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Disturbance rejection and performance analysis for nonlinear systems based on nonlinear equivalent-input-disturbance approach

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Abstract This paper presents a nonlinear equivalent-input-disturbance (NEID) approach to rejecting an unknown exogenous disturbance in a nonlinear system. An NEID compensator has two parts: A conventional equivalent-input-disturbance (EID) estimator and a nonlinear state-feedback term. This design ensures that only the exogenous disturbance is rejected and the useful nonlinearity of the system is retained. Unlike other active disturbance-rejection methods, a Lipschitz condition is not necessary to guarantee the convergence of the observation error. Analysis of control performance provides upper bounds for the evaluation of disturbance-rejection and the degree of nonlinearity retention. Numerical examples show the validity and superiority of this method.

Keywords Nonlinear system · exogenous-disturbance estimation · performance analysis · equivalent input disturbance · local uniformly boundedness.

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1 Introduction

Nonlinear phenomena, such as chaos [1] and bifurcation [2], are common in the real world. While they are complicated, they are important for system behavior, for example, a decrease in cardiac chaos might indicate congestive heart failure [3], chaos in liquid mixing was used to improve mixing efficiency [4], and the idea of anticontrol of chaos was applied to keep a human brain away from a saddle-type equilibrium [5]. Control of nonlinear behavior have received numerous attention, and a considerable number of studies has been made on this topic [5–7]. However, none of them considered the effect of an exogenous disturbance. A disturbance in a nonlinear system significantly influences the behavior of the system [8]. It is more practical to consider the problem of nonlinear control with disturbances. Since nonlinearities, which are the source causing nonlinear phenomena, need to be preserved to feature a system in many control practice, the difficulty of this problem is how to reject an unknown exogenous disturbance while retaining nonlinearities.

Regarding disturbance rejection in a nonlinear system, a series of nonlinear disturbance observers (N-

DOBs) were proposed to estimate and suppress kinds of disturbances. An NDOB was first used to estimate a torque disturbance in a nonlinear robotic manipulator [9]. The error of disturbance estimation was proven to exponentially converge to zero based on Lyapunov stability theory. However, only a constant disturbance was dealt with [9]. Then, a harmonic NDOB were devised for a harmonic disturbance [10], and a high-order disturbance was developed for a general case [11]. The problem with the NDOBs is that they require that the system state is measurable [29]. This may be difficult in control practice. In addition, a priori information, such as the frequency of a disturbance [10], is required in system design, but it is usually not available or may change according to operating conditions.

Other methods of rejecting disturbances in nonlinear systems have also been presented. The sliding-mode control [12,13] and neural networks [14,15] were used to compensate for nonlinearities, exogenous disturbances, and system uncertainties. An adaptive fast finite-time control [16], a mixed H_2/H_∞ fuzzy output feedback control [17], and a robust self-triggered model predictive control algorithm [18] were presented to reduce the influence of nonlinearities and exogenous disturbances, and thus improve control performance. These methods are effective to suppress the effect of nonlinearities and exogenous disturbances. However, they cannot be used to deal with the problem considered in this paper because they also compensate for the effect of nonlinearities.

On the other hand, many methods have been devised for disturbance rejection for a linear system. Among them, since active disturbance rejection methods, such as the disturbance observer (DOB) [19,20], the active disturbance rejection control (ADRC) [21–23], and the equivalent-input-disturbance (EID) approach [24, 25], exhibit better disturbance-rejection performance than conventional one-degree-of-freedom methods (for example, adaptive control [26], predictive control [27], and internal model principle [28]), they have widely been investigated theoretically and used in control engineering [29]. Note that these active disturbance-rejection methods cannot be directly used for the disturbance rejection of such a kind of a nonlinear system considered in this study because they produce a compensation amount for not only disturbances but also nonlinearities. This completely changes the characteristics of a nonlinear system and is not desirable in many applications.

Considering that, unlike the DOB method, the EID approach does not require an inverse model of a plant or any a prior information on disturbances, and unlike the ADRC, the system configuration is simple, this paper extends the EID approach to deal with the disturbance-rejection problem for a nonlinear system and presents a nonlinear EID (NEID) approach to rejecting only an exogenous disturbance and retaining the inherent nonlinearities of a system. An NEID compensator has two parts: a nonlinear term and an EID compensator. The nonlinear term keeps the nonlinearities of the system while the NEID compensator compensated solely for the disturbance. The advantages of this method over others are as follows:

(1) A state observer is used to estimate the state of a nonlinear system, but a Lipschitz condition is not necessary to guarantee the convergence of the observation error.
(2) No a priori information of a disturbance is needed.
(3) While the mechanism is simple, it is effective in rejecting disturbances and retaining nonlinearities.
(4) The stability of the system is guaranteed by the stability of the linear part of a nonlinear system.
(5) Upper bounds for the evaluation of disturbance-rejection and the degree of nonlinearity retention are analyzed and used to show the disturbance-rejection performance of an NEID estimator.

In this paper, for a vector $x(t) = \left[x_1(t) \cdots x_n(t)\right]^{\mathrm{T}}$, $\|x\|_2 := \sqrt{\int_0^\infty x^{\mathrm{T}}(t)x(t)\mathrm{d}t}$, $\|x(t)\|_2 := \sqrt{x^{\mathrm{T}}(t)x(t)}$, and $\|x_i\|_\infty := \sup_t |x_i(t)|$. For any a function $g(t)$, $\|g\|_1 := \int_{-\infty}^{+\infty} |g(\tau)|\mathrm{d}\tau$. For a complex number z, Re z is the real part of z. And for a square matrix A ($\in \mathbb{R}^{n\times n}$), $\lambda_i(A)$ $(i = 1, 2, \ldots, n)$ is its eigenvalues, and $\lambda_{\min}(A)$ is the minimum one and $\lambda_{\max}(A)$ is the maximum one.

2 Configuration of NEID-based disturbance-rejection system

Consider a nonlinear plant

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = Ax(t) + Bf(x(t)) + Bu(t) + Bd(t), & \text{(1a)} \\ y(t) = Cx(t), & \text{(1b)} \end{cases}$$

where $x(t)$ ($\in \mathbb{R}^n$) is the state; $u(t)$ ($\in \mathbb{R}^{n_u}$) is the control input; $y(t)$ ($\in \mathbb{R}^{n_y}$) is the output; $d(t)$ ($\in \mathbb{R}^{n_u}$) is an unknown exogenous disturbance; $f(x(t))$ ($\in \mathbb{R}^{n_u}$) is a state-dependent nonlinearity; and A, B, and C are constant matrices with suitable dimensions. Note that $f(x(t))$ and $u(t)$ are in the same channel.

For convenience, this paper discusses only a single-input, single-output (SISO) plant, which means that $n_u = 1$ and $n_y = 1$. However, the result can easily be extended to a multiple-input, multiple-output (MIMO) one.

