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Temporally Coherent and Artistically Intended Stylization of Feature Lines Extracted from 3D Models

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Abstract

In this paper, we propose a method to maintain the temporal coherence of stylized feature lines extracted from 3D models and preserve an artistically intended stylization provided by the user. We formally define the problem of combining spatio-temporal continuity and artistic intention as a weighted energy minimization problem of competing constraints. The proposed method updates the style properties to provide real-time smooth transitions from current to goal stylization, by assuring first- and second-order temporal continuity, as well as spatial continuity along each stroke. The proposed weighting scheme guarantees that the stylization of strokes maintains motion coherence with respect to the apparent motion of the underlying surface in consecutive frames. This weighting scheme emphasizes temporal continuity for small apparent motions where the human vision system is able to keep track of the scene, and prioritizes the artistic intention for large apparent motions where temporal coherence is not expected. The proposed method produces temporally coherent and visually pleasing animations without the flickering artifacts of previous methods, while also maintaining the artistic intention of a goal stylization provided by the user.

1. Introduction

1.1. Background

A long-standing problem in Non-Photorealistc Rendering (NPR) research has been how to generate and animate line drawings that replicate abstraction and stylistic decisions made by real artists. Polygonal models have recurrently been used to extract meaningful lines and produce highly dynamic animated sequences of stylized line drawings. However, the low degree of freedom to control the appearance of the synthesized line drawings has hindered the proliferation outside the research domain. Recently, some progress into real applications has been achieved by enabling the user to make stylistic annotations to add artistic intention to line drawings generated from 3D models but there is need to address the unique challenges to produce temporally coherent animations. Potential applications include films featuring high-quality off-line rendering, as well as video games where interactive rates are expected.

The process of animating a line drawing sequence differs from drawing standalone images in the sense that artists consider additional aspects such as type of motion and stylistic coherence between frames. In a traditional animation workflow, when an artist draws a sequence of line drawings, information from previously drawn images is taken into account before drawing the next image. This knowledge is a global spatio-temporal information that is essential to produce coherent animated sequences of line drawings. Therefore, an ideal animation system for line drawings should balance between the intended depiction of objects and the temporal continuity in consecutive frames.

1.2. Related Work

Early research in computer-generated line drawings has proposed numerous methods to extract meaningful lines from 3D models. Image-space algorithms tend to produce reasonably appealing line drawings but are prone to noise problems and are not suited for complex stylization [ST90, Dec96, Her99, LMLH07]. Object-space algorithms make use of surface information such as normal and curvature to extract feature line from polygonal meshes. A particularly relevant category is object-space methods for extracting view-dependent feature lines such as occluding contours [K*84], suggestive contours [DFRS03, DFR04] and apparent ridges [JDA07]. As stated by Cardona et al. [CS15], a common problem of line extraction algorithms is that they usually produce either too many lines that would not be drawn by artists or not enough to convey the shape of the objects being depicted.

Recently, a new topic of NPR research called *localized stylization* has focused on allowing the user to make stylistic annotations to represent local features such as material properties or feature sharpness. The main problems of *localized stylization* approached in the literature are how to extrapolate the user-defined stylization of view-dependent lines as the viewpoint changes or the 3D model is animated, and how to replicate global decisions

The definitive version is available at http://diglib.eg.org/ and http://onlinelibrary.wiley.com/ .

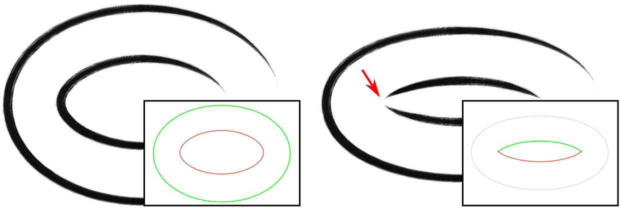
made by artists such as emphasizing or omitting certain lines. Cardona et al. [CS14] proposed a *localized stylization* method based on surface segmentation that uses the sign of the Gaussian curvature to divide *occluding contours* at inflection points and tracks object-space contours to generate additional surface areas. In a more recent work [CS15], the same authors proposed a *localized stylization* method based on line matching that combines image- and object-space techniques to solve some limitations of their previous method such as the terminal points being limited to inflection points, the impossibility of specifying stroke direction and orientation and the lack of support for *suggestive contours*. These methods allow the user to add artistic intention to computer-generated line drawings but do not address the problem of maintaining temporal coherence.

Various methods have been proposed to propagate the style parameterization of feature lines and maintain the temporal coherence of stylized strokes as the viewpoint changes or a 3D model is animated [KDMF03, BFP*11, KH11, BLC*12, XSQ13, LWM15]. An alternative approach proposed by Dalstein et al. [DRvdP15] uses a space-time edition tool that relies on the user to directly model topological events in image-space but requires high number of user operations to produce complex animations. Bénard et al. [BBT11] formalized the problem of temporal coherence in term of three concurrent goals (*flatness*, *motion coherence*, and *temporal continuity*) and compared the methods proposed in recent years. Kalnins et al. [KDMF03] proposed a method to propagate the parameterization from samples generated on the re-projection of previous frame contours. They proposed various schemes for parameterizing a brush path that optimally adapt for the competing objectives of uniform 2D arc-length parameterization and coherence on the 3D shape. Bénard et al. [BLC*12] proposed another temporal coherence method based on active contours that change topology by a number of operators (e.g. splitting, merging). This method supports multiple strokes per active contour and uses the same linear fitting approach as Kalnins et al. [KDMF03]. All previous temporal coherence methods assume a single global style for all feature lines of the same type.

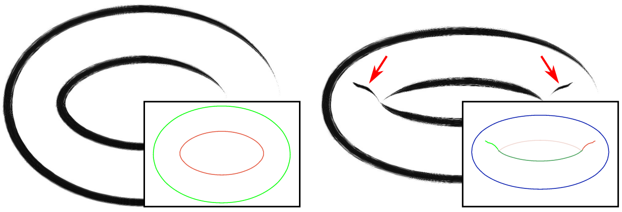
In this paper, we propose a method to maintain temporal coherence by updating the stylization of locally stylized line drawings while preserving the artistic intention of the user. This method is based on the minimization of an energy function that assures first- and second- order temporal continuity as well as spatial continuity along each stroke while approaching the stylization goal. We also introduce a weighting scheme that assures *motion coherence* by controlling the update velocity and acceleration of the stylization update to prioritize artistic intention or temporal continuity depending on the type of motion.

