experimental results of several series of centrifuge loading tests on the footings on sands and glass beads reported by the authors and other researchers. Through the verifications of the proposed method, it was confirmed that the method proposed by the authors provides useful estimation of ultimate bearing capacities attention to stress level effects for surface strip and circular footings on sand. The ultimate bearing capacity of circular footings can be estimated based on the results of slip line methods in the plane condition with the aid of a shape factor of $S_\gamma = 0.6$ for practical use.

The author developed a new simple prediction method of ultimate bearing capacities of surface footings with regard to size effects. Both strip and circular footings were considered in this chapter. The concept of the prediction method proposed is quite simple: a rational

method for determining shear strength constants c and ϕ in accordance with the confining stress range corresponding to footing breadth. In this process the confining stress dependent shear strength is taken into account.

The Finite Element Method which features a confining stress dependent failure criterion was used to derive the appropriate stress range of interest. Consequently, it was shown that the confining stress beneath the footing at failure ranges from $\sigma_m^* = 2\gamma B$ to $10\gamma B$ for strip footings and from $\sigma_m^* = 1\gamma B$ to $\sigma_m^* = 15\gamma B$ for circular footings.

The advantageous features of the proposed method are:

- The stress range to be interested in for the determination of shear strength can be determined directly from footing breadth B and unit weight γ of the foundation ground, without iteration process. Therefore, soil testing for determining shear strength can be easily planned; the proposed confining stress dependent failure criterion is available for examining shear strength properties.
- The proposed method is applicable to strip and circular footings.
- Foundation ground with notable cohesion as well as those without cohesion can be dealt with uniquely; the confining stress dependent failure criterion is incorporated, which is applicable to both cohesionless and cohesive materials.

The applicability of the proposed method was examined with the results of FEM simulation and the experimental results reported by some researchers, which include centrifuge loading tests and an in-situ loading test on the strip, circular and square footings on the cohesive and cohesionless foundation ground. From the verification of the proposed method, it was confirmed that the method proposed by the author gives admissible prediction of ultimate bearing capacities with attention to the size effects for various types of surface footings and foundation ground.

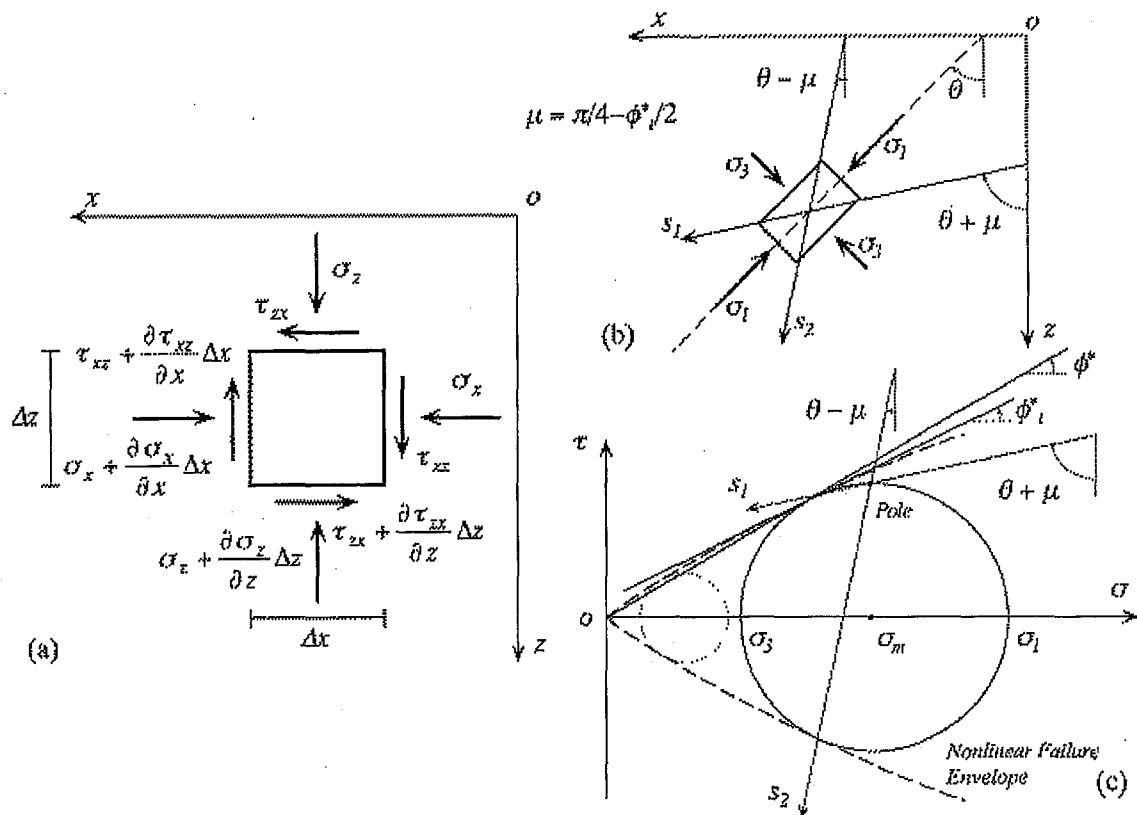


Figure 7.1: Definition of stress condition in a soil element; (a) element in $x-z$ coordinate system, (b) principal stress and slip lines S_1 and S_2 , (c) Mohr's stress circle.

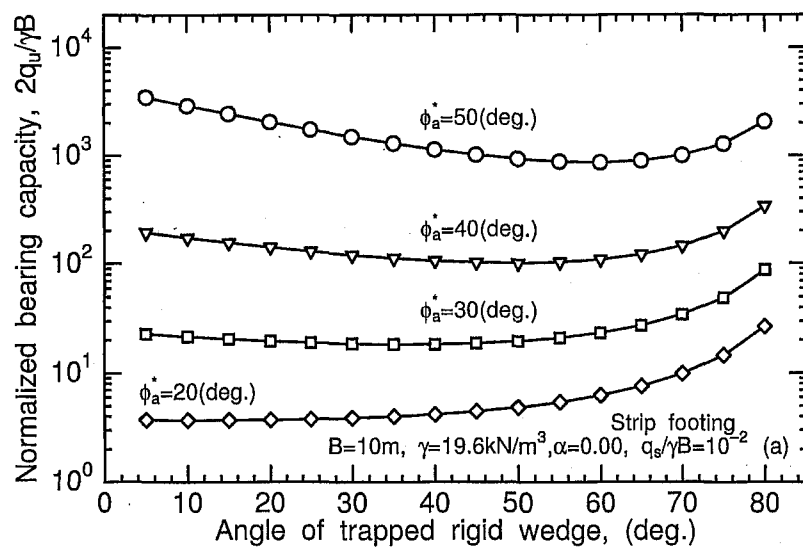


Figure 7.2: The relationships between calculated normalized bearing capacity $2q_u/\gamma B$ and the angle of trapped rigid wedge.

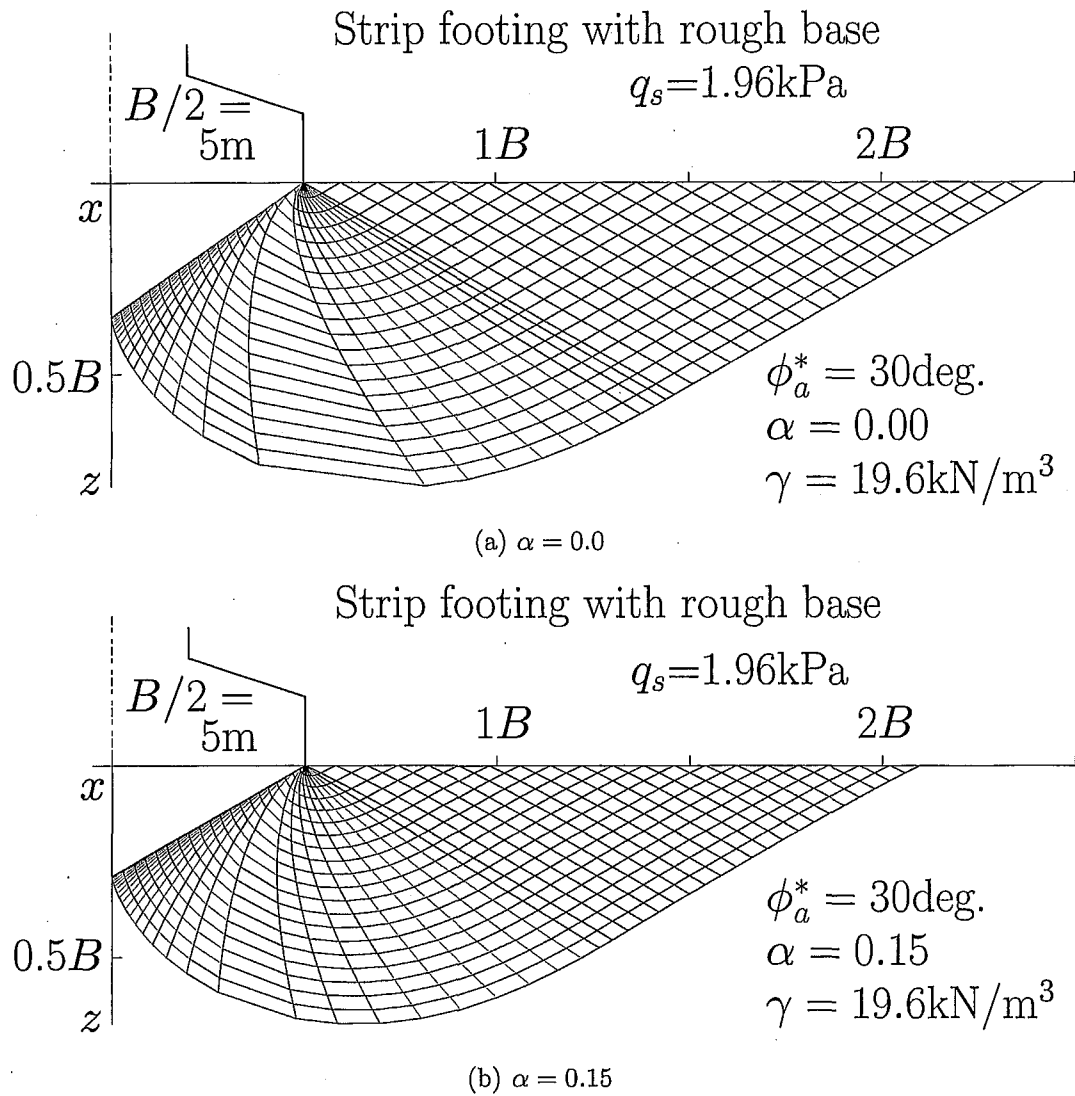


Figure 7.3: Plastic regions and slip lines under a strip footing calculated with extended slip line method with $\phi_a^* = 30 \text{ deg.}$ and surcharge $q_s = 1.96 \text{ kPa}$; (a) with $\alpha = 0.0$, (b) with $\alpha = 0.15$.

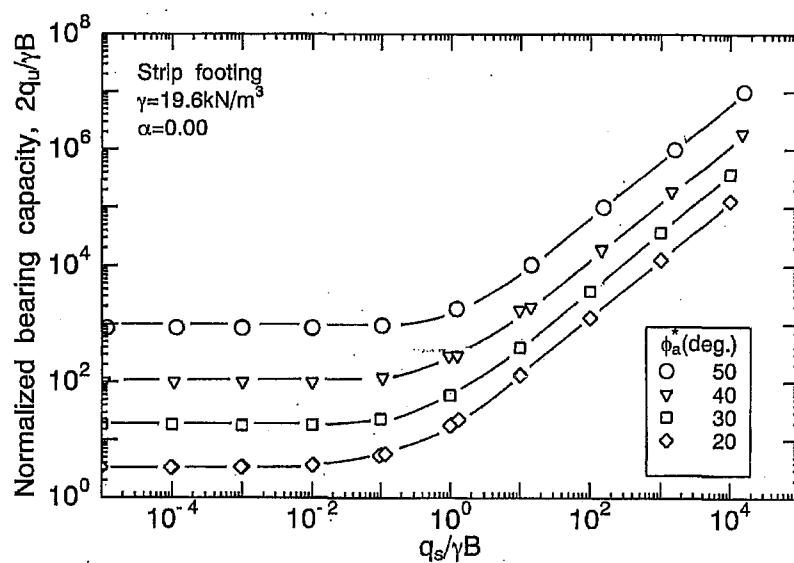


Figure 7.4: Convergence of solution for zero surcharge surface strip footings.