The following assumptions were made in this study:

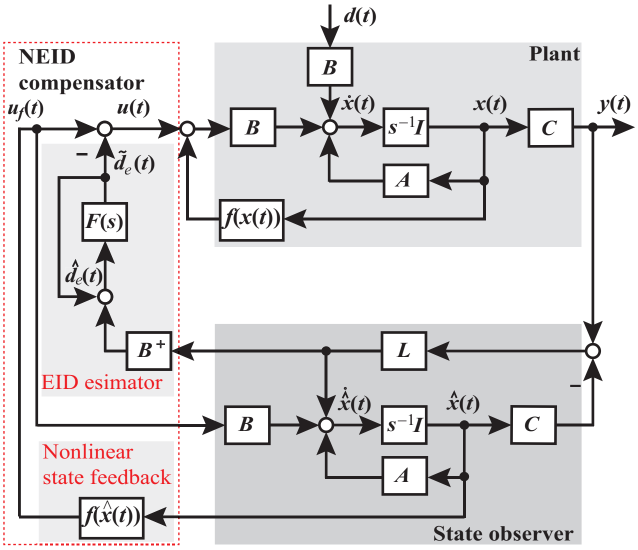

Fig. 1 Configuration of NEID-based disturbance-rejection system.

Assumption 1 *(A, B, C) is controllable and observable.*

Assumption 2 *The disturbance, $d(t)$, satisfies*

$$\|d\|_\infty \leq d_M, \tag{2}$$

where d_M is a positive number.

Assumption 3 *$f(x)$ is a continuous smooth function satisfying*

$$\frac{\|f\|_2}{\|x\|_2} \to 0 \text{ as } \|x\|_2 \to 0. \tag{3}$$

Remark 1 Assumption 3 is used to derive a stability condition for the NEID-based disturbance-rejection system. If nonlinearities do not satisfy Assumption 3 but satisfy

$$\|f\|_\infty \leq \varphi, \tag{4}$$

where φ is a positive constant, the NEID approach is also effective (see the second example in Numerical verification) and the stability is analyzed based on the concept of global uniformly boundedness (see Appendix).

An NEID-based disturbance-rejection system (Fig. 1) contains three parts: The plant, a state observer, and an NEID compensator. The NEID compensator has two parts: An EID estimator and nonlinear state feedback. $F(s)$ in the EID estimator is a low-pass filter, and

$$B^+ = \left(B^{\mathrm{T}}B\right)^{-1} B^{\mathrm{T}}. \tag{5}$$

The improved control input (Fig 1)

$$u(t) = u_f(t) - \tilde{d}_e(t) \tag{6}$$

is used in the NEID compensator.

According to the explanations given by Gao et al. [30], there is an EID, $d_e(t)$, on the control input channel that produces the same effect as the disturbance and the nonlinearity on the output. So, the EID-based system description of the plant is

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = Ax(t) + B[u(t) + d_e(t)], \\ y(t) = Cx(t), \end{cases} \tag{7}$$

where

$$d_e(t) = f(x(t)) + d(t). \tag{8}$$

A full-order observer

$$\begin{cases} \dfrac{\mathrm{d}\hat{x}(t)}{\mathrm{d}t} = A\hat{x}(t) + Bu_f(t) + L[y(t) - C\hat{x}(t)], & \text{(9a)} \\ \hat{y}(t) = C\hat{x}(t) & \text{(9b)} \end{cases}$$

is used to estimate the EID. $\hat{x}(t)$ in (9) is a reconstructed state of $x(t)$.

Define

$$\Delta x(t) = \hat{x}(t) - x(t). \tag{10}$$

An estimate of the EID is (Gao et al. [30])

$$\hat{d}_e(t) = B^+ LC\Delta x(t) + u_f(t) - u(t). \tag{11}$$

Subtracting (1a) from (9a) gives

$$\begin{aligned} \frac{\mathrm{d}\Delta x(t)}{\mathrm{d}t} =& (A - LC)\Delta x(t) + Bu_f(t) - Bu(t) \\ & - Bf(x(t)) - Bd(t). \end{aligned} \tag{12}$$

This paper chooses

$$u_f(t) = f(\hat{x}(t)) \tag{13}$$

to retrieve the nonlinearity of the system and to ensure that only $d(t)$ is rejected by the NEID compensator.

A low-pass filter $F(s)$

$$\begin{cases} \dfrac{\mathrm{d}x_F(t)}{\mathrm{d}t} = A_F x_F(t) + B_F \hat{d}_e(t), & \text{(14a)} \\ \tilde{d}_e(t) = C_F x_F(t) & \text{(14b)} \end{cases}$$

is used to select an angular frequency bandwidth for disturbance estimation. It is selected to satisfy

$$F(j\omega) \approx I, \ \forall \omega \in [0, \omega_r], \tag{15}$$

where ω_r is the highest angular frequency for disturbance estimation.

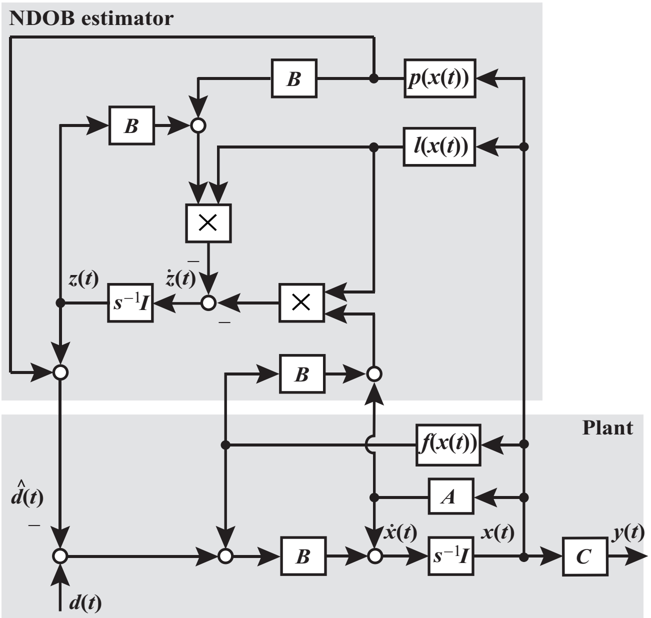

Fig. 2 Configuration of NDOB-based disturbance-rejection system.

Remark 2 The NEID approach is different from the conventional EID approach. $u_f(t)$ is designed in a general way and rejects both $d(t)$ and $f(x(t))$ for the conventional one, but it is chosen to be (13) and rejects only $d(t)$ for the NEID approach. This choice plays a key role and adroitly solves the problem of retaining the characteristics of a nonlinear system.

Next, an NDOB for (1) is [29]

$$\begin{cases} \dfrac{\mathrm{d}z(t)}{\mathrm{d}t} = -l(x(t))Bz(t) - l(x(t))Bp(x(t)) & \text{(16a)} \\ \qquad -l(x(t))\left[Ax(t) + Bf(x(t)) + Bu(t)\right], \\ \hat{d}_D(t) = z(t) + p(x(t)), & \text{(16b)} \end{cases}$$

where $z(t)$ is an auxiliary variable vector, $\hat{d}_D(t)$ is a disturbance estimate, $p(x(t))$ is a function vector for the design and it is chosen to satisfy

$$l(x) = \frac{\partial p(x)}{\partial x}. \tag{17}$$

The NDOB-based disturbance-rejection system (Fig. 2) has two parts: The plant and an NDOB estimator.

