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ESTIMATION METHOD OF TENSILE STRAIN OF LAMINATED RUBBER BEARINGS AND BENDING MOMENT OF FOUNDATION BEAM FOR SEISMICALLY ISOLATED BUILDING

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ABSTRACT

In recent years, seismic isolation buildings are becoming taller and the shape has become more diversified. Therefore, the tensile strain of laminated rubber bearing (LRB) is important. A method of defining the tensile limit of LRB by tensile strain has been proposed, and it is necessary to correctly evaluate the tensile strain generated in LRB due to horizontal and vertical earthquake motions. To evaluate the tensile strain occurring in LRB, it is desirable to perform a time history analysis in which horizontal and vertical are input simultaneously. However, the tensile strain of LRB is affected significantly by the damping setting of the building and the nonlinear characteristics of LRB. Therefore, a detailed examination is required. We propose the estimation method of tensile strain for LRBs using the superposition of horizontal and vertical response values considering that the tensile elastic modulus of LRBs is lower than its compression modulus. In addition, we propose the estimation method of bending moment for a foundation beam when the LRBs are pulled out. The validity of the proposed methods is demonstrated by comparing them with the time-history analysis results using a 24-story isolated base building model.

Keywords: Seismically isolated building; Laminated rubber bearing; Tensile strain; Stress redistribution; Superimposed horizontal and vertical responses

1. INTRODUCTION

In recent years, seismic isolation buildings are becoming taller and the shape has become more diversified. Therefore, the tensile strain of laminated rubber bearing (LRB) is important. Hence, a method of defining the tensile limit of LRB by tensile strain has been proposed (Mori et al. 2015). To evaluate the tensile strain occurring in LRB, it is desirable to perform a time-history analysis in which horizontal and vertical are input simultaneously. However, the tensile strain of LRB is affected significantly by the damping setting of the building and the nonlinear characteristics of LRB. Therefore, a detailed examination is required. In general, because the natural period in the vertical direction is much shorter than that in the horizontal direction, if stiffness-proportional damping are used for analysis, the damping in the vertical direction is evaluated excessively. Furthermore, because an LRB exhibits a nonlinear characteristic in which the tensile modulus becomes lower than the compression modulus (Notomi et al. 2002), advanced analytical techniques that simultaneously perform nonlinear analysis not only in the horizontal direction but also in the vertical direction are required. Therefore, in previous research, many methods have been proposed for analyzing horizontal and

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vertical earthquake motions individually and superimposing the responses (Nishiyama T. et al. 1994, Matsuhira N. et al. 1994, Sato D. et al. 2012, Hongo T. et al. 2014). However, most of these studies model the vertical characteristics of LRB as elastic, and the characteristics of LRB in which the tensile modulus is lower than the compression modulus are not considered. In the case of a seismically isolated building in which the tensile modulus of an LRB is lower than its compressive modulus of elasticity, the authors confirmed that the tensile strain of laminated rubber becomes smaller when using these methods. In addition, the authors investigated how the low tensile modulus of LRBs that is lower than the compression modulus affects the response of the superstructure of an isolated building (Yokota R. et al. 2016). Consequently, it was confirmed that the effect on the maximum response such as acceleration and story drift of the superstructure was small. Meanwhile, it was confirmed that the bending moment of the foundation beam around the pulled-out laminated rubber became larger owing to the influence of stress redistribution. Therefore, when we design for a base-isolated building using the limit of tensile strain of an LRB, it is necessary to consider the bending moment of the foundation beam when an LRB is pulled out.

Based on the study above, we devise an estimation method of tensile strain for LRBs using the superposition of horizontal and vertical response values considering that the tensile elastic modulus of the LRB is lower than its compression modulus. In addition, we devise an estimation method of bending moment for the foundation beam when the LRBs are pulled out.

2. OUTLINE OF THE MODEL AND GROUND MOTIONS

2.1 Superstructure design

The target building is a 24-story steel seismically isolated building whose height is 96.0 m, long-side direction is 32.0 m, short-side direction is 24.0 m, and aspect ratio is 4.0. The reference floor plan and framing elevation of X1 and X6 streets are shown in Fig. 1. A cross section is set based on the allowable stress degree calculation for the horizontal seismic force of $C_0 = 0.2$ using the SM 490 material as a member. The 1st natural period of the superstructure is 2.48 s in the X direction and 2.52 s in the Y direction. The superstructure is elastic in this study. A constant damping ($h = 2\%$) is used such that it is not affected by the difference between the vertical and natural periods of the seismically isolated building. The analysis time step Δt is 1 / 1,000 s.

