

論文 / 著書情報  
Article / Book Information

|                  |                                                                                                               |
|------------------|---------------------------------------------------------------------------------------------------------------|
| Title            | Seismic performance evaluation of single damped - outrigger system incorporating buckling - restrained braces |
| Authors          | Pao-Chun Lin, Toru Takeuchi, Ryota Matsui                                                                     |
| Citation         | Earthquake Engineering and Structural Dynamics, Vol. 47, Issue 12, pp. 2343-2365                              |
| Pub. date        | 2018, 7                                                                                                       |
| DOI              | <a href="https://doi.org/10.1002/eqe.3072">https://doi.org/10.1002/eqe.3072</a>                               |
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## RESEARCH ARTICLE

# Seismic performance evaluation of single damped-outrigger system incorporating buckling-restrained braces

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**Summary**

The outrigger system is an effective means of controlling the seismic response of core-tube type tall buildings by mobilizing the axial stiffness of the perimeter columns. This study investigates the damped-outrigger, incorporating the buckling-restrained brace (BRB) as energy dissipation device (BRB-outrigger system). The building's seismic responses are expected to be effectively reduced because of the high BRB elastic stiffness during minor earthquakes and through the stable energy dissipation mechanism of the BRB during large earthquakes. The seismic behavior of the BRB-outrigger system was investigated by performing a spectral analysis considering the equivalent damping to incorporate the effects of BRB inelastic deformation. Nonlinear response history analyses were performed to verify the spectral analysis results. The analytical models with building heights of 64, 128, and 256 m were utilized to investigate the optimal outrigger elevation and the relationships between the outrigger truss flexural stiffness, BRB axial stiffness, and perimeter column axial stiffness to achieve the minimum roof drift and acceleration responses. The method of determining the BRB yield deformation and its effect on overall seismic performance were also investigated. The study concludes with a design recommendation for the single BRB-outrigger system.

**KEYWORDS**

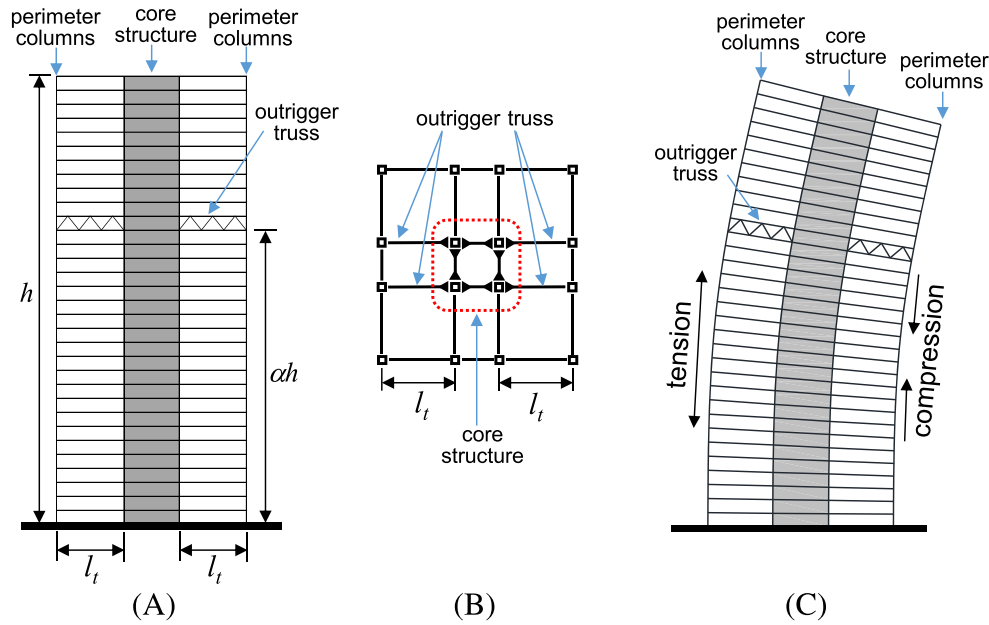
buckling-restrained brace, optimal design, outrigger, parametric analysis, spectral analysis

## 1 | INTRODUCTION

The outrigger system has been widely adopted in tall core-tube type buildings around the world.<sup>1,2</sup> The traditional outrigger mitigates building seismic responses by increasing the system stiffness. However, the increased stiffness may also amplify the acceleration response. Figure 1A and B shows a typical building elevation with a single outrigger, and the floor framing plan on the outrigger floor. The core structure provides most of the lateral force resistance capacity, and the perimeter columns are responsible for supporting the gravity loads. When the building deforms horizontally (Figure 1C), the core structure's flexural deformation triggers the relatively stiff outrigger truss to rotate. The outrigger truss then triggers additional extension or compression on the perimeter columns below the outrigger. By mobilizing the axial stiffness of the perimeter columns, the flexural demand on the core structure can be reduced. However, the elastic design concept of outrigger may result in large force demands on the outrigger members, increasing both complexity and costs in engineering practices.<sup>3</sup>

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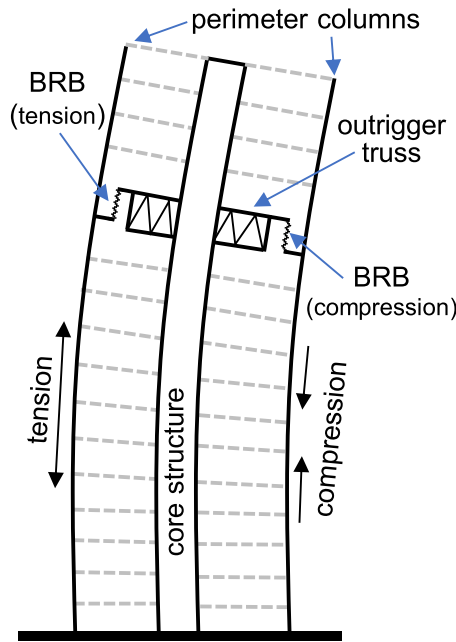


**FIGURE 1** A, Elevation, B, floor framing plan on outrigger floor, and C, outrigger truss mobilizing perimeter column axial stiffness [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

The concept of a damped-outrigger has been proposed<sup>4-7</sup> to increase the damping, instead of increasing the stiffness, by inserting energy dissipation devices at the outrigger truss ends. The dampers dissipate energy through the relative movements between outrigger truss ends and the perimeter columns. The optimal damped-outrigger truss elevation required to achieve a maximum damping ratio has been investigated through complex eigenvalue analysis,<sup>4,5</sup> and dynamic stiffness methods.<sup>6</sup> The performance of a damped-outrigger in reducing the seismic response was also verified experimentally.<sup>8</sup> Huang and Takeuchi<sup>4</sup> indicated that the optimal elevation of a damped-outrigger incorporating viscous dampers ranges from 50% to 80% of the building height. In addition, the seismic performance of multioutrigger was also investigated.<sup>9</sup> The damped-outrigger system adopting viscous dampers as energy dissipation device has been utilized in actual construction projects.<sup>10</sup> In addition, the outrigger truss member, incorporating buckling-restrained brace (BRB),<sup>11</sup> was utilized to limit the maximum forces generated in columns, at connections, and in core walls in recent design practices.<sup>12</sup>

In this study, the BRB is incorporated as an energy dissipation device in the damped-outrigger (BRB-outrigger) system, as shown in Figure 2. The BRBs are arranged vertically between the outrigger truss ends and the perimeter columns. The outrigger truss, BRB, and the perimeter column below the outrigger elevation act in series. Therefore, the maximum force demands in the outrigger truss members and perimeter columns are limited by BRB's axial force capacity. This provides engineers with clear force demands when designing the perimeter columns and outrigger truss members. As shown in Figure 2, when the building deforms toward the right, the right BRB is in compression, and the left in tension. As the BRB's axial deformation exceeds its yield deformation ( $u_{d,y}$ ) as expected during large earthquakes, the BRB dissipates energy through its inelastic deformation, thereby reducing building seismic responses. The stable BRB hysteresis response provides the system with a stable energy dissipation mechanism. During minor earthquakes, a properly designed BRB-outrigger system can behave like a traditional elastic outrigger through BRB's elastic responses. In addition, the feasible BRB strength and stiffness are suitable for various structural configurations.

When viscous dampers are adopted in damped-outrigger system to control responses induced from wind and seismic loads, the design requirements and velocity ranges corresponding to these two demands are usually different.<sup>13</sup> The wide axial force capacity range and feasible stiffness of BRB allows the BRB-outrigger system to be an alternative in resisting seismic loads. However, the wind loads are sometimes greater than seismic lateral loads for high-rise buildings. This could lead to very large axial force and stiffness demands on the BRB because it is inappropriate to allow BRBs to yield due to wind loads. In such circumstances, the BRB might be overdesigned and may not develop satisfactory hysteretic responses during large earthquakes. The combined usage of viscous dampers and BRB in resisting wind and seismic loads respectively could be further investigated in future. This study focuses on the performance of a single BRB-outrigger system in resisting seismic loads.



**FIGURE 2** Buckling-restrained brace-outrigger system [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

This study investigates the optimal outrigger elevation, optimal relationships between the outrigger truss flexural stiffness ( $k_t$ ), BRB's axial stiffness ( $k_d$ ) together with the yield roof drift ratio ( $\theta_r$ ), and perimeter column axial stiffness ( $k_c$ ) to minimize the building seismic responses through spectral analysis (SA), incorporating the equivalent damping concept to consider the BRB's inelastic performance. A nonlinear response history analysis (NLRHA) was also performed to verify the effectiveness of the SA results. The reduction factors for reducing the maximum roof drift ratio and acceleration are adopted as the indicators in the parametric study. This study concludes with a design recommendation for a single BRB-outrigger system.

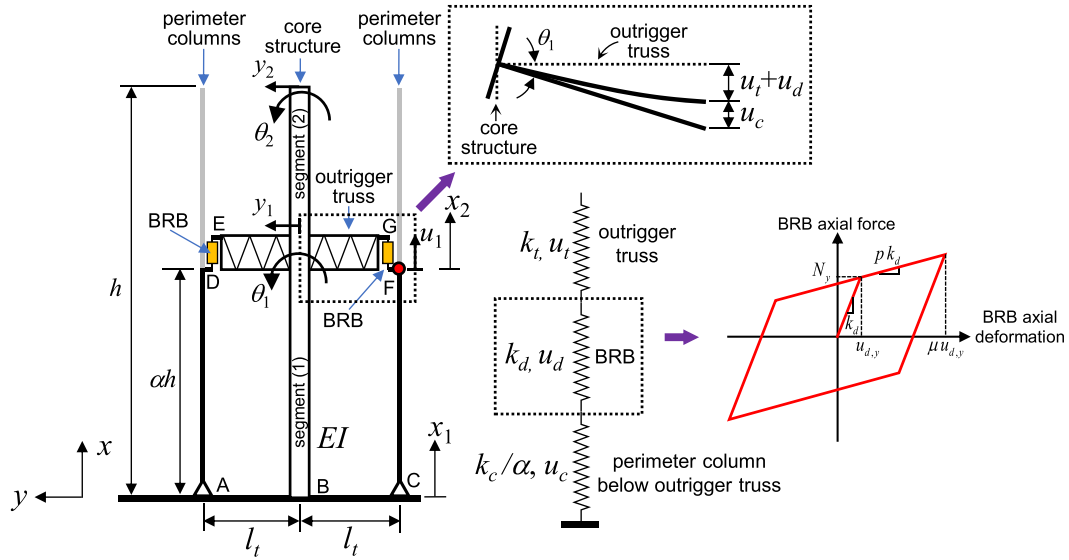
## 2 | ANALYTICAL MODELS

### 2.1 | Simplified structure

A core-tube type structure with a BRB-outrigger system is simplified as shown in Figure 3. For simplicity, it is assumed that the building's lateral stiffness is concentrated on the core structure. The core structure is represented by a cantilever column with uniformly distributed flexural rigidity  $EI$ . Each of the perimeter columns of height  $h$  has an axial stiffness  $k_c$  and a pinned support at the base. The outrigger trusses are located at a height  $\alpha h$  above the ground. The outrigger trusses on both sides of the core structure are assumed identical to each other, and each has a flexural stiffness  $k_t$ . The connections between the outrigger trusses and the core structure have full moment-transfer capacity. The BRB is arranged vertically and connects the outrigger truss end (points G and E in Figure 3) to the perimeter column (points F and D in Figure 3). The axial stiffness of each BRB is  $k_d$ , and both ends of the BRB are free to rotate. The masses are assumed to be concentrated at the core structure and distributed uniformly along the height. The force and deformation relationship of the BRB is bilinear, with a postyield stiffness ratio  $p$ . The other members are assumed to be linearly elastic. The core structure rotation at the outrigger elevation ( $\theta_1$ ) can be expressed as follows:

$$\theta_1 = \frac{1}{l_t}(u_c + u_t + u_d) = \frac{1}{l_t} \left[ 1 + \frac{k_c}{\alpha} \left( \frac{1}{k_t} + \frac{1}{k_d} \right) \right] u_c = \frac{k_d}{l_t} \left( \frac{\alpha}{k_c} + \frac{1}{k_t} + \frac{1}{k_d} \right) u_d \quad (1)$$

where  $l_t$  is the outrigger truss span,  $u_d$  and  $u_c$  are the axial deformations of BRB and perimeter column below outrigger elevation, respectively, and  $u_t$  is the outrigger truss's flexural deformation. The vertical force acting at outrigger truss end ( $F$ ), the corresponding moment applied at the core structure ( $M$ ), and the equivalent rotational spring stiffness induced from outrigger system ( $k_g$ ) can be calculated as follows:



**FIGURE 3** Simplified structure and force and deformation relationship of buckling-restrained brace [Colour figure can be viewed at wileyonlinelibrary.com]

