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論 文 要 旨

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学生氏名： Student's Name	加藤 譲		指導教員（主）： Academic Supervisor(main)	中尾 裕也
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要旨（英文800語程度）
Thesis Summary（approx.800 English Words ）

Synchronization of rhythmic dynamical systems, or nonlinear self-sustained oscillators, is a ubiquitous phenomenon that can be seen in various fields of science and technology, for example, in electrical oscillations, chemical oscillations, biological rhythms, and mechanical vibrations. In analyzing the rhythmic dynamics of classical limit-cycle oscillators, the phase-reduction theory has played a central role. This theory enables us to quantitatively approximate the dynamics of a nonlinear multi-dimensional limit-cycle oscillator by a simple one-dimensional phase equation, which has greatly facilitated systematic analysis of universal properties of limit-cycle oscillators, such as synchronization of oscillators with external periodic forcing, mutual synchronization between coupled oscillators, the collective synchronization transition in a system of globally coupled phase oscillators, and control and optimization of nonlinear oscillators. Moreover, the phase equation has also been helpful for the analysis of engineering applications of synchronization such as the injection locking, ring laser gyroscope, Josephson voltage standard, phase lock loops in electrical circuits, and deep brain stimulation for the treatment of Parkinson's disease.

Experimental realization for synchronization of nonlinear oscillators has recently reached in micro and nano scales and the demand for theoretical studies of synchronization in the quantum regime is rapidly increasing in order to unravel its novel features. Many research papers about quantum synchronization have been published in the last several years, and future potential applications of quantum synchronization in the fields of quantum methodology, quantum standard, and quantum information have also been discussed. In order to systematically analyze quantum synchronization, understanding of the phase dynamics of quantum limit-cycle oscillators is significantly important.

In this thesis, we present several theoretical results using phase dynamics for the analysis of quantum synchronization.

First, we generalize the conventional phase-reduction theory to quantum limit-cycle oscillators in the semiclassical regime. The semiclassical phase-reduction theory enables us to quantitatively approximate a quantum oscillator exhibiting stable limit-cycle oscillations by a simple one-dimensional phase equation, facilitating a systematic analysis of quantum synchronization in this regime. As a simple example, we analyze synchronization properties of a typical model of quantum limit-cycle oscillators, known as the quantum van der Pol oscillator, subjected to a harmonic driving and squeezing, including the case that the squeezing is strong and the oscillation is asymmetric. The proposed semiclassical phase-reduction theory provides a tool for systematic analysis of quantum synchronization in a general class of asymmetric limit-cycle oscillators.

Second, using the semiclassical phase-reduction theory, we consider optimal entrainment of a quantum nonlinear oscillator to a periodically modulated weak harmonic drive in the semiclassical regime. We analyze two types of optimization problems, one for the stability and the other for the phase coherence of the oscillator, and derive optimal waveforms for the periodic modulation by applying

the optimization methods originally developed for classical nonlinear oscillators to a quantum nonlinear oscillator in the semiclassical regime. Using the van der Pol model with squeezing and Kerr effects, we numerically analyze the performance of the optimization schemes and discuss their differences.

Third, we go beyond the semiclassical regime and introduce an asymptotic phase, a fundamental phase variable of quantum nonlinear oscillators, in a fully quantum-mechanical way. This extends the applicability of the phase variable to the strong quantum regime and enables analysis of nontrivial quantum synchronization phenomena. We analyze a quantum van der Pol oscillator with Kerr effect and show that our quantum asymptotic phase yields appropriate results in the strong quantum regime while reproducing the conventional asymptotic phase in the semiclassical regime. We then use the quantum asymptotic phase for analyzing the multiple phase locking of the system with a harmonic drive at several different frequencies, an explicit quantum effect observed only in the strong quantum regime, and clarify that it can be understood as synchronization of the system on a torus rather than on a simple limit cycle, a new interpretation of this quantum phenomenon.

Lastly, we analyze synchronization of a quantum van der Pol oscillator with a harmonic driving signal based on phase dynamics and demonstrate that performing continuous homodyne measurement and applying a feedback control can enhance quantum synchronization. We argue that the phase coherence of the oscillator is increased by the reduction of quantum fluctuations due to the continuous measurement and that a simple feedback policy can suppress the measurement-induced fluctuations by adjusting the frequency detuning between the oscillator and the driving signal.

Quantum synchronization is a burgeoning topic at the boundary between quantum physics and nonlinear dynamics and attracting much attention not only in pure and applied physics but also in information science, applied mathematics, and various engineering fields. The phase dynamics approach to quantum synchronization will not only provide a framework for systematic analysis of quantum synchronization, but also help us to find novel technical applications of quantum synchronization in the growing fields of quantum technologies, such as quantum information, quantum metrology, and quantum standard.