

論文 / 著書情報  
Article / Book Information

|                   |                                                                                                                                                                                             |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
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| Title(English)    | Knot concordance in general 4-manifolds                                                                                                                                                     |
| 著者(和文)            | 佐藤光樹                                                                                                                                                                                        |
| Author(English)   | Kouki Sato                                                                                                                                                                                  |
| 出典(和文)            | 学位:博士(理学),<br>学位授与機関:東京工業大学,<br>報告番号:甲第10390号,<br>授与年月日:2017年3月26日,<br>学位の種別:課程博士,<br>審査員:KALMAN TAMAS,芥川 和雄(芥川 一雄),遠藤 久顕,本多 宣博,村山 光孝,安原 晃                                                  |
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| 学位種別(和文)          | 博士論文                                                                                                                                                                                        |
| Category(English) | Doctoral Thesis                                                                                                                                                                             |
| 種別(和文)            | 論文要旨                                                                                                                                                                                        |
| Type(English)     | Summary                                                                                                                                                                                     |

# 論文要旨

## THESIS SUMMARY

|                          |       |    |                                            |                      |
|--------------------------|-------|----|--------------------------------------------|----------------------|
| 専攻 :<br>Department of    | 数学    | 専攻 | 申請学位 (専攻分野) :<br>Academic Degree Requested | 博士 (理学)<br>Doctor of |
| 学生氏名 :<br>Student's Name | 佐藤 光樹 |    | 指導教員 (主) :<br>Academic Advisor(main)       | カールマン タマシユ           |
|                          |       |    | 指導教員 (副) :<br>Academic Advisor(sub)        |                      |

### 要旨 (英文 800 語程度)

Thesis Summary (approx.800 English Words)

A *knot* is an embedding of the circle into the 3-sphere. *Knot concordance* is an equivalence relation on the set of knots, which generalizes isotopy from a viewpoint of cobordism theory. In particular, the knot concordance class of the standard circle in the 3-sphere is equal to the set of knots which bound a disk in the 4-ball (such knots are called *slice knots*). In knot theory, the classification of knot concordance classes is known as one of the most important open problem.

When two knots are given, there exists a knot such that it is slice if and only if previous two knots are concordant. Hence any concordance problem is translated into sliceness of knots, and it is an essential problem which knots are slice. On the other hand, if we remove an open 4-ball from a given closed 4-manifold, then the boundary is also the 3-sphere, and hence we can consider which knots can bound a disk in such a punctured 4-manifold. In this thesis, titled *Knot concordance in general 4-manifolds*, we study such sliceness of knots in closed 4-manifolds, and consider how the sliceness of knots affects their knot concordance classes.

Although we can define sliceness of knots in an arbitrary closed 4-manifolds, the sliceness is not always interesting. Indeed, it is proved in previous works that there exist infinitely many punctured 4-manifolds in which an arbitrary knot can bound a disk. Hence, in order to find an interesting relationship between general and classical sliceness, we need to choose suitable closed 4-manifolds or set additional conditions. In this thesis, we focus on three types of sliceness, and show that if a given knot is such a slice knot, then its knot concordance invariants must satisfy some algebraic conditions. Here, a *knot concordance invariant* is a map from the set of knot concordance classes to another set. The main tools in this thesis are  $d_1$  and  $\nu^+$ , which are integer-valued knot concordance invariants derived from Heegaard Floer homology theory. We give some estimates of these invariants for our slice knots. While Heegaard Floer homology theory gives very powerful invariants for 3-manifolds and knots, computing those invariants is a difficult problem itself. We show that our estimates of  $d_1$  and  $\nu^+$  contribute not only to study of general sliceness, but also to study of these invariants themselves. The organization of this thesis is as follows.

In Chapters 1 and 2, we show previous works on sliceness of knots in general 4-manifolds, and describe the outline of the following three chapters. Moreover, we give an overview of Heegaard Floer homology theory, and introduce the invariants  $d_1$  and  $\nu^+$ .

In Chapter 3, titled *Characteristic sliceness*, we study disks in punctured 4-manifolds which represent a characteristic homology class. We note that the complement of any such disk has a spin structure. We focus on the cases where the complement has definite intersection form, and give an upper bound of  $d_1$  for their boundary. Furthermore, we use this upper bound to compare  $d_1$  with another concordance invariant  $\tau$ , and give counterexamples to a conjecture on the relationship between these invariants.

In Chapter 4, titled *Null-homologous sliceness*, we consider null-homologous disks in punctured 4-manifolds, and prove that there exist infinitely many knots which cannot bound a disk in any definite 4-manifold. Furthermore, we prove that if we take topologically locally flat embedding (which is weaker than smoothly embedding) into consideration, then all our examples bound a disk in the 4-ball.

In Chapter 5, we give estimates of  $d_1$  and  $\nu^+$  for slice knots in the complex projective plane. As an application, we extend an estimate of  $\nu^+$  for particular positive cable knots to all cable knots.

備考 : 論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。

Note : Thesis Summary should be submitted in either a copy of 2000 Japanese Characters and 300 Words (English) or 1 copy of 800 Words (English).

注意 : 論文要旨は、東工大リサーチリポジトリ(T2R2)にてインターネット公表されますので、公表可能な範囲の内容で作成してください。

Attention: Thesis Summary will be published on Tokyo Tech Research Repository Website (T2R2).