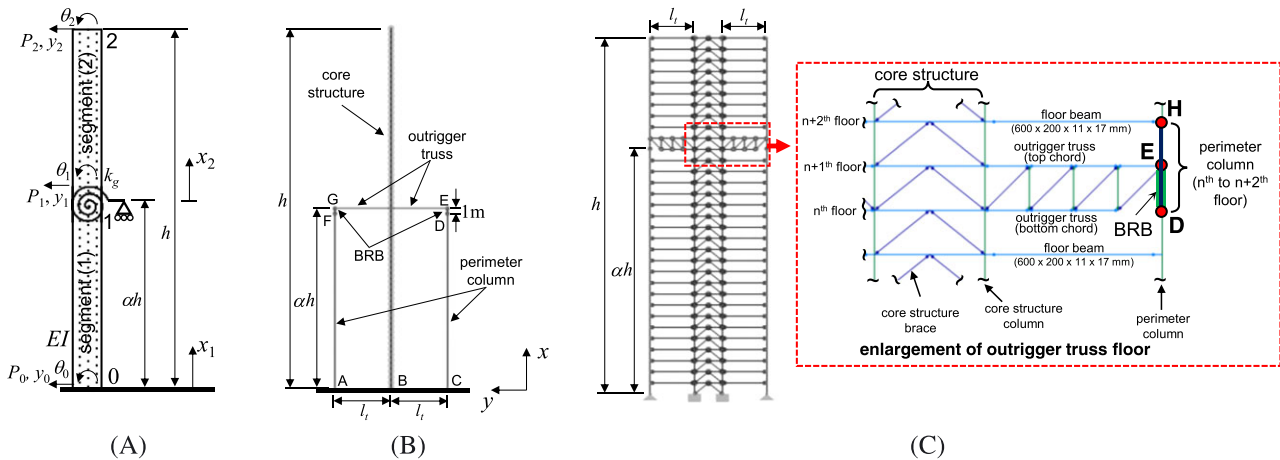
$$F = \frac{u_c k_c}{\alpha} = k_d u_d = k_t u_t \quad (2)$$

$$M = 2Fl_t = \frac{2l_t^2}{\alpha/k_c + 1/k_d + 1/k_t} \theta_1 = \frac{2l_t^2}{1/k_b + 1/k_d} \theta_1 = k_g \theta_1, \quad \text{where } k_b = \frac{1}{\alpha/k_c + 1/k_t} \quad (3)$$

The BRB-outrigger system can be further simplified as a rotational spring with stiffness  $k_g$ , attached to the core structure, as shown in Figure 4A. The core structure below and above outrigger elevation is divided into segments (1) and (2), respectively. The lateral displacement,  $y_N(x_N, t)$ , at distance  $x_N$  from the bottom end of each segment, at time  $t$ , can be solved by applying the D'Alembert principle:

$$EI \frac{\partial^4 y_N(x_N, t)}{\partial x_N^4} + m \frac{\partial^2 y_N(x_N, t)}{\partial t^2} = 0, \quad N = 1 \text{ or } 2 \quad (4)$$

where  $m$  is the mass per unit height. It is assumed that the lateral displacements within segments (1) and (2) ( $y_1$  and  $y_2$ ) are in the form as follows:



**FIGURE 4** Schematic views A, uniform mass, B, discrete mass, and C, member-by-member models [Colour figure can be viewed at wileyonlinelibrary.com]

$$y_N(x_N, t) = Y_N(x_N)Q(t), \quad N = 1 \text{ or } 2 \quad (5)$$

Substituting Equation 5 into Equation 4, the solution of lateral displacement is as follows:

$$Y_N(x_N) = A_{N1} \cosh\left(\frac{\lambda}{h}x_N\right) + A_{N2} \sinh\left(\frac{\lambda}{h}x_N\right) + A_{N3} \cos\left(\frac{\lambda}{h}x_N\right) + A_{N4} \sin\left(\frac{\lambda}{h}x_N\right), \quad \text{where } \lambda^4 = \frac{m\omega^2 h^4}{EI} \quad (6)$$

where  $\omega$  is the angular frequency. By applying the boundary conditions at the ends of segments (1) and (2), Equation 6 can be expressed in matrix form as follows:

$$\mathbf{u}_N = \begin{bmatrix} y_{N-1} \\ \theta_{N-1}h \\ y_N \\ \theta_N h \end{bmatrix} = \begin{bmatrix} Y_N(0) \\ Y'_N(0)h \\ Y_N(L_N) \\ Y'_N(L_N)h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & \lambda & 0 & \lambda \\ C_N & S_N & c_N & s_N \\ \lambda S_N & \lambda C_N & -\lambda s_N & \lambda c_N \end{bmatrix} \begin{bmatrix} A_{N1} \\ A_{N2} \\ A_{N3} \\ A_{N4} \end{bmatrix} = \mathbf{D}_N \mathbf{A}_N, \quad N = 1 \text{ or } 2, \quad (7)$$

where

$$\begin{aligned} L_1 &= \alpha h & C_1 &= \cosh(\alpha\lambda) & C_2 &= \cosh(\lambda - \alpha\lambda) \\ L_2 &= (1 - \alpha)h & S_1 &= \sinh(\alpha\lambda) & S_2 &= \sinh(\lambda - \alpha\lambda) \\ Y'_N &= \frac{dY_N(x)}{dx} & c_1 &= \cos(\alpha\lambda) & c_2 &= \cos(\lambda - \alpha\lambda) \\ & & s_1 &= \sin(\alpha\lambda) & s_2 &= \sin(\lambda - \alpha\lambda) \end{aligned}$$

where  $\mathbf{u}_1$  and  $\mathbf{u}_2$  are the displacement matrices corresponding to both ends of segments (1) and (2), respectively. As shown in Figure 4A, for individual segment  $N$ , the bottom and top ends' shear  ${}_{(N)}P_{N-1}$ ,  ${}_{(N)}P_N$  and bending moments  ${}_{(N)}M_{N-1}$ ,  ${}_{(N)}M_N$  are expressed in matrix  $\mathbf{P}_N$  as follows:

$$\mathbf{P}_N = \begin{bmatrix} {}_{(N)}P_{N-1} \\ {}_{(N)}M_{N-1}/h \\ {}_{(N)}P_N \\ {}_{(N)}M_N/h \end{bmatrix} = \frac{EI}{h^3} \begin{bmatrix} 0 & \lambda^3 & 0 & -\lambda^3 \\ -\lambda^2 & 0 & \lambda^2 & 0 \\ -\lambda^3 S_N & -\lambda^3 C_N & -\lambda^3 s_N & \lambda^3 c_N \\ \lambda^2 C_N & \lambda^2 S_N & -\lambda^2 c_N & -\lambda^2 s_N \end{bmatrix} \mathbf{A}_N \quad (8)$$

$$= \frac{EI}{h^3} \begin{bmatrix} 0 & \lambda^3 & 0 & -\lambda^3 \\ -\lambda^2 & 0 & \lambda^2 & 0 \\ -\lambda^3 S_N & -\lambda^3 C_N & -\lambda^3 s_N & \lambda^3 c_N \\ \lambda^2 C_N & \lambda^2 S_N & -\lambda^2 c_N & -\lambda^2 s_N \end{bmatrix} \mathbf{D}_N^{-1} \mathbf{u}_N = \mathbf{B}_N \mathbf{u}_N$$

Incorporating the rotational spring ( $k_g$ ) and the degree of freedom at points 1 and 2 (Figure 4A), the force and displacement relationship of the system can be expressed as follows:

$$\begin{bmatrix} P_1 \\ M_1/h \\ P_2 \\ M_2/h \end{bmatrix} = \frac{EI}{h^3} \begin{bmatrix} \frac{(S_1 c_1 + C_1 s_1)\lambda^3}{1 - C_1 c_1} + \frac{(S_2 c_2 + C_2 s_2)\lambda^3}{1 - C_2 c_2} & \frac{-\lambda^2 S_1 s_1 + \lambda^2 S_2 s_2}{1 - C_1 c_1 + 1 - C_2 c_2} & \frac{-(S_2 + s_2)\lambda^3}{1 - C_2 c_2} & \frac{(C_2 - c_2)\lambda^2}{1 - C_2 c_2} \\ \frac{-\lambda^2 S_1 s_1 + \lambda^2 S_2 s_2}{1 - C_1 c_1 + 1 - C_2 c_2} & \frac{\lambda(C_1 s_1 - S_1 c_1)}{1 - C_1 c_1} + \frac{\lambda(C_2 s_2 - S_2 c_2)}{1 - C_2 c_2} + \frac{h}{EI} k_g & \frac{(c_2 - C_2)\lambda^2}{1 - C_2 c_2} & \frac{\lambda(S_2 - s_2)}{1 - C_2 c_2} \\ \frac{-(S_2 + s_2)\lambda^3}{1 - C_2 c_2} & \frac{\lambda^2(c_2 - C_2)}{1 - C_2 c_2} & \frac{(S_2 c_2 + C_2 s_2)\lambda^3}{1 - C_2 c_2} & \frac{-\lambda^2 S_2 s_2}{1 - C_2 c_2} \\ \frac{(C_2 - c_2)\lambda^2}{1 - C_2 c_2} & \frac{\lambda(S_2 - s_2)}{1 - C_2 c_2} & \frac{-\lambda^2 S_2 s_2}{1 - C_2 c_2} & \frac{\lambda(C_2 s_2 - S_2 c_2)}{1 - C_2 c_2} \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 h \\ y_2 \\ \theta_2 h \end{bmatrix} = \mathbf{B} \mathbf{u} \quad (9)$$

Matrix  $\mathbf{B}$  expresses the relationships between force and displacement of the core structure's dynamic responses including the outrigger effect. Thus,  $\lambda$ ,  $\omega$ , and the associated vibration periods of the  $n$ th mode can be obtained by solving  $\det \mathbf{B} = 0$ . The  $n$ th mode shape can be obtained by substituting  $\lambda$  into Equations 6 and 7. In this study, the simplified

structure shown in Figure 4A is known as uniform mass (UM) model and is used when performing SA. Figure 4B shows the analytical model with discrete mass (DM) distribution along the height of the core structure, when performing NLRHA. Figure 4C shows the refined member-by-member (MBM) model used for verifying the effectiveness of UM and DM models. The details of DM and MBM models will be introduced in the following sections.

## 2.2 | Modal analysis

As 4 degrees of freedom are considered in the UM model ( $y_1$ ,  $\theta_1$ ,  $y_2$ , and  $\theta_2$  as shown in Figure 4A), the first 4 modes are considered while performing SA. The SA is performed by using the modal analysis results. It is assumed that the mode shapes remain the same and the modal superposition principle is applicable when the BRBs deform inelastically.<sup>14</sup> If  $\phi_r(x)$  is the  $r$ th mode shape, the core structure's lateral displacement  $y(x, t)$  can be expressed by modal superposition as follows:

$$y(x, t) = \sum_{r=1}^4 \phi_r(x) Q_r(t) \quad (10)$$

Substitute Equation 10 into Equation 4, the following can be obtained:

$$m \sum_{r=1}^4 \phi_r(x) \ddot{Q}_r(t) + EI \sum_{r=1}^4 \phi_r''''(x) Q_r(t) = 0, \quad \text{where } \phi_r''''(x) = \frac{d^4 \phi_r(x)}{dx^4} \text{ and } \ddot{Q}_r(t) = \frac{d^2 Q_r(t)}{dt^2} \quad (11)$$

Applying modal orthogonality and integrating with respect to  $x$  from 0 to  $h$ , Equation 11 becomes

$$\ddot{Q}_n(t) \int_0^h m [\phi_n(x)]^2 dx + Q_n(t) \int_0^h EI \phi_n(x) \phi_n''''(x) dx = 0 \quad (12)$$

The modal mass ( $M_n$ ), modal stiffness ( $K_n$ ), and elastic vibration period ( $T_n$ ) are calculated as follows:

$$M_n = \int_0^h m [\phi_n(x)]^2 dx, \quad K_n = \int_0^h EI \phi_n(x) \phi_n''''(x) dx, \quad T_n = 2\pi \sqrt{\frac{M_n}{K_n}}, \quad \text{where } \phi_n''''(x) = \frac{d^4 \phi_n(x)}{dx^4} \quad (13)$$

The detailed SA procedure will be introduced in the following sections.

## 2.3 | Parameter definitions

As shown in Figure 3, the performance of outrigger is affected by  $k_t$ ,  $k_d$ ,  $k_c$ , and the outrigger elevation  $\alpha$ . For the purpose of parametric study, 4 dimensionless parameters ( $S_{bc}$ ,  $R_{dt}$ ,  $R_{dc}$ , and  $R_{db}$ ) are defined to represent the structure's properties. The outrigger stiffness parameter ( $S_{bc}$ ) is adopted to indicate the magnitude of how outrigger affects the structure and is defined by the ratio of rotational stiffness of outrigger ( $k_b l_t^2$ ) when  $k_d$  is infinite to the core structure's rotational stiffness ( $EI/h$ ).  $S_{bc}$  can be computed as follows:

$$S_{bc} = \frac{k_b l_t^2}{EI/h} = \frac{l_t^2 h}{EI(1/k_t + \alpha/k_c)} \quad (14)$$

A larger  $S_{bc}$  value indicates a more significant outrigger effect. A longer outrigger truss span ( $l_t$ ) while  $k_b$  remains constant, or a stiffer outrigger truss (greater  $k_t$ ), or a stiffer perimeter column (greater  $k_c$ ) can enhance the outrigger effect. However, when  $EI$ ,  $h$ ,  $l_t$ ,  $k_t$ , and  $k_c$  are kept constant while increasing  $\alpha$ , the outrigger effect would be smaller. For taller structures,  $EI/h$  would be larger because of greater seismic lateral force demands. Thus, in this study, smaller  $S_{bc}$  value is adopted to represent taller structures. In addition to  $S_{bc}$ , the ratio of BRB axial stiffness to outrigger truss's flexural stiffness ( $R_{dt}$ ) and the BRB stiffness parameters ( $R_{db}$ ,  $R_{dc}$ ) are defined as follows:

$$R_{dt} = \frac{k_d}{k_t}, \quad R_{dc} = \frac{k_d}{k_c}, \quad R_{db} = \frac{k_d}{k_b} = k_d \left( \frac{1}{k_t} + \frac{\alpha}{k_c} \right) = R_{dt} + \alpha R_{dc} \quad (15)$$

In the design practice, the perimeter column sizes are usually determined by gravity load demands; thus, it is possible that BRB has to be designed based on perimeter column sizes. The  $R_{dc}$  describes the stiffness relationship between

BRB and the perimeter column, and  $R_{db}$  describes the relationship between BRB and the combination of perimeter column and outrigger truss. The larger  $R_{db}$  or  $R_{dc}$  value indicates the BRB is stiffer. The optimal  $R_{db}$  or  $R_{dc}$  value could provide an easy and straightforward way for engineers to roughly design BRB in the preliminary design stage. In addition, a smaller  $R_{dt}$  value should be preferred because it results in larger BRB axial deformation demand.