Define the error of the disturbance estimation to be

$$e_D(t) = \hat{d}_D(t) - d(t). \tag{18}$$

Then, the dynamics of the disturbance estimation error is

$$\begin{aligned} \dot{e}_D(t) &= \dot{\hat{d}}_D(t) - \dot{d}(t) \\ &= \dot{z}(t) + \frac{\partial p(x)}{\partial x}\dot{x}(t) - \dot{d}(t) \\ &= -l(x(t))Be_D(t) - \dot{d}(t). \end{aligned} \tag{19}$$

If

$$\dot{d}(t) = 0, \tag{20}$$

the NDOB is effective to estimate constant disturbances for a suitable $l(x(t))$.

There are two main differences between the NDOB method and the NEID approach:

1) The disturbance-rejection mechanism is different. The NDOB uses the nonlinear dynamics of the disturbance estimation error to construct a disturbance observer. Moreover, it requires that the system state is measurable. However, the NEID approach first constructs a state observer and then it elaborates the information of the observer to estimate a disturbance.
2) The NDOB method requires $\dot{d}(t) = 0$ to ensure the convergence of the disturbance estimation error, which means that it is only effective to estimate and compensate for constant disturbances in theory. But the NEID approach does not need the requirement. Thus, it is effective to estimate and compensate for not only constant disturbances but also time-varying disturbances.

According to the above explanation, it is known that the application range is wider for the NEID approach than for the NDOB method.

It is not easy to select a nonlinear function, $p(x(t))$, in an NDOB estimator [9]. For a time-varying disturbance, the NDOB method needs the frequency of the disturbance to design a disturbance compensator [10]. Thus, the problem of rejecting a time-varying disturbance in a nonlinear system is much easier to be handled for the NEID approach than for the NDOB method.

3 Stability of NEID-based disturbance-rejection system

This section shows the local uniformly boundedness of the NEID-based disturbance-rejection system. The following definitions and lemmas are employed in the stability analysis.

Definition 1 (Definition 4.2 [32]) A continuous function $\alpha : [0, a) \to [0, \infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$, and belongs to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$.

Definition 2 (Definition 4.3 [32]) A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ belongs to class $\mathcal{KL}$ if the following conditions hold: 1) for each fixed q, the mapping $\beta(p, q)$ belongs to class $\mathcal{K}$ with respect to p, 2) for each fixed p, the mapping $\beta(p, q)$ decreases with respect to q, and $\beta(p, q) \to 0$ as $q \to \infty$.

Definition 3 (Definition 4.6 [32]) The solutions of an autonomous system

$$\frac{\mathrm{d}x}{\mathrm{d}t} = h(x) \tag{21}$$

are uniformly bounded if there exists a positive constant c that is independent of t_0 (≥ 0), and for every $a \in (0, c)$, there is $\beta = \beta(a) > 0$ that is independent of t_0, such that

$$\|x(t_0)\|_2 \leq a \ \Rightarrow\ \|x(t)\|_2 \leq \beta,\ \forall t \geq t_0. \tag{22}$$

Lemma 1 (Theorem 4.7 [32]) *Let $x = 0$ be an equilibrium point of the system (21), where $h : D \to \mathbb{R}^m$ is continuously differentiable and D is a neighborhood of the origin and defined as $D = \{x \in \mathbb{R}^m | \ \|x\|_2 < r\}$ (r is a positive constant). Let*

$$M = \frac{\mathrm{d}h(x)}{\mathrm{d}x}\Big|_{x=0}. \tag{23}$$

Then,

(1) the origin is asymptotically stable if $\mathrm{Re}\ \lambda_i < 0$ *for all eigenvalues of M,*
(2) the origin is unstable if $\mathrm{Re}\ \lambda_i > 0$ *for one or more of the eigenvalues of M.*

Lemma 2 (Corollary 4.3 [32]) *Let $x = 0$ be an equilibrium point of the system (21), where $h(x)$ is continuously differentiable in a neighborhood of $x = 0$. Then, $x = 0$ is an exponentially stable equilibrium point of the system (21) if and only if (23) is Hurwize.*

Lemma 3 (Theorem 4.14 [32]) *Let $x = 0$ be an equilibrium point of a nonlinear system*

$$\frac{\mathrm{d}x}{\mathrm{d}t} = h(t, x), \tag{24}$$

where $h : [0, \infty) \times D \to \mathbb{R}^m$ is continuously differentiable, a neighborhood of the origin D, and the Jacobian matrix $\dfrac{\partial h(t, x)}{\partial x}$ is bounded on D and uniformly in t. Let κ, λ^*, and r_0 be positive constants with $r_0 < \dfrac{r}{\kappa}$. Let $D_0 = \{x \in \mathbb{R}^m | \ \|x\|_2 < r_0\}$. Assume that the trajectories of the system satisfy

$$\begin{cases} \|x(t)\|_2 \leq \kappa \|x(t_0)\|_2 e^{-\lambda^*(t-t_0)}, \\ \forall x(t_0) \in D_0,\ \forall t \geq t_0 \geq 0. \end{cases} \tag{25}$$

Then, there exists a function $V : [0, \infty) \times D_0 \to \mathbb{R}$ that satisfies

$$\begin{cases} c_1\|x\|_2^2 \leq V(t, x) \leq c_2\|x\|_2^2, \\ \dfrac{\partial V(t, x)}{\partial t} + \dfrac{\partial V(t, x)}{\partial x} h(t, x) \leq -c_3\|x\|_2^2, \\ \left\| \dfrac{\partial V}{\partial x} \right\|_2 \leq c_4\|x\|_2 \end{cases} \tag{26}$$

for positive constants c_1, c_2, c_3, and c_4. Moreover, if $r = \infty$, then the origin is globally exponentially stable and $V(t, x)$ is well defined and satisfies (26) globally. Furthermore, if the system is autonomous, $V(t, x)$ can be chosen independent of t.

Lemma 4 (Theorem 4.18 [32]) *Let $V : [0, \infty) \times D \to \mathbb{R}$ be a continuously differentiable function such that*

$$\begin{cases} \alpha_1(\|x\|_2) \leq V(t, x) \leq \alpha_2(\|x\|_2), \\ \dfrac{\partial V(t, x)}{\partial t} + \dfrac{\partial V(t, x)}{\partial x} h(t, x) \leq -W_3(x), \\ \forall \|x\|_2 \geq \mu > 0,\ \forall\ t \geq 0,\ \forall\ x \subset D, \end{cases} \tag{27}$$

where α_1 and α_2 are class $\mathcal{K}$ functions and $W_3(x)$ is a continuous positive-definite function. Take $r > 0$ such that $B_r \subset D$ and suppose that

$$\mu < \alpha_2^{-1}(\alpha_1(r)). \tag{28}$$

Then, there exists a class $\mathcal{KL}$ function β and, for every initial state $x(t_0)$ that satisfies $\|x(t_0)\|_2 \leq \alpha_2^{-1}(\alpha_1(r))$, there is T (> 0) [dependent on $x(t_0)$] and μ such that the solution of (24) satisfies

$$\begin{cases} \|x(t)\|_2 \leq \beta(\|x(t_0)\|_2, t - t_0),\ \forall t_0 \leq t \leq t_0 + T, \\ \|x(t)\|_2 \leq \alpha_1^{-1}(\alpha_2(\mu)),\ \forall t \geq t_0 + T, \end{cases} \tag{29}$$

which means that the system (24) is uniformly bounded. Moreover, if $D = \mathbb{R}^m$ and α_1 belongs to class $\mathcal{K}_\infty$, then (29) hold for any initial state $x(t_0)$ with no restriction on how large μ is.