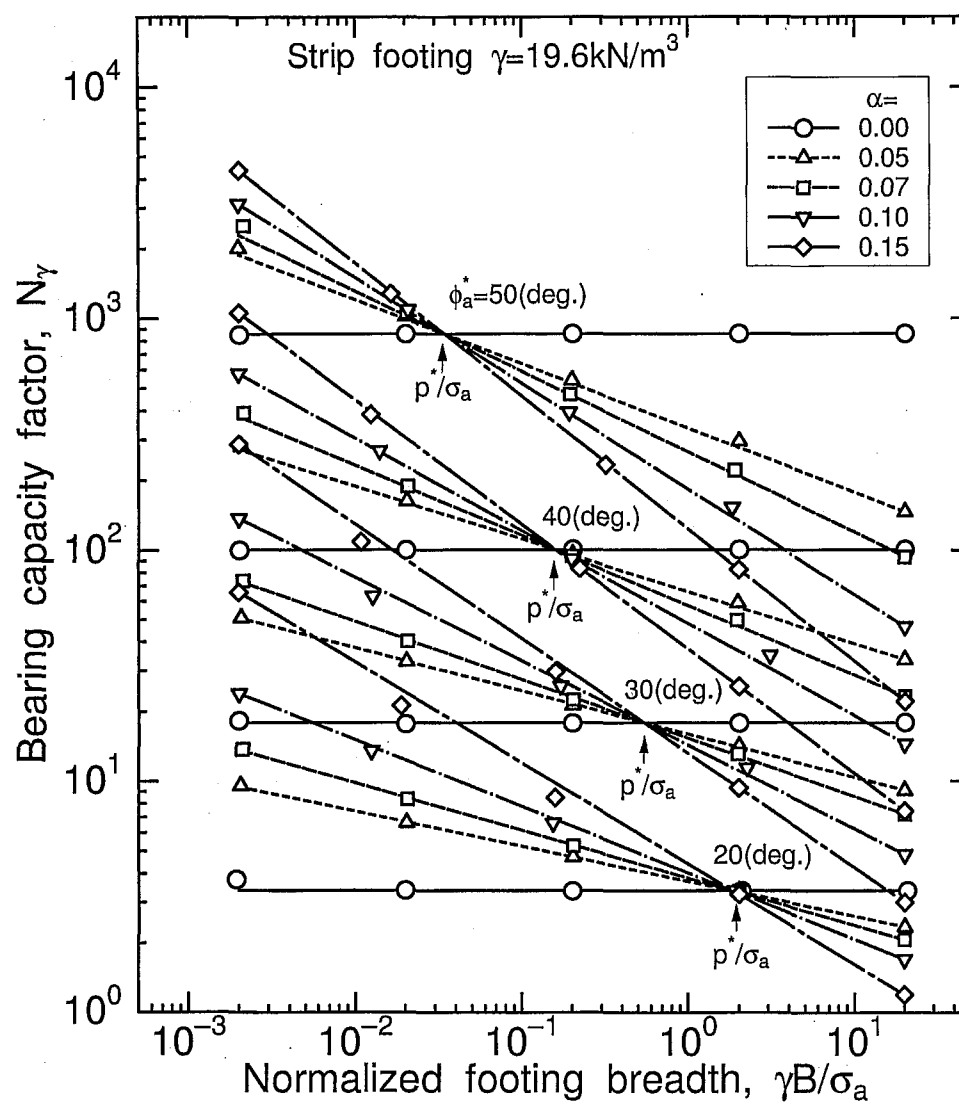


Figure 7.5: Calculated relationships between $N_\gamma (= 2q_u/\gamma B)$ and γB .

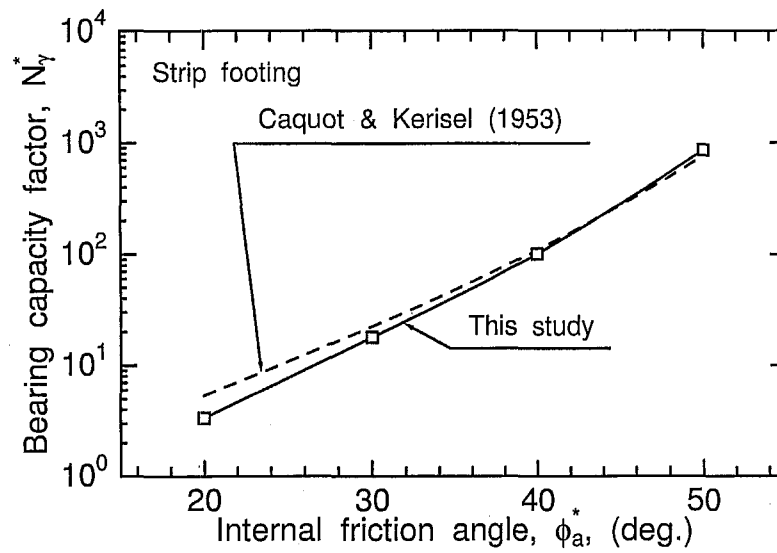


Figure 7.6: Relationship of reference bearing capacity factor N_γ^* with reference secant internal friction angle ϕ_a^* .

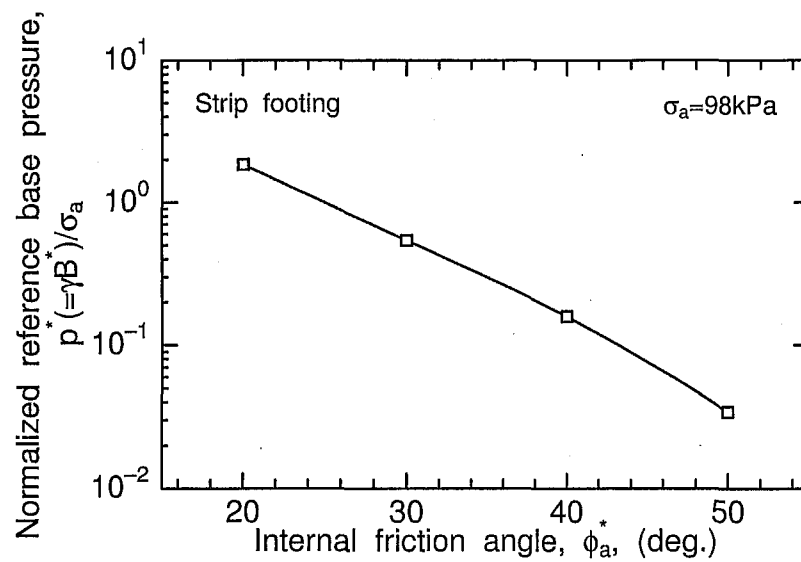


Figure 7.7: Relationship of reference base pressure p^* with reference secant internal friction angle ϕ_a^* .

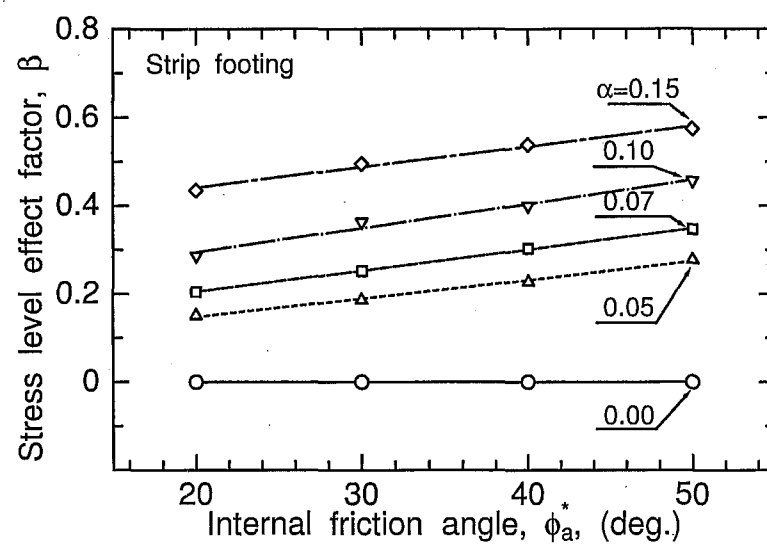
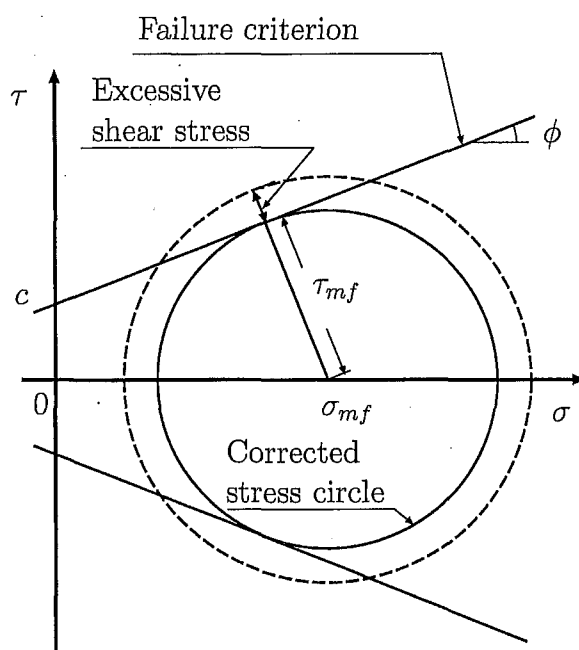
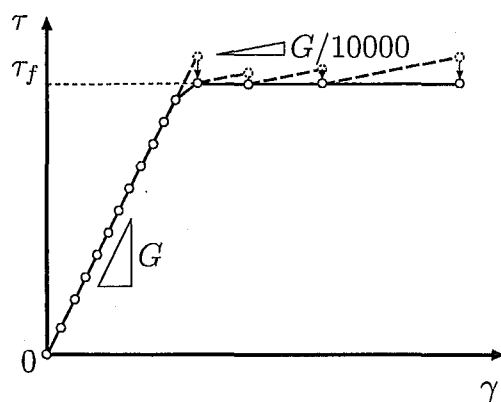


Figure 7.8: Relationship of stress level effect factor β with reference secant internal friction angle ϕ_a^* .



(a) Definition of excessive shear stress with respect to failure criterion.



(b) Correction of excess shear stress in the FEM.

Figure 7.9: Explanation of excessive shear stress.