For each set of analyses,  $\alpha$  varies from 0 to 1 under given  $k_c$ ,  $k_d$ , and  $k_t$ . As indicated in Equation 3,  $k_g$  decreases with increasing  $\alpha$  when  $k_t$ ,  $k_d$ , and  $k_c$  are kept as constant. Two methods were developed for the parametric study. Method I (Met. I) sets  $k_b$  and  $k_g$  as constants while changing  $\alpha$  under a given  $R_{db}$  in each analysis set. As indicated in Equation 3,  $k_b$  is kept constant by fixing both  $k_t$  and  $k_c/\alpha$ . Thus,  $k_c$  is proportional to  $\alpha$ . This suggests that  $k_c$  increases with increasing  $\alpha$  in Met. I. Method II (Met. II) sets  $k_c$  as a constant by specifying  $R_{dc}$  in each analysis set. Thus,  $k_g$  decreases with increasing  $\alpha$ . Met. I provides a straightforward analysis procedure because only  $\alpha$  is changed, while Met. II would be more realistic for practical design purpose because  $k_c$  and  $\alpha$  are set to be independent to each other. Table 1 summarizes the variation of parameters in each analysis set when  $\alpha$  varies from 0 to 1 in Met. I and Met. II.

The BRB axial yield deformation ( $u_{d,y}$ ) is crucial as it determines the start of energy dissipation. From Equation 1,  $u_{d,y}$  and the corresponding core structure rotation at outrigger elevation when BRB yields ( $\theta_y$ ) can be expressed as follows:

$$\theta_y = \frac{k_d}{l_t} \left( \frac{\alpha}{k_c} + \frac{1}{k_t} + \frac{1}{k_d} \right) u_{d,y} = \frac{1}{l_t} (\alpha R_{dc} + R_{dt} + 1) u_{d,y} = \frac{1}{l_t} (R_{db} + 1) u_{d,y} \quad (16)$$

In this study, when the roof drift ratio in the first mode shape reaches a given yield drift ratio ( $\theta_r$ ), the corresponding BRB axial deformation is referred to as  $u_{d,y}$ .  $\theta_r$  should be properly selected so that  $u_{d,y}$  lies in a reasonable range based on the actual BRB configuration. The investigation on relationship between  $\theta_r$  and  $u_{d,y}$ , and its effects on the seismic performance, will be presented in the following sections.

## 2.4 | Numerical models

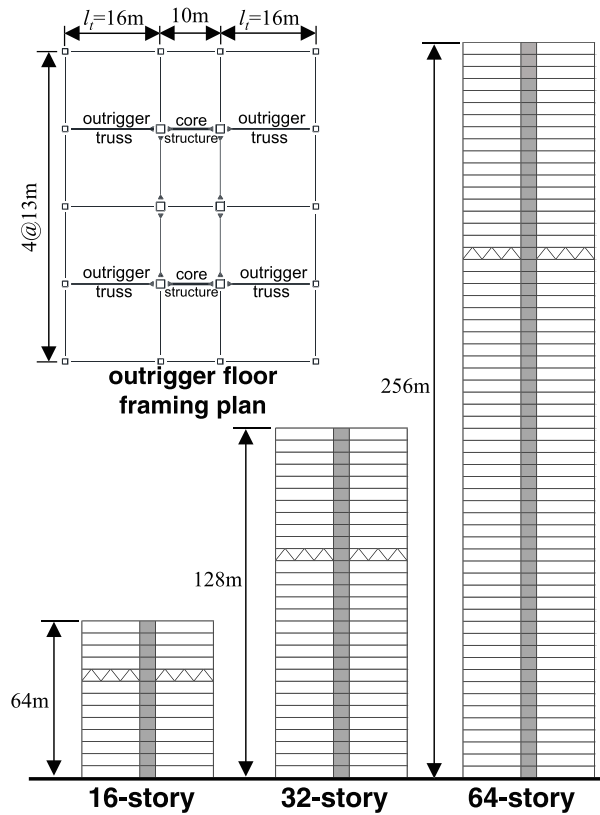
To perform NLRHA by using OpenSees,<sup>15</sup> the DM model (Figure 4B) was developed. The core structure, BRBs, and the perimeter columns are all included in the DM model. The masses are concentrated at the nodes that are uniformly distributed along the core structure height with an equal spacing of either 1 m (DM1) or 4 m (DM4). Each of the outrigger trusses is modeled as a beam element with a flexural stiffness  $k_t$ . The core structure, outrigger truss, and perimeter columns are modeled by using the Elastic Beam Column element.<sup>16</sup> The BRBs are modeled as truss elements, with a fixed length of 1 m. The bottom end of the core structure (point B in Figure 4B) is fixed, and the bottom ends of perimeter columns (points A and C in Figure 4B) are free to rotate about the  $z$ -axis. The 2 ends of the BRBs are free to move in the  $x$ -direction and free to rotate about the  $z$ -axis.

Figure 4C shows the MBM model that was analyzed by using the PISA3D program.<sup>17</sup> The details of story levels, core structure, outrigger truss, and floor beams are included in the MBM model. The analytical results of MBM model were used to compare with the results of UM, DM1, and DM4 models, to verify the effectiveness of those simplified models. The beams and columns of MBM model are modeled by using the beam column elements,<sup>17</sup> the braces and outrigger truss members are modeled as truss elements. Each story is 4 m high, the mass at each floor level is concentrated at the midspan of beam of core structure. The core structure is represented by a braced frame with a lateral stiffness close to  $EI$ . The bottom end of the perimeter columns is pinned. Figure 4C also shows enlarged details of the outrigger truss. The outrigger truss's top and bottom chords are located at the  $n$ th + 1 and  $n$ th floors, respectively. The BRB's upper and lower ends connect to the outrigger's top chord end (point E) and to the perimeter column at the  $n$ th floor, respectively. No rigid diaphragm was assigned in the MBM model.

Figure 5 shows the elevations and floor framing plan of the outrigger floor. It is assumed that the core structure provides sufficient lateral force resistance in horizontal directions, so that the core structure remains elastic. The outrigger truss span ( $l_t$ ) is 16 m, and the building heights ( $h$ ) are 64, 128, and 256 m for the 16, 32, and 64-story models, respectively. Each floor area is 2184 m<sup>2</sup> (52 m × 42 m), with a uniformly distributed dead load of 0.8 tonf/m<sup>2</sup>, including the

**TABLE 1** Parameter variations while  $\alpha$  increases from 0 to 1 in each analysis set in Met. I and Met. II

|         | $S_{bc}$  | $k_g$     | $k_b$     | $k_t$ | $k_c$     | $k_d$ |
|---------|-----------|-----------|-----------|-------|-----------|-------|
| Met. I  | Fixed     | Fixed     | Fixed     | Fixed | Increased | Fixed |
| Met. II | Decreased | Decreased | Decreased | Fixed | Fixed     | Fixed |



**FIGURE 5** Elevation and outrigger floor framing plan of analytical models

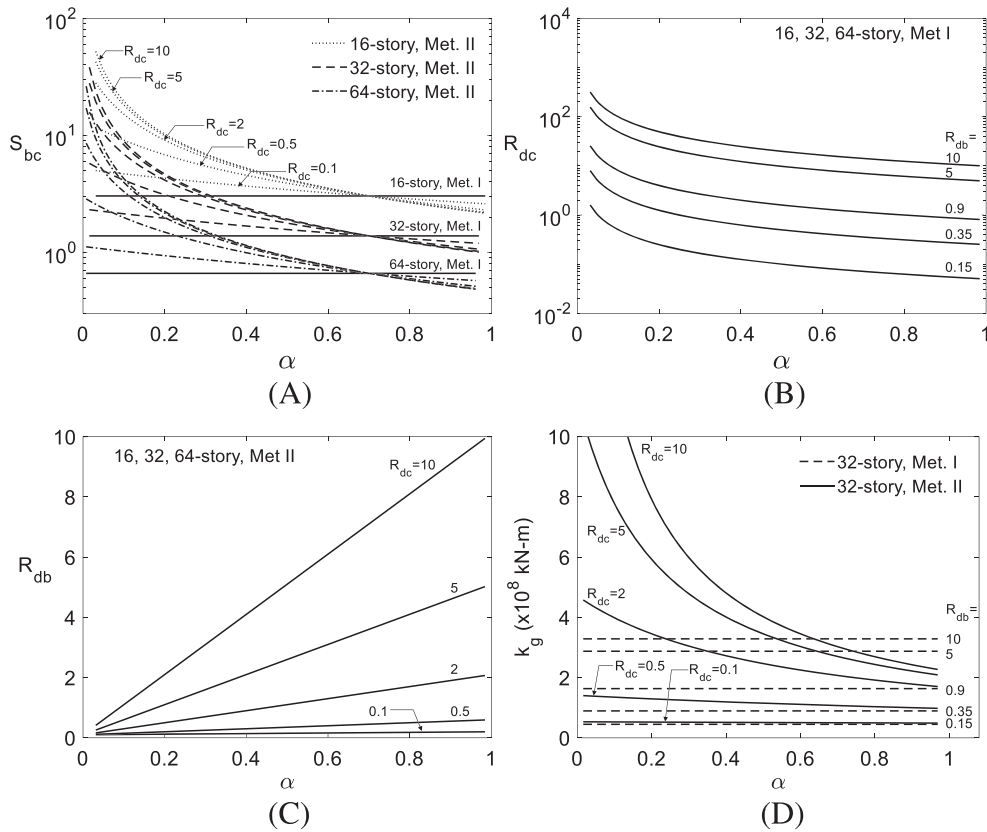
member self-weight. The mass is 900 ton on each floor of the MBM models, 225 and 900 ton concentrated at each node on the core structure of the DM1 and DM4 model, respectively, and 225 ton/m uniformly distributed along the core structure height for the UM model. For simplicity, the force and deformation relationships are bilinear for BRBs and linearly elastic for all the other members in UM, DM1, DM4, and MBM models. The secondary effects due to gravity loads are excluded. Table 2 shows  $h$ ,  $EI$ , and the ranges of  $R_{db}$  for Met. I and  $R_{dc}$  range for Met. II.  $EI$  is determined by setting the fundamental mode vibration period to be approximately  $0.03 h$ .  $S_{bc}$  is set to be 3.03, 1.38, and 0.66 for the 16, 32, and 64-story models in the Met. I analysis, respectively. In Met. II,  $S_{bc}$  varies with changing  $\alpha$  but is set to be 3.03, 1.38, and 0.66 for the 16, 32, and 64-story models, respectively, when  $\alpha$  equals 0.7. The relationships between  $S_{bc}$  and  $\alpha$  for Met. I and Met. II are shown in Figure 6A. Figure 6B shows the relationship between  $\alpha$  and the corresponding  $R_{dc}$  under given  $R_{db}$  values for Met. I. Figure 6C shows the relationship between  $\alpha$  and the corresponding  $R_{db}$  under given  $R_{dc}$  values for Met. II. Figure 6D shows the relationship between  $k_g$  and  $\alpha$  for the 32-story model.

## 2.5 | Comparison between UM, DM1, DM4, and MBM models

The modal analysis and NLRHA results of MBM model were used to verify the effectiveness of UM (used for performing SA), DM1 (used for performing NLRHA), and DM4 models. In addition, the analyses on MBM models without outrigger (core structure only) and with the elastic outrigger (by setting  $u_{dy}$  to be infinity) were performed to demonstrate the prominent performance of BRB-outrigger system. The 32-story model ( $S_{bc} = 1.38$ ) with  $\alpha = 0.7$ ,  $R_{dt} = 0.1$ ,  $R_{dc} = 5.0$ ,

**TABLE 2** Parameters of analytical models for Met. I and Met. II analyses

| Model    | $h$ (m) | $EI$ (kN-m <sup>2</sup> ) | Met. I    |          | Met. II  |
|----------|---------|---------------------------|-----------|----------|----------|
|          |         |                           | $R_{db}$  | $S_{bc}$ | $R_{dc}$ |
| 16-story | 64      | $4.1 \times 10^9$         | 0.15-20.0 | 3.03     | 0.1-20.0 |
| 32-story | 128     | $1.6 \times 10^{10}$      | 0.15-20.0 | 1.38     | 0.1-20.0 |
| 64-story | 256     | $6.5 \times 10^{10}$      | 0.15-20.0 | 0.66     | 0.1-20.0 |



**FIGURE 6** Relationships between A,  $S_{bc}$ , B,  $R_{dc}$ , C,  $R_{db}$ , and D,  $k_g$  with  $\alpha$

and  $\theta_r = 1/750$  was chosen as the example model. Table 3 shows the member sizes and material properties. The perimeter columns were designed according to the axial force demand in the first story ( $0.8 \text{ tonf} \times 104 \text{ m}^2$  [tributary area]  $\times 32$  stories  $+ 1.1 \times 1.3 \times N_y$ ), where  $N_y$  is the BRB core yield force capacity and 1.1 and 1.3 are factors of overstrength and strain hardening, respectively.<sup>18,19</sup> According to the member sizes,  $k_c$ ,  $k_d$ , and  $k_t$  are  $0.486 \times 10^6$ ,  $2.43 \times 10^6$ , and  $24.3 \times 10^6$  kN/m, respectively.  $u_{d,y}$  is 5.2 mm, and  $N_y (=k_d \times u_{d,y})$  is 12368 kN. Thus, the perimeter column axial force demand in the first story (44190 kN) can be calculated. Considering a strength reduction factor of 0.9,<sup>20</sup> the axial force demand-to-capacity ratio of the perimeter column is 0.47.