2. Problem

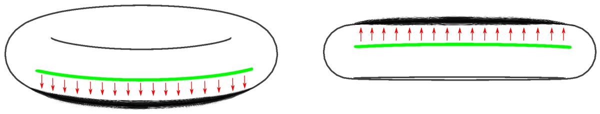
This paper focuses on the problem of how to achieve smooth interpolation of artistically intended line drawings. In this section, we classify the events of sudden changes of stylization and define the problem of combining *temporal coherence* and *artistic intention* as balance of competing constraints.



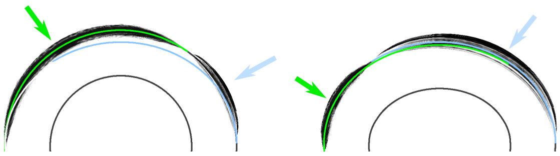
(a) Sudden change in stylization after the merging / splitting of feature lines.



(b) Sudden change in stylization after the appearance / disappearance of feature lines.



(c) Sudden change in stylization after a stroke registered by the user (green line) is matched to completely different locations.



(d) Sudden change in stylization after binary decisions of the overlapping policy to prioritize a stroke over another overlapped stroke.

Figure 1: ***Events of sudden changes in stylization.***

2.1. Events of Stylization Discontinuity

In common line drawing rendering frameworks, sudden changes in stylization in consecutive frames occur as consequence of the following topological events:

- Splitting and merging of lines (Figure 1a).
- Appearance and disappearance of lines (Figure 1b)

Localized stylization methods may produce additional events of stylization discontinuities. In our case, we use Cardona et al. method [CS15], which is based on line matching and produces the following additional types of events:

- Changes in location of strokes (Figure 1c).
- Changes in stroke trimming (Figure 1d).

The interpolation method proposed in this paper is general in the sense that it is independent of the *localized stylization* method used to set the artistic intention of the user. Therefore, the events in Figures 1c and 1d are only provided for explanation purposes.

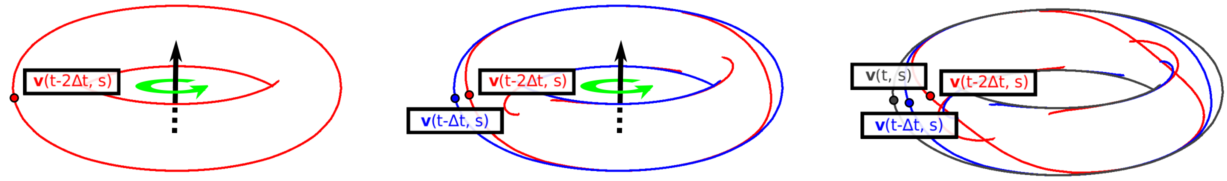


Figure 2: ***Parametric feature lines.*** *The parametric feature lines at time $t-\Delta 2t$ (red lines) (left); at time $t-\Delta t$ (blue) after a change of view by rotation (center); and at current time t after another change of view by rotation (right).*

2.2. Stylization Objectives

The problem of maintaining *temporal coherence* while approaching a stylization goal can be divided into the following three concurrent objectives:

Artistic intention. *localized stylization* adds artistic intention through an ideal stylization goal, provided by the user, that best depicts the scene from the current view.

Temporal continuity. In a sequence of line drawings, *temporal continuity* prevents sudden changes in the appearance of strokes in a short time interval.

Spatial continuity. At a given time instant, *spatial continuity* prevents abrupt changes of stylization along each stylized stroke on the feature lines that compose the line drawing.

Let $\mathcal{L} = \{\mathbf{v}(t,s)\}$ be a parametric line where t is the time parameter, $s \in [0,1]$ is the normalized arc-length parameter and $p(t,s)$, $g(t,s)$ are respectively the current and goal style parameterizations at point $\mathbf{v}(t,s)$. In this paper, we assume that the goal style parameterization $g(t,s)$ is already defined using a *localized stylization* method such as Cardona et al. [CS15]. For more details on this process, refer to the original paper. Previous temporal coherence methods [KDMF03, BLC*12] support multiple strokes per feature line through topological operators that determine when adjacent strokes should be merged or split. In contrast, our method derives directly this information from the stylization goal by subdividing the feature lines into separated intervals for each stroke. The proposed update method then provides smooth transitions between different goal stylizations. We formally define the objectives described above as the following constraints:

Artistic intention:

$$p(t,s) - g(t,s) = 0 \tag{1}$$

First-order temporal continuity:

$$\frac{dp(t,s)}{dt} = 0 \tag{2}$$

Second-order temporal continuity:

$$\frac{d^2p(t,s)}{dt^2} = 0 \tag{3}$$

Second-order spatial continuity:

$$\frac{d^2p(t,s)}{ds^2} = 0 \tag{4}$$

These constraints usually can not be satisfied simultaneously and therefore require a trade-off to be made such that the importance of each constraint is determined depending on the motion characteristics in the animated sequence. Formally, this balance problem can be expressed as the minimization of following energy:

$$\begin{aligned} E(t) = \int_0^1 \frac{1}{2} \Big(& w_{AI} E_{AI}\big(p(t,s)\big) + w_{TC1} E_{TC1}\big(p(t,s)\big) \\ & + w_{TC2} E_{TC2}\big(p(t,s)\big) + w_{SC2} E_{SC2}\big(p(t,s)\big) \Big)\, ds \end{aligned} \tag{5}$$