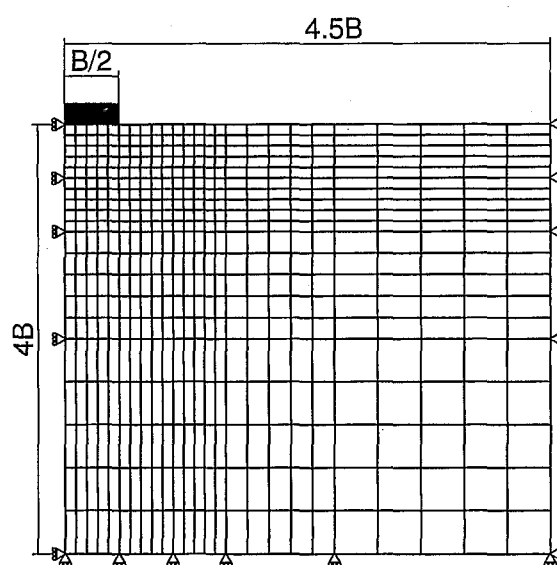
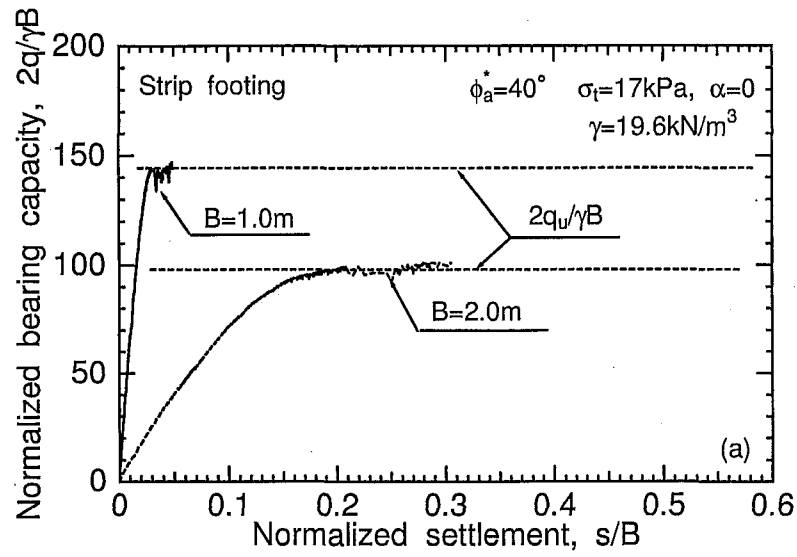
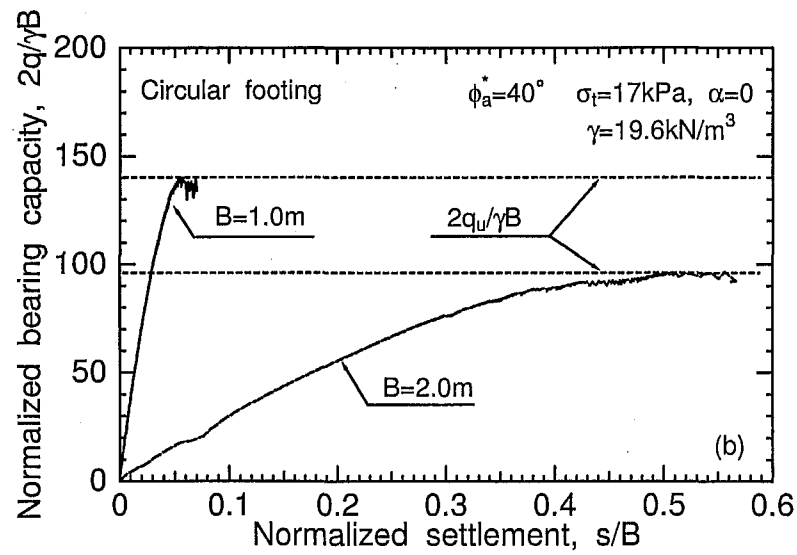


Figure 7.10: Geometry of numerical model ground and footing used for the calculation by FEM.



(a) Strip footing



(b) Circular footing

Figure 7.11: Typical calculation results by FEM. (b) circular footings.

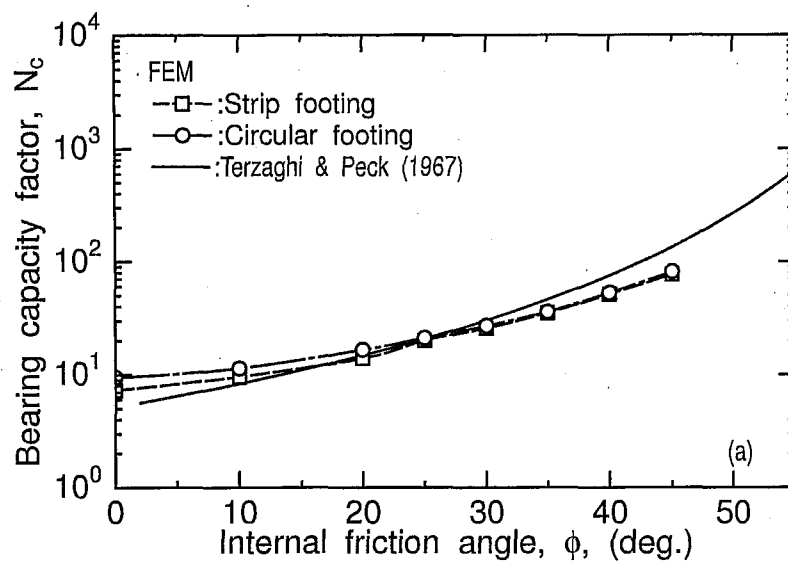
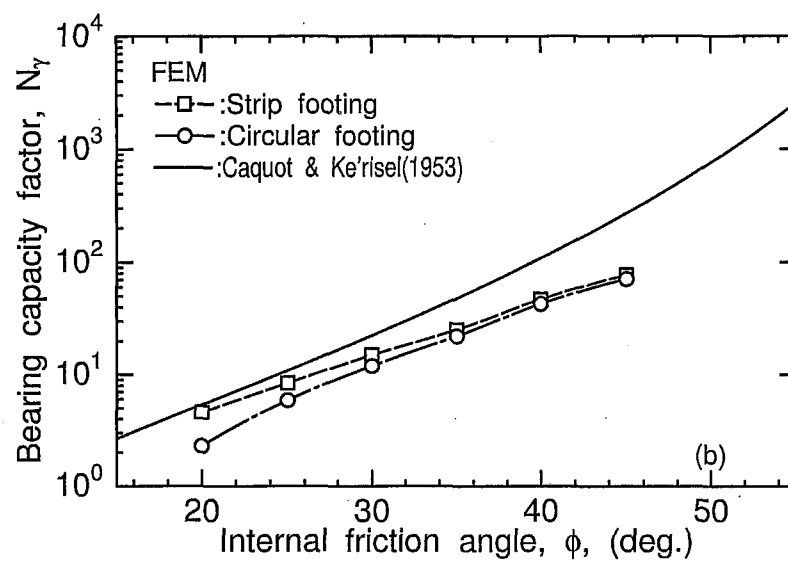
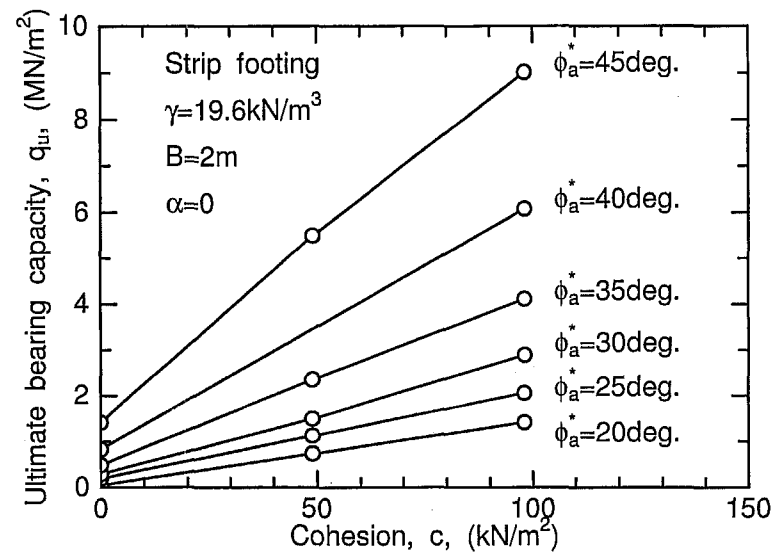
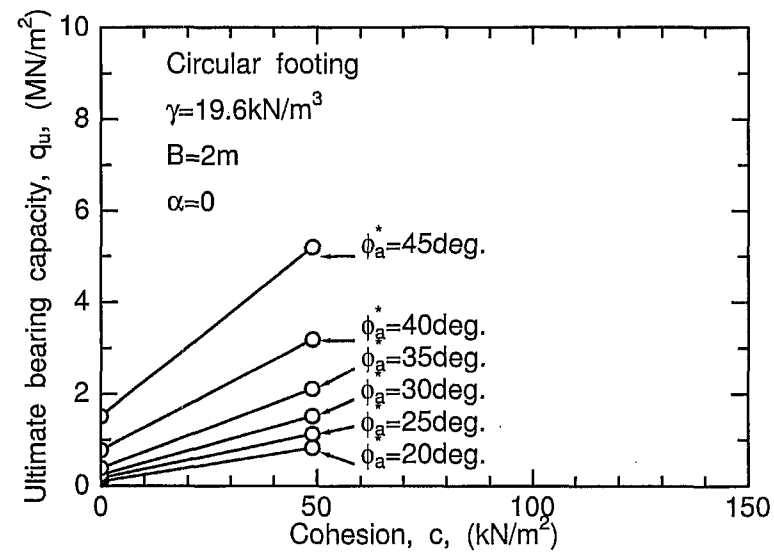
(a) N_c (b) N_γ

Figure 7.12: Comparisons of bearing capacity factors obtained from FEM analysis and calculated from Terzaghi and Peck (1967) and Caquot and Kérisel(1953).



(a) Strip footing.



(b) Circular footing.

Figure 7.13: Ultimate bearing capacity q_u under various conditions.

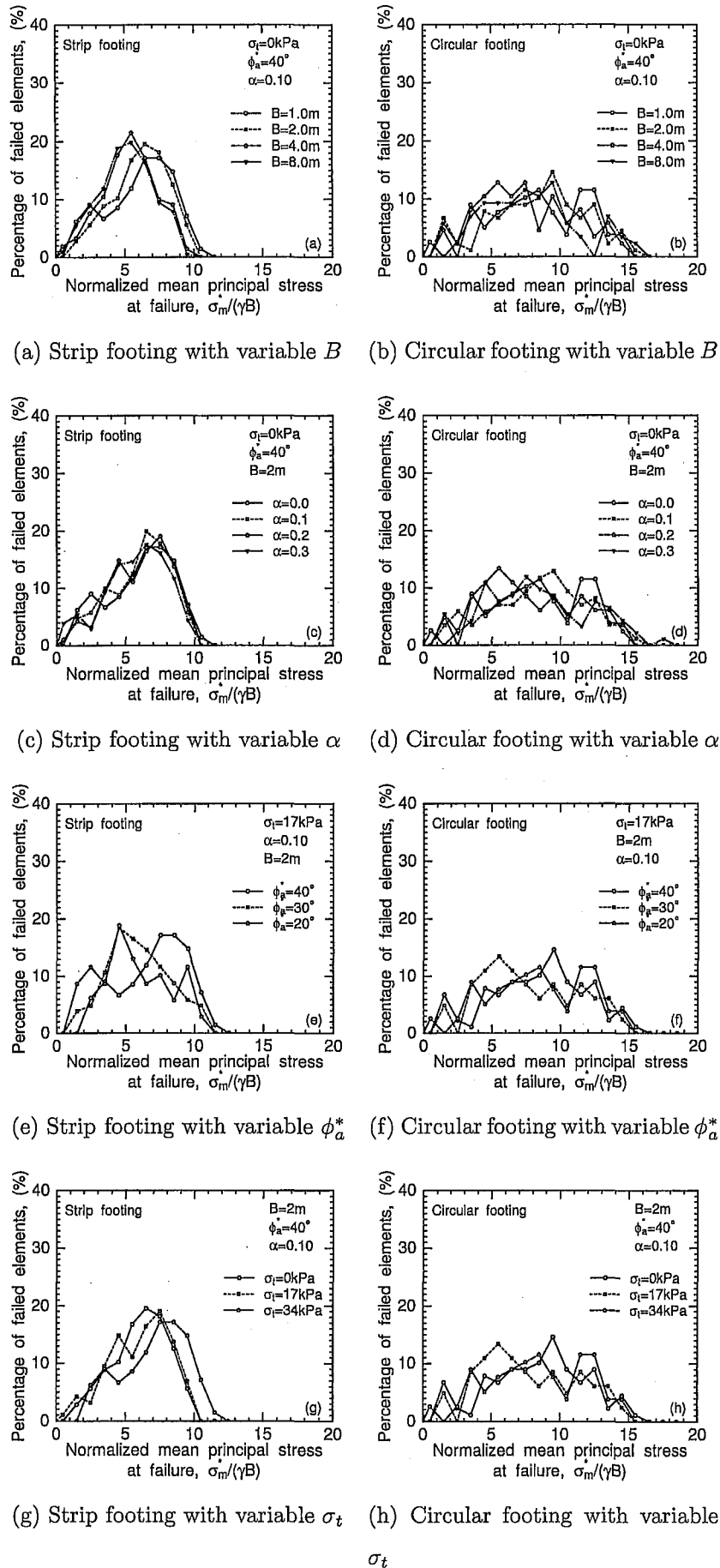


Figure 7.14: Histograms for distribution of effective mean principal stress in failed elements.