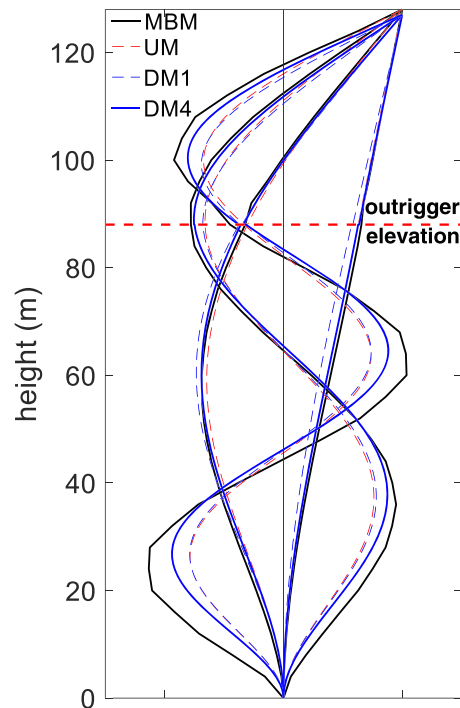
Table 4 and Figure 7 show the vibration periods and mode shapes of the first 4 modes. The vibration periods of UM and DM1 models are close to each other. This suggests that the DM1 model developed in OpenSees with mass spacing of 1 m is a good representation of the UM model. As the masses are concentrated at nodes with 4-m spacing in MBM and DM4 model, the vibration periods are longer if compared with UM and DM1 models. In addition, there are vibration period differences between DM4 and MBM models because the core structure span of 10 m was not included in the DM4 model, and the behavior of the braced frame to represent core structure in MBM model may not accurately resemble a cantilever column. Table 4 also shows the vibration periods of the MBM model when the BRBs remain elastic (elastic-outrigger) and also without outrigger (no outrigger). The much longer first to fourth mode vibration periods of MBM (no outrigger) as compared with MBM with outrigger system show significant outrigger effect. In addition, NLRHA on the aforementioned analytical models was also performed. Table 5 shows the information of 8 ground motions adopted for NLRHA. Figure 8A and B shows the 2% damping response spectra of the original observed and

**TABLE 3** 32-story MBM model member sizes ( $\alpha = 0.7$ ,  $R_{dt} = 0.1$ ,  $R_{dc} = 5.0$ ,  $\theta_r = 1/750$ )

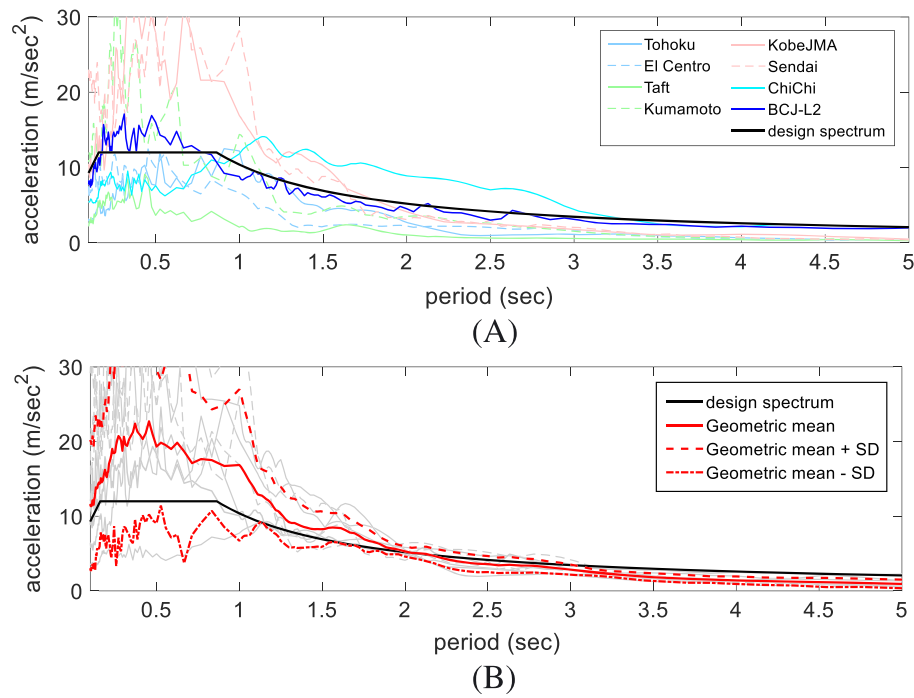
| Member           | Size                                                                 | Material Property                                     |
|------------------|----------------------------------------------------------------------|-------------------------------------------------------|
| Perimeter column | Box 1 000 $\times$ 1 000 $\times$ 85 mm                              | Linearly elastic (SN490)                              |
| BRB              | +775 $\times$ 32 mm (cross-sectional area = 48 576 mm <sup>2</sup> ) | Bilinear (SN490, yield stress = 345 MPa, $p = 0.01$ ) |

**TABLE 4** The modal analysis results of the example UM, DM1, DM4, and MBM models

| Model                   | Vibration Period (s) |             |            |             |
|-------------------------|----------------------|-------------|------------|-------------|
|                         | First Mode           | Second Mode | Third Mode | Fourth Mode |
| UM                      | 2.498                | 0.512       | 0.198      | 0.100       |
| DM1                     | 2.498                | 0.515       | 0.199      | 0.100       |
| DM4                     | 2.540                | 0.529       | 0.204      | 0.102       |
| MBM                     | 2.587                | 0.581       | 0.253      | 0.142       |
| MBM (elastic-outrigger) |                      |             |            |             |
| MBM (no outrigger)      | 3.770                | 0.646       | 0.253      | 0.145       |

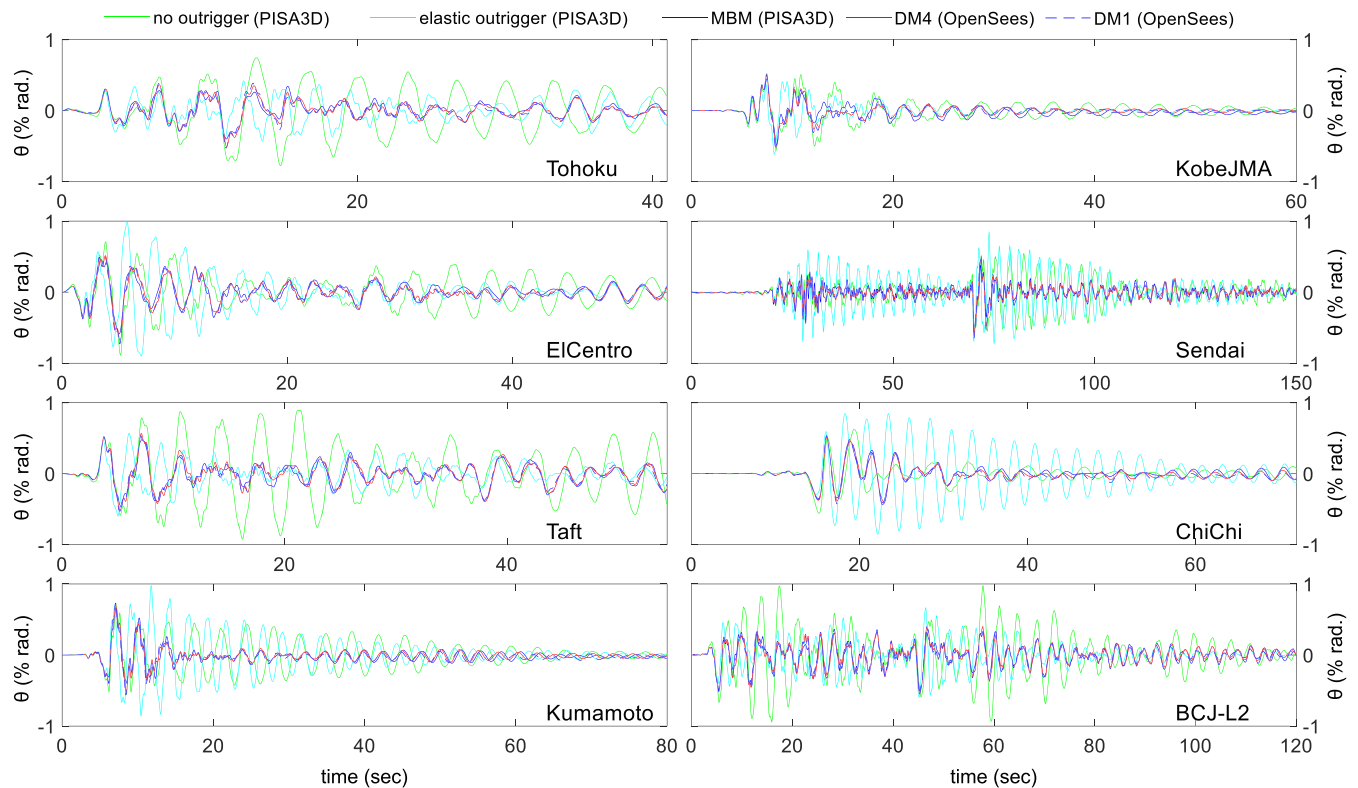
**FIGURE 7** The first to the fourth mode shapes of uniform mass, discrete mass (1-m mass spacing), discrete mass (4-m mass spacing, and member-by-member 32-story models [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]**TABLE 5** The ground motions used for NLRHA

| Ground Motion | Earthquake Event | Date               | Magnitude    | Depth (km) | PGA (gal) | Scale Factor |
|---------------|------------------|--------------------|--------------|------------|-----------|--------------|
| Tohoku        | Miyagi           | June 12, 1978      | <i>M</i> 7.7 | 44         | 258       | 2.03         |
| El Centro     | El Centro        | May 18, 1940       | <i>M</i> 6.9 | 16         | 342       | 2.54         |
| Taft          | Kern Country     | July 21, 1952      | <i>M</i> 7.3 | 16         | 176       | 4.90         |
| Kumamoto      | Kumamoto         | April 16, 2016     | <i>M</i> 7.0 | 10         | 627       | 1.82         |
| KobeJMA       | Great Hanshin    | January 17, 1995   | <i>M</i> 6.9 | 18         | 821       | 1.19         |
| Sendai        | Tohoku           | March 11, 2011     | <i>M</i> 9.0 | 29         | 1517      | 1.33         |
| ChiChi        | ChiChi           | September 21, 1999 | <i>M</i> 7.3 | 33         | 439       | 0.66         |
| BCJ-L2        | Artificial       | —                  | —            | —          | 356       | 1.14         |

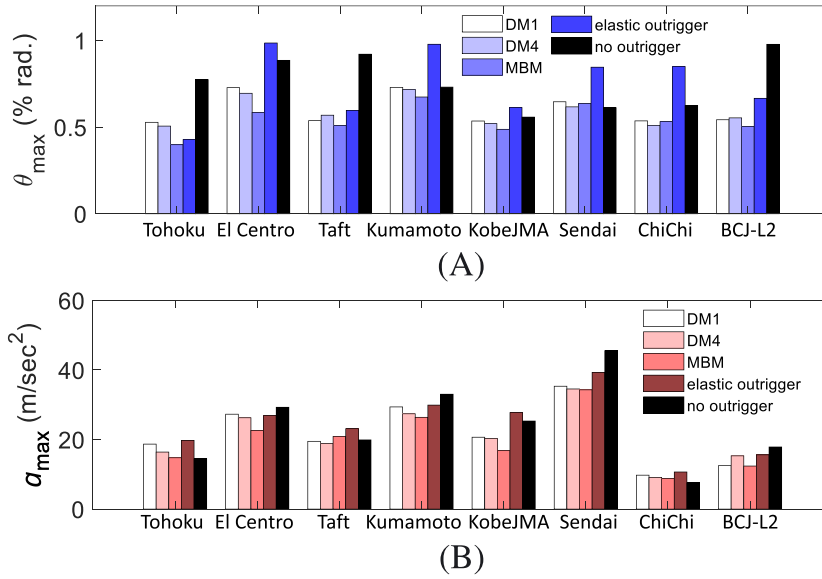


**FIGURE 8** The response spectra of the A, original observed and B, scaled ground motions and design spectrum [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

scaled ground motions. The response spectra of the original observed ground motions were scaled to fit the design spectral acceleration within the range of  $0.2T_1$  to  $1.5T_1$ , where  $T_1$  is the first mode period (2.5 s). Rayleigh damping of 0.02 for the first and second modes was adopted for all NLRHA. In this study, the maximum roof drift ratio ( $\theta_{\max}$ ) and the



**FIGURE 9** The nonlinear response history analysis results of roof displacement histories of the 32-story model under each ground motion [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]



**FIGURE 10** The A,  $\theta_{\max}$  and B,  $a_{\max}$  responses from nonlinear response history analysis results [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

maximum roof acceleration response ( $a_{\max}$ ) are assigned as measures of seismic performance. Figures 9 and 10 show the roof drift histories, the  $\theta_{\max}$  and  $a_{\max}$  under each NLRHA. If the cases without outrigger, with elastic outrigger, and with BRB-outrigger are compared, the elastic outrigger reduces the displacement responses only in some cases, but the peak accelerations are greater than the BRB-outrigger cases (DM1, DM4, and MBM). Figure 9 shows close trends and marginal differences among DM1, DM4, and MBM models. Although the peak responses among DM1, DM4, and MBM models are slightly different, this may not affect the parametric analysis in investigating optimal parameters. Thus, the DM1 model is used when performing NLRHA using OpenSees for verifying the SA results. For simplicity, the BCJ-L2, which best matches the design spectrum, was adopted for NLRHA in the following sections.

### 3 | ANALYSIS PROCEDURES

To investigate the effects of  $\alpha$  on the seismic performance and study the relationships among  $k_d$ ,  $k_c$ , and  $k_t$ , the SA and NLRHA were performed. The analysis procedures are described in the following sections.

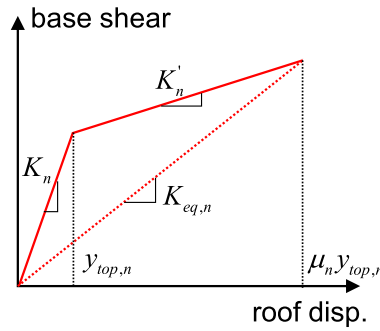
#### 3.1 | Parameter computation in each analysis set

In each analysis set, when  $\alpha$  varies from 0 to 1, Met. I sets  $R_{db}$  as a constant with respect to  $\alpha$  and Met. II sets  $R_{dc}$  as a constant. In Met. I, the first step is to calculate  $k_b$  according to the given  $h$ ,  $S_{bc}$ , and  $l_t$ .  $k_d$  can then be calculated from the given  $R_{db}$  in each analysis set. Finally,  $k_t$  and  $k_c$  are calculated from the given  $R_{dt}$ . In Met. II,  $k_d$ ,  $k_c$ , and  $k_t$  are calculated by solving the simultaneous equations (Equations 14 and 15) with the given  $R_{dc}$  and  $R_{dt}$  and setting  $\alpha$  to 0.7 in Equation 15.