where

$$E_{AI}(t,s) = \big|p(t,s) - g(t,s)\big|^2 \tag{6}$$

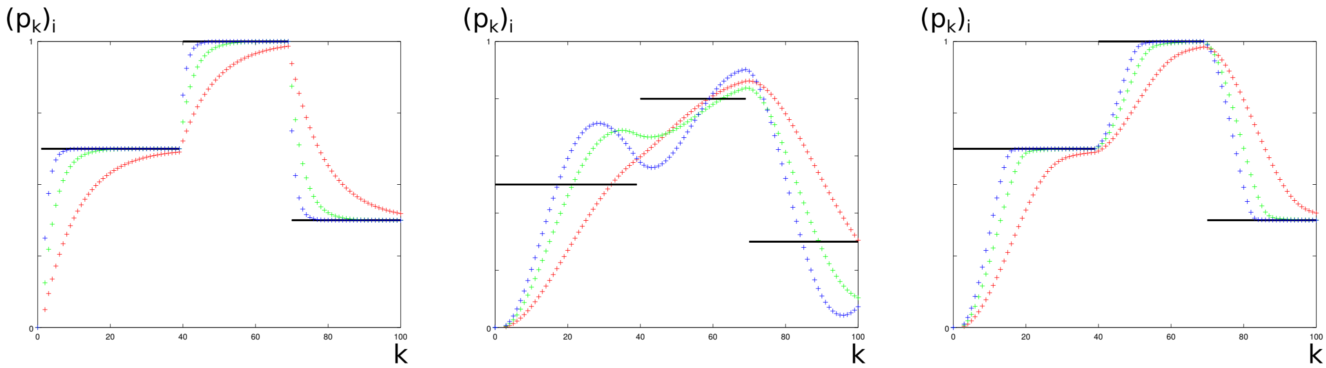
$$E_{TC1}(t,s) = \left|\frac{dp(t,s)}{dt}\right|^2 \tag{7}$$

$$E_{TC2}(t,s) = \left|\frac{d^2p(t,s)}{dt^2}\right|^2 \tag{8}$$

$$E_{SC2}(t,s) = \left|\frac{d^2p(t,s)}{ds^2}\right|^2 \tag{9}$$

and w_{AI}, w_{TC1}, w_{TC2}, and w_{SC2} are the weights for each constraint.

Since the topology and location in image and object space of the feature lines changes as the viewpoint is modified or the 3D model is animated, there is no trivial correspondence between points of feature lines in different instants in times (Figure 2). Computing the time derivatives in Eq. 2 and 3 with finite differences requires the parameterization to be propagated by matching the feature lines in consecutive frames. In our implementation, we use Kalnins et al. [KDMF03] method, which propagates the parameterization by searching in the direction of the surface normal. We refer to the original paper for more details on the matching and propagation process. In this paper, we simplify the notation to represent a single parameterization but the same process is used for stylizations composed of multiple style properties such as texture coordinates and blending weights, tapering, width and color. For each style property, its parameterization is updated and smoothed in time and space as explained in the following section.



(a) $w_{AI} = 1$ with $w_{TC1} = 0.1$ (red), $w_{TC1} = 0.04$ (green) and $w_{TC1} = 0.02$ (blue).

(b) $w_{AI} = 1$ with $w_{TC1} = 0.10$, $w_{TC2} = 200$ (red), $w_{TC1} = 5$, $w_{TC2} = 100$ (green) and $w_{TC1} = 3$, $w_{TC2} = 65$ (blue).

(c) $w_{AI} = 1$ with $w_{TC1} = 10$, $w_{TC2_{max}} = 200$ (red), $w_{TC1} = 5$, $w_{TC2_{max}} = 100$ (green) and $w_{TC1} = 3$, $w_{TC2_{max}} = 65$ (blue).

Figure 3: ***Comparison of parameter update schemes.*** *The style parameterization is updated using Eq. 10 to approach the goal parameterization (black bars) in three discontinued intervals. The evolution of the style parameterization with artistic intention and first-order continuity constraints (a); with artistic intention and first-order continuity and second-order continuity weights using constant weights (b); and with artistic intention and first-order continuity and second-order continuity constraints using the dynamic second-order weight weighting scheme of Eq.12. The vertical axis represents the discredited style parameterization p_k while the horizontal axis represents the frame index k.*

3. Proposed Method

In this section, we explain a parameter update scheme that makes a trade-off between the constraints in Eq. 1 through 4. The proposed method updates the style parameterization so that *temporal continuity* and *spatial continuity* along each stroke are maintained while approaching the stylization goal provided by the user.

3.1. Spatio-Temporal Optimization

Let $\mathcal{L}_k = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}^T$ be the discretization of a feature line at frame k where $\mathbf{p}_k = \{p_1, \ldots, p_n | p_i \in [0,1]\}^T$ and $\mathbf{g}_k = \{g_1, \ldots, g_n | g_i \in [0,1]\}^T$ are respectively the current and goal parameterizations. For each feature line, we update the parameterization at current time by minimizing Eq. 5 through its Euler-Lagrange equation. The numerical solution yields the following sparse linear system:

$$A\mathbf{p}_k = w_{AI}\mathbf{g}_k + w_{TC1}\mathbf{p}_{k-1} - w_{TC2}(\mathbf{p}_{k-2} - 2\mathbf{p}_{k-1}) \qquad (10)$$

where $\mathbf{p}_{k-1}$ and $\mathbf{p}_{k-2}$ are the parameterizations at frames $k-1$ and $k-2$ respectively, and A is a sparse $n \times n$ matrix such that $A = (1 + w_{TC1} + w_{TC2})I + w_{SC2}B$, where B is the 4th order derivative coefficient matrix using finite differences as follows:

$$\begin{bmatrix} 1 & -4 & 6 & -4 & 1 & & & & \\ 1 & -4 & 6 & -4 & 1 & & & & \\ 1 & -4 & 6 & -4 & 1 & & & & \\ & 1 & -4 & 6 & -4 & 1 & & & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ & & & & 1 & -4 & 6 & -4 & 1 \\ & & & & 1 & -4 & 6 & -4 & 1 \\ & & & & 1 & -4 & 6 & -4 & 1 \end{bmatrix} \qquad (11)$$

For details on how to obtain Eq. 10, refer to the Appendix below.

