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2 Sway-rocking spring system applicable to short-rigid monopile foundations

3

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23

24 **Abstract**

25 Sway-Rocking (SR) spring system is a foundation modelling method in which a 2×2 spring
26 stiffness matrix represents the foundation stiffness at seabed level. Since each individual
27 element of the spring stiffness matrix is a function of a few physical properties of the soil and
28 pile, it is the simplest and computationally cheapest method to estimate the deformation at
29 the seabed level and the natural frequency of an offshore wind turbine structure. Formulae
30 currently available are only applicable to conventional long-flexible piles or perfectly rigid
31 foundations such as footings or caissons. This study aims to extend the application of the SR
32 spring system to a so-called short-rigid pile range, which lays between the two ranges: long-
33 flexible and perfectly rigid. Monopiles, the most commonly selected type of offshore wind
34 foundations, are categorized in the short-rigid pile range. By conducting three-dimensional
35 finite element analyses, the short-rigid pile behaviour is investigated. Based on the results, a
36 new set of formulae of the SR spring stiffness, which is applicable to a wide range (long-flexible,
37 short-rigid and perfectly rigid) of piles, is proposed. The new formulae also improve the
38 accuracy of frequency estimation for practical use cases.

39

40

41 **Keywords**

42 offshore wind turbine structure; monopile; short-rigid pile; Sway-Rocking spring system;
43 natural frequency

44

45 1. Introduction

46 In the last decade, the offshore wind industry has vastly developed to become one of the
47 largest renewable energy sectors. Especially in the last years, commercial offshore wind farms
48 are under development not only in Europe but also in America and Asia. Along with the
49 acceleration of worldwide development of the offshore wind farms, the size of wind turbine
50 generators (WTG) is exponentially increasing as pictured in **Fig. 1** (Data source: [Wind-Europe](#)
51 [2017](#); [4C Offshore 2018](#); and [IRENA 2019](#)). It is expected that the 12 MW size turbines will
52 become the world standard in the upcoming 5 years.

53 The most commonly used supporting structure type till now is the pile foundation due to
54 its relatively low construction cost and ease of installation in shallow to intermediate water
55 depths. This type of foundation is composed of a monopile, which is installed into the seabed,
56 and a transition piece, which connects a WTG tower and the monopile. Along with the
57 development of WTG generators with larger energy yielding capacities, the diameter of
58 monopiles has increased up to 10 m. As a result, the slenderness ratio (L_p/D_p) of state-of-the-
59 art monopiles decreases to less than 10 while most conventional friction piles have a
60 slenderness ratio of more than 20. Hence, these monopiles are no longer considered as long-
61 flexible foundations but as short-rigid structures closer to caissons. Short-rigid structures
62 show different behaviour than conventional long-flexible piles. If the size effect of rigid piles
63 with lower slenderness ratio (L_p/D_p) such as base shear and moment and a distributed
64 moment along the pile is taken into account as studied in the PISA project ([Byrne 2020](#)), a
65 more economical and optimized design will be possible. Hence, the short-rigid pile behaviour
66 should be further studied and novel design methods should be developed.

67 Monopiles are exposed to cyclic lateral loads due to the rotating blades and environmental
68 loads such as wind and waves. According to [Arany et al. \(2016\)](#), the natural frequency of WTG

69 structures is designed to be between the rotor frequency (1P) and blade passing frequency
70 (3P). To accurately estimate the natural frequency, the foundation stiffness should be
71 considered. Finite element analyses and frame calculations with Winkler springs along the pile
72 is recommended by [DNVGL-RP-C212 \(2017\)](#) and often-used by WTG foundation designers.
73 However, these methods are computationally more expensive. For Service Limit State (SLS),
74 the pile deflection at seabed level and the natural frequency of a tower-foundation system
75 should be assessed. A simpler method, which is called the Sway-Rocking (SR) spring system, is
76 available as introduced by [Bhattacharya \(2019\)](#).

77 The SR spring system is a foundation modelling method where the soil-pile interaction is
78 modelled with a lateral spring (K_L), a rotational spring (K_R) and coupling springs (K_{LR} and K_{RL})
79 at the seabed as shown in **Fig. 2**. Since the vertical spring K_V is an independent element which
80 does not interact with the SR spring system, discussion on the vertical spring is avoided in this
81 paper. For estimation of K_V value, see [Fleming et al. \(1992\)](#) and [DNV \(2002\)](#). The spring
82 stiffness can be used to estimate lateral deformation: u and the rotation: ϑ at the seabed level
83 from the excitation horizontal loading: H and the moment: M and vice versa as shown in **Eqn.**
84 **(1)**.

$$85 \begin{bmatrix} H \\ M \end{bmatrix} = \begin{bmatrix} K_L & K_{LR} \\ K_{RL} & K_R \end{bmatrix} \begin{bmatrix} u \\ \vartheta \end{bmatrix} \quad (1)$$

86 Using this method, the soil-pile interaction can be easily elaborated in the natural frequency
87 estimation and the dynamic analysis of the WTG structure during the preliminary design phase.