#### 3.2 | Spectral analysis

The SA is used to evaluate  $\theta_{\max}$ . The response of each mode is calculated separately, and then superposed by using the square root of the sum of the squares rule. For the  $n$ th mode response, as shown in Equation 16, when the core structure rotation at outrigger elevation reaches  $\theta_y$ , the BRB initially yields, and the corresponding roof displacement is adopted as the yield roof displacement ( $y_{\text{top},n}$ ). In this study,  $\theta_y$  is computed from the first mode shape when the roof drift reaches  $\theta_r$ . When the BRB deforms inelastically, the postyield modal stiffness  $K'_n$  of  $n$ th mode can be computed as follows:

$$K'_n = \left( \frac{T_n}{T'_n} \right)^2 K_n = p_n K_n \quad (17)$$



**FIGURE 11** Relationship among  $K_n$ ,  $K'_n$ , and  $K_{eq,n}$  [Colour figure can be viewed at wileyonlinelibrary.com]

where  $T'_n$  is the  $n$ th mode vibration period of the system after BRB has yielded and the  $k_d$  has been replaced by postyield stiffness ( $pk_d$ ). If  $y_{\max,n}$  is the maximum roof displacement and  $\mu_n (=y_{\max,n}/y_{\text{top},n})$  is the ductility ratio in the  $n$ th mode, the equivalent stiffness ( $K_{eq,n}$ ), the equivalent vibration period ( $T_{eq,n}$ ), and the equivalent damping ratio ( $h_{eq,n}$ ) can be calculated as follows<sup>19,21,22</sup>:

$$K_{eq,n} = K'_n + \frac{K_n - K'_n}{\mu_n}, \quad T_{eq,n} = T_n \sqrt{\frac{K_n}{K_{eq,n}}}, \quad h_{eq,n} = h_0 + \frac{2}{\pi p_n \mu_n} \ln \left( \frac{1 - p_n + p_n \mu_n}{\mu_n^{p_n}} \right), \quad \mu_n \geq 1 \quad (18)$$

where  $h_0$  (0.02) is the inherent damping ratio. The relationships among  $K_n$ ,  $K'_n$ , and  $K_{eq,n}$  are shown in Figure 11. If  $S_{pv}(T, h_0)$  is the pseudo velocity spectrum with vibration period  $T$  and damping ratio  $h_0$ , and  $D_{h,n}$  is the reduction factor for computing  $S_{pv}(T, h_{eq,n})$  because of the increased damping ratio ( $h_{eq,n}$ ), the relationship between  $S_{pv}(T, h_0)$  and  $S_{pv}(T, h_{eq,n})$  can be expressed as follows<sup>21</sup>:

$$S_{pv}(T, h_{eq,n}) = D_{h,n} S_{pv}(T, h_0), \quad \text{where } D_{h,n} = \sqrt{\frac{1 + \gamma h_0}{1 + \gamma h_{eq,n}}}, \quad \gamma = 75 \text{ for artificial waves} \quad (19)$$

The displacement response after the yielding of BRB ( $S_d(T_{eq,n}, h_{eq,n})$ ) is

$$S_d(T_{eq,n}, h_{eq,n}) = \frac{T_{eq,n}}{2\pi} D_{h,n} S_{pv}(T_{eq,n}, h_0) \quad (20)$$

The maximum displacement of each mode is calculated by an iterative process. The first step is to obtain the maximum roof displacement ( $S_d(T_n, h_0)$ ) by using the elastic vibration period ( $T_n$ ) and inherent damping ratio. Then, calculate the corresponding ductility ratio ( $S_d(T_n, h_0)/y_{\text{top},n}$ ) and the equivalent damping ratio  $h_{eq,n}$  (Equation 18). The updated displacement because of increased damping ratio and the equivalent vibration period ( $S_d(T_{eq,n}, h_{eq,n})$ ) is computed by Equation 20. The iterations should be performed until the displacements, before and after being updated, are satisfactorily close. Table 6 shows the SA results of the example model in Section 2.4. The first mode dominates the response, and only slight inelastic response in the second mode response. The  $\theta_{\max}$  computed from SA by using square root of the sum of the squares is 0.588% rad., and the average  $\theta_{\max}$  obtained from NLRHA described in section 2.5 are 0.560%, 0.586%, and 0.541% rad. for the DM1, DM4, and MBM models, respectively.

**TABLE 6** SA results of example model

|             | Mass Participation Ratio | Yield Roof Drift Ratio ( $y_{\text{top},n}/h$ , % rad.) | $\theta_{\max}$ (% rad.) | $\mu_n$ | $h_{eq,n}$ |
|-------------|--------------------------|---------------------------------------------------------|--------------------------|---------|------------|
| First mode  | 72.6%                    | 0.133                                                   | 0.586                    | 4.41    | 0.085      |
| Second mode | 16.5%                    | 0.052                                                   | 0.052                    | 1.02    | 0.020      |
| Third mode  | 7.2%                     | 0.281                                                   | 0.005                    | 0.02    | 0.020      |
| Fourth mode | 3.7%                     | 0.021                                                   | 0.001                    | 0.05    | 0.020      |

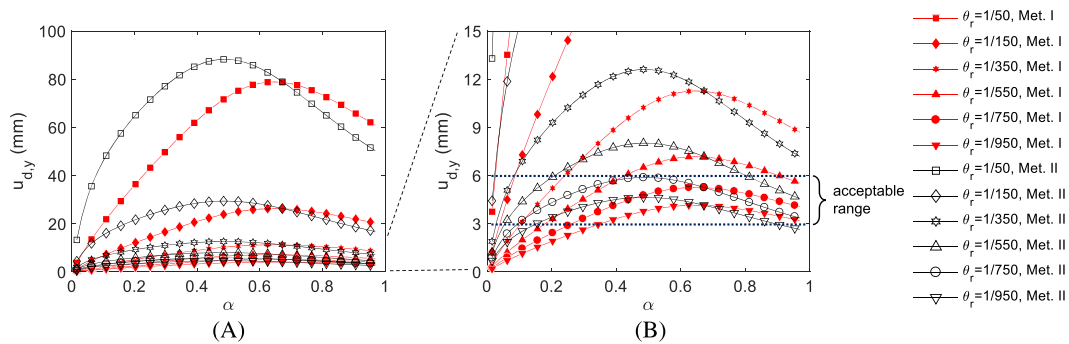
### 3.3 | Nonlinear response history analysis

The results of NLRHA using BCJ-L2 ground motion are used to verify the SA results. In this study, OpenSees was utilized to perform NLRHA by using the DM1 models. The elastic design acceleration spectrum used by SA and the BCJ-L2 acceleration spectrum are shown in Figure 8A. In each analysis, the BCJ-L2 acceleration spectrum was scaled according to the  $S_a(T_1)$  method,<sup>23</sup> so that the spectral acceleration at the linearly elastic fundamental period of DM1 model matches the target design spectrum.

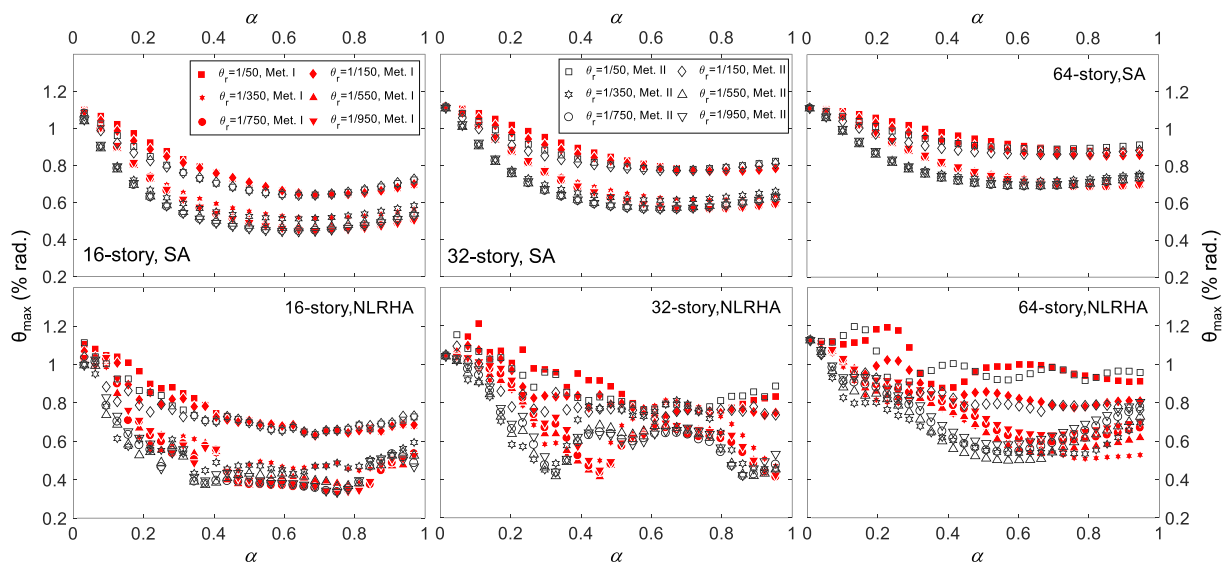
## 4 | ANALYSIS RESULTS

### 4.1 | Investigation of $\theta_r$ and $R_{dt}$

Prior to investigating the optimal values of  $\alpha$ ,  $R_{db}$ , and  $R_{dc}$  to minimize the building's seismic response, the effects of  $\theta_r$  and  $R_{dt}$  on the analytical results have been studied.  $\theta_r$  controls the BRB performance. A larger  $\theta_r$  suggests that the BRB starts dissipating energy at a relative large roof drift ratio and thus may reduce the BRB's energy dissipation efficiency. However, if  $\theta_r$  is too small, the BRB may yield in a minor earthquake or use up its ductility capacity during a large earthquake. Smaller  $R_{dt}$  indicates that the outrigger truss is stiffer than BRB and would result in a larger  $u_{dy}$  and vice versa. In this section, 2 sets of analyses are performed. The first set of analyses was performed by fixing  $R_{dt} = 0.1$ ,  $R_{db} = 3.5$  (Met. I), and  $R_{dc} = 5.0$  (Met. II) but varying  $\theta_r$  between 1/50 and 1/950. In the second set of analyses,  $\theta_r$  is fixed at 1/750, and the other parameters are the



**FIGURE 12** A, Relationships between  $u_{dy}$  and  $\alpha$  for different  $\theta_r$  of 32-story model and B, enlargement [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

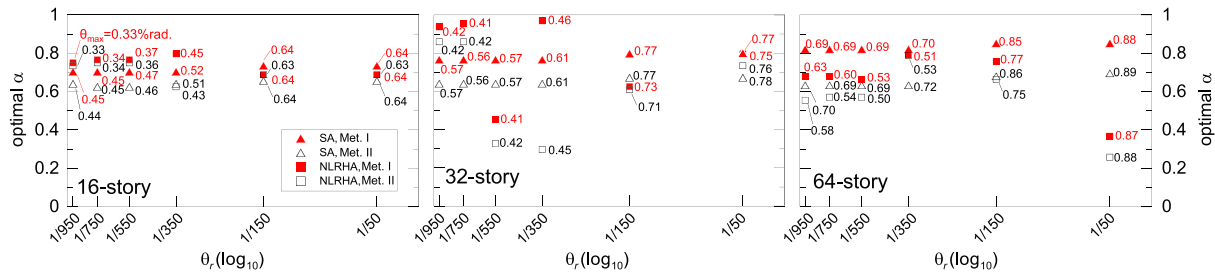


**FIGURE 13** Relationship between  $\theta_{max}$  and  $\alpha$  for different  $\theta_r$  of 16, 32, and 64-story models as obtained from spectral analysis and nonlinear response history analysis [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

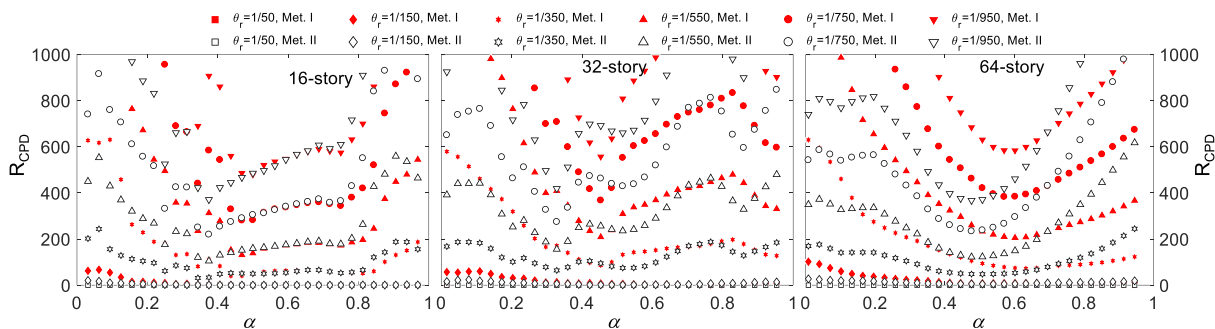
same as used in the first analysis, except that  $R_{dt}$  varies between 0.05 and 0.9. As  $k_g$  remains constant with varying  $\alpha$  in Met. I, varying  $R_{dt}$  does not affect the analysis result. The second analyses were performed by using Met. II only.

Figure 12 shows the relationship between  $u_{d,y}$  and  $\alpha$ . A larger  $\theta_r$  leads to a larger  $u_{d,y}$ . For a given  $\theta_r$ , the maximum value of  $u_{d,y}$  is obtained when  $\alpha$  is approximately 0.6 to 0.7 in Met. I and 0.5 in Met. II. In design practices,  $u_{d,y}$  should lie in a reasonable range (eg, 1/1000 of the BRB length). In the example structure, the BRB is arranged vertically in a story with a height of 4 m; therefore,  $u_{d,y}$  should be approximately 3 to 6 mm. Figure 13 shows the relationship between  $\theta_{max}$  and  $\alpha$  for different  $\theta_r$  as obtained from SA and NLRHA. The trends of maximum roof drift ratio variations with respect to  $\alpha$  obtained from both SA and NLRHA are similar. However, these trends differ slightly from the 32-story model's analytical results in certain period ranges. This could be because, for periods other than the first mode, the BCJ-L2 spectral accelerations are considerably greater than the design spectral acceleration. Figure 14 shows the relationship between  $\theta_r$  and  $\alpha$  when minimum  $\theta_{max}$  is achieved. The corresponding  $\theta_{max}$  of each point is also shown in Figure 14. Although the SA results correlated well with the NLRHA results for the 16- and 64-story models, as shown in Figures 13 and 14, both the SA and NLRHA results suggest that a smaller  $\theta_r$  best reduces  $\theta_{max}$ , and the BRBs start dissipating energy in a relatively small lateral deformation of the building. However, for  $\theta_r$  values smaller than 1/550, the reduction in  $\theta_{max}$  was not large enough to significantly improve the performance. The ratio of BRB's cumulative plastic deformation to axial yield deformation ( $R_{CPD}$ )<sup>18</sup> is adopted to indicate the ductility demand for the BCJ-L2 ground motion. Figure 15 shows the  $R_{CPD}$  obtained from the NLRHA results. When  $\theta_r$  is greater than 1/150, the relatively small  $R_{CPD}$  suggest that the BRBs have low energy dissipation efficiency or deform elastically (when  $R_{CPD} = 0$ ) as a traditional outrigger system. However, when  $\theta_r$  is smaller than 1/950, the large  $R_{CPD}$  may not be practically achieved in conventional BRBs. Therefore, from the above analysis results,  $\theta_r = 1/750$  results in an acceptable range of  $u_{d,y}$  (from 3 to 6 mm, for example) and  $R_{CPD}$  values for the example model. For the design practices,  $\theta_r$  can be determined by code specified maximum allowable elastic roof drift ratio. As indicated in Equation (16),  $u_{d,y}$  is affected not only by  $\theta_r$  but also by  $\alpha$  and  $l_r$ . Thus, in design practices,  $\theta_r$  should be properly defined based on the actual building configuration.