Figure 1. Building model

2.2 Outline of the base isolation layer

Fig. 2 shows the arrangement of the laminated rubber bearings, and Fig. 3 shows the arrangement of the dampers. The base isolation layer consists of natural rubber bearings and hysteresis dampers. Natural rubber bearings are located under each column, whose shear modulus of elasticity $G = 0.392$ N/mm², and second shape factor $S_2 = 5.0$. The diameter of the natural rubber bearing is determined such that the compressive stress against the long-term axial force of the column becomes 10 to 15 N/mm². Steel dampers with initial stiffness of 19.2 kN/mm, yield load of 608 kN, and yield displacement of 31.7 mm are used. The yield load is $\sim 2.5\%$ of the building weight. The 1st natural period of the seismically isolated building at a 250% shear strain is 4.38 s in the X direction and 4.40 s in the Y direction. Two types of vertical tensile properties for laminated rubbers are used and compared with each other. Fig. 4 shows the relationship between vertical stress σ and strain ε .

Type-0 is a model with the same elastic modulus K_v on the compression and tension sides. Type-1 is a nonlinear elastic restoring force characteristic in which the tensile elastic modulus is $1/50$ of the compressive elastic modulus (Mori et al. 2015).

Figure 2. Placement of laminated rubber bearings

Figure 3. Placement of steel dampers

Figure 4. Vertical stress (σ)–strain (ϵ) relationship

2.3 Earthquake ground motions

In this study, a horizontal 45° direction and vertical input are used simultaneously. For the study, we used two simulated waves with a constant pseudo-speed response spectrum after the corner period (0.64 s) and four waves with two observation waves. The vertical movement in the simulated wave was set by multiplying the horizontal motion response spectrum by the vertical motion component coefficient using the method. The phase characteristics of the simulated wave were obtained using the 1995 Hyogo ken Nanbu Earthquake NS component, the UD component of the Kobe Maritime Atmosphere record (JMA Kobe), and the EW and UD components of the 1968 Tokachi-oki earthquake (Hachinohe). Hereinafter, the simulated earthquake using the JMA Kobe phase is called Art Kobe, and the simulated earthquake using Hachinohe's phase is called Art Hachi. The Kobe Maritime Atmosphere record of the 1995 Hyogo ken Nanbu earthquake NS component and UD component (JMA Kobe), and the EW and UD components of the 1940 El Centro earthquake were used as observation waves. Herein, they are denoted JMA Kobe and El Centro, respectively. Figs. 5(a), (b) show the pseudo-velocity response spectrums (pS_v) (Damping ratio $h = 5\%$) and time history of acceleration of input ground motions used in this study. As an example, the simulated wave result with $pS_v = 80$ cm/s after the corner cycle is shown. Further, the result of the observed wave with maximum velocity $V_{max} = 50$ cm/s is shown. In the subsequent investigation, only the laminated rubber at the corners of the laminated rubber is pulled out, and the seismic motion is multiplied by a constant such that the maximum tensile strain becomes 5% or less and is used as the input wave.

Figure 5. Input ground motions

3. CONSIDERATION WHEN TENSILE MODULUS OF LAMINATED RUBBER IS EQUAL TO COMPRESSIVE MODULUS

In this section, we examine the sum of the horizontal and vertical time history (STH) method when the tensile modulus of the laminated rubber is equal to the compressive elastic modulus (Type - 0). First, we compare the tensile strain of the laminated rubber with the time history sum method and the laminated rubber at the time of simultaneous input horizontally and vertically in the tensile strain of the laminated rubber. Subsequently, the bending moment of the foundation beam is examined when the tensile strain of the laminated rubber becomes a maximum deformation.