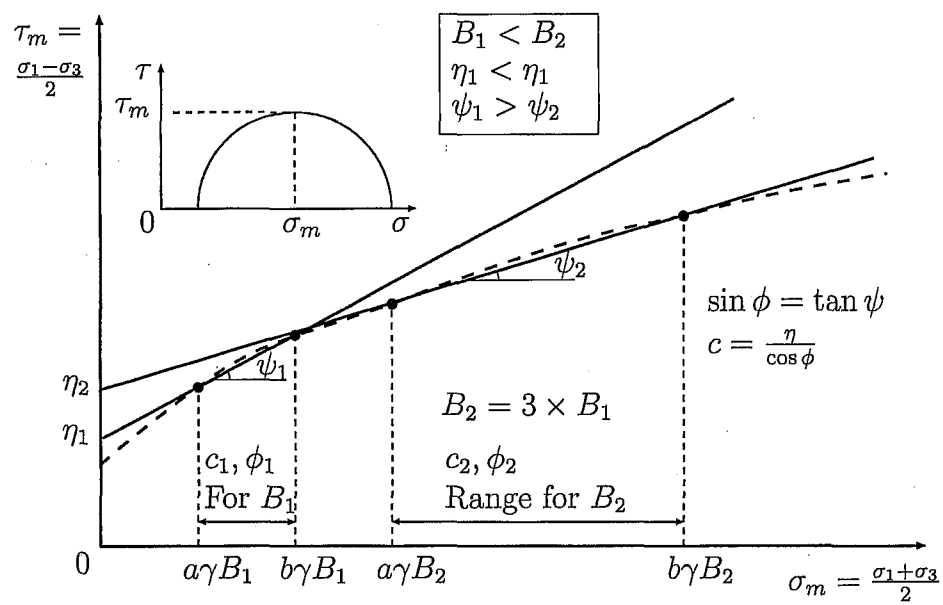


Figure 7.15: Conceptual illustration of size effect on bearing capacity related to confining stress dependency of shear strength of ground.

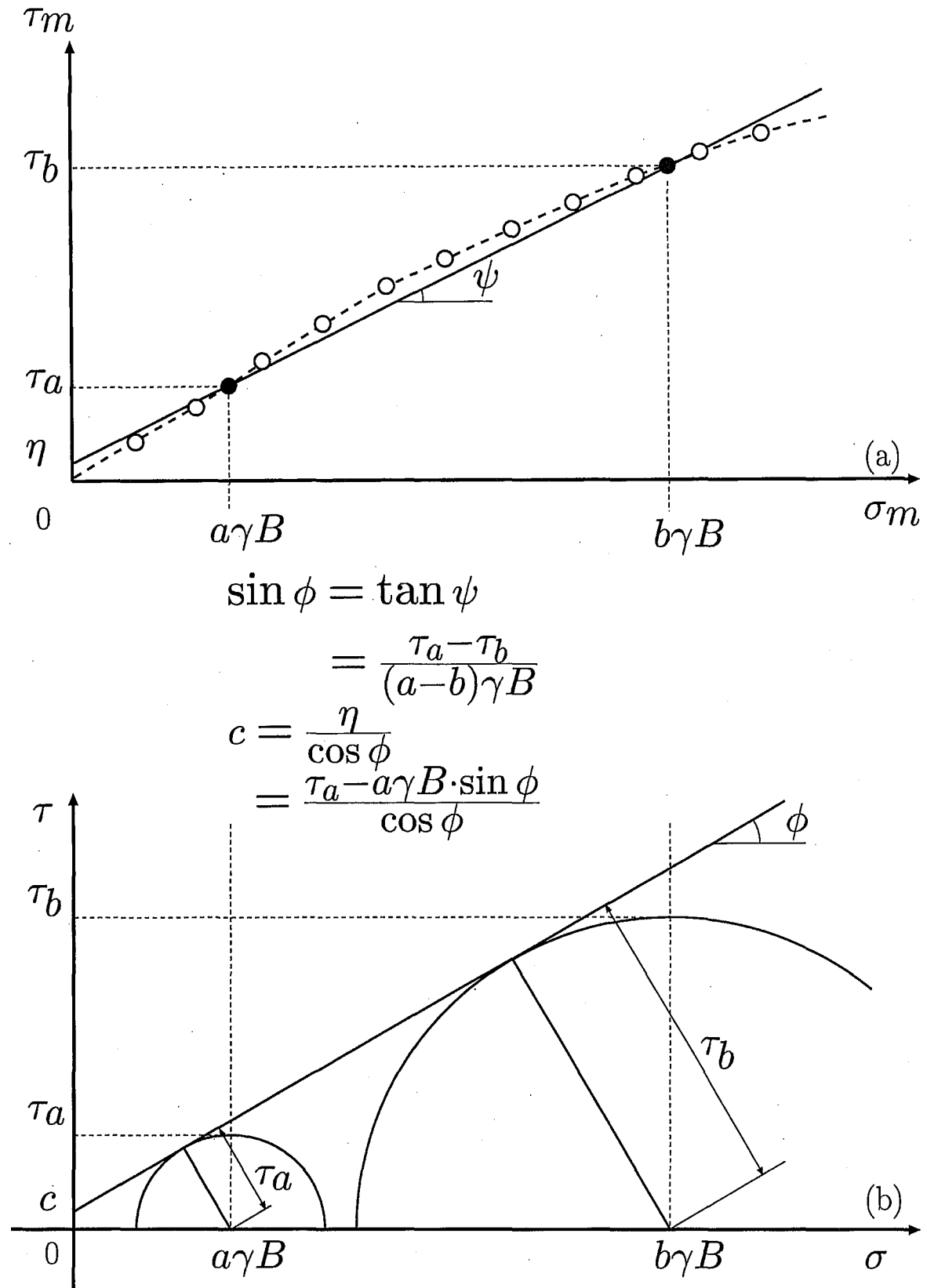


Figure 7.16: Illustration of determination method of c and ϕ ; (a) in $\tau_m - \sigma_m$ plane, (b) in $\tau - \sigma$ plane.

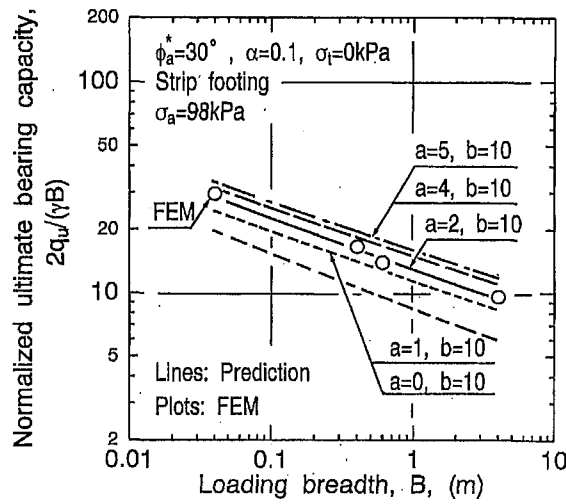
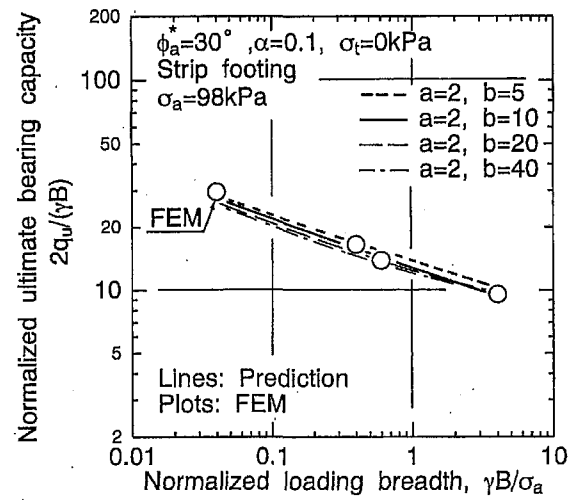
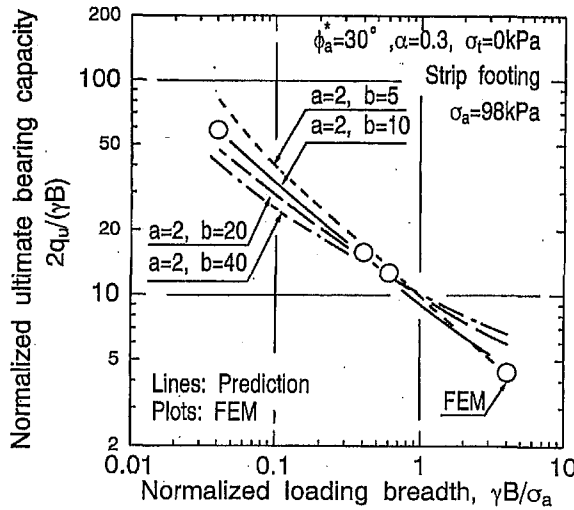
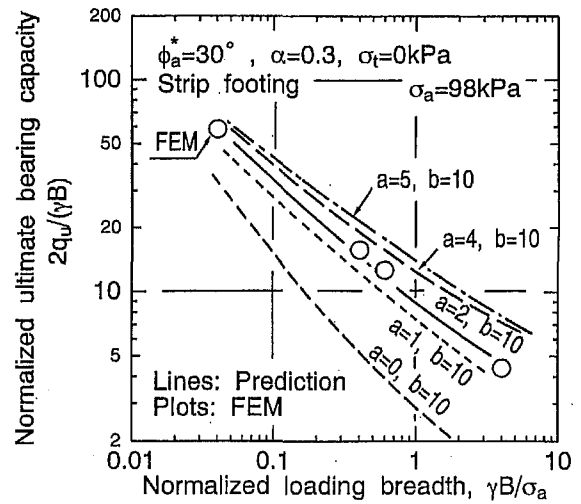
(a) $\alpha = 0.1$ with variable a (b) $\alpha = 0.1$ with variable b (c) $\alpha = 0.3$ with variable a (d) $\alpha = 0.3$ with variable b

Figure 7.17: Sensitivity of calculated ultimate bearing capacities for strip footing with respect to parameters α and β .