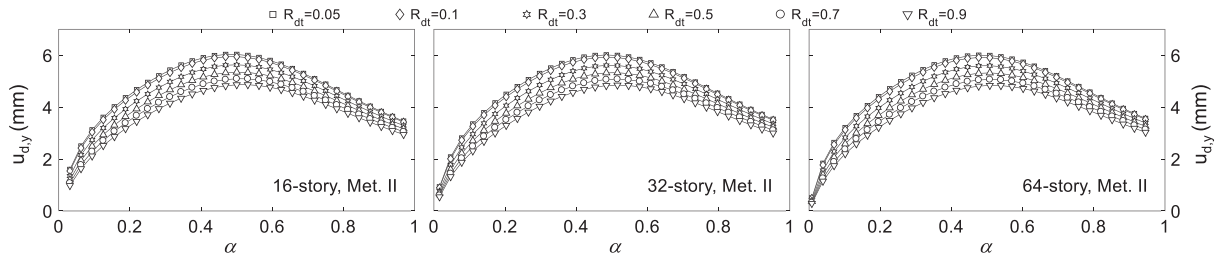
Figure 16 shows the relationship between  $u_{d,y}$  and  $\alpha$  under different  $R_{dt}$ . The ranges of  $u_{d,y}$  differences because of varying  $R_{dt}$  (around 3 mm) are much smaller than varying  $\theta_r$  (more than 20 mm, Figure 12). Figure 17 shows the relationships between the first to fourth mode vibration periods and  $\alpha$  for different  $R_{dt}$ . Figure 18 shows the relationship between  $\theta_{max}$  and  $\alpha$  as obtained from SA and NLRHA. Because  $k_t$  is much greater than  $k_d$  and  $k_c$ , changing  $R_{dt}$  only slightly affects  $k_g$ . Thus, the changes in dynamic characteristics and the maximum responses of the overall system because of the variation



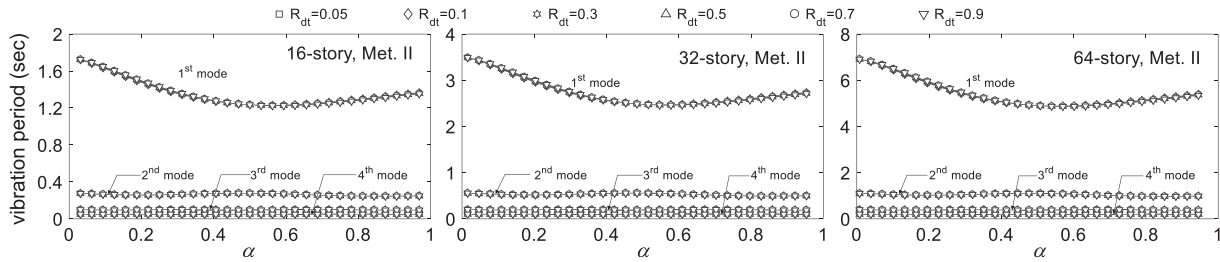
**FIGURE 14** Relationships between optimal  $\alpha$  and  $\theta_r$  of 16, 32, and 64-story models as obtained from spectral analysis and nonlinear response history analysis [Colour figure can be viewed at [wileyonlinelibrary.com](#)]



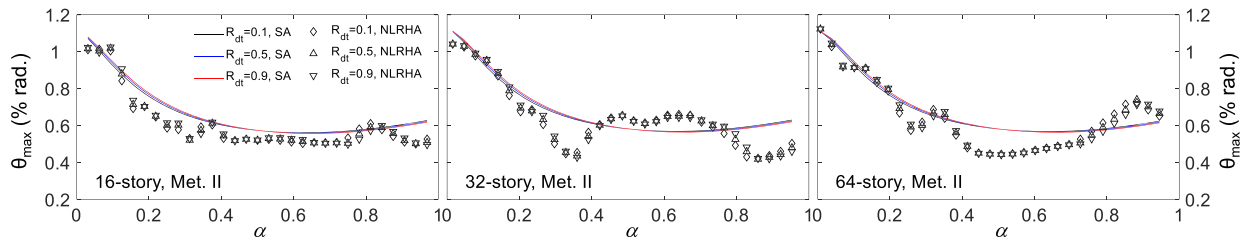
**FIGURE 15** Relationships between  $R_{CPD}$  and  $\alpha$  for different  $\theta_r$  of 16, 32, and 64-story models as obtained from nonlinear response history analysis [Colour figure can be viewed at [wileyonlinelibrary.com](#)]



**FIGURE 16** Relationships between  $u_{d,y}$  and  $\alpha$  for different  $R_{dt}$  of 16, 32, and 64-story models



**FIGURE 17** Relationships between vibration period and  $\alpha$  for different  $R_{dt}$  of 16, 32, and 64-story models



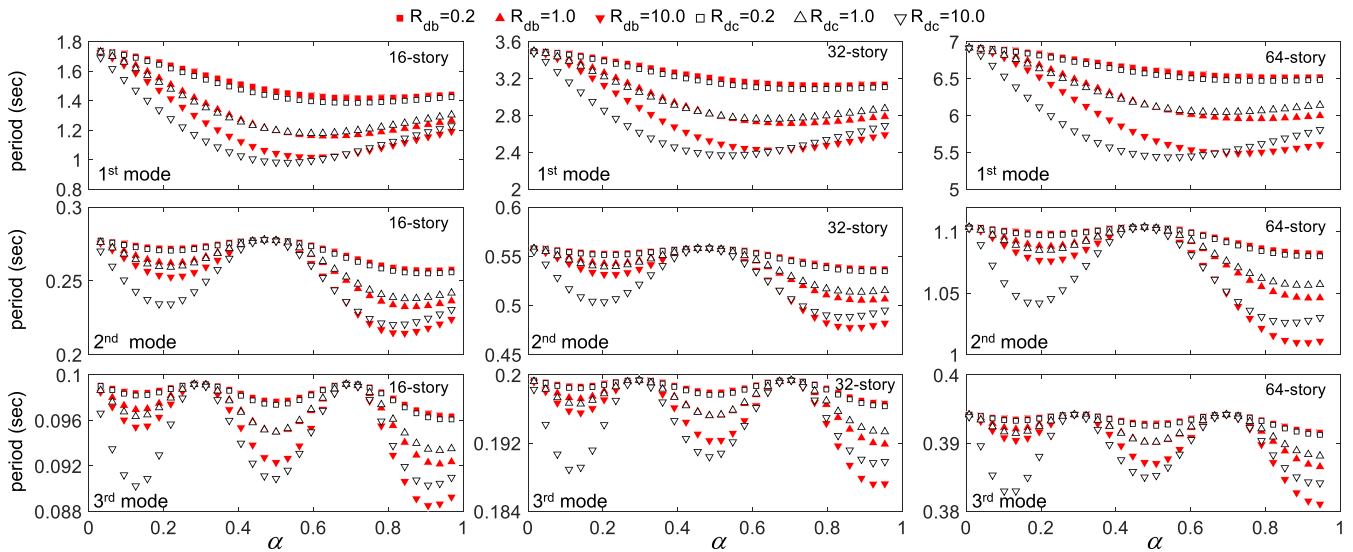
**FIGURE 18** Relationships between  $\theta_{\max}$  and  $\alpha$  for different  $R_{dt}$  of 16, 32, and 64-story models as obtained from spectral analysis and nonlinear response history analysis [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

in  $R_{dt}$  (ranging from 0.05 to 0.9) are insignificant. For the design practices, the  $u_{d,y}$  can be fine-tuned by changing  $R_{dt}$  so that  $u_{d,y}$  lies within a desirable range.  $R_{dt}$  is fixed at 0.1 for further analysis in the following sections.

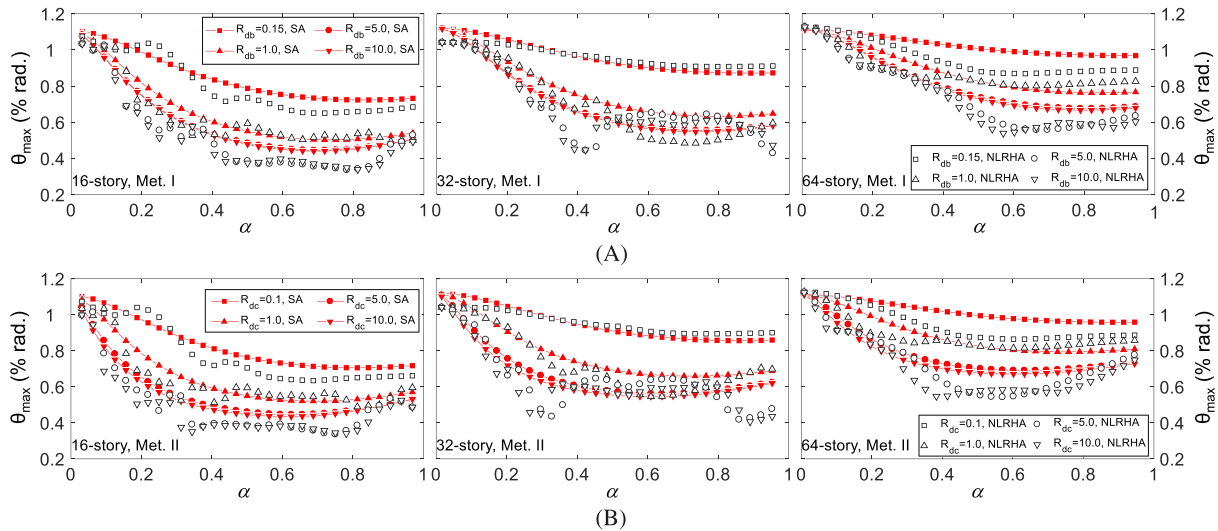
## 4.2 | Investigation of optimal $\alpha$ , $R_{db}$ , and $R_{dc}$

In this section, the optimal  $\alpha$ ,  $R_{db}$ , and  $R_{dc}$  to achieve the minimum  $\theta_{\max}$  are investigated. Figure 19 shows the relationship between the first to the third mode periods and  $\alpha$  for different  $R_{db}$  (Met. I) and  $R_{dc}$  (Met. II). It should be noted that Met. I fixes  $R_{db}$  as constant while changing  $\alpha$ , so the corresponding  $R_{dc}$  varies with the changing  $\alpha$ . Likewise, the Met. II fixes  $R_{dc}$  as constant, and the corresponding  $R_{db}$  varies with  $\alpha$ . Increasing  $R_{db}$  (Met. I) or  $R_{dc}$  (Met. II) stiffens the system and causes the vibration periods to decrease. Under the same set of  $S_{bc}$ , and  $R_{db}$  or  $R_{dc}$ , when  $\alpha$  is between 0.5 and 0.8, the outrigger effect on the system is significant because the first mode period becomes minimum within this  $\alpha$  range. The  $\alpha$  that results in the smallest first mode period is higher for taller structure model in Met. I (when  $S_{bc}$  is smaller) but almost remains unchanged in Met. II. In addition, increasing  $R_{db}$  or  $R_{dc}$  enhances the outrigger effect and thus results in smaller vibration periods.

Figure 20 shows the relationships between  $\theta_{\max}$  and  $\alpha$  under various  $R_{db}$  (Met. I) and  $R_{dc}$  (Met. II) as obtained from SA and NLRHA. Both the SA and NLRHA results show similar trends. Based on the analytical results, for a given set of  $R_{db}$  or  $R_{dc}$ ,  $\theta_{\max}$  becomes minimum when  $\alpha$  is approximately between 0.7 and 0.8 for Met. I, and from 0.5 to 0.7 for Met. II. In addition, the Met. I results indicate that the optimal  $\alpha$  value is higher for taller structure model (when  $S_{bc}$  is smaller). The trend of  $\theta_{\max}$  with respect to  $\alpha$  is similar to the first mode period trend as shown in Figure 19. This suggests that the outrigger elevation that has greatest outrigger effect on the system is also the optimal elevation to achieve



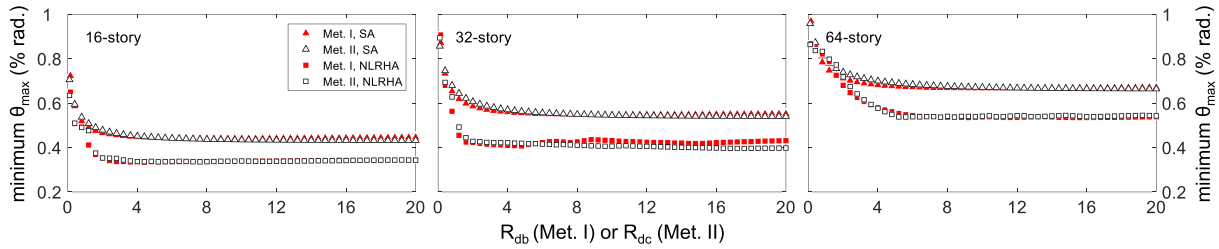
**FIGURE 19** Relationships between the first to third mode periods and  $\alpha$  for different  $R_{db}$  and  $R_{dc}$  of 16, 32, and 64-story models [Colour figure can be viewed at [wileyonlinelibrary.com](#)]



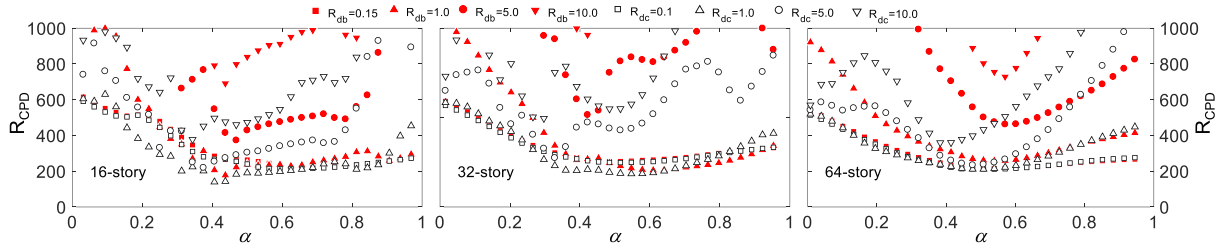
**FIGURE 20** Relationships between  $\theta_{\max}$  and  $\alpha$  for various A,  $R_{db}$  and B,  $R_{dc}$  of 16, 32, and 64-story models as obtained from spectral analysis and nonlinear response history analysis. [Colour figure can be viewed at [wileyonlinelibrary.com](#)]

minimum  $\theta_{\max}$ . Figure 20 also shows that larger  $R_{db}$  and  $R_{dc}$  lead to smaller  $\theta_{\max}$ . However, both SA and NLRHA show that  $\theta_{\max}$  obtained when  $R_{db}$  equals 5.0 and 10.0, and when  $R_{dc}$  equals 5.0 and 10.0, are very close to each other. This suggests that a stiffer BRB (larger  $R_{db}$  or  $R_{dc}$ ) does not guarantee a better performance in reducing  $\theta_{\max}$ .