3.1 Study on tensile strain of laminated rubber

The tensile strain $\varepsilon_e^{(n)}$ of the n -step laminated rubber by the STH method is calculated from Eq. (1).

$$\varepsilon_e^{(n)} = {}_{X,Y}\varepsilon_e^{(n)} + {}_Z\varepsilon_e^{(n)} + {}_Z\varepsilon_0 \quad (1)$$

Here, ${}_{X,Y}\varepsilon_e^{(n)}$, ${}_Z\varepsilon_e^{(n)}$ are the variation distortions of the analysis in which the horizontal direction alone and the vertical direction alone are inputted, respectively, and ${}_Z\varepsilon_0$ is vertical strain due to long-term loading. Fig. 6 shows the strain time history of the laminated rubber calculated from the STH method $\varepsilon_e^{(n)}$ and time history of the strain of the laminated rubber bearings obtained from the simultaneous inputs of the horizontal and vertical (SimHV) method $\varepsilon^{(n)}$ at position X1-Y1 (Fig. 2), where the tensile strain became the largest. As shown, when the tensile modulus of the laminated rubber bearing is equal to the compressive elastic modulus (Type-0), the tensile strain of the laminated rubber at the time of simultaneous horizontal and lateral inputs can be evaluated by the STH method.

Figure 6. Time history of tensile strain of laminated rubber bearing

3.2 Examination of bending moment of foundation beam

The bending moment of the foundation beam at the step where the tensile strain of the laminated rubber became the maximum is compared. In the subsequent study, we investigate the bending moment of the X1 and Y1 basic beams immediately above the laminated rubber where tensile strain has occurred. The bending moment M_e calculated from the time history sum method is calculated from Eq. (2) using the bending moment at step n' where the tensile strain $\varepsilon_e^{(n)}$ (Eq. (1)) becomes the maximum.

$$M_e^{(n')} = {}_{X,Y}M_e^{(n')} + {}_ZM_e^{(n')} + {}_ZM_0 \quad (2)$$

Here, $_{x,y}M_e^{(n)}$, ${}_zM_e^{(n)}$ are the bending moments of the foundation beam of analysis, ${}_zM_0$ is the bending moment of the foundation beam due to long-term loading. Figs. 7 and 8 show the bending moment $M_e^{(n)}$ of the foundation beam at X1 and Y1 calculated from the STH method (Eq. (2)), and the bending moment $M^{(n)}$ of the foundation beam obtained from the SimHV method. As shown in Figs. 7 and 8, when the tensile elastic modulus of the laminated rubber is equal to the compressive elastic modulus (Type-0), the bending moment of the foundation beam can be evaluated by the STH method.

Figure 7. Maximum bending moment of the foundation beam at X1

Figure 8. Maximum bending moment of the foundation beam at Y1

4. INVESTIGATION WHEN TENSILE MODULUS OF LAMINATED RUBBER IS LOWER THAN COMPRESSIVE MODULUS

As shown in the previous section, the tensile strain of the laminated rubber and the bending moment of the foundation beam can be evaluated using the STH method when the tensile modulus of the laminated rubber is equal to the compressive elastic modulus (Type - 0). In this section, we investigate whether the tensile strain of the laminated rubber and the bending moment of the foundation beam can be evaluated by the STH method even when the tensile modulus of the laminated rubber is lower than the compressive elastic modulus (Type 1).

4.1 Study on tensile strain of laminated rubber

Fig. 9 shows the strain of the laminated rubber calculated from the time history sum method when the tensile modulus of the laminated rubber is Type-1 ($\varepsilon_e^{(n)}$), and the time history of the strain of laminated rubber at horizontal and vertical simultaneous inputs when the tensile modulus of the laminated rubber is Type-1 ($\varepsilon^{(n)}$). Fig. 10 shows the time history of strain of the LRB when earthquake ground motion is input independently in the horizontal direction and vertical directions. From Fig. 9, it is confirmed that $\varepsilon^{(n)}$ is greater than $\varepsilon_e^{(n)}$ when tensile strain occurs. In addition, when the tensile modulus of the laminated rubber is lower than the compressive modulus of elasticity, it is confirmed that the tensile strain of the laminated rubber cannot be evaluated by the STH method. At the time of simultaneous horizontal and vertical inputs, the tensile elastic modulus becomes lower after pulling out the laminated rubber, such that the tensile strain increases. However, as shown in Fig. 10, most of the horizontal and vertical single inputs are not pulled out or, even if it occurs, it is a small value. That is, when the tensile modulus of the laminated rubber is lower than the compressive elastic modulus, the tensile strain of the laminated rubber calculated by the STH method is evaluated to be small.

