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種別(和文)	論文要旨
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論 文 要 旨

THESIS SUMMARY

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要旨（英文 800 語程度）
Thesis Summary (approx.800 English Words)

This thesis concerns the optimization over the efficient set (OE) of a multiobjective linear programming problem and its applications. We propose that the optimal solutions to these problems can be obtained by solving an equivalent mixed integer programming (MIP) problem with a MIP solver.

1. Research Background

When making a decision in the real world, we usually need to consider several objectives simultaneously. Therefore, the multiobjective programming, which is to minimize all objective functions over the feasible region at one time, is proposed to deal with this situation. If a feasible solution to this problem satisfies that there is no other feasible solution that can improve all objective values simultaneously, then this solution is called an efficient solution. The set of efficient solutions is called the efficient set. There is a problem that we cannot compare different efficient solutions simply since there exists a trade-off between them. The OE problem, which is to minimize an additional objective function over the efficient set, is proposed to solve this situation.

The OE problem plays an important role in theoretical research in multiobjective programming and global optimization for the nonconvexity of the efficient set. The OE problem is called a milestone of difference-of-convex (DC) functions programming, which is one of the most important research themes of global optimization. Recently, Sun proposed a new approach to the OE problem, which transforms the OE problem into an MIP problem under some strong assumptions. Since the MIP has been well researched, it is promising for solving the OE problem efficiently with a state-of-the-art MIP solver. However, since one of the assumptions, which needs one of the objective functions is injective over the feasible region, is too strong, the application of this approach is limited.

2. Research Approach

In this thesis, we aim to propose a new MIP approach that can relax the strong assumption of Sun's approach and reduce the size of the MIP problem so that we can improve the computational performance and solve more application problems. By applying Wendell and Lee's Theorem 1, we define the efficient set as a linear complementarity system, which implies the OE problem is a mathematical program with equilibrium constraints. Then we transform the problem into an equivalent MIP problem and compute an optimal solution with an MIP solver. Theoretically, compared with the previous MIP approach by Sun, the proposed approach relaxes the strong assumption and reduces the numbers of constraints and binary variables of the MIP problem. By conducting the numerical experiments, we show that the proposed approach is faster than the previous MIP approach.

3. Applications

There are many applications of the OE problem in the real world. In this thesis, we consider the minimum maximal flow (MMF) problem and the least distance problem (LDP).

3.1. The Minimum Maximal Flow Problem

First, we propose a new approach to solve the MMF problem, which is a network flow problem to find the maximal flow with the minimum flow value.

We give an upper bound for this problem and prove this problem satisfies the assumption of our approach. Since the MMF problem is known to be an OE problem, the proposed MIP approach can be applied.

Based on them, an equivalent simplified MIP problem of the MMF problem is proposed to compute the MMF problem with the MIP solver. By performing computational experiments, we find that: The previous MIP approach can solve 3% of test problems, while the proposed approach's accuracy is 100%; The proposed approach is faster than previous researches mostly. The growth rate of the running time of our approach

is slower than the rates of previous works when the sizes of the instances grow; The proposed approach is efficient to the MMF problem even for relatively large instances, where the number of edges is up to 15,000.

3.2. The Least Distance Problem

Then, we show that the LDP in data envelopment analysis (DEA) can be modeled as an OE problem. DEA is an empirical method to measure relative productive efficiency of decision making units (DMU). Here, a DMU is an enterprise or organization which uses some input to produce some output. Basically, it compares the assessed DMU with an efficient point which cannot be improved without increasing some input or reducing output. The traditional DEA models measure efficiency without considering the difficulty of improving efficiency. The LDP model, which is to find the closest efficient point from assessed DMU, is considered to deal with this point. However, because of the nonconvexity of the feasible region, it is difficult to find a global optimal solution. In this thesis, we transform the LDP into an OE problem by defining the efficient set with a multiobjective programming problem. Then, the proposed MIP approach can apply to the LDP. Compared with previous research, this approach can obtain the closest efficient target over the whole efficient frontier instead of part of the frontier under the variable returns-to-scale assumptions with a theoretical guarantee.