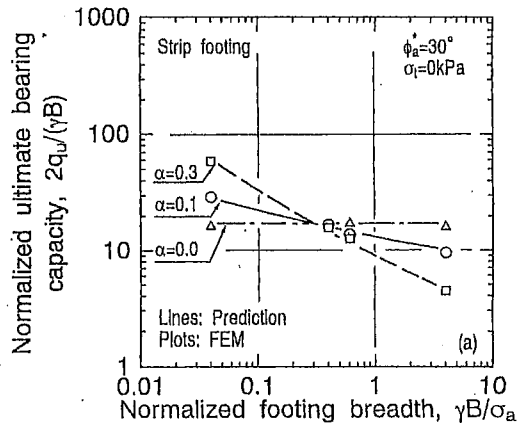
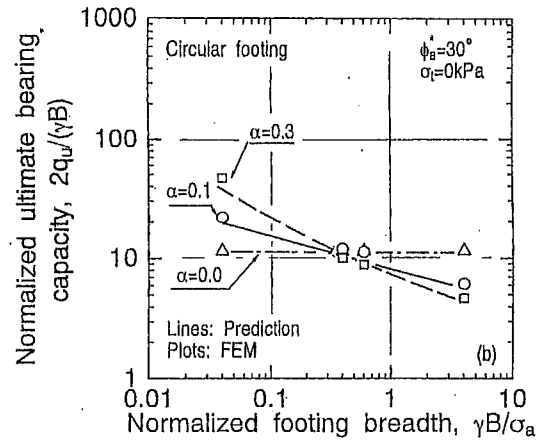
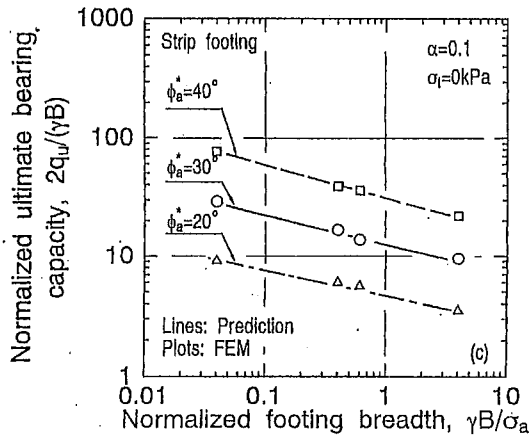
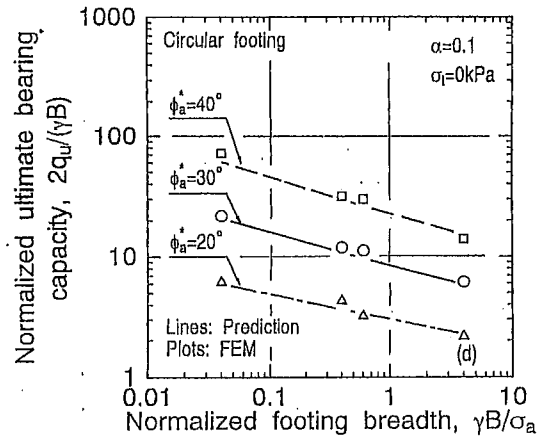
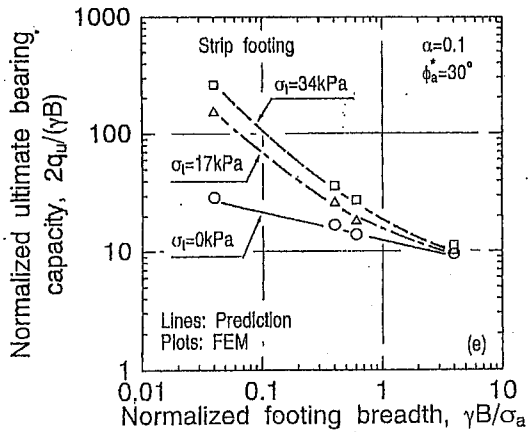
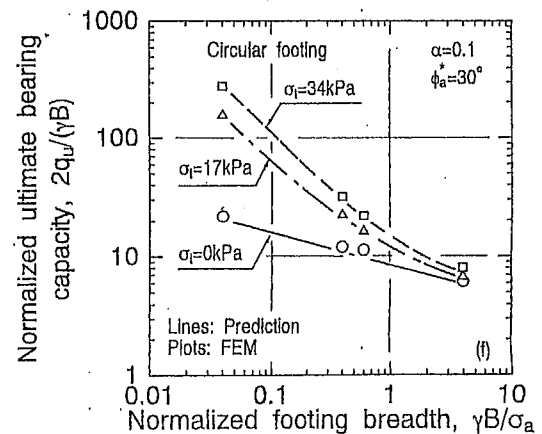
(a) Strip footing with variable α (b) Circular footing with variable α (c) Strip footing with variable ϕ_a^* (d) Circular footing with variable ϕ_a^* (e) Strip footing with variable σ_t (f) Circular footing with variable σ_t

Figure 7.18: Comparisons between FEM analysis and proposed method.

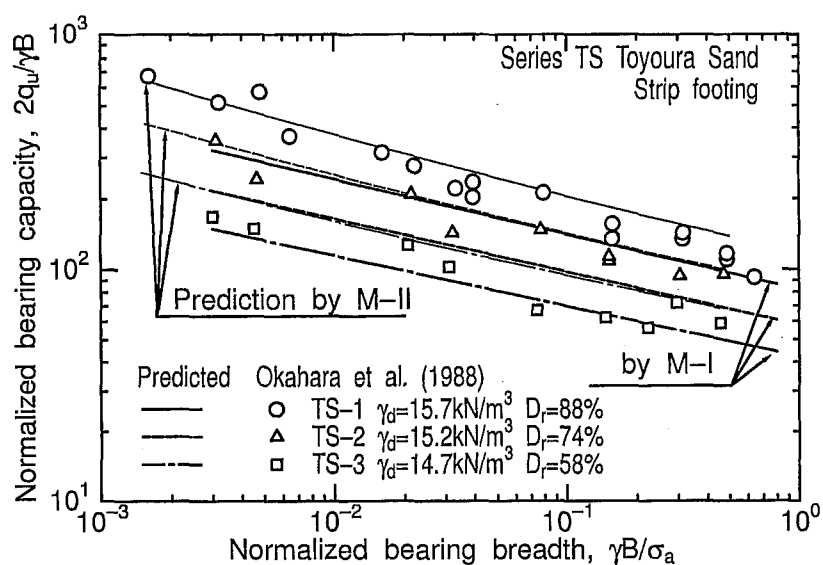


Figure 7.19: Comparison of estimated and measured ultimate bearing capacities for Series TS; strip footing.

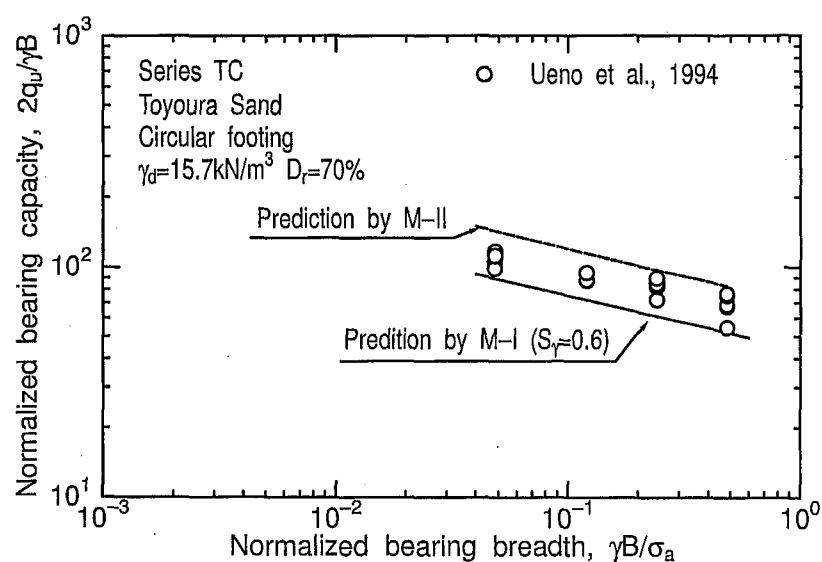


Figure 7.20: Comparison of estimated and measured ultimate bearing capacities for Series TC; circular footing.

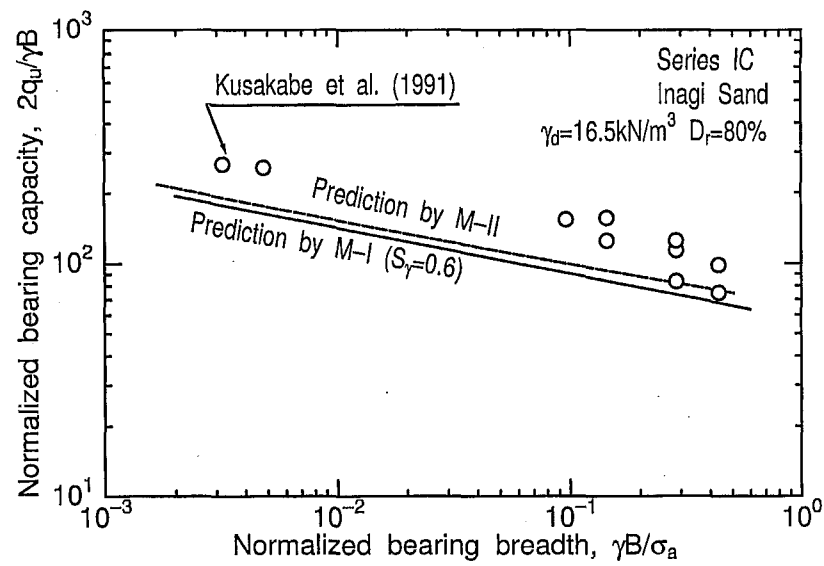


Figure 7.21: Comparison of estimated and measured ultimate bearing capacities for Series IC; circular footing.

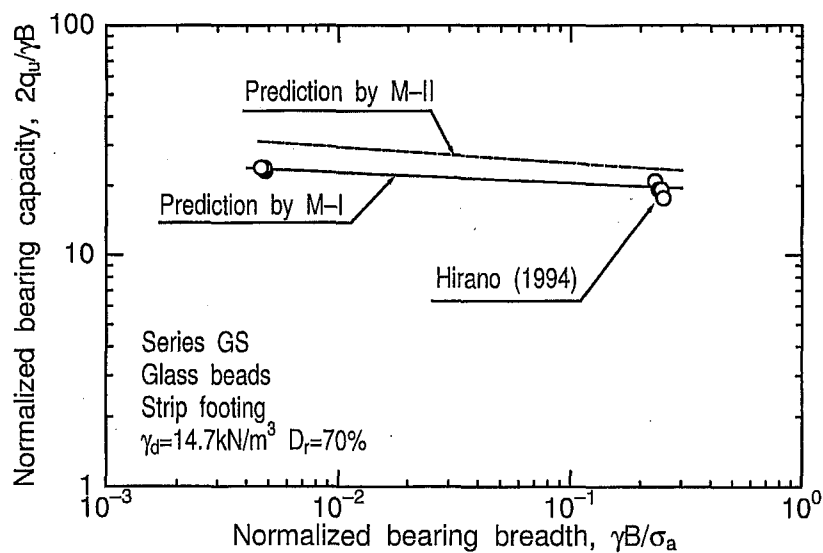


Figure 7.22: Comparison of estimated and measured ultimate bearing capacities for Series GS; strip footing.

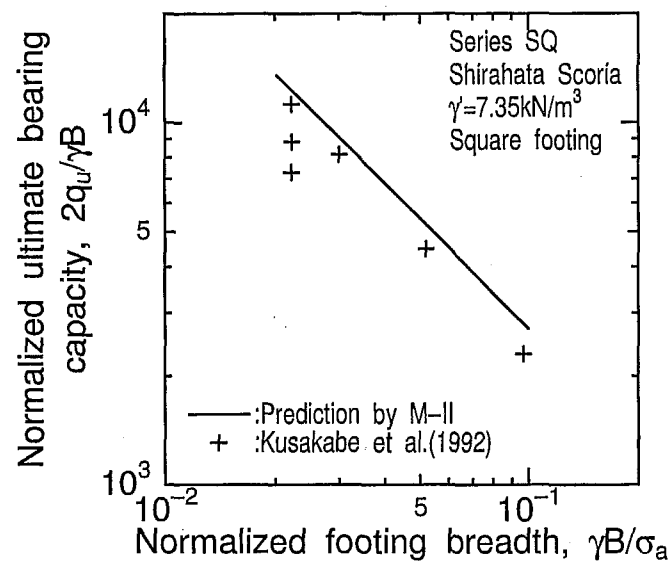


Figure 7.23: Comparison of estimated and measured ultimate bearing capacities for Series SQ; square footing.

Table 7.1: Conditions of performed parametric FEM analysis.

Footing shape	strip and circle
Footing breadth $B(\text{m})$	0.2 – 20
Unit weight $\gamma(\text{kN/m}^3)$	19.6
E_a	1000
m	0.5
Poisson's ratio ν	0.3, 0.4
Friction angle $\phi^*(deg)$	0 – 45
Tensile strength $\sigma_t(\text{kPa})$	0 – 98
Stress dependency α	0 – 0.5

Table 7.2: Stress ranges of interest for determining shear strength constants c and ϕ , for strip footing and circular footing.

Footing	$\sigma'_m/\gamma B$	
	a	b
Strip footing	2	10
Circular footing	1	15

Table 7.3: Loading tests used for evaluation

Series	Footing	Method	Soil	$D_r(\%)$	Citation
TS-1	Strip	Centrifuge	Toyoura sand	88%	Okahara <i>et al.</i> (1988)
TS-2				74%	
TS-3				58%	
TC	Circular	Centrifuge	Toyoura sand	70%	Chapter6, Ueno <i>et al.</i> (1994)
IC	Circular	Centrifuge	Inagi sand	80%	Kusakabe <i>et al.</i> (1991)
GS	Strip	Centrifuge	Glass beads	70%	Chapter6, Hirano (1994)
BS	Circular	Centrifuge	Monterey 0/30 sand	93–95%	Kutter <i>et al.</i> (1988)
SQ	Square	<i>in-situ</i>	Shirahata Scoria	—	Kusakabe <i>et al.</i> (1992)

Table 7.4: Parameters for method M-I.