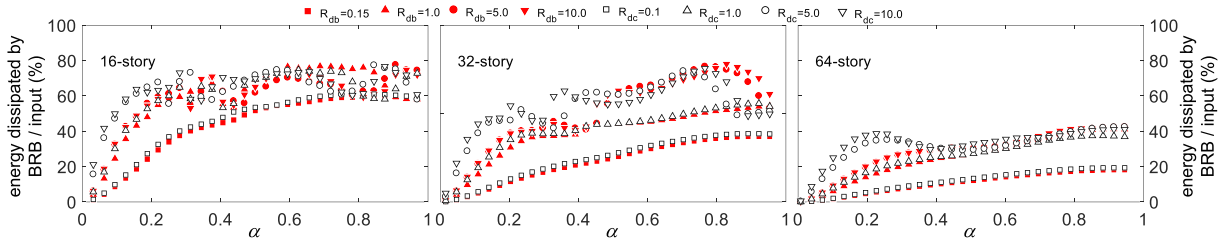
Figure 21 shows the minimum  $\theta_{\max}$  and its corresponding  $R_{db}$  or  $R_{dc}$  from each analysis set. Larger  $R_{db}$  or  $R_{dc}$  value results in greater reductions in  $\theta_{\max}$ ; however, the reductions stop increasing when  $R_{db}$  or  $R_{dc}$  is greater than around 2, 3, and 5 for the 16, 32, and 64-story model, respectively. Figures 22 and 23 show  $R_{CPD}$  and the relationship between ratios of energy dissipated by BRBs to the total input energy with respect to  $\alpha$ , computed from the NLRHA results. The  $R_{CPD}$  begins increasing significantly when  $R_{db}$  and  $R_{dc}$  are greater than 5.0. For the models with large  $R_{db}$  or  $R_{dc}$  values, once the BRB yields, the drop in BRB stiffness from  $k_d$  to relatively small postyield stiffness ( $pk_d$ ), if compared with  $k_c$  and  $k_t$ , can result in large deformation concentration in the BRB. In addition, as illustrated in section 2.3, large  $R_{db}$  or  $R_{dc}$  would result in smaller  $u_{dy}$ . Thus, the energy dissipation efficiencies for the models with large  $R_{db}$  or  $R_{dc}$  values (small  $u_{dy}$ ) accompanied with large BRB axial deformation may be similar to those with small  $R_{db}$  or  $R_{dc}$  value that lies between 3 and 5. This explains that when  $R_{db}$  or  $R_{dc}$  is larger,  $\theta_{\max}$  cannot be proportionally reduced and the



**FIGURE 21** Relationships between  $\theta_{\max}$  and  $R_{db}$  (Met. I) or  $R_{dc}$  (Met. II.) of 16, 32, and 64-story models [Colour figure can be viewed at wileyonlinelibrary.com]



**FIGURE 22** Relationships between  $R_{CPD}$  and  $\alpha$  for various  $R_{db}$  or  $R_{dc}$  of 16, 32, and 64-story models as obtained from nonlinear response history analysis [Colour figure can be viewed at wileyonlinelibrary.com]



**FIGURE 23** Relationships between the ratio of energy dissipated by buckling-restrained brace to input energy and  $\alpha$  for various  $R_{db}$  or  $R_{dc}$  of 16, 32, and 64-story models as obtained from nonlinear response history analysis [Colour figure can be viewed at wileyonlinelibrary.com]

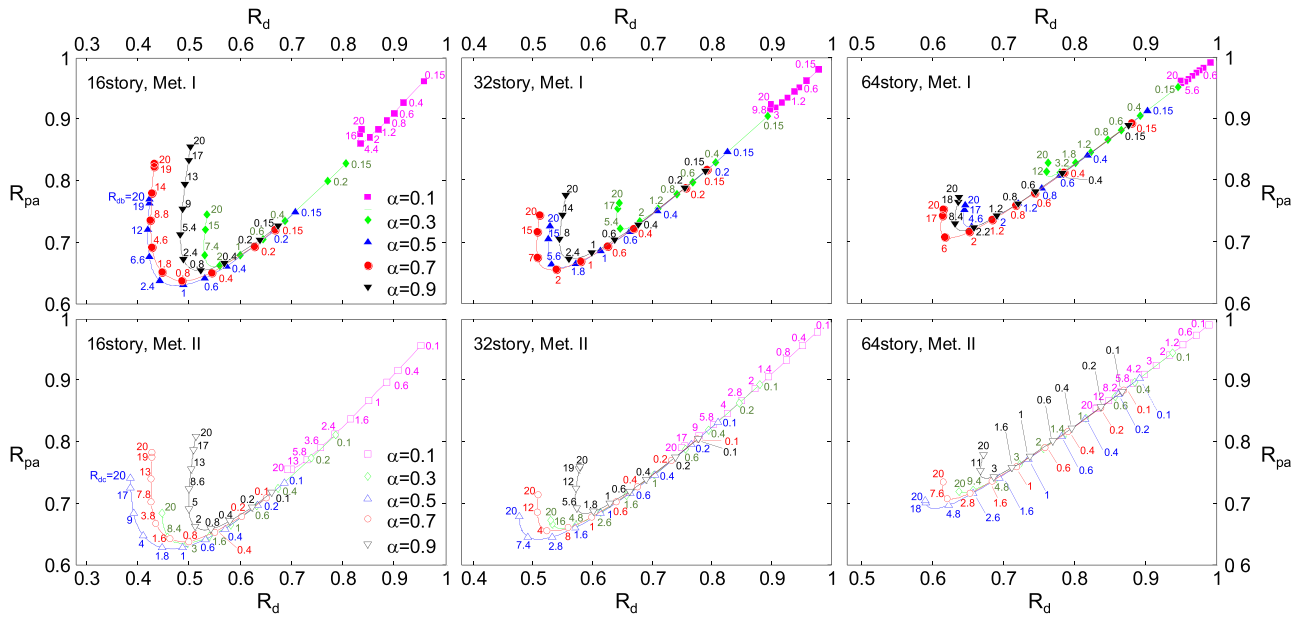
BRB energy dissipation efficiency remains almost the same. This also explains the large  $R_{CPD}$  found from the models with large  $R_{db}$  or  $R_{dc}$  values. However, a very large  $R_{CPD}$  value indicates that the BRB may use up its ductility capacity and eventually fracture before the end of earthquake, which is not desirable for engineering practices. For the design purpose, increasing  $R_{db}$  or  $R_{dc}$  also increases the cost of BRB and may reduce the BRB use life. Based on the analysis results, to reduce  $\theta_{\max}$  and let the BRB to properly function at the same time, it is suggested that the  $R_{db}$  and  $R_{dc}$  should be greater than 1 and smaller than 5.

The reduction factors of  $\theta_{\max}(R_d)$  and  $a_{\max}(R_{pa})$ , if compared with the structure without the outrigger, can be calculated as follows:

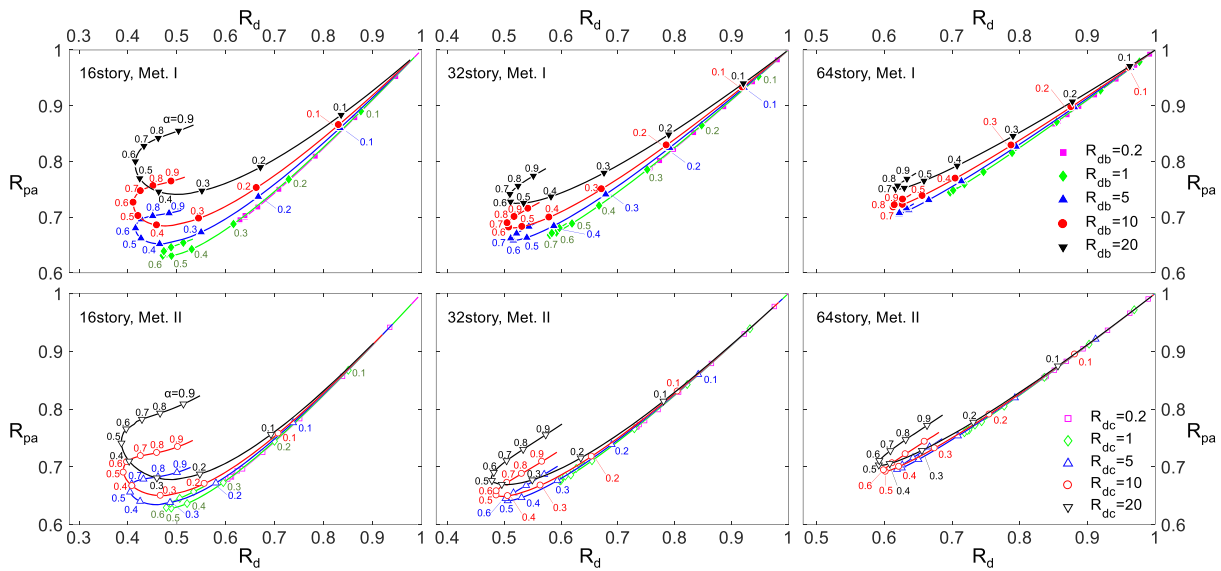
$$R_d = \frac{S_d(T_{eq,1}, h_{eq,1})}{S_d(T_0, h_0)} = D_{h,1} \frac{T_{eq,1} S_{pv}(T_{eq,1}, h_0)}{T_0 S_{pv}(T_1, h_0)}, \quad R_{pa} = \frac{S_{pa}(T_{eq,1}, h_{eq,1})}{S_{pa}(T_n, h_0)} = D_{h,1} \frac{T_0 S_{pv}(T_{eq,1}, h_0)}{T_{eq,1} S_{pv}(T_0, h_0)} \quad (21)$$

where  $T_0$  is the linearly elastic fundamental period of the core structure only. As the fundamental mode dominates the building responses, the  $R_d$  and  $R_{pa}$  calculations consider the contribution of first mode only. Figures 24 and 25 show the relationships between  $R_{pa}$  and  $R_d$  (performance curves) with fixed  $\alpha$  and fixed  $R_{db}$  or  $R_{dc}$ . The numbers in Figure 24 indicate the corresponding  $R_{db}$  (Met. I) or  $R_{dc}$  (Met. II), and the numbers in Figure 25 indicate the corresponding  $\alpha$ .

From both Figures 24 and 25, both  $R_d$  and  $R_{pa}$  reach minima when  $\alpha$  is between 0.5 and 0.7. Met. I results show that the optimal  $\alpha$  value, to achieve minimum  $R_{pa}$  and  $R_d$ , increases from around 0.5 to 0.7 with increasing building height



**FIGURE 24** Performance curves of Met. I and Met. II with fixed  $\alpha$  of 16, 32, and 64-story models [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]



**FIGURE 25** Performance curves of Met. I and Met. II with fixed  $R_{db}$  or  $R_{dc}$  of 16, 32, and 64-story models [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

(decreasing  $S_{bc}$ ). In addition, for any fixed  $\alpha$ , the  $\theta_{\max}$  can be reduced by increasing  $R_{db}$  or  $R_{dc}$ . However, if  $R_{db}$  or  $R_{dc}$  is too large resulting in a very stiff system,  $R_{pa}$  could be significantly amplified, and  $R_d$  could increase again. As shown in Figure 25, when  $R_{db}$  or  $R_{dc}$  is fixed and when the outrigger elevation is higher than optimal  $\alpha$ , both  $\theta_{\max}$  and  $a_{\max}$  could increase. Table 7 shows the  $\alpha$ ,  $R_{db}$ , or  $R_{dc}$  when minimum  $R_d$  or  $R_{pa}$  is achieved. The optimal  $\alpha$  required to achieve minimum  $R_d$  and  $R_{pa}$  is approximately 0.6 (16-story), 0.65 (32-story), and 0.7 (64-story) for Met. I and 0.5 for Met. II. As shown in Figure 24, The  $R_{db}$  or  $R_{dc}$  for achieving minimum  $R_{pa}$  could result in  $R_d$  that is close to its minimum. Thus, the optimal  $R_{db}$  and  $R_{dc}$  could be approximated from the minimum  $R_{pa}$  value. If compared the analysis results with 3 types of  $S_{bc}$  values in Met. I, increasing  $S_{bc}$  (increasing  $k_b$ ) should be more efficient than increasing  $R_{db}$  or  $R_{dc}$  to enhance the outrigger effect because the large  $R_{db}$  and  $R_{dc}$  could amplify  $a_{\max}$ . Based on the analytical results, the design with  $\alpha$

**TABLE 7**  $\alpha$ ,  $S_{bc}$ ,  $R_{db}$ , and  $R_{dc}$  at minimum  $R_d$  and  $R_{pa}$  with  $R_{db}$  or  $R_{dc}$  ranges from 0.1 to 20.0