Figure 3 illustrates the evolution of the current parameterization using the update method of Eq. 10 with different weights as the goal parameterization abruptly changes in three time intervals. Figure 3a shows the evolution of the current parameterization using only the artistic intention and first-order continuity constraints (the weights for the other constraints are set to zero). In this case, we notice that the resulting parameterization is not differentiable at time instants coinciding with style discontinuities with respect to time in the goal parameterization. Figure 3b shows the evolution of a parameter using the artistic intention, first-order continuity and second-order continuity constraints. As illustrated in the figure, the update scheme is differentiable at any instant in time but has an "overshooting" problem where the parameterization goes beyond the stylization goal. To prevent this problem, we dynamically set the second-order temporal continuity weight by reducing its value as the parameterization approaches the goal stylization as follows:

$$\hat{w}_{TC2} = \frac{w_{TC2_{max}}}{n} \sum_{i=1}^{n} |(\mathbf{g}_k)_i - (\mathbf{p}_{k-1})_i|) \qquad (12)$$

where w_{TC2max} is the maximum second-order continuity. Figure 3c shows the evolution of a parameter using Eq. 10 with the dynamic second-order continuity weight defined in Eq. 12. Figure 4 compares the contribution of each temporal continuity constraint to the parameterization update after an discontinuity event.

Spatial continuity assures that the parameter distribution remains smooth along each stroke as the style parameterization transitions between the goal stylizations before and after a discontinuity event occurs as explained in Section 2.1. Figure 5 illustrates the spatial continuity of the style parameterization of a feature line for a sequence of time instants.

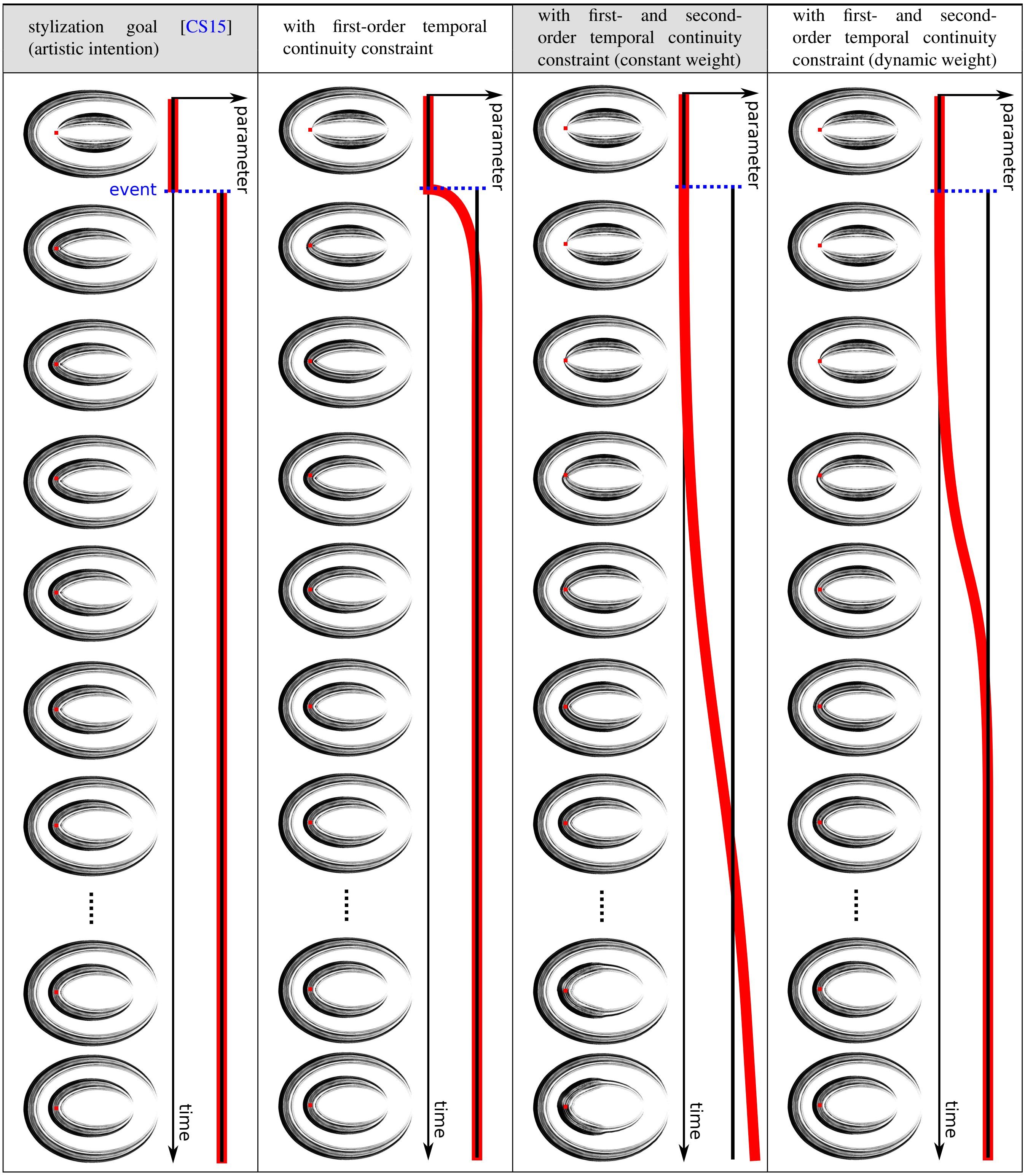


Figure 4: ***Temporal continuity constraints.*** *Contribution of each constraint to the parameterization update at a contour line point (red dot) after a merge event: the stylization goal [CS15] (black bars) adds artistic intention but is discontinuous in time; the first-order temporal continuity constraint slightly smooths the style transition but produces fast rate of change after the event; the second-order temporal continuity constraint with constant weights adds "acceleration" to further smooth the transition but suffers from overshooting problem where the stylization goal is exceeded; the weighting scheme of Eq. 12 solves the overshooting problem while maintaining the smooth style transitions.*

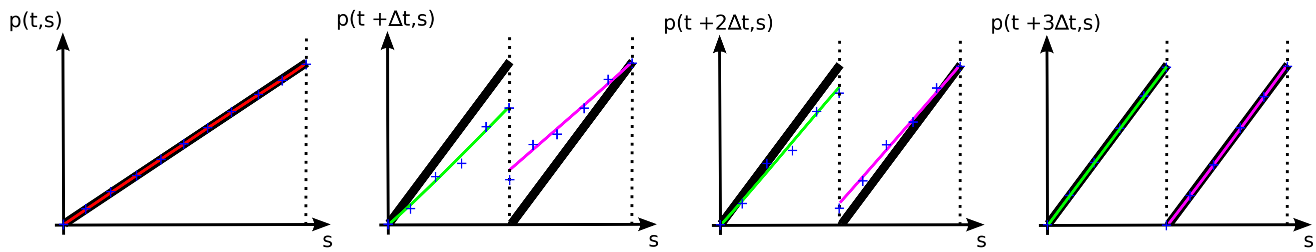
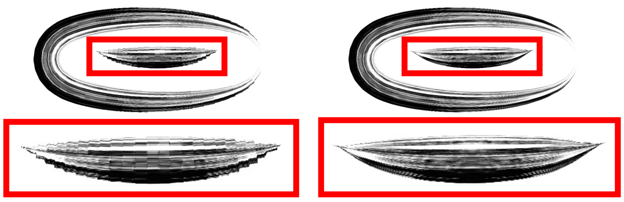