Series	ϕ_a^* (deg.)	$\tan \phi_a^*$	α	Rough base		β	S_γ
				N_γ^*	p^*/σ_a		
TS-1	41.2	0.874	0.050	132.	0.143	0.237	1.0
TS-2	39.2	0.815	0.050	85.5	0.176	0.228	1.0
TS-3	36.7	0.747	0.050	57.5	0.240	0.218	1.0
GS	30.4	0.587	0.019	19.1	0.513	0.048	1.0
TC	40.7	0.861	0.050	115.	0.143	0.235	0.6
IC	41.9	0.897	0.038	145.	0.120	0.197	0.6
BS	49.7	1.177	0.089	780.	0.0347	0.415	0.6

Table 7.5: Predictions obtained from M-II

Series	B (m)	c (kPa)	ϕ (Deg.)	Bearing factors		q_u (MPa)
				N_c	N_γ	
TS-1	0.03	0.05	47.3	181.	428.	0.109
$\gamma =$	0.30	0.46	44.0	118.	224.	0.582
15.7kN/m ³	3.00	4.59	40.7	81.2	124.	3.28
TS-2	0.03	0.04	45.2	138.	284.	0.071
$\gamma =$	0.30	0.45	41.9	93.0	154.	0.391
15.2kN/m ³	3.00	4.41	38.7	65.6	87.3	2.28
TS-3	0.03	0.04	42.7	101.	176.	0.043
$\gamma =$	0.30	0.43	39.4	70.8	98.9	0.249
14.7kN/m ³	3.00	4.21	36.2	51.6	58.3	1.50
TC	0.03	0.03	46.4	160.	358.	0.061
$\gamma =$	0.30	0.33	43.1	106.	190.	0.336
15.7kN/m ³	3.00	3.28	39.8	74.0	106.	1.95
IC	0.03	0.03	44.5	126.	247.	0.043
$\gamma =$	0.30	0.26	42.0	113.	156.	0.278
16.5kN/m ³	3.00	2.63	39.5	71.6	101.	1.84
SQ	0.30	118.	38.2	62.8	81.1	13.4
$\gamma =$	0.40	118.	38.2	62.7	80.8	13.4
7.35kN/m ³	0.70	118.	38.2	62.3	80.0	13.3
	1.30	117.	38.1	61.7	78.7	12.2

Table 7.6: Prediction and its comparison for Series BS

	q_u (MPa)	
	B=0.96m	B=1.91m
Measured ¹⁾	2.01, 2.06	3.05, 3.09
Kutter ¹⁾	2.32	2.99
Prediction by M-I ($S_\gamma = 0.6$)	2.23	3.33
Prediction by M-II	2.01	2.92
Note: ¹⁾ cited from Kutter <i>et al.</i> (1988)		

Chapter 8

Conclusions

8.1 Development of a measuring method of displacement field for sand by image processing

The author developed a measuring method of displacement field for sand by image processing. In this thesis, the method, named CCIP method, was introduced in Chapter 3. CCIP method consists of two parts. The first part, named CC method, is pattern recognition. A cross correlation of two images is used to evaluate the similarity between the original and the destination images. The second part, named IP method, is a newly developed coordinate transformation method. Using IP method, we can transform image coordinate system to real coordinate systems, and correct distortion of images occurred during exposing and digitizing sequences. With an aid of IP method, CCIP method requires no target installed in the model to measure displacement field on sand. Only coordinates of four fixed points covering the analyzing zone are required to determine real coordinates. These features can reduce labor in the preparation of model sand and the analysis of displacement fields.

According to evaluation of accuracy based on a series of numerical experiments, the following conclusions are drawn from the results.

1. The size of the referential image is required at least 11×11 pixels.
2. The accuracy becomes suddenly worse as the size of referential image becomes smaller than the size of sand particles. Therefore, the size of referential image should be grater than the size of sand particles.

3. Applicable range of deformation of CC Method between two images is 15%. Within this range of deformation no significant differences in accuracy appear among different sizes of referential image.
4. Considering above three points, the author can provide the following recommendation. Setting the size of referential image equal to the size of sand particles, exposing photographs of each 15% of deformation, and employing CC Method, we can measure displacement on model sand within an accuracy of two pixels.

8.2 Influence of boundary and initial conditions

8.2.1 On friction and lubrication

The author examined the friction acting on lubricated and non-lubricated metal by means of the direct shear test in Chapter 4. The influence of the friction on vertical stress distributions was evaluated. The following comments were derived from the results.

1. Lubrication made from a thin latex membrane and silicon grease sufficiently reduced friction resistance of the interface between sand and metal. The friction resistance of lubricated interface tended to be smaller as latex membrane became thin. Thinner latex membrane is also preferable in consideration of the plane strain condition.
2. Roughness of the interface affected friction resistance even the interfaces were lubricated. It is preferable to polish the wall of the container to reduce friction. The Mohr-Coulomb criterion can be used to represent the friction resistance on the interfaces. On lubricated interfaces, a small amount of cohesion intercept, which was attributed to viscosity of silicon grease, was observed.
3. Vertical stress distribution in a container can be estimated by using Terzaghi's theory of silo effect. Estimated values of vertical stress were well agree with measured values.
4. Reproduction of ideal stress distribution under the gravity acceleration is very difficult, even container walls are lubricated. Cohesion intercepts on lubricated interfaces significantly affect stress distribution in the container, because stress levels in 1g model tests are, generally, extremely small comparing the cohesion intercepts.

5. If a surcharge was applied on the surface of model sand, or the model tests were conducted under 12.5g or higher accelerations, the lubrication technique using silicon grease and a latex membrane can be utilized because the stress level in the model sand becomes much higher than the cohesion intercepts. Combination of centrifuge testing technique and the lubrication technique can provide an ideal stress distribution in the model sand.

8.2.2 On initial condition

To investigate the influence of initial conditions affecting bearing characteristics of sand, a series of compression tests and loading tests were carried out in Chapter 4. The following conclusions were drawn from these test results.

1. Settlements due to centrifuge acceleration cycles were observed during virgin centrifugal accelerating operation. Precise experiments in dry sand will require either a measurement of deformation or a correction of void ratio.
2. Load-settlement curves were independent from over-compression and the number of centrifuge accelerating/deaccelerating cycles.
3. Increasing K_0 leads to increasing normalized ultimate bearing capacity under the acceleration larger than 50g. On the other hand under the acceleration of 10g, no influence of K_0 exists, even OCR equals ten. Neglecting the influence of K_0 on overconsolidated soil, as is done in current design methods, would result in a conservative estimation of bearing capacity.

8.3 Particles and shear bands

The shear deformation and the shear banding in granular materials were observed in direct shear box. Using CCIP method developed in Chapter 3, the author demonstrated shear banding in granular media. By means of the stereo-graphic microscope, the width of shear zone and the thickness of shear band were measured for the several granular materials used in this study. The findings about the influencing factors of shear zone and shear band can be summarized as follows:

1. The width of shear zone is influenced by testing condition of direct shear test as well as by physical properties of granular materials. The size of shear zone on the specimen surface was determined by the dimension and relative density of the specimen. The size of shear zone is, however, rather independent of the size and shape of constituent particles.
2. The thickness of shear band is independent of the testing condition. The thickness in different three types of shear bands in lens-shape pattern, and it was constant independently of the amount of shearing specimen. The initial relative density of the specimen was not an influence factor on the thickness. The ratio of the thickness of shear band to mean grain size (D_b/D_{50}) ranges for 18 to 28; the ratio is determined as a function of angularity of constituent particles A_{2D} or void ratio extent ($e_{\max} - e_{\min}$); the ratio (D_b/D_{50}) decreases with an increase of A_{2D} and ($e_{\max} - e_{\min}$).

8.4 Size effects on bearing characteristics

The author conducted a series of loading tests by using a centrifuge to examine size effects on bearing characteristics of granular materials. Size effects on load intensity-settlement curves, settlement at failure, subgrade reaction factors and ultimate bearing capacities were discussed in Chapter 6. The following conclusions were derived from the test results.

1. Clear size effects appeared in normalized load intensity - settlement curves except in a series using glass beads.
2. The values of S_f/B tend to increase with increasing $\gamma B/\sigma_a$. Comparing the settlement at failure obtained from Series INSS and Series INLS, we can see that settlement at failure in Series INLS tends to be smaller than that in Series INSS. On the other hand, settlement at failure obtained from Series GS shows extremely smaller values in comparison with the values from other series.
3. In all cases, both of subgrade reaction factors K_i and K_f decrease along logarithmic linear lines and they can be expressed by power functions. The index expressing the reduction tendency for K_i ranged from 0.294 for Series GS to 0.617 for Series INSS. On the other hand for K_f , the index ranged from 0.275 for Series GS to 0.508 for Series INLS.

4. In all series, normalized bearing capacity N_γ decrease along logarithmic linear lines and they can be expressed by the following power function; however a value of 0.060 of β for Series GS, which is rather small in comparison among other series, might be attributed to a weak stress dependency in shear strength (see **Fig. 6.20**), as Oda and Furuto (1982) mentioned.
5. Series INSS provides somewhat higher values of ultimate bearing capacity than Series INLS. There is no clear differences about particle size effects according to the test results.
6. The values of β for cohesionless materials are ranging from 0.060 for Series GS to 0.402 for Series BS. Most of the values of β tends to concentrate within a range from 0.2 to 0.3, as Shiraishi (1990) recommended.

8.5 Proposal of design methods

Two prediction methods for the ultimate bearing capacities of surface footings were derived in Chapter 7. In these methods, the stress level effects were taken into account. The concept of the predication methods proposed is that the stress level dependency of the ultimate bearing capacity is attributed to the confining stress dependency of shear strength of ground material.

The relationship between stress level dependency of the ultimate bearing capacity and confining stress dependency of shear strength was investigated through a comprehensive parametric examination by means of the extended slip line method. The following modified bearing capacity formula was derived from various independent loading tests:

$$q_u = S_\gamma \frac{\gamma B}{2} N_\gamma^* \left(\frac{B}{B^*} \right)^{-\beta} \quad (8.1)$$

The parameters N_γ^* , p^* , B^* and β were associated with the parameters in nonlinear failure criterion with confining stress dependency, ϕ_a^* and α . Thus, the parameters N_γ^* , p^* , B^* and β can be determined from a series of laboratory element tests for shear strength under variable confining stress using **Figs. 7.6, 7.7 and 7.8**. From the test results on shear strength of various sands, it can be said that the parameter β in the formula would be from 0.2 to 0.3 for ordinary natural sands. Also, results obtained from a series of in situ loading tests on footings with variable size can be used to determine the parameters in **Eq. (8.1)**.

The applicability of the proposed method was verified with experimental results of several series of centrifuge loading tests on the footings on sands and glass beads reported by the authors and other researchers. Through the verifications of the proposed method, it was confirmed that the method proposed by the authors provides useful estimation of ultimate bearing capacities attention to stress level effects for surface strip and circular footings on sand. The ultimate bearing capacity of circular footings can be estimated based on the results of slip line methods in the plane condition with the aid of a shape factor of $S_\gamma = 0.6$ for practical use.

The author developed a new simple prediction method of ultimate bearing capacities of surface footings with regard to size effects. Both strip and circular footings were considered in this chapter. The concept of the prediction method proposed is quite simple: a rational method for determining shear strength constants c and ϕ in accordance with the confining stress range corresponding to footing breadth. In this process the confining stress dependent shear strength is taken into account.

The Finite Element Method which features a confining stress dependent failure criterion was used to derive the appropriate stress range of interest. Consequently, it was shown that the confining stress beneath the footing at failure ranges from $\sigma_m^* = 2\gamma B$ to $10\gamma B$ for strip footings and from $\sigma_m^* = 1\gamma B$ to $\sigma_m^* = 15\gamma B$ for circular footings.

The advantageous features of the proposed method are:

- The stress range to be interested in for the determination of shear strength can be determined directly from footing breadth B and unit weight γ of the foundation ground, without iteration process. Therefore, soil testing for determining shear strength can be easily planned; the proposed confining stress dependent failure criterion is available for examining shear strength properties.
- The proposed method is applicable to strip and circular footings.
- Foundation ground with notable cohesion as well as those without cohesion can be dealt with uniquely; the confining stress dependent failure criterion is incorporated, which is applicable to both cohesionless and cohesive materials.

The applicability of the proposed method was examined with the results of FEM simulation and the experimental results reported by some researchers, which include centrifuge loading

tests and an in- situ loading test on the strip, circular and square footings on the cohesive and cohesionless foundation ground. From the verification of the proposed method, it was confirmed that the method proposed by the author gives admissible prediction of ultimate bearing capacities with attention to the size effects for various types of surface footings and foundation ground.