If Assumptions 2 and 3 are satisfied, the lumped disturbance including $f(x(t))$ has influences on the stability of the NEID-based disturbance-rejection system. The local uniformly boundedness of the NEID-based disturbance-rejection system is analyzed below. Combing (1), (6), (12), (13), and (14) yields

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = Ax(t) + Bf(x(t)) - BC_F x_F(t) \\ \qquad\qquad +Bd(t) + Bf(\hat{x}(t)), & \text{(30a)} \\ \dfrac{\mathrm{d}\Delta x(t)}{\mathrm{d}t} = (A - LC)\Delta x(t) + BC_F x_F(t) \\ \qquad\qquad -Bf(x(t)) - Bd(t), & \text{(30b)} \\ \dfrac{\mathrm{d}x_F(t)}{\mathrm{d}t} = -B_F B^+ LC\Delta x(t) \\ \qquad\qquad +(A_F + B_F C_F)x_F(t). & \text{(30c)} \end{cases}$$

Define the state of the disturbance-rejection system (Fig. 1) to be

$$\xi(t) = [x^{\mathrm{T}}(t)\ \ \Delta x^{\mathrm{T}}(t)\ \ x_F(t)]^{\mathrm{T}}. \tag{31}$$

Thus,

$$\frac{\mathrm{d}\xi(t)}{\mathrm{d}t} = \bar{A}\xi(t) + \bar{B}f(\hat{x}(t)) + \bar{B}_b f(x(t)) + \bar{B}_d d(t), \tag{32}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 & -BC_F \\ 0 & A - LC & BC_F \\ 0 & -B_F B^{+} LC & A_F + B_F C_F \end{bmatrix}, \tag{33}$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix},\ \bar{B}_b = \begin{bmatrix} B \\ -B \\ 0 \end{bmatrix},\ \bar{B}_d = \begin{bmatrix} 0 \\ -B \\ 0 \end{bmatrix}. \tag{34}$$

Letting

$$\sigma(t) = \bar{B}_d d(t) \tag{35}$$

and

$$\begin{aligned} \eta(\xi(t)) &= \bar{B}f(\hat{x}(t)) + \bar{B}_b f(x(t)) \\ &= \begin{bmatrix} Bf(x(t)) + Bf(\hat{x}(t)) \\ -Bf(x(t)) \\ 0 \end{bmatrix} \end{aligned} \tag{36}$$

gives

$$\frac{\mathrm{d}\xi(t)}{\mathrm{d}t} = \bar{A}\xi(t) + \eta(\xi(t)) + \sigma(t). \tag{37}$$

The stability analysis of the system (37) is divided into two steps. First, show the asymptotically stable of the system when $d(t) = 0$. Then, show that the system is uniformly bounded for $d(t) \neq 0$.

First, let $d(t) = 0$. Then (37) becomes

$$\frac{\mathrm{d}\xi(t)}{\mathrm{d}t} = g(\xi(t)) = \bar{A}\xi(t) + \eta(\xi(t)). \tag{38}$$

It is clear that the origin is an equilibrium point of (38). According to Lemma 1, stability of the origin can be characterized by the locations of the eigenvalues of the matrix $\bar{A}$. Thus, if $\bar{A}$ is Hurwitz, then, for any positive-definite symmetric matrix Q, the solution, P, of

$$P\bar{A} + \bar{A}^{\mathrm{T}}P = -Q \tag{39}$$

is positive definite. Let

$$V_1(\xi(t)) = \xi^{\mathrm{T}}(t)P\xi(t) \tag{40}$$

be a Lyapunov function candidate of (38). The derivative of $V_1(\xi(t))$ along the trajectories of (38) is given by

$$\frac{\mathrm{d}V_1(\xi(t))}{\mathrm{d}t} = -\xi^{\mathrm{T}}(t)Q\xi(t) + 2\xi^{\mathrm{T}}(t)P\eta(\xi(t)). \tag{41}$$

The first term on the right side of (41) is negative definite while the second term is indefinite. According to Assumption 3, the function $\eta(\xi(t))$ satisfies

$$\frac{\|\eta\|_2}{\|\xi\|_2} \to 0 \text{ as } \|\xi\|_2 \to 0. \tag{42}$$

Therefore, for any $\gamma > 0$, there exists $\nu > 0$ such that

$$\|\eta\|_2 < \gamma\|\xi\|_2,\ \forall\|\xi\|_2 < \nu. \tag{43}$$

Hence,

$$\frac{\mathrm{d}V_1(\xi(t))}{\mathrm{d}t} < -\xi^{\mathrm{T}}(t)Q\xi(t) + 2\gamma\|P\|_2\|\xi\|_2^2,\ \forall\|\xi\|_2 < \nu. \tag{44}$$

However, $\xi^{\mathrm{T}}(t)Q\xi(t) \geq \lambda_{\min}(Q)\|\xi\|_2^2$. Note that $\lambda_{\min}(Q)$ is real and positive because Q is symmetric and positive definite. Thus,

$$\frac{\mathrm{d}V_1(\xi(t))}{\mathrm{d}t} < -\left[\lambda_{\min}(Q) - 2\gamma\|P\|_2\right]\|\xi\|_2^2,\ \forall\|\xi\|_2 < \nu. \tag{45}$$

Choosing

$$\gamma < \frac{\lambda_{\min}(Q)}{2\|P\|_2} \tag{46}$$

ensures that $\mathrm{d}V_1(\xi(t))/\mathrm{d}t$ is negative definite. Thus, if $\bar{A}$ is Hurwitz then the origin of (38) is asymptotically stable.

Now, we are ready to analyze the local boundedness of the system for $d(t) \neq 0$.

The origin of (38) is exponentially stable based on Lemma 2. Thus, there exist constants κ and λ^* such that

$$\begin{cases} \|\xi(t)\|_2 \leq \kappa\|\xi(t_0)\|_2 e^{-\lambda^*(t-t_0)}, \\ \forall\|\xi(t_0)\|_2 < \nu,\ \forall t \geq t_0 \geq 0. \end{cases} \tag{47}$$

Choosing

$$\nu_0 < \min\{\nu,\ \nu/\kappa\}, \tag{48}$$

all the conditions of Lemma 3 are satisfied: (47) corresponds to (25) and (48) corresponds to r_0. Thus, there is a Lyapunov function $V_2(\xi(t))$ of (38) for $\|\xi\|_2 < \nu_0$ that satisfies

$$\begin{cases} c_1\|\xi\|_2^2 \leq V_2(\xi(t)) \leq c_2\|\xi\|_2^2, \\ \dfrac{\mathrm{d}V_2(\xi)}{\mathrm{d}\xi} g(\xi(t)) \leq -c_3\|\xi\|_2^2, \\ \left\|\dfrac{\mathrm{d}V_2}{\mathrm{d}\xi}\right\|_2 \leq c_4\|\xi\|_2. \end{cases} \tag{49}$$

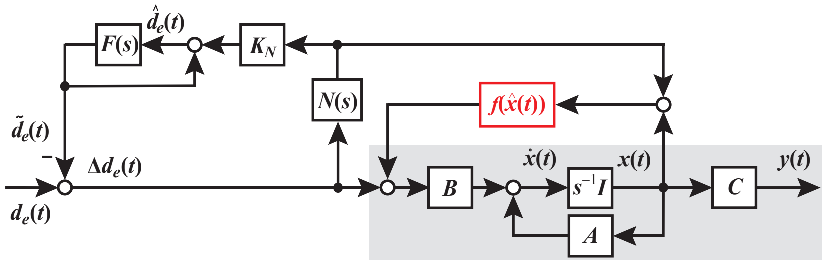

Fig. 3 Equivalent description of Fig. 1.

