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Effects of a contact energy for surfaces

by

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Summary

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Introduction

In this paper, we consider surfaces associated to an energy structure. The energy is defined as the following: Let n be a positive integer and $\Omega \subset \mathbb{R}^n$ a domain whose boundary is smooth. Assume $\theta \in [0,\pi]$ is a constant and $\mathcal{H}^k$ is the k dimensional Hausdorff measure for $0 \leq k < n$. Then for any subset $A \subset \Omega$ the energy of A is defined by

$$E(A) := \mathcal{H}^{n-1}(\partial^* A \cap \Omega) + \cos\theta \mathcal{H}^{n-1}(\partial^* A \cap \partial\Omega), \tag{1}$$

where $\partial^* A$ is the reduced boundary of A. Here we regard $E(A) = \infty$ if A is not of locally finite perimeter in $\mathbb{R}^n$. This energy has a relation with capillarity phenomena, in more details, we may regard Ω, A and $\Omega \backslash A$ as a container, a fluid and another fluid or gas, respectively (cf. [4]). The first term and the second term of the right hand side of (1) are called the surface energy and the contact energy, respectively. Now, we define

$$\Sigma_m := \{A \subset \Omega : \mathcal{L}^n(A) = m\}$$

for $m \in (0, \mathcal{L}^n(\Omega))$, where $\mathcal{L}^n$ is the Lebesgue measure on $\mathbb{R}^n$. If a set $A \in \Sigma_m$ is critical point of E in Σ_m, then $M := \partial^* A \cap \Omega$ satisfies

$$\begin{cases} h_M = \text{constant} & \text{on} \quad M, \\ \langle \nu_M, \nu_\Omega \rangle = \cos\theta & \text{on} \quad \partial M \end{cases} \tag{2}$$

under the assumptions of the smoothness for M and ∂M, where h_M is the mean curvature of M, ν_M is the outer unit normal to A and ν_Ω is the outer unit normal to Ω. When $\Omega = \mathbb{R}^n$, $\partial\Omega$ is empty, thus the energy E consists of only surface energy (or surface area). In this case, the constant mean curvature property appears for critical points of the surface energy under the restriction of fixed volume condition. The second one of (2) means the angle between ν_M and ν_Ω is θ, thus the contact energy of (1) has an effect for the contact angle condition of the surface between A and $\Omega \backslash A$.

In the case of $\Omega = \mathbb{R}^n$, mean curvature flow is well known as the formal gradient flow of the surface energy, that is, for an initial set $A(0) \subset \mathbb{R}^n$ if $A(t) \subset \mathbb{R}^n$ moves to the inverse direction of the "gradient" of the surface energy, then the moving surface $M(t) := \partial A(t)$ is called the mean curvature flow. Thus the surface area of $M(t)$ is non-increasing with respect to the parameter t and we may expect the surface $M(t)$ is asymptotic to a point. In the case of $n = 2$, we see the conjecture is true by combining the results shown by Gage and Hamilton [6] and Grayson [8], that is, $M(t)$ converges to a point in finite time. However, in the other case, i.e., $n > 2$, Grayson [7] also construct a counter-example for the conjecture. The example is formed as dumbbell and the neck develops a singularity. Now, we focus on the "gradient" flow with the fixed volume condition, namely, in the case that $A(t)$ moves to the inverse direction of the "gradient" of the surface energy in the set Σ_m with a fixed constant m. This "gradient" flow is introduced by Gage [5] in the case of $n = 2$, which is called the area preserving curvature flow. If the moving curve $M(t)$ is an area preserving curvature flow, the area $\mathcal{L}^2(A(t))$ is preserved and the length $\mathcal{H}^1(M(t))$ is non-increasing, thus we may expect the moving curve is asymptotic to a circle. In Gage's paper [5], he showed the flow $M(t)$ converges to a circle as the time tend to infinity if $A(0)$ is a convex set. Furthermore, he also constructed a area preserving curvature flow which has a self-intersecting point in finite time in the case that $A(0)$ is not convex. In the general dimension, Escher and Simonett [3] considered the volume preserving mean curvature flow ("gradient" flow of surface area with fixed volume condition) and showed the local exponential stability of spheres for the flow. In Section 2, we consider the "gradient" flow of the energy E with the fixed area condition on the domain $\Omega = \{(x, y) \in \mathbb{R}^2 : y > 0\}$ including the case that E is a non-uniform energy, which is based on [11]. The non-uniformness means that the contact energy density depends on the position. When the energy is uniform, energy minimizers exist and they are arcs, on the other hand, when the energy is non-uniform, we can show the non-existence of energy minimizers. Thus the asymptotic behavior of solutions for the "gradient" flow of the non-uniformly energy is not obvious, and we show the existence and the local exponential stability of traveling waves for the "gradient" flow in Section 2.