Figure 5: ***Spatial continuity.*** *Style parameterization (e.g texture coordinates) sampled along the normalized arc-length parameter s of each stroke (blue points) approach the goal values (black). The smoothing spline (red) adds spatial continuity as the style parameterization transitions from a single stroke to a two stroke style parameterization after a split event.*



(a) without spatial continuity (b) with spatial continuity

Figure 6: ***Spatial continuity constraint.*** *Stair-step artifacts along strokes can be avoided by the spatial continuity constraint.*

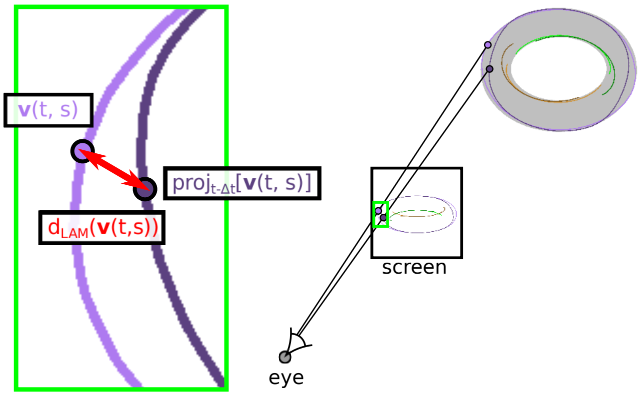


Figure 7: ***Local Apparent Motion (LAM) distance.*** *For each point* $\mathbf{v}(t,s)$ *of a feature line, a measure of apparent motion is obtained by the distance to its re-projection at time* $t-\Delta t$.

3.2. Motion Coherence

One important fact to note is that, while flicker artifacts caused by the lack of temporal coherence are less apparent in low frame rate renderings, they are still perceived in animations with subtle motions. In general, time discontinuities of style parameterizations can be perceived when the human vision system is able to make a correspondence between the lines composing the drawing in consecutive frames. Therefore, we argue that if the motion is subtle enough so that the viewer can keep track of the scene, then the velocity at which the style parameterization tends to the stylization goal should be also small. Conversely, for large motions where the viewer can not keep track of the scene, temporal continuity is not expected, and therefore the parameterization should tend to the stylization goal at faster rate. The concept of proportionality between the motion magnitude and rate of change of the style parameterization is referred as *motion coherence*. We propose a weighting scheme that assures *motion coherence* by adjusting the weights for the artistic intention and the temporal continuity constraints in Eq. 1 and 3 to prioritize *artistic intention* or *temporal continuity* depending on the type of motion.

Let $r(\mathbf{v}(t,s)) \in \mathbb{R}^3$ be the 3D position of point $\mathbf{v}(t,s)$ before its projection on the image space. Then, we define the *Local Apparent Motion (LAM)* distance for point $\mathbf{v}(t,s)$ of a feature line as follows:

$$d_{LAM}(\mathbf{v}(t,s)) = \|proj_{t-\Delta t}[r(\mathbf{v}(t,s))] - \mathbf{v}(t,s)\| \quad (13)$$

where $\|\cdot\|$ is the L^2 norm and $proj_{t-\Delta t}[\cdot] \in \mathbb{R}^2$ is the projection into the image plane at time $t-\Delta t$. This distance definition is a measure analogous to the magnitude of the optical flow and is illustrated in Figure 7.

Adjusting the *artistic intention* and the *temporal continuity* weights for each point $\mathbf{v}(t,s)$ separately would produce different update velocities along the same stroke and therefore loose its linear relationship with respect to the arc-length parameter. Therefore, we define the *Feature Line Apparent Motion (FLAM)* distance for a feature line $\mathcal{L}$ as follows:

$$d_{FLAM}(\mathcal{L}) = \int_0^1 d_{LAM}(\mathbf{v}(t,s))\, ds \quad (14)$$

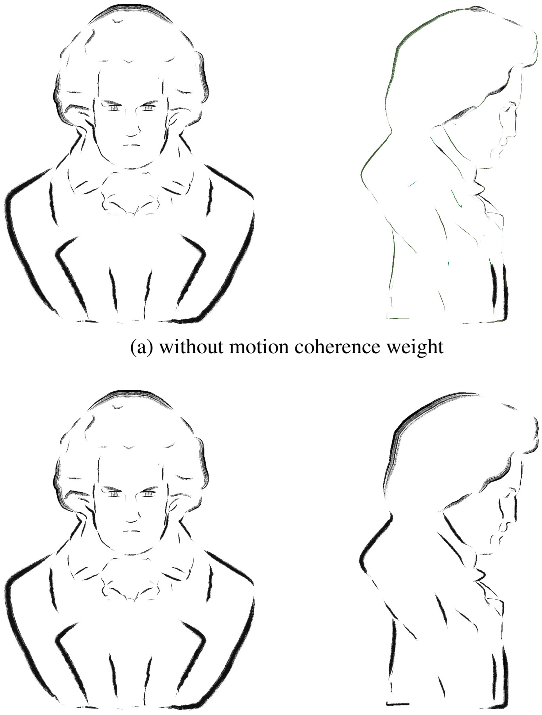
In the discrete case, where two frames could be arbitrarily different, we set the *artistic intention* and *temporal continuity* weights as follows:

$$w_{AI} = (1 - w_{MC}) w_{AI_{max}} \quad (15)$$

$$w_{TC1} = w_{MC} w_{TC1_{max}} \quad (16)$$

$$w_{TC2} = w_{MC} w'_{TC2} \quad (17)$$

where $w_{AI_{max}}$ and $w_{TC1_{max}}$ are respectively, the artistic intention and the first-order continuity weight maximum values, w'_{TC2} is the dy-



