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論文 / 著書情報
Article / Book Information

題目(和文)	量子系と古典系における実時間と虚時間の動的な 効果
Title(English)	Study on real and imaginary time dynamical effects in quantum and classical systems
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Category(English)	Doctoral Thesis
種別(和文)	論文要旨
Type(English)	Summary

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（博士課程）
Doctoral Program

論 文 要 旨

THESIS SUMMARY

系・コース： Department of, Graduate major in	物理学系 物理学	系 コース	
学生氏名： Student's Name	奥山 真佳		

申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of （ 理学 ）
指導教員（主）： Academic Supervisor(main)	西森 秀稔
指導教員（副）： Academic Supervisor(sub)	

要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

The present thesis investigates the real and imaginary time dynamical effects of quantum and classical systems, inspired by quantum annealing. In particular, we have examined mean-field quantum spin systems and the quantum speed limit.
The first main purpose of this thesis is to treat exactly the imaginary-time dependence of the partition function of mean-field quantum spin systems. Mean-field quantum spin systems are one of the simplest quantum spin systems, and there are many studies on them from the view point of quantum annealing. However, mean-field quantum spin systems had been analyzed with the static approximation, which ignores the imaginary-time dependence of the partition function, and had not been solved exactly.
In order to solve mean-field quantum spin systems, we have proposed a new method using the optimal control theory. We have mapped a problem of obtaining the exact partition function in the thermodynamic limit to solving the corresponding optimal control problem in the imaginary-time Schrodinger equation. In the optimal control problem, finding the optimal solution is equivalent to solving the corresponding classical Hamilton equations with special initial and terminal conditions. We have applied this identification to a wider class of mean-field quantum spin systems which generalizes the Hopfield model. Although it is very difficult to find an analytical solution of the optimal control problem in general, we can solve the optimal solution and show that the optimal solution coincides with the static approximate solution, that is, the static approximation is exact for a wider class of mean-field quantum spin systems in general. This is a first systematic result for the exactness of the static approximation.
Next, we have applied our result to the previous studies on quantum annealing. We have verified their analysis where the non-stoquastic Hamiltonian and the inhomogeneous transverse field accelerate the computational time exponentially for mean-field quantum spin systems.
Furthermore, we have investigated why our optimal control problem can be solvable because, in general, it is very difficult to solve the optimal control problem exactly. We have examined the corresponding classical Hamiltonian in detail and, as a result, we have found that the corresponding classical Hamiltonian belongs to classical nonlinear integrable systems. To the best of my knowledge, this classical nonlinear integrable system has not been discovered so far. This means that the new class of classical nonlinear integrable systems is hidden in mean-field quantum spin systems. It is an interesting and urgent problem to obtain further understanding of this classical nonlinear integrable system.
The second main purpose of this thesis is to obtain fundamental speed limits corresponding to the quantum speed limit in classical systems. The quantum speed limit gives an upper bound to the speed of state change of the quantum system and is not only a formal concept but also a useful idea for quantum annealing. Since QSL originally developed in a similar context to Heisenberg's uncertainty principle, previous studies have regarded the quantum speed limit as a pure quantum phenomenon.
On the contrary, we have revealed that classical mechanics also has a similar speed limit. Using the Hilbert space theory for the classical Liouville equation, we have shown that classical mechanics also has fundamental speed limit corresponding to the quantum speed limit. Therefore, the quantum speed limit is not a purely quantum phenomenon but a universal dynamical property of the Hilbert space.
Next, we have extended another type of quantum speed limit, which is the Kieu bound, to the imaginary-time Schrodinger equation. In the Grover problem, the Kieu bound is used to prove that the optimal computational time is of order $\sqrt{M}$ in quantum annealing. We have generalized the Kieu bound to the imaginary-time Schrodinger equation and found that the optimal computational time of the Grover problem is bounded below order $\log M$, which is consistent with the previous study. We also note that the imaginary time Schrodinger equation is the dynamics on a classical computer and the physical system corresponding to the imaginary time Grover problem cannot exist in real world.

備考 ： 論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。
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