Acknowledgments

The author would like to express his sincere gratitude to Professor Osamu Kusakabe of Tokyo Institute of Technology and Associate Professor Kinya Miura of Asia Institute of Technology for their continual guidance and encouragement given through the study and their critical reading of this manuscript. The author also would like to express his sincere gratitude to Professor Shosuke Toki of Hokkaido Institute of Technology for valuable advice and continual encouragement given through the study in the author's bachelor and master courses. The author wishes to express his indebtedness to Professor Yukimitsu Yokoyama and Professor Shigeyoshi Imaizumi of Utsunomiya University and Professor Akitoshi Mochizuki of The University of Tokushima for their kind support and encouragement.

The author would like to emphasize that the present study has been supported by many colleagues and graduates of Hokkaido University, Utsunomiya University and The University of Tokushima. In this respect the author is grateful to Associated Professor Kenichi Maeda and Mr. Migitoshi Nishimura for their support. The author was vastly indebted to Messrs. Toshihiko Nakatomi, Tomoyasu Yamashita, Nobuyuki Akiyama, Hiroyuki Hirano, Takehito Nishida, Kimio Maruyama and Yasuhiko Ofuji who conducted a part of the experiments in the present study.

REFERENCES

- 1) Allersma, G. Henderikus (1997): "Using imaging technologies in experimental geotechnics," Imaging Technologies: Techniques and Applications in Civil Engineering, pp. 1–9.
- 2) Bishop, A. W. (1966): "The strength of soils as engineering material," Géotechnique, Vol. 16, No. 2, pp. 91–130.
- 3) Bishop, A. W. and Clugh, G. B. (1968): "Behavior of granular materials under high stresses," J. of Soil Mechanics and Foundation Div. , Proc. of ASCE, Vol. 94, No. SM3, pp. 661–688.
- 4) Bolton, M. D. (1986): "The strength and dilatancy of sands," Géotechnique, Vol. 36, No. 1, pp. 65–78.
- 5) Caquot, A. and Kérisel, J. (1953): "Ultimate bearing capacity of a foundation lying on the surface of a cohesionless soil," Proc. 3rd Int. Conf. on Soil Mechanics and Foundation Engineering, Vol. 1, pp. 336–337 (in French).
- 6) Chiken, K., Okahara, M., Asai, K.-i., Mori, H. and Tatsuoka, F. (1992): "Influence of over-consolidation on bearing capacity of strip footings on sand," The 47th annual meeting of JSCE, Vol. 3, pp. 528–529 (in Japanese).
- 7) Cundall, P. A. and Strack, O. D. L. (1979): "A discrete numerical model for granular assemblies," Géotechnique, Vol. 29, No. 1, pp. 47–65.
- 8) D'Appolonia, D. and Lambe, T. (1970): "Method for predicting initial settlement," Proc. of ASCE, Vol. 96, No. SM2, pp. 523–544.
- 9) Davies, M. and Jones, A. (1998): "Image acquisition using an on-board film camera," Proceedings of the international conference CENTRIFUGE 98, Vol. 1, pp. 67–71.
- 10) De Beer, E. E. (1963): "The scale effect in the transposition of the results of deep sounding tests on the ultimate bearing capacity of piles and caisson foundations," Géotechnique,

- Vol. 13, No. 1, pp. 39–75.
- 11) De Beer, E. E. (1965): “Bearing capacity and settlement of shallow foundations on sand,” Proc. symposium on Bearing capacity and settlement of foundations, pp. 15–33.
 - 12) De Beer, E. E. (1970): “Experimental determination of the shape factors and the bearing capacity factors of sand,” *Géotechnique*, Vol. 20, No. 4, pp. 387–411.
 - 13) Desai, C. S. and Zaman, M. M. (1984): “Thin-layer element for interfaces and joints,” *International Journal for Numerical Method in Geomechanics*, Vol. 8, No. 1, pp. 19–43.
 - 14) Feld, T., Boolmquist, D. and Townsend, F. C. (1991): “Investigation of geotstatic stresses in a limited sized centrifuge model container,” *Proc. of the Int. Conf. Centrifuge 91*, pp. 569–573.
 - 15) Gibson, R. E. (1967): “Some results concerning displacements and stresses in a non-homogeneous elastic half-space,” *Géotechnique*, Vol. 17, pp. 58–67.
 - 16) Golder, H. Q. (1941): “The ultimate bearing pressure of rectangular footings,” *Journal of Institute for civil Engineers*, Vol. 17, pp. 161–174.
 - 17) Graham, J. and Stuart, J. (1971): “Scale and boundary effects in foundation analysis,” *Journal of Soil Mechanics and Foundations Division*, Vol. 97, No. SM11, pp. 1533–1548.
 - 18) Guler, M., Edil, T. and Bosscher, P. (1999): “Measurement of particle movement in granular soils using image analysis,” *Jour. of Computing in Civil Engineering*, Vol. 13, No. 2, pp. 116–122.
 - 19) Haibara, O. and Tatsuoka, F. (1984): “Effect of side wall-condition in model test on sand ground,” *The 19th annual meeting of JSSMFE*, pp. 967–970 (in Japanese).
 - 20) Hettler, A. and Gudehus, G. (1988): “Influence of the foundation width on the bearing capacity factor,” *Soils and Foundations*, Vol. 28, No. 4, pp. 81–92.
 - 21) Hirano, H. (1994): “Loading tests of shallow foundation using centrifuge,” *Master’s thesis*, Utsunomiya University (in Japanese).
 - 22) Horii, H., Takamatsu, K., Inoue, J. and Sasaki, N. (1997): “Measurement of displacement field by “matching method” and observation of strain localization in soft rock,” *Imaging Technologies: Techniques and Applications in Civil Engineering*, pp. 10–19.

- 23) Iizuka, A., Yoshimine, T., Ohta, H. and Hata, S. (1987): "Bearing capacity and stress distribution in k_0 -consolidated clayey ground," Journal of JSCE, Vol. 382, No. III-7, pp. 255–263 (in Japanese).
- 24) Imamura, S., Nomoto, T., Saito, T., Ueno, K. and Fujii, N. (1996): "Design and performance of compact load cell for measuring earth pressure in a centrifuge," Proc. of JSCE, Vol. VI-31, No. 540, pp. 203–213 (in Japanese).
- 25) Jáký, J. (1944): "The coefficient of earth pressure at rest," Jour. of Society of Hungarian Architects and Engineering, pp. 355–358.
- 26) Khoo, E., Okumura, T. and Lee, F. (1994): "Side friction effects in plane strain models," Proc. of the Int. Conf. Centrifuge 94, pp. 115–120.
- 27) Kimura, T., Fujii, N., Saitoh, K. and Kusakabe, O. (1982): "An investigation on the bearing capacity of shallow footings on sand," Proc. of JSCE, No. 319, pp. 97–104 (in Japanese).
- 28) Kimura, T., Kusakabe, O. and Saitoh, K. (1985): "Geotechnical model tests of bearing capacity problems in a centrifuge," Géotechnique, Vol. 35, No. 1, pp. 33–45.
- 29) Ko, H. and Scott, R. (1973): "Bearing capacities by plasticity theory," Journal of the Soil Mechanics and Foundations Division, Vol. 99, No. SM1, pp. 25–43.
- 30) Kobayashi, I., Ohta, H., Hirata, M. and Iizuka, A. (1999): "Shear strength mobilization in shear box test under constant volume," Proc. of Slope Stability Engineering, pp. 763–768.
- 31) Kusakabe, O., Maeda, Y. and Ohuchi, M. (1992): "Large-scale loading tests of shallow footings in pneumatic caisson," Jour. of Geotechnical Engineering, ASCE, Vol. 118, No. 11, pp. 1681–1696.
- 32) Kusakabe, O., Yamaguchi, H. and Morikage, A. (1991): "Experiment and analysis of scale effect of N_γ for circular and rectangular footings," Proc. of the Int. Conf. Centrifuge 91, pp. 179–186.
- 33) Kutter, B. L., Abghari, A. and Cheney, J. A. (1988): "Strength parameters for bearing capacity of sand," Jour. of Geotechnical Engrg. , ASCE, Vol. 114, No. 4, pp. 491–498.
- 34) Lees, G. (1964a): "The measurement of particle shape and its influence in engineering materials," Jour. Brit. Granite and Whinestone Federation, Vol. 4, No. 2.

- 35) Lees, G. (1964b): "A new method for determining the angularity of particles," *Sedimentology*, Vol. 3.
- 36) Maeda, K.-i. and Miura, K. (1999): "Confining stress dependency of mechanical properties of sands," *Soils and Foundations*, Vol. 39, No. 1, pp. 53–67.
- 37) Maeda, K. (1994): *A Study on the Deformation-Failure Behavior of Sands with different Primary Properties*, Doctoral thesis, Hokkaido University.
- 38) Meyerhof, G. G. (1950): *The bearing capacity of sand* Doctoral thesis, University of London.
- 39) Meyerhof, G. (1961): "The ultimate bearing capacity of wedge-shaped foundations," *Proc. of Fifth ICSMFE*, Vol. 2, pp. 105–109.
- 40) Miura, K. (1981): "Stress-strain-dilatancy relationship of isotropically consolidated sand," Master's thesis, Hokkaido University (in Japanese).
- 41) Miura, K., Maeda, K., Furukawa, M. and Toki, S. (1997): "Physical characteristics of sands with different primary properties," *Soils and Foundations*, Vol. 37, No. 3, pp. 53–64.
- 42) Miura, K., Maeda, K., Furukawa, M. and Toki, S. (1998): "Mechanical characteristics of sands with different primary properties," *Jour. of Soils and Foundations*, Vol. 38, No. 4, pp. 159–172.
- 43) Miura, K., Ueno, K. and Toki, S. (1991): "Simulation of cyclic mobility by dem," *Proc. of the 7th Int. Conf. on computer Methods and Advances in Geomechanics*, pp. 839–844.
- 44) Miura, N., Murata, H. and Yasufuku, N. (1979): "Particle-crushing of a decomposed granite soil under shear stresses," *Soils and Foundations*, Vol. 19, No. 3, pp. 1–14.
- 45) Mochizuki, A. and Mikasa, M. (1984): "Deformation measurement of slope models in centrifuge," *Proc. of Int. Sympo. on Geotechnical Centrifuge Model Testing*, pp. 139–148.
- 46) Morgenstern, N. and Tchalenko, J. (1967): "Microscopic structures in kaolin subjected to direct shear," *Géotechnique*, Vol. 17, pp. 309–328.
- 47) Mühlhaus, H. B. and Vardoulakis, I. (1987): "The thickness of shear bands in granular materials," *Géotechnique*, Vol. 37, No. 3, pp. 271–283.

- 48) Nakatomi, T. (1992): "An investigation of bearing capacity and compressibility of over-consolidated sand," Master's thesis, Utsunomiya University (in Japanese).
- 49) Nishida, T., Ueno, K. and Yokoyama, Y. (1996): "The influence of side friction on overburden pressures in model grounds," Proc. of the 23 rd Technical Meeting of Kanto branch, JSCE, pp. pp. 512-513 (in Japanese).
- 50) Oda, M. (1974): "A mechanical and statistical model of granular material," Soils and Foundations, Vol. 14, No. 1, pp. 13-27.
- 51) Oda, M. and Furuto, Y. (1982): "Meaning of progressive failure in bearing capacity problem of shallow footing," Proc. of JSCE, No. 321, pp. 113-122 (in Japanese).
- 52) Oda, M., Nemat-Nasser, S. and Konishi, J. (1985): "Stress-induced anisotropy in granular masses," Soils and Foundations, Vol. 25, No. 3, pp. 85-97.
- 53) Okahara, M., Takagi, S., Obata, H., Mori, K. and Tatsuta, M. (1988): "Centrifuge tests on scale effect of bearing capacity," Proc. of 42nd Japan Annual Conference of Civil Engineers, Vol. III, pp. 250-251 (in Japanese).
- 54) Ovesen, N. K. (1975): "Centrifugal testing applied to bearing capacity problems of footings on sand," Géotechnique, Vol. 25, No. 2, pp. 394-401.
- 55) Ovesen, N. K. (1979): "The use of physical models in design, panel discussion in session 9," Proc. of seventh ECSMFE, Vol. 4, pp. 318-323.
- 56) Park, C.-S., Goto, S. and Tatsuoka, F. (1992): "Lubrication of specimen ends for sands and gravels in laboratory shear tests," SEISANKENKYU, Vol. 44, No. 5, pp. 238-242 (in Japanese).
- 57) Perkins, S. W. (1995): "Bearing capacity of highly frictional material," Geotechnical Testing Journal, GTJODJ, Vol. 18, No. 4, pp. 450-462.
- 58) Perkins, S. W. and Madson, C. R. (1996): "Scale effects of shallow foundations on lunar regolith," Proc. 5th Int. Conf. on Engin. Construction and Operations in Space, SPACE '96, Vol. 2, pp. 963-972.
- 59) Perkins, S. W. and Madson, C. R. (2000): "Bearing capacity of shallow foundations on sand: A relative density approach," Jour. of Geotechnical and Geoenvironmental Engineering, ASCE, Vol. 126, No. 6, pp. 521-530.