(a) without motion coherence weight

(b) with motion coherence weight

Figure 8: ***Comparison of results with and without the proposed weighting scheme.*** *Constants weights fail to keep up with the artistic intention (a) while the proposed motion coherence weight prioritizes the stylization goal over temporal continuity to maintain the artistic intention provided by the user (b).*

namic second-order *temporal continuity* weight defined in Eq. 12 and w_{MC} is a *motion coherence* weight defined as follows:

$$w_{MC} = \begin{cases} \frac{\bar{d}_{FLAM}}{d_{max}} & \bar{d}_{FLAM}(\mathcal{L}_K) < d_{max} \\ 1 & otherwise \end{cases} \tag{18}$$

where d_{max} is the maximum distance where the *temporal continuity* is no longer expected ($d_{max} = 25 pixels$ in our implementation) and $\bar{d}_{FLAM}$ is the average *Feature Line Apparent Motion* distance:

$$\bar{d}_{FLAM}(\mathcal{L}_k) = \frac{1}{n}\sum_{i=1}^{n} d_{LAM}(\mathbf{v}_i) \tag{19}$$

where $\mathbf{v}_i \in \mathbb{R}^2$ is i-th sample point of feature line $\mathcal{L}_k$ at frame k.

Figure 8 shows a comparison of a line drawing generated using our method with and without the proposed *motion coherence* weight. Under large motions, the absence of *motion coherence* weight causes the style parameterization to not be updated enough to represent the *artistic intention*. In contrast, the proposed *motion coherence* weight prioritizes *artistic intention* over *temporal continuity* in the case of large motions and therefore correctly maintains the *artistic intention* for the current view. The absence of motion coherence would also mean that the parameterization continues to be updated even when the surface is not in motion.

4. Results

Our results show that our method produces temporally coherent animations that maintain the artistic intention of the user, even for challenging transformations such as rotation, zoom in / out and morphing. Figure 9 shows the result for a complex animated sequence that combines rotation along two different axis and radical close-up. Figures 11 and 10 respectively show the results for other moderately complex surfaces and an animated model. Unfortunately, the impact of stylization discontinuities and temporal coherence is difficult to convey in static images. Therefore, we refer to the video provided in the supplementary materials, which was rendered at 24 fps. For comparison purposes, we also provide the results obtained by directly using the stylization goal of a previous localized stylization method [CS15], which is by nature discontinuous in time. This video shows how the proposed method produces temporally coherent and visually pleasing animations without flickering artifacts of previous localized stylization methods while maintaining the artistic intention of the user. Our current implementation achieves interactive rates only for moderately complex models but performance improvements could be achieved with parallel implementations of each step of the stylization pipeline.

5. Conclusion and Future Work

In this paper we proposed a temporal coherence method for artistically intended stylization of feature lines extracted from 3D models. We introduced a new interpolation method that prevents sudden changes of parameterization by guaranteeing *temporal continuity* while respecting the *artistic intention* of the user, and maintains *spatial continuity* along each stroke by penalizing curvature in parameter space. An important characteristic of our method is that it does not need to explicitly determine topology events because the composition of strokes along each feature line is directly derived from the goal stylization. Furthermore, the proposed weighting scheme provides *motion coherence* by controlling the parameterization update to prioritize temporal continuity for small apparent motions and artistic intention for large apparent motions where temporal coherence is not expected. Our method provides a general temporal coherence framework for artistically intended stylization that produces visually pleasing transitions compared with previous localized stylization methods.

While the proposed method is independent on the animation complexity, morphing between completely different shapes remains a challenging problem because state-of-the-art localized stylization methods assume that the surface remains relatively similar. A limitation of the proposed method is its dependence on the topology of the object-space feature lines, which in the case of complex models results in over-fragmented small strokes. Therefore, a remaining problem is how to simplify the over-segmented line topology o of complex models. A partial solution to this problem is to simplify the line topology by removing high-frequency components of the surface through mesh smoothing. Future work may build on this idea by controlling the local amount of smoothing in accordance to user annotations to achieve temporally coherent level of detail that emulates how artists depict far and close objects with different degrees of abstraction.

