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論 文 要 旨

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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

Matching under preferences is a problem involving matching people with items or with other people while each person has a list that ranks items or other people in his/her order of preference. This problem models many important real-world situations including assignment of medical residents to hospitals and families to government-subsidized housing. The main objective of this problem is to find an optimal matching in each situation. Various definitions of optimality have been proposed, such as Pareto optimality and rank-maximality. However, two most well-studied properties of matchings are stability and popularity.

The Stable Marriage Problem is one of the most actively studied problems in computer science, mathematics, and economics. In the original bipartite setting called Marriage Problem (MP), a set of n/2 men and a set of n/2 women are given. Each person has a preference list that ranks all people of the opposite gender in strict order of preference. A man and a woman are called a blocking pair w.r.t. a matching M if they are not matched with each other in M but prefer each other to their own partners in M. A matching is called stable if it does not admit any blocking pair. Gale and Shapley proved that a stable matching always exists in any instance and developed an $O(n^2)$ time algorithm to find one. The Stable Roommates Problem is a generalization of the original Stable Marriage Problem to a non-bipartite setting called Roommates Problem (RP), where each person can be matched with anyone regardless of gender.

Apart from stability, another less restrictive property of a preferable matching is popularity. For a pair of matchings X and Y, let $\phi(X,Y)$ denote the number of people who prefer a person they get matched by X to a person they get matched by Y. A matching M is called popular if $\phi(M,N) \geq \phi(N,M)$ for any other matching N. Besides MP and RP settings, popular matchings were also studied in a setting of one-sided preference lists (matching people with items, where each person has a list that ranks items but each item does not have a list that ranks people) called House Allocation Problem (HAP). Note that the relation $\phi(X,Y) \geq \phi(Y,X)$ is not transitive, so a popular matching may or may not exist depending on the preference lists of people. The Random Popular Matching Problem (RPMP) is a probabilistic problem involving the probability of existence of a popular matching in a random instance, where each person's preference list is independently and uniformly generated at random.

While a popular matching may not exist in some instances, several measures of badness of a matching that is not popular have been introduced, including unpopularity factor and unpopularity margin. The unpopularity factor u(M) of a matching M is the maximum ratio $\phi(N,M)/\phi(M,N)$ among all other possible matchings N, while the unpopularity margin g(M) is the maximum difference $\phi(N,M)-\phi(M,N)$ among all other possible matchings N.

Finally, besides constraints on the preferences, geometric constraints are also important factors to consider in many real-world situations involving matchings, such as in the VLSI layout design. In general, the Noncrossing Matching Problem (NMP) deals with a set of 2n vertices lying on two parallel lines, with some edges joining vertices on the opposite lines. The goal of NMP is to find a noncrossing matching, a matching whose edges do not cross one another, subject to different objectives such as maximum size, maximum weight, etc.

In this thesis, we analyze three open problems in matching under preferences using graph-theoretic characterizations.

In the first problem, we study RPMP in an HAP instance with n_1 people and $n_2 \geq n_1$ items, with every person's preference list being strict (containing no tie) and having the same length of a constant k. We discover a phase transition of probability when the ratio $\alpha = n_2/n_1$ is around α_k, where $\alpha_k \geq 1$ is the root of equation $xe^{-1/2x} = 1-(1-e^{-1/x})^{k-1}$. In particular, we prove that for k≥4, if $\alpha > \alpha_k$, then a popular matching exists with high probability (1-o(1) probability); and if $\alpha < \alpha_k$, then a popular matching exists with low probability (o(1) probability). For k≤3, where the equation does not have a solution in $[1, \infty)$, a popular matching always exists with high probability for any value of $\alpha \geq 1$ without a phase transition. We also perform a simulation to help illustrate and verify the discovered phase transition.

In the second problem, we develop the first polynomial-time algorithm to compute the unpopularity factor of a given matching in an MP or RP instance with n people. The algorithm runs in $O(m\sqrt{n}\log n)$ time for MP and in $O(m\sqrt{n}\log^2 n)$ time for RP, where m is the total length of people's preference lists. We also generalize the notion of unpopularity factor to the weighted setting where people are given different voting weights, and show that our algorithm can be slightly modified to support that setting with the same running time.

In the third problem, we investigate an NMP instance with n men and n women represented by points lying on two parallel lines, each line containing n people of one gender. Each person has a strict preference list that ranks a subset of people of the opposite gender. A noncrossing blocking pair w.r.t. a matching M is a blocking pair w.r.t. M that does not cross any edge in M. Our goal is to find a noncrossing matching that does not admit any noncrossing blocking pair, called a weakly stable noncrossing matching (WSNM). We constructively prove that a WSNM always exists in any instance by developing an $O(n^2)$ time algorithm to find one in a given instance.