- 60) Pietruszczak, S. and Niu, X. (1993): "On the description of localized deformation," International Journal for Numerical Method in Geomechanics, Vol. 17, No. 11, pp. 791–805.
- 61) Ponce, V. M. and Bell, J. M. (1971): "Shear strength of sand at extremely low pressure," Journal of Soil Mechanics and Foundation Div. , Proc. of ASCE, Vol. 97, No. SM4, pp. 625–638.
- 62) Pope, R. G. (1975): "Non-dimensional chart for the ultimate bearing capacity of surface and shallow foundations," Géotechnique, Vol. 25, No. 3, pp. 593–597.
- 63) Prandtl, L. (1921): "Über die eindringungsfestigkeit (härte) plastischer baustoffe und die festigkeit von schneiden," Zeitschrift für angewandte Mathematik und Mechanik, Vol. 1, No. 1, pp. 15–20 (in German).
- 64) Reissner, H. (1924): "Zum erddruckproblem," Proc. 1st Int. Congr. of Applied Mechanics, pp. 295–311 (in German).
- 65) Roscoe, K. H. (1970): "the influence of strains in soil mechanics," Géotechnique, Vol. 20, No. 2, pp. 129–170.
- 66) Santamarina, J. and Goodings, D. (1989): "Centrifuge modeling: a study of similarity," Geotechnical Testing Journal, Vol. 12, pp. 163–166.
- 67) Sato, G., Tatsuoka, F., Ikuhara, O. and Yamada, S. (1983): "Development and performance of a two directional load cell," Proc. of the 38th anual meeting of JSCE, Vol. III, pp. 285–286 (in Japanese).
- 68) Scarpelli, G. and Wood, D. M. (1982): "Experimental observations of shear band patterns in direct shear tests," IUTAM Conf. on Deformation and Failure of Granular Materials, pp. 473–484.
- 69) Shibuya, S., Mitachi, T. and Tamate, S. (1997): "Interpretation of direct shear box testing of sands as quasi-simple shear," Géotechnique, Vol. 47, No. 4, pp. 769–790.
- 70) Shimizu, M. and Konaigai, K. (1994): *Failure of ground and strain localization*, Chapter 2. 4 Measuring technologies, , pp. 42–49, JSSMFE (in Japanese).
- 71) Shiraishi, S. (1990): "Variation in bearing capacity factors of dense sand assessed by model loading tests," Soils and Foundations, Vol. 30, No. 1, pp. 17–26.

- 72) Siddiquee, M. S. A., Tanaka, T., Tatsuoka, F., Tani, K. and Morimoto, T. (1999): "Numerical simulation of bearing capacity characteristics of strip footing on sand," *Soils and Foundations*, Vol. 39, No. 4, pp. 93–109.
- 73) Sokolovskii, V. V. (1965): *Statics of Granular Media* New York: Pergamon Press J. K. Luscher, translator.
- 74) Takeda, A. (1992): "An experimental study on dynamic deformation and strength of a few volcanic ash soils in Hokkaido," Master's thesis, Hokkaido University (in Japanese).
- 75) Tanaka, T. (1992): *Deformation and stability analysis by finite element method*, , pp. 109–154, JSSMFE (in Japanese).
- 76) Tani, Y. (1982): "Loadcells used in cutting work," *SEISANKENKYU*, Vol. 34, No. 6 (in Japanese).
- 77) Tatsuoka, F. and Fukushima, S. (1984): "Strength and deformation characteristics of saturated sand at extremely low pressures," *Soils and Foundations*, Vol. 24, No. 4, pp. 30–48.
- 78) Tatsuoka, F., Molenkamp, F., Torii, T. and Hino, T. (1984): "Behaviour of lubrication layers of platens in element tests," *Soils and Foundations*, Vol. 24, No. 1, pp. 113–128.
- 79) Tatsuoka, F., Nakamura, S., Ching-Chuang, H. and Tani, K. (1990): "Strength anisotropy and shear band direction in plane strain tests of sand," *Soils and Foundations*, Vol. 30, No. 1, pp. 35–54.
- 80) Tatsuoka, F., Okahara, M., Tanaka, T., Tani, T., Morimoto, T. and Siddeque, M. (1991): "Progressive failure and particle size effect in bearing capacity of a footing on sand," *Geotechnical Engineering Congress –ASCE, Geotechnical Special Publication 27*, Vol. 2, pp. 788–802.
- 81) Tatsuoka, F., Sakamoto, M., Kawamura, T. and Fukushima, S. (1986): "Strength and deformation characteristics of sand in plane strain compression at extremely low pressures," *Soils and Foundations*, Vol. 26, No. 1, pp. 65–84.
- 82) Tatsuoka, F. and Haibara, O. (1984a): "Influence of side wall friction of sand box in bearing capacity test on model sand ground II–measurements of friction forces between sand mass and platens –," *SEISANKENKYU*, Vol. 36, No. 1, pp. 25–28 (in Japanese).

- 83) Tatsuoka, F. and Haibara, O. (1984b): "Influence of side wall friction of sand box in bearing capacity test on model sand ground III—measurements of friction forces between sand mass and platens —," SEISANKENKYU, Vol. 36, No. 3, pp. 158–161 (in Japanese).
- 84) Tatsuoka, F. and Haibara, O. (1984c): "Influence of side wall friction of sand box in bearing capacity test on model sand ground VI—effects of various side wall conditions in small and medium-size sand boxes —," SEISANKENKYU, Vol. 37, No. 7, pp. 329–332 (in Japanese).
- 85) Tatsuoka, F. and Haibara, O. (1984d): "Influence of side wall friction of sand box in bearing capacity test on model sand ground VII—relationship between surface friction on side wall and measured bearing capacity —," SEISANKENKYU, Vol. 36, No. 8, pp. 346–349 (in Japanese).
- 86) Tatsuoka, F., Haibara, O. and Sato, T. (1984): "Influence of side wall friction of sand box in bearing capacity test on model sand ground IV—measurements of stress distribution on footing base with separate load cells —," SEISANKENKYU, Vol. 36, No. 4, pp. 181–184 (in Japanese).
- 87) Tatsuoka, F., Siddiquee, M. S. A., Tanaka, T. and Okahara, M. (1992): "A new aspect of a very old issue: Bearing capacity of footing on sand," Proceedings of 9th Asian Regional Conference of Soil Mechanics and Foundation Engineering, Vol. 2, pp. 358.
- 88) Tatsuoka, F., Toh, N., Haibara, O. and Sato, T. (1983): "Influence of side wall friction of sand box in bearing capacity test on model sand ground I—test results showing influence of side wall friction —," SEISANKENKYU, Vol. 35, No. 12, pp. 543–546 (in Japanese).
- 89) Taylor, R., Robson, S. and Kuwano, J. (1998): "An image analysis system for determining plane and 3-D displacements in soil models," Proceedings of the international conference CENTRIFUGE 98, Vol. 1, pp. 73–78.
- 90) Terzaghi, K. (1942): *Theoretical Soil Mechanics*, Chapman and Hall, Ltd.
- 91) Terzaghi, K. and Peck, R. B. (1967): *Soil Mechanics in Engineering Practice*, John Wiley and Sons.
- 92) Ueno, K. (1991): "A study on the prediction method for the ultimate bearing capacity of spread footing with regard to the confining stress dependency of shear strength characteristics of sand," Master's thesis, Hokkaido University, Sapporo, Japan (in Japanese).

- 93) Ueno, K., Miura, K. and Maeda, Y. (1998): "Prediction of ultimate bearing capacity of surface footings with regard to size effects," *Soils and Foundations*, Vol. 38, No. 3, pp. 165–178.
- 94) Ueno, K., Miura, K., Tanise, A. and Maeda, K.-i. (1994): "Determinations of dimension of shear bands produced in granular media," *Proc. of Sympo. on failure and strain localization of soil*, pp. 49–56 (in Japanese).
- 95) Ueno, K., Nakatomi, T., Mito, K. and Kusakabe, O. (1994): "Influence of initial conditions on bearing characteristics of sand," *Proc. of the Int. Conf. Centrifuge 94*, pp. 541–546.
- 96) Uesugi, M., Kishida, H. and Tsubakihara, Y. (1988): "Behavior of sand particles in sand-steel friction," *Soils and Foundations*, Vol. 28, No. 1, pp. 107–118.
- 97) Vardoulakis, I. G. and Graf, B. (1982): "Imperfection sensitivity of the biaxial test on dry sand," *Proc. IUTAM Conf. on Deformation and Failure of Granular Materials*, pp. 485–491.
- 98) Vardoulakis, I. G. and Graf, B. (1985): "Calibration of constitutive models for granular materials using data from biaxial experiments," *Géotechnique*, Vol. 33, No. 3, pp. 299–317.
- 99) Vardoulakis, I. G., Graf, B. and Hettler, A. (1985): "Shear-band formation in a fine-grained sand," *Proc. 5th Int. Conf. on Numerical Method in Geomechanics*, Vol. 1, pp. 517–521.
- 100) Vesić, A. S. (1973): "Analysis of ultimate loads of shallow foundations," *Journal of the Soil Mechanics and Foundations Division, ASCE*, Vol. 99, No. SM1, pp. 45–73.
- 101) Vesić, A. S. (1975): "Bearing capacity of shallow foundations," In *Foundation Engineering Handbook*, , pp. 121–147.
- 102) Vogel, C. and Baracos, A. (1973): "Discussion of 'analysis of ultimate loads of shallow foundations', by Vesić, A. S.," *Journal of Soil Mechanics and Foundations Division, ASCE*, Vol. 99, No. SM11, pp. 1016–1018.
- 103) Yamaguchi, H. (1969): "Chapter 7, soil plasticity," In *Soil Mechanics* Tokyo, Japan: Gihodo (in Japanese).
- 104) Yamaguchi, H., Kimura, T. and Fujii, N. (1977): "On the scale effect of footings in dense sand," *Proc. 9th ICSMFE*, Vol. 1, pp. 795–798.

- 105) Yamaguchi, H., Kimura, T. and Fujii, N. (1976): "On the influence of progressive failure on the bearing capacity of shallow foundation in dense sand," *Soils and Foundations*, Vol. 16, No. 4, pp. 11–22.
- 106) Yoshida, T., Goto, S., Kamegai, Y., Tatsuoka, F., Kohata, Y. and Dong, J. (1994): "Stress-deformation relationships of shear band in plane strain compression tests of sands and gravels," *Proc. of domestic symposium on raptures and strain localizations of grounds*, pp. 189–196 (in Japanese).
- 107) Yoshida, T., Tatsuoka, F., Siddiquee, M. S. A. and Kamegai, Y. (1994): "Shear banding in sands observed in plane strain compression," *Proc. of the third International Workshop on Localization and Bifurcation Theory for Soils and Rocks*, pp. 165–179.
- 108) Zadoroga, B. (1994): "Bearing capacity of shallow foundations on non-cohesive soils," *Jour. of Geotechnical Engineering*, Vol. 120, No. 11, pp. 1991–2008.
- 109) Zienkiewicz, O. C. and Corneau, K. M. (1974): "Viscoplasticity and creep in elastic soils – a unified numerical method in engineering," *International Journal of Numerical Method in Engineering*, Vol. 8, pp. 821–845.