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Title(English)	Non-Stoquastic Effects on Phase Transitions in Spin Systems for Quantum Annealing
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種別(和文)	論文要旨
Type(English)	Summary

（博士課程）
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論 文 要 旨

THESIS SUMMARY

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学生氏名： Student's Name	高田　珠武己	指導教員（主）： Academic Supervisor(main)	西森　秀稔
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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

Quantum annealing is a metaheuristic, a generic approximate algorithm, for solving a combinatorial optimization problem, i.e., the problem of determining the arguments which minimize a function of discrete variables. In quantum annealing, we control the quantum Ising model in a time-dependent manner to obtain the ground state of the Ising Hamiltonian which encodes the solution to a given combinatorial optimization problem. Traditionally, a transverse field has been used as a quantum driver term appended to the Ising Hamiltonian.

The presence of a first-order phase transition is a bottleneck of quantum annealing, because it makes the required annealing time exponentially long as a function of the system size. One of the possible candidates for avoidance of first-order transitions is introduction of non-stoquasticity. Here, a Hamiltonian whose every off-diagonal matrix element is real and non-positive is called to be stoquastic, while a non-stoquastic Hamiltonian has a positive or imaginary off-diagonal element. Although non-stoquastic Hamiltonians are known to be difficult to classically simulate due to the sign problem, it does not necessarily mean a speedup of quantum annealing. In this thesis, we study three quantum Ising models to reveal effects of non-stoquastic interactions on phase transitions and to compare them with those of stoquastic interactions and inhomogeneity in the transverse field.

First, we investigate the mean-field weak-strong cluster model. This model is a two-body interacting quantum Ising model which consists of the weak cluster and the strong cluster. There are longitudinal fields which have opposite directions in the two clusters, and all-to-all ferromagnetic interactions. The strength of the longitudinal field in the weak cluster is smaller than that in the strong cluster. This structure of the problem causes a first-order transition when the uniform transverse-field driver is applied. Analyzing the model by a semi-classical method, we show that the first-order transition disappears if a non-stoquastic transverse interaction is appended between the clusters with an appropriate strength, while the transition persists if the transverse interaction is stoquastic. We also calculate the energy gap between the ground and first excited states in the thermodynamic limit and determine the optimal strength of the non-stoquastic transverse interaction between the clusters which maximizes the minimum energy gap in quantum annealing. Furthermore, we find that the removal of the first-order transition is achieved by a non-stoquastic transverse interaction within the strong cluster, a stoquastic transverse interaction within the weak cluster, or inhomogeneous transverse-field driving in which the transverse field in the weak cluster is stronger than that in the strong cluster.

Second, we analyze the weak-strong cluster model which has dense interactions within each cluster and sparse interactions between clusters. We evaluate the partition function in the thermodynamic limit and the zero-temperature limit analytically by the path-integral technique and the saddle-point method. Then, we show that a non-stoquastic or stoquastic transverse interaction and inhomogeneous transverse-field driving can remove the first-order transition if appropriately implemented. The behavior of the system generally resembles that in the previous case with all-to-all interactions, except that the first-order transition can be eliminated by a stoquastic transverse interaction between the clusters as well as a non-stoquastic one.

Third, we study the frustrated Ising ladder in a transverse field with transverse interactions. This model is a quasi-one-dimensional system and has only spatially local two-body interactions and magnetic fields, which indicates that the model is much closer to experimental realization than the former two models. It is known in the case without the transverse interactions that a first-order transition which originates in the topological nature of the model occurs as the transverse field is varied with the strength of the frustration fixed to a large value. To explore the effects of non-stoquastic interactions, we analyze the model with the transverse interactions by the real-space renormalization group method in which the projector onto a coarse-grained space is variationally determined. The resulting phase diagram is in qualitative agreement with numerical methods and shows that neither non-stoquastic nor stoquastic transverse interactions remove the first-order transition in the presence of the large frustration. Nevertheless, we make a rough prediction that the driver with the non-stoquastic transverse interactions might have a slight advantage in quantum annealing over the stoquastic drivers, in the sense that the former possibly reduces the exponential decay rate of the energy gap at the first-order transition as a function of the system size.

Although it is generally difficult to choose the optimal strength and position of a non-stoquastic interaction to enhance the performance of quantum annealing for a given real-world optimization problem, our results imply that introduction of non-stoquastic transverse interactions with tunable strengths as well as stoquastic transverse interactions and an inhomogeneous transverse field is likely to be useful in the design of hardware of quantum annealing. To better understand the non-stoquastic effects, analytical and numerical studies of many other problems are highly desired, especially in the cases of sparse connectivity to represent realistic situations.