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論 文 要 旨

THESIS SUMMARY

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学生氏名： Student's Name	湯淺亘	指導教員（主）： Academic Supervisor(main)	遠藤久顕 教授
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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

In this thesis, I treated two objects; the Goldman-Turaev Lie bialgebra and the colored Jones polynomials.

First, I defined the oriented and unoriented Goldman Lie algebra for an oriented 2-orbifold with isolated singularities. The Turaev cobracket was also introduced for the oriented Goldman Lie algebra on it. It is made into a Lie bialgebra. Furthermore, I defined the oriented Goldman Poisson algebra and Turaev co-Poisson coalgebra for an oriented bordered surface. The unoriented Goldman Poisson algebra was also defined and its skein quantization was given. An oriented bordered surface (S,M) is an oriented surface S with a finite set M on the boundary of S. The underlying algebra of the (un)oriented Goldman Poisson algebra for (S,M) is the symmetric algebra of free $\mathbf{Z}$-module spanned by the set of homotopy classes of (un)oriented free loops and boundary-fixing homotopy classes of (un)oriented arcs whose ends lie in M. The Poisson bracket and Poisson cobracket are generalizations of the Goldman bracket and the Turaev cobracket. The oriented Goldman Poisson algebra for a bordered disk coincides with the swapping algebra defined by Labourie. I constructed the skein quantization of unoriented Goldman Poisson algebra for (S,M) by using the Kauffman bracket skein algebra for (S,M) defined by Muller. Muller showed a localization of the Kauffman bracket skein algebra of (S,M) is coincides with a quantum cluster algebra obtained by a triangulation of (S,M) with some conditions. I also observed a relationship between oriented Goldman Lie algebras of oriented surfaces S and T when S is a finite Galois covering space of T. The covering transformation group acts on the Goldman Lie algebra of S naturally. The invariant submodule of the Goldman Lie algebra of S is made into a Lie subalgebra of the Goldman Lie algebra of S. I showed that the invariant Lie subalgebra is embedded into one of T as a Lie algebra.

The second topic is a graphical calculus for colored Jones polynomials. I calculated A_1 and A_2 webs with clasps and obtained some formulas: for the A_1 and the A_2 web spaces, the skein relations with clasps, the m-full twists formula for two strands with clasps, and the bubble skein formula. These formulas are useful to compute the sl_2 and sl_3 colored Jones polynomials. Especially, it is difficult to compute the sl_3 colored Jones polynomial and known a few examples. The formula for trefoil knot was given by Lawrence. More generally for the torus knot T(2,m) was given by Garoufalidis, Morton and Vuong through a method of the representation theory. By using our formulas, I explicitly computed the sl_2 and sl_3 colored Jones polynomials for 2-bridge knots and links. Moreover, I explicitly gave the tail of the sl_3 colored Jones polynomial for the (2, 2m)-torus link T(2,2m). The tail is a q-series which is a kind of limit of the colored Jones polynomial. For the sl_2 colored Jones polynomial, the existence of sl_2 tails for some class of links was known, and gave some explicit formulas of sl_2 tails. However, an existence theorem and explicit formulas for sl_3 tails were unknown. Precisely, I gave two explicit formulas for the sl_3 tail for T(2,2m). One of the two formulas was obtained by using the m-full twists formula for the A_2 web space with clasps. The other is obtained by calculation using sl_3 colored trivalent graphs. The method of graphical calculus using colored trivalent graphs are well-known for A_1 webs but not for A_2 webs. To be precise, the value of the theta-graph and the quantum 6-j symbols for sl_3 are unknown. These values are necessary for calculations of colored Jones polynomials using colored trivalent graph. I evaluated the theta-graph and the quantum 6-j symbols for A_2 webs in a special case and computed the sl_3 colored Jones polynomial for the T(2,2m). I derived two formulas for the sl_3 tail for the T(2, 2m) from the explicit presentations of the sl_3 colored Jones polynomial. Of course, these two sl_3 tails are identical. Thus, I obtained a q-series identity. This q-series identity is a knot theoretical generalization of the Andrews-Gordon identities for the Ramanujan false theta function. Recently, Bringmann, Kaszian, and Milas confirmed that the left-hand side of the q-series identity is written by a "summand" of the false theta function for sl_3.

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