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論 文 要 旨
THESIS SUMMARY

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要旨（英文 800 語程度）
Thesis Summary (approx.800 English Words)

A mixed-integer second-order cone (MI-SCOP) has been increasingly utilized in different important areas. An important of its application is arising from tree breeding in which the goal is to provide the forestry sector with the best possibility plant candidates under genetic diversity consideration. Therefore, an optimal contribution selection (OCS) method is required to generate the best candidates.

This study is concerned with OCS in equal deployment problem (EDP) that requires a specified number of selected candidates to give either the fixed size or zero contribution. General solvers like GENCONT and dsOpt (embedded in OPSEL) were proposed to obtain the solution of EDP. However, those existing solvers somehow require either large memory size or long computation time to handle the certain EDP.

On the other hand, EDP formulation involves quadratic constraint from the genetic diversity and integer constraint from the candidate contributions, that can be composed to the MI-SOCP formulation. Thus, the main target of this project is to develop efficient methods for MI-SCOP in tree breeding field, using three main approaches that involve a conic relaxation and a steepest-ascent method, a lifted polyhedral programming relaxation, and a cone decomposition method.

We begin with a discussion of conic relaxation for EDP. One of conic relaxation applications can be found in a well-known paper due to Goemans and Williamson which applied SDP to max-cut problems. More precisely, they derived an SDP problem by ignoring the rank-one constraint, in which this SDP relaxation can lead to favorable approximate solutions. Following this achievement, the SDP relaxation approach has been widely applied to combinatorial optimization problems. This research direction has been extended to conic relaxation with the use of LP, SOCP and SDP.

Since relaxation problems give good approximation of the optimal value, one research direction might be the use of the relaxation in branch-and-bound frameworks to generate an exact solution. However, pursuing an exact solution demands heavy computation costs, so our focus is to acquire a favorable solution that is available in a practical computation time.

Our focus is followed by the development of a steepest-ascent method that employs the solution obtained from the conic relaxation problems as a starting point. The quadratic constrains is embedded into the objective function as a penalty term with a weight computed from the Lagrange multiplier. Numerical experiments show that the steepest-ascent method generates qualified solutions for the EDP. In particular, the steepest-ascent method starting with the SOCP relaxation problem attains the best performance among the LP, SOCP, and SDP relaxation problems. It even performs better than the general solvers from the viewpoints of both solution quality and computation time. For instance, the result of EDP with the largest size of candidates present that the steepest-ascent method starting with SOCP slightly reduces computation time into 4.72 seconds while the general solvers face insufficient memory size problem and requires more than 3 hours to obtain the solution. This fact gives interpretation to propose another efficient implementation using SOCP as a basic concept since the above implementation is only an iterative approach.

We consider another implementation proposed by Vielma et al.,2008, called lifted polyhedral programming (LPP). LPP relaxation solves mixed-integer conic quadratic problems (MIQCP) that have similar structure with our EDP problem. We conducted preliminary experiment to implement the relaxation to EDP. However, a larger number of constrains generated by polyhedral relaxation lead to a heavy computation. We then developed an improvement to the LPP by utilizing active constraint selection method (LPP-ACSM) that can remove the non-linearity. Surprisingly, the performance of LPP-ACSM is somehow worse than LPP itself so that we need other second-order cone approach for the EDP.

A cone decomposition method (CDM) is proposed which the basic concept also draws the second-order cone properties. CDM utilizes different decomposition with LPP based on a geometric cut in combination with a lagrangian multiplier method. A cutting plane is a geometric cut if the plane is computed with an orthogonal projection. Cone decomposition itself has already been used in a general MI-SOCP solver (CPLEX), but it depends on an outer approximation. Therefore, the proposed CDM generates a different linear approximation.

The last approach to reduce heavy computation burden is utilizing the sparsity found in the inverse of Wright's numerator relationship matrix. A combination of this sparsity and the geometric cuts into a sparse linear approximation, strongly enhances the performance of CDM. In addition, we prove that the Lagrangian multiplier method in the framework of CDM gives an analytical form for the geometric cut, therefore, the proposed CDM and its sparse variant generate the linear cuts without relying on iterative methods.

We conducted a numerical experiment to compare all proposed method with the general solver of OCS. Along the experiment, LPP and LPP-ASCM lead to a heavy computation time due to larger size of constraints. The methods need a tight epsilon which is required to attach larger number of hyperplanes for getting better approximation. Thus, the generated constraints for the hyperplanes consume larger memory size. Both methods failed to obtain the solution, even for small problem, for the chosen tight epsilon.

Contrary to the LPPs, CDM gives better performance, especially for its enhancement. CDM with sparsity structure significantly decrease a heavy computation problem. More precisely, the largest size problem with a tighter gap setting reduces 10 computation times of CDM itself. Thus, it is verified that sparse CDM is the most efficient approach to solve EDP. In addition, the implementation of sparse CDM is not only limited to the EDP but also for other problems with similar structure.