Let $V_2(\xi(t))$ be a Lyapunov function candidate of (37). The derivative of $V_2(\xi(t))$ along the trajectories of (37) is given by

$$\begin{aligned}\frac{\mathrm{d}V_2(\xi(t))}{\mathrm{d}t} =&\frac{\partial V_2}{\partial \xi}\left[g(\xi(t))+\sigma(t)\right]\\ \leq&(-c_3\|\xi(t)\|_2+c_4\|\bar{B}\|_2 d_M)\|\xi(t)\|_2\\ \leq&c_3(-\|\xi(t)\|_2+r)\|\xi(t)\|_2,\end{aligned} \tag{50}$$

where

$$r=\frac{c_4\|\bar{B}\|_2 d_M}{c_3}. \tag{51}$$

If

$$\mu<\frac{c_1 r}{c_2}, \tag{52}$$

conditions in Lemma 4 are satisfied: (49) and (52) correspond to (27) and (28), respectively. Thus,

$$\|\xi(t)\|_2\leq r,\ \forall t\geq t_0. \tag{53}$$

According to Definition 3, (37) is local uniformly boundedness. Thus, the following theorem is obtained.

Theorem 1 *The system (37) is local uniformly boundedness for (53) if (52) is satisfied and $\bar{A}$ is Hurwitz.*

If (52) is not satisfied, we can tune $\bar{A}$ to increase ν and finally ensure that (52) is satisfied.

Remark 3 This paper presented the stability analysis of the NEID-based disturbance-rejection system for a class of nonlinearities satisfying Assumption 3. For such a class of nonlinearities, the stability-analysis method presented in the original EID approach [24] and the linear-matrix-inequality-based approach used in [30] can hardly be extended to obtain a simple stability condition for the system design. Thus, in this paper, we derived the stability conditions based on the concept of local uniformly boundedness.

4 Performance analysis for NEID-based disturbance-rejection system

Since the nonlinear term is bounded for a stable NEID-based disturbance-rejection system, it is reasonable to assume that

$$\|f\|_\infty\leq f_M,\ \forall\|\xi\|_2\leq r, \tag{54}$$

where f_M is a positive constant. Thus,

$$\begin{aligned}\|d_e\|_\infty&=\|f+d\|_\infty\leq\|f\|_\infty+\|d\|_\infty\\&\leq f_M+d_M.\end{aligned} \tag{55}$$

Using $\Delta x(t)$ to describe the dynamics (9) [31] and redrawing Fig. 1 yield Fig. 3, in which

$$K_N=-B^+LC,\ N(s)=-[sI-(A-LC)]^{-1}B. \tag{56}$$

The figure clearly illustrates the feature of the NEID estimator: It provides us a new degree of freedom to add a compensation amount directly to the control input channel to compensate for the EID. The bound analysis is carried out by making use of this feature.

Define

$$\Delta d_e(t)=\tilde{d}_e(t)-d_e(t). \tag{57}$$

Let $d_e(t)$ be the input and $\Delta d_e(t)$ be the output. The transfer function from $d_e(t)$ to $\Delta d_e(t)$ is (Fig. 3)

$$G_{\Delta d}(s)=[I-F(s)G_L(s)]^{-1}[I-F(s)], \tag{58}$$

where

$$G_L(s)=I-K_N N(s). \tag{59}$$

Thus,

$$\Delta d_e(t)=g_{\Delta d}(t)*d_e(t),\ g_{\Delta d}(t)=\ell^{-1}[G_{\Delta d}(s)], \tag{60}$$

where $g_{\Delta d}(t)$ is the inverse Laplace transform of $G_{\Delta d}(s)$, $*$ is the convolution operator, and ℓ^{-1} is the inverse Laplace transform.

Substituting (8) and (14b) into (30b) yields

$$\frac{\mathrm{d}\Delta x(t)}{\mathrm{d}t}=(A-LC)\Delta x(t)+B\Delta d_e(t). \tag{61}$$

Note that there are no nonlinear terms in (61). Thus, a Lipschitz condition is not necessary for the convergence of Δx, which means that our method is effective no matter the nonlinear term satisfying the Lipschitz condition or not. Moreover,

$$\Delta x(t) = g_{\Delta x}(t) * d_e(t), \tag{62}$$

where

$$\begin{aligned} &g_{\Delta x}(t) \\ &= \ell^{-1}\{-N(s)[I - F(s)G_L(s)]^{-1}[I - F(s)]\}. \end{aligned} \tag{63}$$

Remark 4 The Lipschitz condition usually is used to ensure the convergence of the observation error for nonlinear systems. It is difficult to do that without such a condition. This paper uses the NEID approach to successfully construct the error system (61) that does not contain the nonlinearity. The convergence of the observation error is guaranteed by the linear part. Thus, it avoids the Lipchitz condition.

According to Table 2.2 [33], (60) and (62) become

$$\|\Delta d_e\|_\infty = \|g_{\Delta d}\|_1 \|d_e\|_\infty \tag{64}$$

and

$$\|\Delta x_i\|_\infty = \|g_{\Delta x_i}\|_1 \|d_e\|_\infty,\ 1 \le i \le n. \tag{65}$$

Substituting (55) into (64) and (65) yields

$$\|\Delta d_e\|_\infty \le \|g_{\Delta d}\|_1 (f_M + d_M), \tag{66}$$

and

$$\|\Delta x_i\|_\infty \le \|g_{\Delta x_i}\|_1 (f_M + d_M),\ 1 \le i \le n. \tag{67}$$

Let the nonlinearity reconstruction error to be

$$e_f(t) = f(x(t)) - f(\hat{x}(t)). \tag{68}$$

According to the mean value theorem,

$$e_f(t) = \phi\left(\hat{x}(t) - x(t)\right) = \phi \Delta x(t), \tag{69}$$

where

$$\begin{cases} \phi = \mathrm{D}F(\delta),\ \mathrm{D}F = \left[\dfrac{\partial f}{\partial x_1}, \dfrac{\partial f}{\partial x_2}, ..., \dfrac{\partial f}{\partial x_n}\right], \\ \delta = x(t) + \theta \Delta x(t),\ \theta \in (0, 1). \end{cases} \tag{70}$$

Thus,

$$\begin{aligned} \|e_f\|_\infty = \|\phi \Delta x\|_\infty &\le \|\phi\|_\infty \sum_{i=1}^{n} \|\Delta x_i\|_\infty \\ &\le \left(\sum_{i=1}^{n} \|g_{\Delta x_i}\|_1\right) \|\phi\|_\infty (f_M + d_M). \end{aligned} \tag{71}$$

Note that, since $f(x)$ is known and the stable region is given in Section 3, $\|\phi\|_\infty$ is easy to obtain.

