

論文 / 著書情報
Article / Book Information

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種別(和文)	論文要旨
Type(English)	Summary

論文要旨

THESIS SUMMARY

系・コース： Department of, Graduate major in	数学 数学	系 コース	申請学位 (専攻分野)： Academic Degree Requested	博士 Doctor of	(理学)
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要旨 (英文 800 語程度)

Thesis Summary (approx.800 English Words)

The contents of this thesis are divided into these two parts roughly:

1. cocycle-level constructions of characteristic classes of fiber bundles and their relations to cohomology of Lie algebras of derivations, and
2. a relation between the dgl of symplectic derivations on a semi-free dgl and a certain graph complex.

The former includes two constructions of characteristic classes using de Rham homotopy theory. One of the constructions gives representative differential forms of characteristic classes of a fiber bundle like Chern-Weil theory fixing a Riemannian metric of a fiber bundle. According to Chen's de Rham homotopy theory, this metric defines the map from the base space to a certain moduli space of homotopy algebra models of a fiber. Under a certain condition, we can construct a flat connection on the moduli space, whose coefficient Lie algebra consists of equivalence classes of derivations on a Chen model of a fiber. Existence of the flat connection is proved by the fundamental theory of homotopy algebras. Applying the Chern-Weil theory for a flat bundle, we can obtain characteristic classes of a fiber bundle satisfying a suitable condition as the image of the characteristic map from the Chevalley-Eilenberg complex of a Lie algebra of derivations on a Chen's model of a fiber to the cohomology of the base space through the map above. As an example of such characteristic classes, we have Morita-Miller-Mumford classes of surface bundles. It is a joint work with Y.Terashima and H.Kajiura.

Another construction is an attempt to construct simplicial enhancement of the construction above. In the construction, we define the associated simplicial bundle of homotopy algebra models of fibers for a fiber bundle. The space of connected components of a fiber of simplicial bundle is the moduli space used in the construction above. The obstruction theory of simplicial bundles yields the obstruction class of the simplicial bundle. A characteristic class of a fiber bundle is obtained for each invariant anti-symmetric form on the homotopy group of a fiber of the simplicial bundle. This discussion includes two cases: the fiber of the simplicial bundle is connected or not. The non-connected case is related to the construction above, while the connected case captures characteristic classes that could not be obtained in the construction above. For a surface bundle, as non-connected case, this class is equivalent to the obstruction class known as twisted Morita-Miller-Mumford classes for a surface bundle. Thus, the characteristic classes include Morita-Miller-Mumford classes of a surface bundle. As an example of connected case, we can obtain the Euler class of a specific sphere bundle by direct calculus.

The latter gives a way to represent characteristic classes of fibrations by linear combinations of graphs. Kontsevich constructed a chain complex generated by decorated graphs, which is called the graph complex, and a chain map from the graph complex to the Chevalley-Eilenberg complex of the Lie algebra of symplectic derivations on a free algebra given symplectic form on generators. On the other hand, it is known that the (total) cohomology classes of the dgl of symplectic derivations on a semi-free dgl (given symplectic form on generators) give characteristic classes of simply-connected fibrations whose fiber has a sphere boundary. So, in the thesis, we construct a double complex as a generalization of Kontsevich's graph complex, which is called the double graph complex, and a chain map from the double graph complex to the Chevalley-Eilenberg double complex of the dgl of symplectic derivations on a semi-free dgl. It means graph cocycles yield characteristic classes of certain fibrations. A generator of the double graph complex is described as a decorated graph which has two types of vertices and graded half-edges. One type of vertices

corresponds to vertices of Kontsevich's decorated graphs, while another type corresponds to the differential of the given semi-free dgl. Using these structures, we can endow the new differential with our graph complex corresponding to the differential of the semi-free dgl through the chain map defined in the same way as Kontsevich's one. In the thesis, we give two examples of specific relations between graph cocycles and characteristic classes of fibrations whose fiber is a punctured low-dimensional complex projective space. As other application, we can observe the part of the chain map gives Hamiton-Lazarev's characteristic classes of a cyclic homotopy algebra. In this sense, our chain map can be regarded as a mixture of Kontsevich's map and Hamilton-Lazarev's map.

備考：論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。

Note：Thesis Summary should be submitted in either a copy of 2000 Japanese Characters and 300 Words (English) or 1copy of 800 Words (English).

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