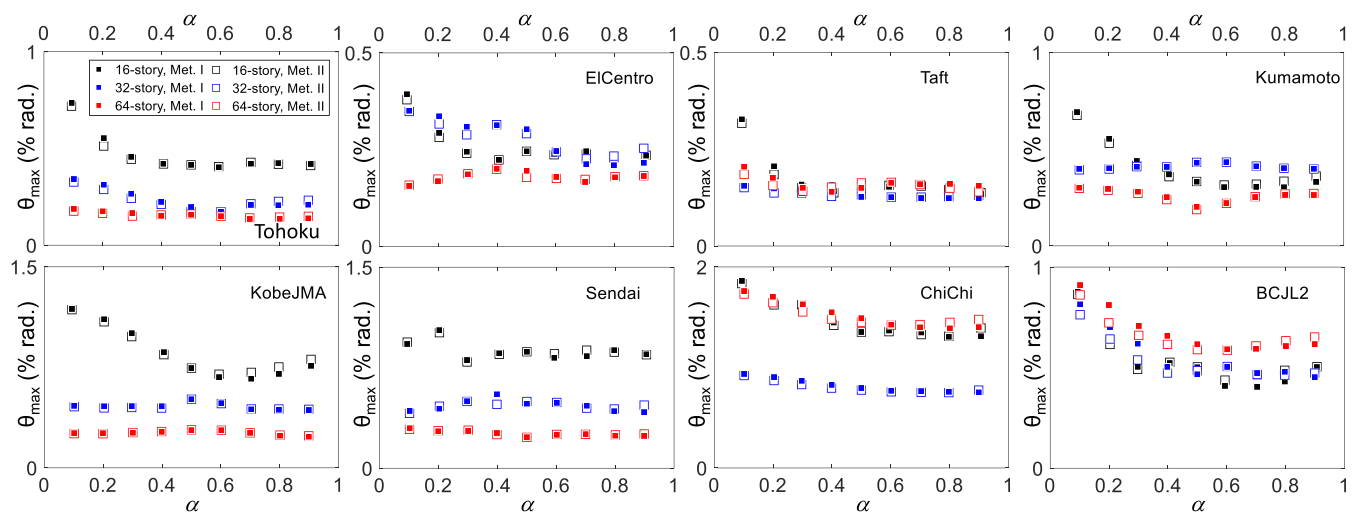
| Model    | Met. | To Achieve Minimum $R_d$ |          |                      |       |          | To Achieve Minimum $R_{pa}$ |          |                      |       |          |
|----------|------|--------------------------|----------|----------------------|-------|----------|-----------------------------|----------|----------------------|-------|----------|
|          |      | $\alpha$                 | $S_{bc}$ | $R_{db}$ or $R_{dc}$ | $R_d$ | $R_{pa}$ | $\alpha$                    | $S_{bc}$ | $R_{db}$ or $R_{dc}$ | $R_d$ | $R_{pa}$ |
| 16-story | I    | 0.58                     | 3.03     | 10.2                 | 0.41  | 0.72     | 0.50                        | 3.03     | 1.2                  | 0.48  | 0.63     |
|          | II   | 0.50                     | 4.23     | 20.0                 | 0.39  | 0.74     | 0.45                        | 4.48     | 1.6                  | 0.47  | 0.62     |
| 32-story | I    | 0.64                     | 1.38     | 11.0                 | 0.50  | 0.69     | 0.61                        | 1.38     | 2.6                  | 0.53  | 0.65     |
|          | II   | 0.52                     | 1.89     | 20.0                 | 0.48  | 0.68     | 0.48                        | 1.97     | 4.6                  | 0.51  | 0.64     |
| 64-story | I    | 0.71                     | 0.66     | 11.2                 | 0.61  | 0.72     | 0.70                        | 0.66     | 4.4                  | 0.62  | 0.70     |
|          | II   | 0.52                     | 0.89     | 20.0                 | 0.59  | 0.71     | 0.51                        | 0.90     | 8.4                  | 0.60  | 0.69     |

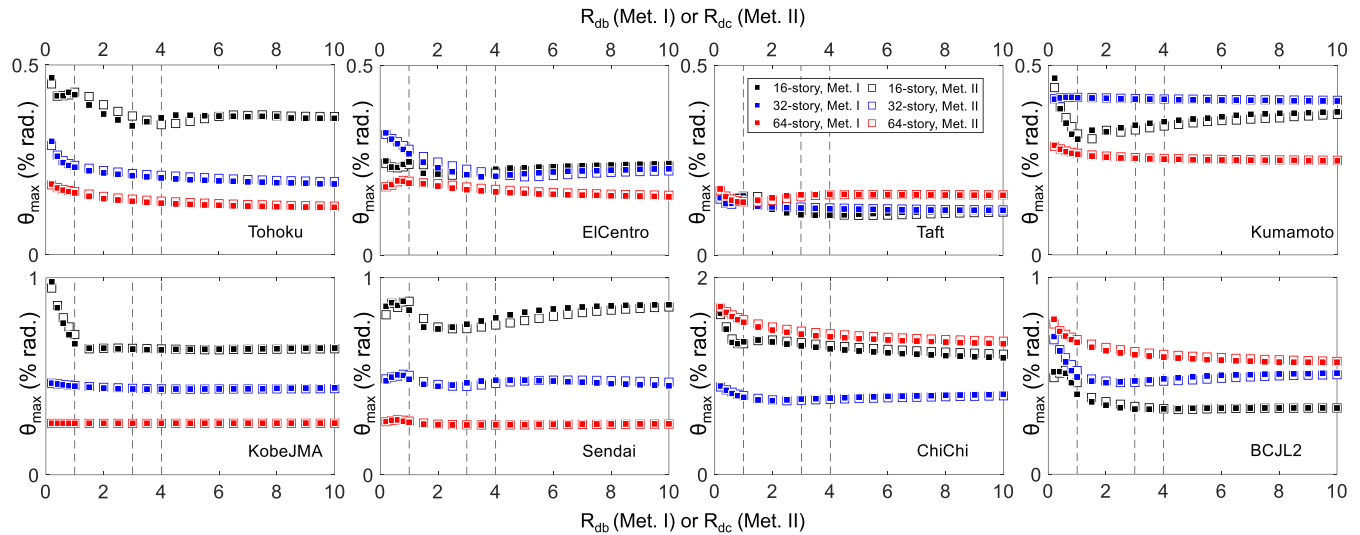
that lies between 0.5 and 0.8,  $R_{db}$  (Met. I) and  $R_{dc}$  (Met. II) that lie around 1 when  $S_{bc}$  is between 2 and 5, 3 when  $S_{bc}$  is between 1 and 2, and 4 when  $S_{bc}$  is less than 1 could achieve satisfactory results in reducing both  $\theta_{\max}$  and  $a_{\max}$  responses.

### 4.3 | Verification of optimal design by NLRHA

The proposed optimal design parameters are examined by performing NLRHA with the original observed ground motions listed in Table 5. The response spectra are shown in Figure 8A. Figure 26 shows the relationship between  $\theta_{\max}$  and  $\alpha$ .  $R_{db}$  and  $R_{dc}$  are set to 1, 3, and 4, for the 16, 32, and 64-story model, respectively. In most cases, the optimal  $\alpha$  lies between 0.5 and 0.8. However, the optimal  $\alpha$  for the 32 and 64-story models correspond to the building top and bottom for Kumamoto, KobeJMA, and Sendai ground motions. This could be because the second mode responses are amplified because of the relatively large spectral accelerations of those ground motions corresponding to the second mode period, if compared with the design spectrum. And the optimal outrigger elevations (optimal  $\alpha$ ) are close to the  $\alpha$  that results in the smallest second mode period as shown in Figure 19. Based on the NLRHA results, when the outrigger is placed at the optimal location, the  $\theta_{\max}$  is found to be reduced by 42% for the 16-story model, 24% for the 32-story model, and 22% for the 64-story model, respectively, on average.

Figure 27 shows the relationships between  $\theta_{\max}$  and  $R_{db}$  or  $R_{dc}$ , where the  $\alpha$  is set to 0.7 in each analysis. Because both  $R_{db}$  and  $R_{dc}$  directly affect structure stiffness, the relationships between  $\theta_{\max}$  and  $R_{db}$  or  $R_{dc}$  vary with ground motions. Figure 27 also shows trend similar to Figure 21;  $\theta_{\max}$  cannot be further reduced or even amplified when  $R_{db}$  or  $R_{dc}$  is too large. The optimal  $R_{db}$  and  $R_{dc}$  values for 16-story model are larger than 1.0 for the Tohoku, Taft, and Sendai ground motions. This could be because 16-story model with larger  $S_{bc}$  and shorter fundamental period is more sensitive to the variations in ground motions. However, the optimal  $R_{db}$  and  $R_{dc}$  values for all analytical results lie in the range of 1 to 5.

**FIGURE 26** Relationships between  $\theta_{\max}$  and  $\alpha$  of nonlinear response history analysis results with original observed earthquakes [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

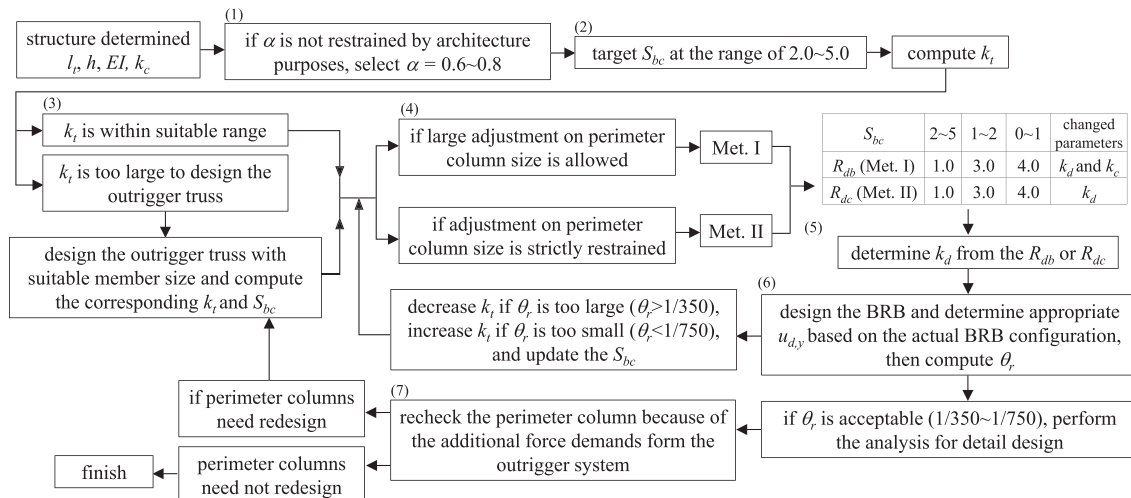


**FIGURE 27** Relationships between  $\theta_{\max}$  and  $R_{db}$  or  $R_{dc}$  of nonlinear response history analysis results with original observed earthquakes [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

#### 4.4 | Design recommendation

The optimal values of  $\alpha$ ,  $R_{db}$ , and  $R_{dc}$  to minimize  $R_d$  and  $R_{pa}$  were investigated. Figure 28 shows a recommended design flow chart. For design practice, the building lateral stiffness (core structure flexural rigidity,  $EI$ ) should be determined based on the code specifications. The perimeter column sizes ( $k_c$ ) should be determined according to floor framing plan and gravity load demands. Thus,  $k_d$  and  $k_t$  may be less restrained. Smaller  $k_d$  (smaller  $R_{db}$  and  $R_{dc}$ ) and  $k_t$  are desirable because they reduce material usage and cost. Based on the analytical results, the recommended design procedure is as follows:

- (1) If  $\alpha$  is not restricted for architectural reasons, select  $\alpha$  between 0.6 and 0.8.
- (2) Target  $S_{bc}$  to lie in the range of 2.0 to 5.0, as larger  $S_{bc}$  leads to smaller optimal  $R_{db}$  and  $R_{dc}$ .
- (3) Compute  $k_t$  according to the  $S_{bc}$  determined in the previous step. If  $k_t$  is too large to design the outrigger truss members, reduce  $k_t$  until the outrigger truss member sizes are reasonable and recompute  $S_{bc}$ .
- (4) Select Met. I if the perimeter column sizes are adjustable; otherwise, select Met. II. Target the optimal  $R_{db}$  (Met. I) or  $R_{dc}$  (Met. II) according to the  $S_{bc}$ .
- (5) Compute  $k_d$  based on the selected optimal  $R_{db}$  or  $R_{dc}$ .



**FIGURE 28** Flow chart of design recommendation

- (6) Design the BRB based on the  $k_d$  computed in Step (5). Determine appropriate  $u_{d,y}$  based on the actual BRB configuration and calculate the corresponding  $\theta_r$  by performing pushover analysis. Decrease  $k_t$  if  $\theta_r$  is too large ( $\theta_r > 1/350$ ) or increase  $k_t$  if  $\theta_r$  is too small ( $\theta_r < 1/750$ ) until the  $\theta_r$  is within a reasonable range (eg, 1/350 to 1/750).
- (7) After all the parameters are determined, perform the analysis and proceed to member design. As the outrigger would lead to additional force demands on the perimeter column, the perimeter column design should include the effect of possible maximum BRB axial force.

## 5 | CONCLUSIONS

This study proposed a simplified structure to evaluate the seismic behavior of buildings with BRB-outrigger systems. The methods to determine the BRB yield deformation, and the optimal stiffness relationships among the core structure, BRB, outrigger truss, and the perimeter columns were discussed. Based on the analytical results, the following conclusions can be drawn:

- (1) Three types of analytical models were adopted in this study. The UM was used when performing SA, and DM1 was used when performing NLRHA. The analytical results of simplified structure using UM, DM1, and DM4 models were in good agreement with the results obtained from MBM models.
- (2) SA was adopted to investigate the optimal  $\alpha$ ,  $R_{db}$ , and  $R_{dc}$  values to achieve minimum roof drift ratio and acceleration responses. The SA incorporated the equivalent damping induced from BRB's inelastic deformation, and the results were verified by performing NLRHA. Both the SA and NLRHA results showed similar trends with various  $\alpha$ ,  $R_{db}$ , and  $R_{dc}$ .
- (3) The BRB's yield deformation should be appropriately determined. A very large  $u_{d,y}$  would reduce the BRB's energy dissipation efficiency. The BRB with very small  $u_{d,y}$  could easily yield during minor earthquakes or use up its ductility capacity before the end of a large earthquake.
- (4) Based on the analytical results, the first mode response dominates the seismic behavior. It is suggested to determine  $u_{d,y}$  from the BRB's axial deformation in the first mode when roof drift reaches elastic deformation limit (1/750 for example).
- (5) Two methods were developed in this study. In Met. I, the rotational stiffness resulting from the outrigger is kept constant while changing  $\alpha$ . In Met. II,  $k_c$  remains constant, and thus,  $k_g$  decreases with increasing  $\alpha$ . Based on the analysis results,  $\theta_{\max}$  can be reduced best when  $\alpha$  is between 0.6 and 0.8 for Met. I and between 0.5 and 0.6 for Met. II.
- (6) Based on the analytical results, the optimal  $\alpha$  values were not found to be significantly affected by either  $R_{db}$  or  $R_{dc}$ . Larger  $R_{db}$  and  $R_{dc}$  resulted in greater efficiency in reducing  $\theta_{\max}$ . However, the maximum acceleration response would be amplified if  $R_{db}$  and  $R_{dc}$  are too large. The optimal  $R_{db}$  and  $R_{dc}$  values to achieve minimum  $\theta_{\max}$  and  $a_{\max}$  are approximately 1 for  $S_{bc}$  ranging from 2 to 5, 3 for  $S_{bc}$  ranging from 1 to 2, and 4 for  $S_{bc}$  smaller than 1.
- (7) According to the results of NLRHA on the models with and without optimal design parameters, the ones with optimal  $\alpha$  could reduce the  $\theta_{\max}$  by 42%, 24%, and 22% for the 16, 32, and 64-story models, respectively. The optimal  $R_{db}$  and  $R_{dc}$  values lie within the range of 1 to 5.

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**How to cite this article:** Lin P-C, Takeuchi T, Matsui R. Seismic performance evaluation of single damped-outrigger system incorporating buckling-restrained braces. *Earthquake Engng Struct Dyn*. 2018;1-23. <https://doi.org/10.1002/eqe.3072>