Next, we are ready to calculate the upper-bound of the estimate error for the NEID estimator. Note that the function of the NEID estimator is to produce an estimate of the exogenous disturbance. The exogenous-disturbance estimate is

$$\hat{d}(t) = \tilde{d}_e(t) - f(\hat{x}(t)). \tag{72}$$

The error of the disturbance estimation is

$$e_d(t) = \hat{d}(t) - d(t) = \tilde{d}_e(t) - f(\hat{x}(t)) - d(t). \tag{73}$$

That is,

$$\begin{aligned} e_d(t) &= [\tilde{d}_e(t) - d_e(t)] + [f(x(t)) - f(\hat{x}(t))] \\ &= \Delta d_e(t) + e_f(t). \end{aligned} \tag{74}$$

As a result,

$$\begin{aligned} \|e_d\|_\infty &\le \|\Delta d_e\|_\infty + \|e_f\|_\infty \\ &\le \|g_{\Delta d}\|_1 (f_M + d_M) + \|e_f\|_\infty \\ &\le \|g_{\Delta d}\|_1 (f_M + d_M) \\ &+ \left(\sum_{i=1}^{n} \|g_{\Delta x_i}\|_1\right) \|\phi\|_\infty (f_M + d_M) \\ &\le \left[\|g_{\Delta d}\|_1 + \left(\sum_{i=1}^{n} \|g_{\Delta x_i}\|_1\right) \|\phi\|_\infty\right] (f_M + d_M). \end{aligned} \tag{75}$$

Summarizing the above results gives the following theorem.

Theorem 2 *The upper bound of the disturbance estimation error is*

$$\begin{aligned} &\sup \|e_d\|_\infty \\ &= \left[\|g_{\Delta d}\|_1 + \left(\sum_{i=1}^{n} \|g_{\Delta x_i}\|_1\right) \|\phi\|_\infty\right] (f_M + d_M), \end{aligned} \tag{76}$$

and the upper bound of the nonlinearity reconstruction error is

$$\sup \|e_f\|_\infty = \left(\sum_{i=1}^{n} \|g_{\Delta x_i}\|_1\right) \|\phi\|_\infty (f_M + d_M). \tag{77}$$

5 Numerical verification

First, a numerical example is used to illustrate the design procedures and demonstrate the validity of the method. The parameters of the plant (1) are

$$\begin{aligned} &A = \begin{bmatrix} 0 & 1 \\ -1 & -0.5 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 0 \ 1 \end{bmatrix}, \\ &d(t) = \sin t, \end{aligned} \tag{78}$$

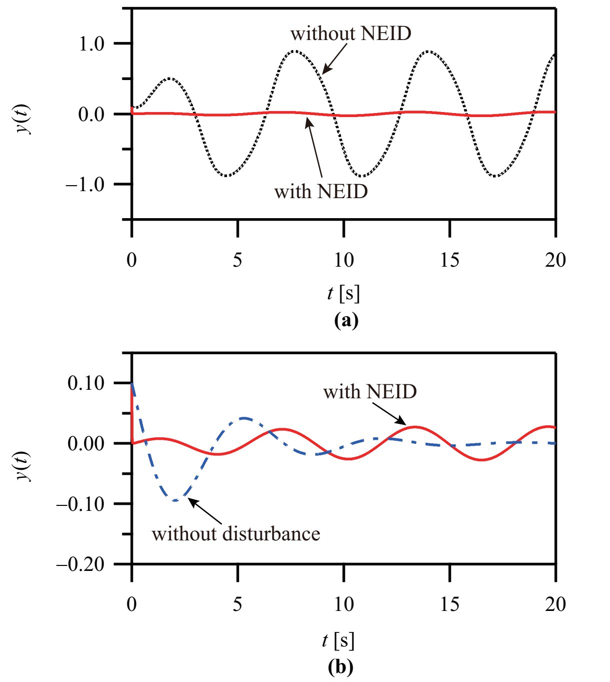

Fig. 4 Outputs of (78) and (79) to initial condition (89). (a) With and without an NEID compensator. (b) With NEID and without disturbance.

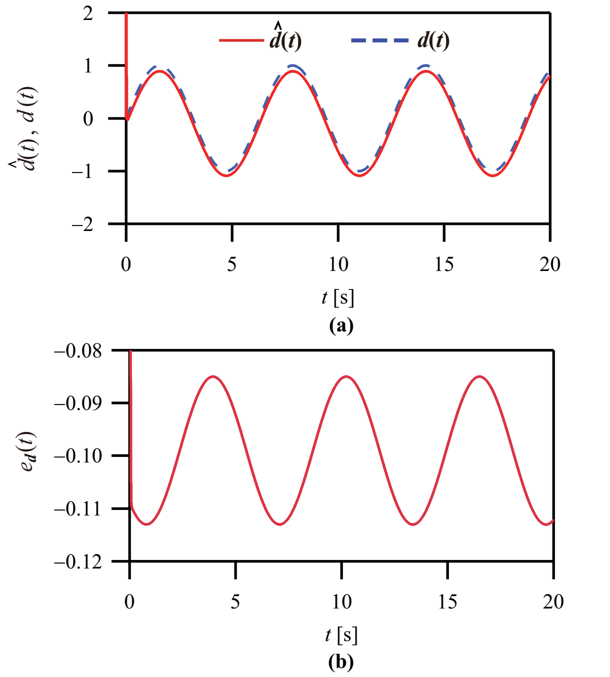

Fig. 5 Responses of (78) and (79) to initial condition (89) for $\hat{d}(t)$ and $d(t)$ [(a)], and $e_d(t)$ [(b)].

and

$$f(x(t)) = -x_2^3(t). \tag{79}$$

Since only $x_2(t)$ is measurable, the conventional N-DOB approach cannot be applied in this example. Set

$$u_f(t) = -\hat{x}_2^3(t). \tag{80}$$

A first-order filter

$$\begin{cases} F(s) = \dfrac{100}{s+101}, \\ A_F = -101,\ B_F = 100,\ C_F = 1 \end{cases} \tag{81}$$

was used in this study.

Optimizing the performance index [24]

$$J_L = \int_0^\infty \{\rho x_L^{\mathrm{T}}(t) Q_L x_L(t) + u_L^{\mathrm{T}}(t) R_L u_L(t)\}\mathrm{d}t \tag{82}$$

yields

$$L = \left(R_L^{-1} C P_\rho\right)^{\mathrm{T}}, \tag{83}$$

where P_ρ is a positive symmetrical solution of the Riccati equation

$$AP_\rho + P_\rho A^{\mathrm{T}} - P_\rho C^{\mathrm{T}} R_L^{-1} C P_\rho + \rho Q_L = 0. \tag{84}$$

The selection of

$$Q_L = I,\ R_L = 1,\ \rho = 10^5 \tag{85}$$

gives

$$L = [-999.90\ 1000.00]^{\mathrm{T}}. \tag{86}$$

The above selection results

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ -1 & -0.5 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1000.99 & 0 \\ 0 & 0 & -1 & -1000.50 & 1 \\ 0 & 0 & 0 & -100000 & -1 \end{bmatrix}. \tag{87}$$

The eigenvalues of (87) are

$$-886.43,\quad -115.06,\quad -0.01,\quad -0.25 \pm 0.97\mathrm{j}. \tag{88}$$

It is clear that $\bar{A}$ is Hurwitz.