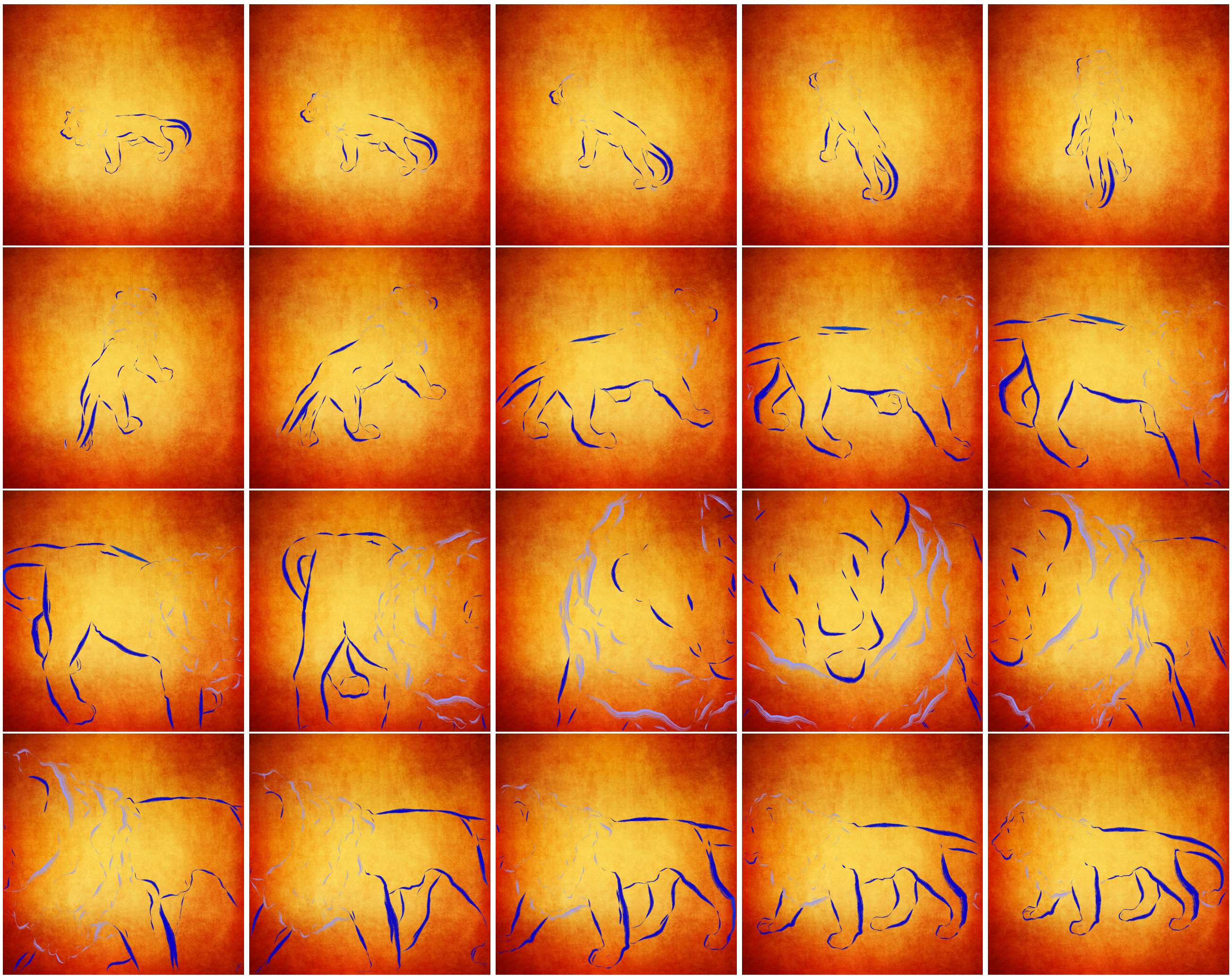


Figure 9: ***Line drawing sequence obtained by our method (see video).*** *This example shows how the proposed method allows complex animations combining challenging transformations such as rotation and zoom in and out.*

Appendix

Minimizing the energy in Eq. 5 requires a robust estimation of time derivatives in a real-time scenario where only the parameterizations from previous frames is known. We found that numerical stability can be assured by approximating the time derivatives with finite difference and treating them as polynomial functions with $p(t,s)$ as the dependent variable:

$$\frac{dp(t,s)}{dt} \approx \frac{p(t,s) - p(t-\Delta t)}{\Delta t} \tag{20}$$

$$\frac{d^2p(t,s)}{dt^2} \approx \frac{p(t-2\Delta t,s) - 2p(t-\Delta t) + p(t,s)}{(\Delta t)^2} \tag{21}$$

For $\Delta t = 1$ the above approximations reduce to:

$$\frac{dp(t,s)}{dt} \approx p(t,s) - p(t-\Delta t) \tag{22}$$

$$\frac{d^2p(t,s)}{dt^2} \approx p(t-\Delta t,s) - 2p(t-\Delta t) + p(t,s) \tag{23}$$

The energy in Eq. 5 can be rewritten as the following functional:

$$\begin{aligned} S(p(t,s)) &= \int_0^1 L(s,p(t,s),p_s,p_{ss})\,ds \\ &= \int_0^1 \frac{1}{2}\Bigg(w_{AI}\Big|p(t,s)-g(t,s)\Big|^2 \\ &\quad + w_{TC1}\Big|p(t,s)-p(t-\Delta t)\Big|^2 \\ &\quad + w_{TC2}\Big|p(t-2\Delta t,s)-2p(t-\Delta t)+p(t,s)\Big|^2 \\ &\quad + w_{SC2}\Big|\frac{d^2p(t,s)}{ds^2}\Big|^2\Bigg)\,ds \end{aligned} \tag{24}$$

where $p_s = \frac{dp(t,s)}{ds}$ and $p_{ss} = \frac{d^2p(t,s)}{ds^2}$.

The corresponding Euler-Lagrange equation is given by:

$$\frac{\partial L}{\partial p(t,s)} - \frac{\partial}{\partial s}\left(\frac{\partial L}{\partial p_s}\right) + \frac{\partial^2}{\partial s^2}\left(\frac{\partial L}{\partial p_{ss}}\right) = 0 \tag{25}$$

The partial derivatives of L are:

$$\begin{aligned}\frac{\partial L}{\partial p} =& w_{AI}\big(p(t,s)-g(t,s)\big)\\ &+w_{TC1}\big(p(t,s)-p(t-\Delta t)\big)\\ &+w_{TC2}\big(p(t-2\Delta t,s)-2p(t-\Delta t)+p(t,s)\big)\end{aligned} \tag{26}$$

$$\frac{\partial}{\partial s}\left(\frac{\partial L}{\partial p_s}\right) = 0 \tag{27}$$

$$\frac{\partial^2}{\partial s^2}\left(\frac{\partial L}{\partial p_{ss}}\right) = \frac{\partial^4 p(t,s)}{\partial s^4} \tag{28}$$

By substituting the partial derivatives of L into the Euler-Lagrange equation, we get:

$$\begin{aligned}w_{AI}\big(p(t,s)-g(t,s)\big)+w_{TC1}\big(p(t,s)-p(t-\Delta t)\big)\\ +w_{TC2}\big(p(t-2\Delta t,s)-2p(t-\Delta t)+p(t,s)\big)\\ +w_{SC2}\frac{\partial^4 p(t,s)}{\partial s^4} = 0\end{aligned} \tag{29}$$

We then discretize for current frame k as follows:

$$\begin{aligned}w_{AI}(\mathbf{p}_k-\mathbf{g}_k)+w_{TC1}(\mathbf{p}_k-\mathbf{p}_{k-1})+w_{TC2}(\mathbf{p}_{k-2}-2\mathbf{p}_{k-1}+\mathbf{p}_k)\\ +w_{SC2}B\mathbf{p}_k = 0\end{aligned} \tag{30}$$

Finally, we rearrange the above equation to factor $\mathbf{p}_k$:

$$A\mathbf{p}_k = w_{AI}\mathbf{g}_k + w_{TC1}\mathbf{p}_{k-1} - w_{TC2}(\mathbf{p}_{k-2}-2\mathbf{p}_{k-1}) \tag{31}$$

where $\mathbf{p}_{k-1}$ and $\mathbf{p}_{k-2}$ are the parameterizations at frames $k-1$ and $k-2$ respectively, and A is a sparse $n \times n$ matrix such that $A = (w_{AI}+w_{TC1}+w_{TC2})I + w_{SC2}B$, where B is the 4th order derivative coefficient matrix in Eq. 11. In our implementation, we use Compressed Row Storage (CRS) for memory efficient storage of sparse matrices and Pre-conditioned Bi-Conjugate Gradient (PBCG) to solve the above sparse linear system with great efficiency.