Figure 9. Time history of tensile strain of the laminated rubber bearing

Figure 10. Time history of tensile strain of laminated rubber bearing in each direction

4.2 Examination of bending moment of foundation beam

As in the previous section, the bending moment of the foundation beam in the step where the tensile strain of the laminated rubber became the maximum is compared. Figs. 11 and 12 show the bending moment M_e of the foundation beam calculated from the STH method of X 1 and Y 1 (Eq. (2)), and the bending moment M of the foundation beam at the time of horizontal and vertical simultaneous inputs. From Figs. 11 and 12, it is confirmed that when the tensile modulus of the laminated rubber is lower than the compressive elastic modulus (Type-1), the bending moment of the foundation beam cannot be evaluated by simultaneous horizontal and vertical inputs by the STH method. In particular, a large difference in bending moment is shown between the foundation beams Y1-Y2 and X1-X2 immediately above the laminated rubber in which tensile strain occurs. This is because the bending moment generated in the foundation beam owing to the redistribution of stress at the time of pulling out the laminated rubber cannot be considered in the STH method.

Figure 11. Maximum bending moment of the foundation beam at X1

Figure 12. Maximum bending moment of the foundation beam at Y1

5. ESTIMATION OF TENSILE STRAIN USING CONSTANT ENERGY LAW

5.1 Estimation method of tensile strain

From previous studies (Osawa Y. et al. 1961, Shibata A 2010), it is considered that in the short period, elastic and elasto-plastic systems are almost equal in the short-period region regardless of the yield strength (energy constant law, ECL). We herein propose a method for estimating tensile strain in Type - 1 to which the ECL is applied mutatis mutandis. The following estimation method will be described (Fig. 13). First, time history analysis in the horizontal direction and vertical directions are performed separately using Type-0, followed by analysis using $x, y \varepsilon_e^{(n)}$, $z \varepsilon_e^{(n)}$, and long-term distortion $z \varepsilon_0$ by the horizontal single input on Type-0. The tensile strain $\varepsilon_e^{(n)}$ can be obtained from Eq. (1). The tensile strain $\varepsilon_{eq}^{(n)}$ is set such that the elastic strain energy in the vertical direction of the laminated rubber for Type-0 and the elastic strain energy of Type-1 that is lower than the compression elastic modulus are equal, as shown in Fig. 14 (Eq. (3)).

$$\begin{cases} \varepsilon_{eq}^{(n)} = \varepsilon_e^{(n)} & (\varepsilon_e^{(n)} \leq 0) \\ \varepsilon_{eq}^{(n)} = \varepsilon_e^{(n)} \sqrt{\frac{1}{\alpha}} & (\varepsilon_e^{(n)} \geq 0) \end{cases} \quad (3)$$

Here, α is the rate of decrease in tensile elastic modulus to compressive modulus, and $\alpha = 1/50$ in Type-1 as in the previous section. Subsequently, the next step is performed and the same procedure is repeated.

Figure 12. Calculation flow of tensile strain in Type-1

Figure 13. Outline of energy constant law (ECL)

5.2 Study on tensile strain of laminated rubber

Fig. 15 shows the time history of strain $\varepsilon_{eq}^{(n)}$ of the laminated rubber calculated by the ECL method, and the strain $\varepsilon^{(n)}$ of the laminated rubber at the horizontal and vertical simultaneous inputs. As shown in Fig. 15, the distortion $\varepsilon_{eq}^{(n)}$ calculated by the ECL exhibits good correspondence with $\varepsilon^{(n)}$. Fig. 16 shows the relationship between the maximum value of tensile strain of the laminated rubber at the time of simultaneous horizontal and vertical inputs using Type-1 (ε_{max}), and the energy constant rule using Type-0 of the tensile strain ($\varepsilon_{eq, max}$) when the level of the input ground motion is changed. From Fig. 16, it is confirmed that $\varepsilon_{eq, max}$ exhibits good correspondence to ε_{max} at any earthquake motion and level. Further, it is confirmed that this method is suitable even when the tensile modulus of the laminated rubber is lower than the compressive elastic modulus.

Figure 15. Time history of tensile strain of the laminated rubber bearing

Figure 16. Comparison of maximum tensile strain of laminated rubber bearing

6. ESTIMATION OF BENDING MOMENT OF FOUNDATION BEAM USING FORCED DISPLACEMENT

6.1 Estimation method of bending moment of foundation beam

The procedure for estimating the bending moment of the foundation beam considering the stress redistribution at the time of withdrawing the laminated rubber is described below. First, the bending moment of the foundation beam for Type-0 in the maximum tensile strain generating step n' calculated from the STH method (M_e) shown in Eq. (2) is calculated. Next, using the total rubber thickness of the laminated rubber (h_R), the forced displacement (FD) Δu calculated from Eq. (4) is calculated.

$$\Delta u = (\varepsilon_{eq}^{(n')} - \varepsilon_e^{(n')}) h_R \quad (4)$$

Δu is given as the forced displacement to the node (Fig. 17) of the laminated rubber where extraction occurred. Finally, by adding M_f (the bending moment occurring on the foundation beam when Δu is given) to M_e , we estimate the bending moment of the foundation beam (M_{eq}) considering the stress redistribution at the time of withdrawing the laminated rubber (Eq.(5)).

$$M_{eq} = M_e + M_f \quad (5)$$

Hereinafter, it is known as the forced displacement method.