Simulations were carried out for the initial state

$$x_1(0) = 0.1,\ x_2(0) = 0.1. \tag{89}$$

Fig. 4 (a) shows the simulation results for the plant (78) and (79), with and without using the NEID compensator. Observing from Fig. 4 (a), the disturbance had a big effect on the output when the NEID compensator was not used. The output oscillated at a vibration angular frequency of 1 rad/s. On the other hand, incorporating the NEID compensator in the system dramatically reduced the influence of the disturbance. More specifically, the largest output was reduced by more than 99.5% (from 0.89 to 0.03). Fig. 4 (b) shows simulation results for the plant without the disturbance and the

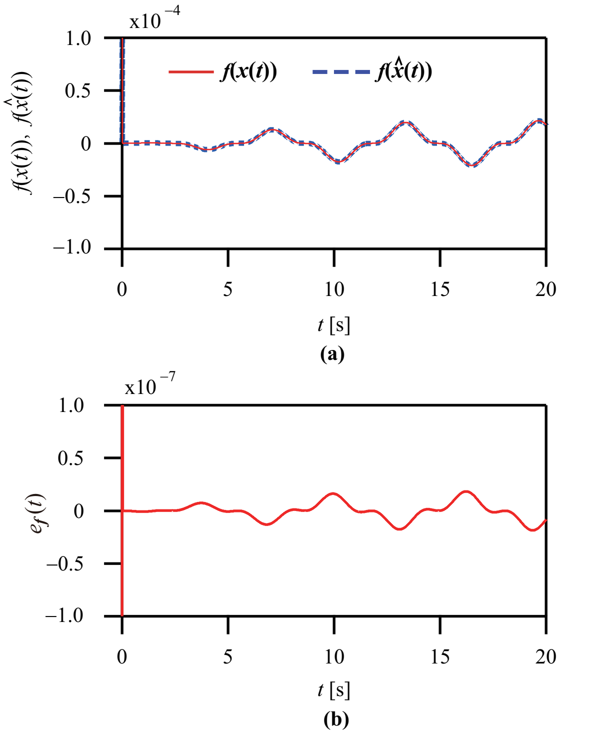

Fig. 6 Responses of (78) and (79) to initial condition (89) for $f(x(t))$ and $f(\hat{x}(t))$ [(a)], and $e_f(t)$ [(b)].

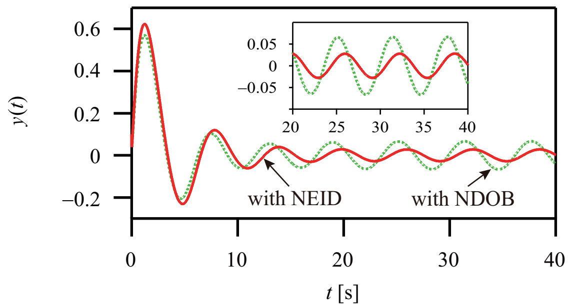

Fig. 7 Outputs of (78) and (92) to initial condition (89) for $y(t)$.

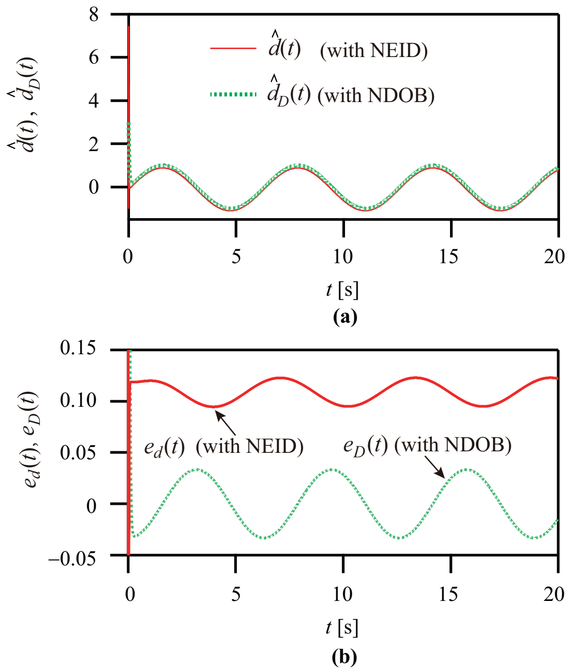

Fig. 8 Repones of (78) and (92) to initial condition (89) for $\hat{d}(t)$ and $\hat{d}_D(t)$ [(a)], and $e_d(t)$ and $e_D(t)$ [(b)].

plant using the NEID compensator. In Fig. 4 (b), the output of the system with the NEID compensator is almost the same as that without the disturbance. This shows that the nonlinear behavior was recovered to that without the disturbance using the NEID compensator.

Fig. 5 shows the effect of the NEID compensator for disturbance estimation. The NEID estimator was produced with a very high precision [Fig. 5 (a)]. The largest relative estimation error

$$\frac{|\max|\hat{d}(t)| - \max|d(t)||}{\max|d(t)|} \times 100\% \tag{90}$$

is 8.8% [Fig. 5 (b)]. Fig. 6 shows the effect of the nonlinear state feedback term for nonlinearity reconstruction. And the nonlinearity was well reconstructed as shown in Fig. 6 (a). The largest relative reconstruction error

$$\frac{|\max|f(x(t))| - \max|f(\hat{x}(t))||}{\max|f(x(t))|} \times 100\% \tag{91}$$

is 0.1% [Fig. 6 (b)]. Those show the effectiveness of the NEID compensator.

Next, another numerical example is used to compare with the NDOB method. In this example, the nonlinear term is chosen to be

$$f(x(t)) = \cos x_2(t). \tag{92}$$

Note that the nonlinear term (92) satisfies the Lipschitz condition and (4) rather than Assumption 3.

The low-pass filter and the observer gain were chosen to be the same as (81) and (86), respectively, and the control law was chosen to be

$$u_f(t) = \cos\hat{x}_2(t) \tag{93}$$

to design an NEID compensator for (78) and (92).

To design an NDOB for (78) and (92), we assume that the system state is available. The parameters of the NDOB were selected to be

$$p(x(t)) = 30x_2(t),\ l(x(t)) = \begin{bmatrix} 0 & 30 \end{bmatrix}. \tag{94}$$

Then, the disturbance estimate is

$$\hat{d}_D(t) = z(t) + 30x_2(t) \tag{95}$$

according to (16b).

Simulations were carried out for the initial state (89). This paper uses the steady-state peak-to-peak value (PPV) to evaluate the disturbance-rejection performance and the disturbance-estimation accuracy of the two methods.

Fig. 7 shows the outputs for (78) and (92) using the NEID compensator and the NDOB. The steady-state PPV of the output using the NEID compensator is 0.05 which is about 33.3% of that using the NDOB method (0.15). This means that the disturbance-rejection performance is better for the NEID approach than for the NDOB method.

Fig. 8 (a) shows the disturbance estimates calculated by the NEID estimator [$\hat{d}(t)$] and the NDOB [$\hat{d}_D(t)$], and Fig. 8 (b) shows their estimation errors [$e_d(t)$ and $e_D(t)$]. The PPV of $e_d(t)$ is 0.03 which is about 42.9% of that of $e_D(t)$ (0.07). This means that the disturbance-estimation accuracy is higher for the NEID approach than for the NDOB method.