On the other hand, the energy structure has been considered by using a phase separation model. An energy structure of the phase separation model was introduced by Modica [14] according to the van der Waals-Cahn-Hilliard theory [2] and Cahn's approach [1] as the following:

$$E_\varepsilon(u) = \int_\Omega \frac{\varepsilon|\nabla u|^2}{2} + \frac{W(u)}{\varepsilon}\, dx + \int_{\partial\Omega} \sigma(u)\, d\mathcal{H}^{n-1},$$

where $\varepsilon \in (0, 1)$ is a small parameter, $\Omega \subset \mathbb{R}^n$ is a bounded domain, u is a function defined on $\bar{\Omega}$, W is a double well potential with strict minima at ± 1 and σ is a function on $\mathbb{R}$. This energy describe the stable configurations of the fluid, and the function u, the strict minima of W and the function σ correspond to the normalized density of a multi-phase fluid, stable fluid phases and a contact energy density between the fluid and the container wall $\partial\Omega$, respectively. Now, we consider this energy under the restriction of the fixed total mass

$$\int_\Omega u\, dx = l$$

of the fluid in Ω for some constant $l \in (-|\Omega|, |\Omega|)$. If ε is sufficiently small, then the effect of $|\nabla u|^2$ is weak and the effect of $W(u)$ is strong, and the smallness of $|\nabla u|^2$ and $W(u)$ means the smoothness of u and the stability of the fluid, respectively. According to this energy effect, if u_ε satisfies uniform boundedness of the energy $E_\varepsilon(u_\varepsilon)$ with respect to $\varepsilon \in (0, 1)$, we may expect that the domain Ω is mostly divided into two regions $\{u_\varepsilon \approx 1\}$ and $\{u_\varepsilon \approx -1\}$ for sufficiently small ε. The correctness of the perturbation E_ε of the energy E was also shown by Modica [14] within the framework of the Γ-convergence, namely, a family of energy minimizers $\{u_\varepsilon\}_{\varepsilon\in(0,1)}$ subconverges to a bounded variation function u_0 in $L^1(\Omega)$ such that $u_0 = \pm 1$ a.e. in Ω and the set $\{u_0 = 1\}$ is a minimizer of E in Σ_m with

$$\theta = \arccos\left(\frac{\sigma(1) - \sigma(-1)}{c_0}\right), \quad m = \frac{l + |\Omega|}{2},$$

where

$$c_0 = \int_{-1}^{1} \sqrt{2W(s)}\, ds.$$

In Section 5, we expand this energy convergence theory for critical points in a weak sense, that is, for the limit u_0 of critical points u_ε for E_ε, the surface $\partial\{u_0 = 1\} \cap \Omega$ satisfies the property (2) in a weak sense. This section is based on [13].

This line of the research has been carried out by introducing a natural varifold associated with u_ε (cf. [9, 10, 15, 16]), and the surface $\partial\{u_0 = 1\} \cap \Omega$ was often described as the limit varifold in a weak sense. We note some notations related to varifold in Section 3. The generalized mean curvature of varifold was already introduced and also the first condition of (2) for the limit varifold was shown in our singular limit problem. On the other hand, as far as I know, general contact angle condition for varifolds has not been studied, thus we need to introduce a notion of contact angle condition for varifolds to prove the second condition of (2) for the limit varifold. In Section 4, we study general varifolds with a fixed contact angle condition, which is based on [12].

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