Fig. 18 shows the bending moment of the foundation beam M_f when forced displacement Δu calculated from the result at the time of Art Kobe input in Type-1 is imposed to position (X1 - Y1) of the laminated rubber where extraction occurred. From Fig. 18, it is confirmed that not only Y 2 and X 2 but also Y 3 and X 3 are influenced by forced displacement.

Figure 17. Outline of forced displacement (FD)

Figure 18. Bending moment of foundation beam (Art Kobe)

6.2 Examination of bending moment of foundation beam

Figs. 19 and 20 show the bending moment M_{eq} of the foundation beam calculated from the forced displacement method on the beams of X1 and Y1 and the bending moment M of the foundation beam at the time of simultaneous horizontal and vertical inputs. From Figs. 19 and 20, it is confirmed that the bending moment M_{eq} of the foundation beam calculated from the forced displacement method can be used to evaluate the bending moment generated in the foundation beam by the stress redistribution when the tensile strain of the laminated rubber occurs.

Particularly, even for the foundation beam directly above the laminated rubber where tensile strain cannot be evaluated by the STH method (Eq. (2)) occurred (Figs. 10 and 11), the forced displacement calculated from the ECL, Δu , can be evaluated by adding a bending moment when imposing it. Hence, it is shown that the bending moment of the foundation beam can be calculated by this method when the tensile modulus of the laminated rubber is lower than the compressive elastic modulus. However, herein, the response level at which the adjacent laminated rubber cannot be pulled out simultaneously is regarded as the application range of this method. A study on the simultaneous withdrawal of two or more laminated rubbers will be reported in another paper.

Figure 19. Maximum bending moment of the foundation beam at X1

Figure 20. Maximum bending moment of the foundation beam at Y1

7. CONCLUSIONS

We herein proposed a method for estimating the tensile strain of laminated rubber using the ECL when the tensile elastic modulus of laminated rubber decreases with compressive elastic modulus input horizontally and vertically in two seismic isolation buildings. Furthermore, a method for estimating the bending moment of the foundation beam considering the stress redistribution at the time of drawing was described. The findings obtained are as follows:

1. A method of adding two analysis results analyzed individually in the horizontal and vertical directions on the time history was verified. In this method, it was found that the tensile strain of the laminated rubber and the bending moment of the foundation beam could not be evaluated when the tensile elastic modulus of the laminated rubber was lower than the compressive elastic modulus.
2. Two time-history analyses were conducted in the horizontal and the vertical directions of a model in which the tensile elastic modulus and compressive elastic modulus were equal. The results indicated that the tensile strain could be approximated when the tensile elastic modulus of the laminated rubber was lower than the compressive elastic modulus, using the constant energy constant.
3. The bending moment of the foundation beam could be calculated by the sum of the bending moment calculated from the STH method and the bending moment of analysis that yielded the tensile strain calculated from the ECL as the forced displacement.

Using this method, investigation was enabled by setting an independent attenuation in the horizontal and the vertical directions. Even when the tensile modulus of elasticity was lower than the compressive modulus of elasticity, it was highly advantageous that the upper and lower responses could be studied by elastic analysis

with equal tensile elastic modulus and compressive elastic modulus. However, the laminated rubber proposed by Mori et al. (2015) was a nonlinear elastic restoring force characteristic with the tensile elastic modulus being 1/50 of the compressive elastic modulus. Furthermore, the response level at which adjacent laminated rubbers could not be pulled out simultaneously was regarded as the application range in the present method. The case of the bilinear type restoring force characteristic on the tensile side of the laminated rubber and that of changing the ratio of the tensile elastic modulus to the compressive elastic modulus will be reported in another paper. In addition, the discussion on the simultaneous withdrawal of two or more laminated rubbers will be reported in another paper.

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