Furthermore, the above presentation shows that our method is effective not only for a nonlinear system satisfying Assumption 3 but also for a nonlinear system satisfying (4).

6 Conclusion

This paper presented a method of compensating for an unknown exogenous disturbance in a class of nonlinear systems. It has two parts: An equivalent-input-disturbance (EID) estimator and a nonlinear term. The nonlinear term is used to reconstruct the actual nonlinearities of the system. The combination of the nonlinear term and the EID estimator ensures that only the exogenous disturbance is rejected but the nonlinearities of the system is retained. The configuration of the nonlinear EID (NEID) compensator is simple, and the design of the NEID-based disturbance-rejection is easy. Unlike other disturbance-rejection methods for nonlinear systems, a Lipschitz condition is not necessary to guarantee the convergence of the observation error, which extends the application range of the NEID compensator. The upper bounds of disturbance-estimation error and the degree of nonlinearity retention were presented for this method.

Since the characteristics of a nonlinear system do not change after inserting the NEID compensator into the system, exploring the use of this method by utilizing this characteristic to control other nonlinear phenomena with disturbances, such as limit cycle, chaos, and bifurcation, is of great significance and will be carried out in the future.

This study first considered matched nonlinearities and exogenous disturbances. However, many practical control systems contain mismatched nonlinearities and exogenous disturbances. Thus, extending the NEID approach to deal with mismatched nonlinearities and exogenous disturbances is meaningful, and will be carried out in the future.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest regarding the publication of this paper.

Appendix

Stability of the NEID-based disturbance-rejection system for (4) is analyzed below.

The state-space representation of the NEID-based disturbance-rejection system is

$$\frac{\mathrm{d}\xi(t)}{\mathrm{d}t} = \bar{A}\xi(t) + \bar{B}f(\hat{x}(t)) + \bar{B}_b f(x(t)) + \bar{B}_d d(t), \quad \text{(A.1)}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 & -BC_F \\ 0 & A - LC & BC_F \\ 0 & -B_F B^+ LC & A_F + B_F C_F \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \ \bar{B}_b = \begin{bmatrix} B \\ -B \\ 0 \end{bmatrix}, \ \bar{B}_d = \begin{bmatrix} 0 \\ -B \\ 0 \end{bmatrix}.$$

If $\bar{A}$ is Hurwitz, then, for any positive-definite symmetric matrix K, the solution, P_g, of

$$P_g \bar{A} + \bar{A}^{\mathrm{T}} P_g = -K \quad \text{(A.2)}$$

is positive definite. Let

$$V_g(\xi(t)) = \xi^{\mathrm{T}}(t) P_g \xi(t) \quad \text{(A.3)}$$

be a Lyapunov function candidate of (A.1). The derivative of $V_g(\xi(t))$ along the trajectories of (A.1) is given by

$$\frac{\mathrm{d}V_g(\xi(t))}{\mathrm{d}t} = -\xi^{\mathrm{T}}(t) K \xi(t) + \Pi_1 + \Pi_2 + \Pi_3, \quad \text{(A.4)}$$

where

$$\Pi_1 = 2\xi^{\mathrm{T}}(t)P_g\bar{B}f(\hat{x}(t)),\ \Pi_2 = 2\xi^{\mathrm{T}}(t)P_g\bar{B}_bf(x(t)),$$
$$\Pi_3 = 2\xi^{\mathrm{T}}(t)P_g\bar{B}_dd(t).$$

Since

$$\begin{aligned}\Pi_1 &\le \frac{1}{\theta_1}\xi^{\mathrm{T}}(t)\Gamma_1\xi(t) + \theta_1 f^{\mathrm{T}}(\hat{x}(t))f(\hat{x}(t)) \\ &\le \frac{1}{\theta_1}\xi^{\mathrm{T}}(t)\Gamma_1\xi(t) + \theta_1\|f\|_\infty^2 \\ &\le \frac{1}{\theta_1}\xi^{\mathrm{T}}(t)\Gamma_1\xi(t) + \theta_1\varphi^2,\end{aligned} \tag{A.5}$$

$$\begin{aligned}\Pi_2 &\le \frac{1}{\theta_2}\xi^{\mathrm{T}}(t)\Gamma_2\xi(t) + \theta_2 f^{\mathrm{T}}(x(t))f(x(t)) \\ &\le \frac{1}{\theta_2}\xi^{\mathrm{T}}(t)\Gamma_2\xi(t) + \theta_2\|f\|_\infty^2 \\ &\le \frac{1}{\theta_2}\xi^{\mathrm{T}}(t)\Gamma_2\xi(t) + \theta_2\varphi^2,\end{aligned} \tag{A.6}$$

and

$$\begin{aligned}\Pi_3 &\le \frac{1}{\theta_3}\xi^{\mathrm{T}}(t)\Gamma_3\xi(t) + \theta_3 d^{\mathrm{T}}(t)d(t) \\ &\le \frac{1}{\theta_3}\xi^{\mathrm{T}}(t)\Gamma_3\xi(t) + \theta_3 d_M^2,\end{aligned} \tag{A.7}$$

(A.4) becomes

$$\begin{aligned}&\frac{\mathrm{d}V_g(\xi(t))}{\mathrm{d}t} \\ &\le -\xi^{\mathrm{T}}(t)K\xi(t) + \frac{1}{\theta_1}\xi^{\mathrm{T}}(t)\Gamma_1\xi(t) + \frac{1}{\theta_2}\xi^{\mathrm{T}}(t)\Gamma_2\xi(t) \\ &+ \frac{1}{\theta_3}\xi^{\mathrm{T}}(t)\Gamma_3\xi(t) + \theta_1\varphi^2 + \theta_2\varphi^2 + \theta_3 d_M^2 \\ &\le -[\lambda_{\min}(K) - \frac{1}{\theta_1}\lambda_{\max}(\Gamma_1) - \frac{1}{\theta_2}\lambda_{\max}(\Gamma_2) \\ &- \frac{1}{\theta_3}\lambda_{\max}(\Gamma_3)]\|\xi\|_2^2 + \theta_1\varphi^2 + \theta_2\varphi^2 + \theta_3 d_M^2,\end{aligned}$$

where $\theta_1,\ \theta_2,\ \theta_3$ are positive numbers and

$$\Gamma_1 = P_g\bar{B}\bar{B}^{\mathrm{T}}P_g,\ \Gamma_2 = P_g\bar{B}_b\bar{B}_b^{\mathrm{T}}P_g,\ \Gamma_3 = P_g\bar{B}_d\bar{B}_d^{\mathrm{T}}P_g.$$

It is easy to check that there exist positive numbers $\theta_1,\ \theta_2,\ \theta_3$, such that

$$\lambda_{\min}(K) > \frac{\lambda_{\max}(\Gamma_1)}{\theta_1} + \frac{\lambda_{\max}(\Gamma_2)}{\theta_2} + \frac{\lambda_{\max}(\Gamma_3)}{\theta_3} \tag{A.8}$$

is true. Thus,

$$\frac{\mathrm{d}V_g(\xi(t))}{\mathrm{d}t} < 0,$$

which means that the NEID-based disturbance-rejection system is global uniformly boundedness for (4).

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