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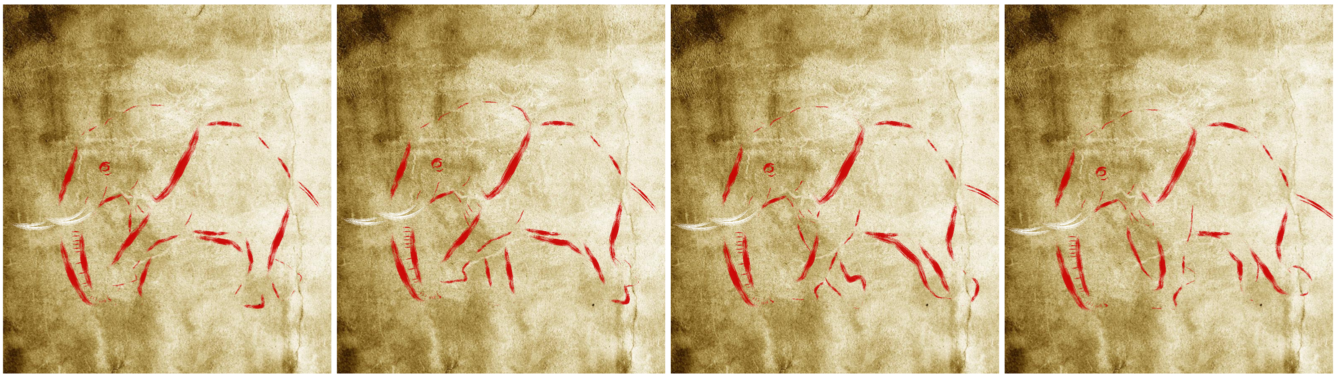

Figure 10: ***Temporally coherent result in a line drawing sequence of an animated model (see video).***

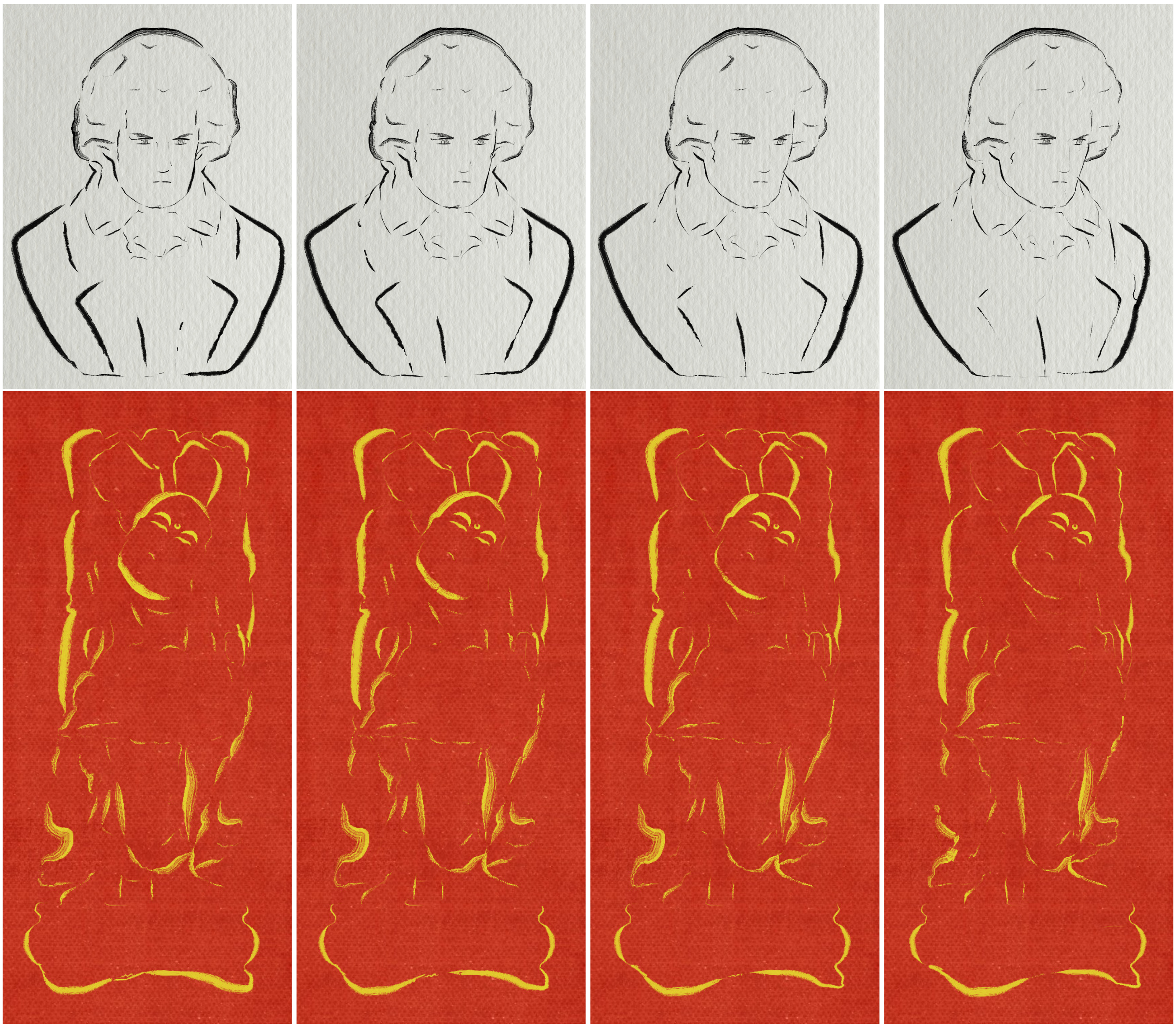

Figure 11: ***Temporally coherent results for complex models as the view is rotated (see video).***