88 The SR spring stiffness for long-flexible piles is determined by the soil stiffness (E_s), pile
89 diameter (D_p) and soil-pile stiffness ratio (E_s/E_p) but not the slenderness ratio (L_p/D_p)
90 because the conventional piles are longer than a critical pile length, that means these piles
91 would have the same behaviour as infinitely long piles. [Randolph \(1981\)](#) proposed stiffness

92 formulae for long-flexible piles embedded in a linear elastic homogeneous soil. Several
93 researchers have proposed modified formulae for not only homogeneous soils, which aims at
94 over-consolidated clay, but also parabolic inhomogeneous (sandy soil) and linear
95 inhomogeneous (normally consolidated clay) according to [Shadlou et al. \(2016\)](#). The ones
96 proposed by [Gazetas \(1984\)](#) are recommended in [Eurocode 8 \(2004\)](#). According to the
97 references [Randolph \(1981\)](#) and [Gazetas \(1984\)](#), the non-dimensional spring stiffness η , which
98 is defined in **Eqns. (2) ~ (5)**, is a power function of the stiffness ratio (pile effective stiffness /
99 soil stiffness).

$$100 \quad \eta_L = K_L/E_s D_p \quad (2)$$

$$101 \quad \eta_{LR} = K_{LR}/E_s D_p^2 \quad (3)$$

$$102 \quad \eta_{RL} = K_{RL}/E_s D_p^2 \quad (4)$$

$$103 \quad \eta_R = K_R/E_s D_p^3 \quad (5)$$

104 On the other hand, as for perfectly rigid piles such as footing or caisson type of foundations,
105 the mechanical behaviour is influenced by the soil stiffness (E_s), pile diameter (D_p) and
106 slenderness ratio (L_p/D_p) but not the stiffness ratio (E_s/E_p), as the deflection is negligibly
107 small. [Carter et al. \(1992\)](#) proposed stiffness formulae for shaft foundations in linear elastic
108 homogeneous grounds as a power function of the slenderness ratio (L_p/D_p), that became the
109 foundation of perfectly rigid pile modelling with the SR spring system. [Higgins et al. \(2013\)](#)
110 have recently extended the SR modelling method to perfectly rigid pile foundations in soil with
111 stiffness proportional to depth as well as in double-layered soil with two different
112 homogeneous profiles.

113 State-of-the-art monopiles are categorized between conventional long-flexible piles and
114 perfectly rigid piles and exhibit characteristics of the both. In this study, these types of piles
115 are named as short-rigid. The SR spring stiffness of the short-rigid type of piles will be defined

116 as a function of both stiffness ratio (E_s/E_p) and slenderness ratio (L_p/D_p). Despite that most
117 monopiles are categorized in this short-rigid range, there is no research which thoroughly
118 investigated this range to propose SR spring stiffness formulae depicting the behaviour of piles
119 influenced by both stiffness ratio (E_s/E_p) and slenderness ratio (L_p/D_p).

120 This study aims to expand the applicable range of the SR spring system to the short-rigid
121 piles and to contribute to a more accurate estimation of the first natural frequency of WTG
122 structures with this simple analytical method. In this study, finite element (FE) models on a
123 commercial software PLAXIS 3D are used as a reference. First of all, FE static load-
124 displacement analyses are conducted for several monopile foundations with various
125 slenderness ratio (L_p/D_p) embedded into a single elastic homogeneous soil stratum with
126 different stiffness (E_s). Regarding the analytical results, a single set of SR spring stiffness
127 formulae as a function of both the stiffness ratio (E_s/E_p) and the slenderness ratio (L_p/D_p) is
128 proposed. The newly proposed formulae are applicable to conventional long-flexible, short-
129 rigid and perfectly rigid piles. The first natural frequency of a structure considering the soil-
130 pile interaction effect is theoretically estimated using the SR spring stiffness calculated from
131 the formulae. Finally, a FE free vibration analysis is performed to validate the estimated first
132 natural frequencies.

133

134 **2. FE analysis model**

135 **2.1 Target structure**

136 Three example WTG structures from three different wind farms with different generation
137 capacities in the United Kingdom (Blyth: 2 MW, Barrow 3 MW and Walney1: 3.6 MW)
138 presented in [Arany et al. \(2016\)](#) are investigated to make a reference model for the FE analysis.
139 As is the case with the most typical WTG structures, the towers and the monopiles of the three

140 wind turbines are made of hollow steel pipes. Hereafter, for simplification of the structural
141 model, the ‘pile’ is defined as the part of the monopile below the seabed whereas the ‘tower’
142 is the rest part of the structure above the seabed. The pile slenderness ratio (L_p/D_p), tower-
143 pile length ratio (L_t/L_p), tower-pile diameter ratio (D_t/D_p), tower-pile thickness ratio (t_t/t_p)
144 and tower-pile bending stiffness ratios (EI_t/EI_p) are firstly fixed to create the reference model,
145 representing the commercially designed monopile and tower. The effective pile stiffness E_p is
146 used as the parameter representing the pile stiffness. E_p is given by **Eqn. (6)** expressed by the
147 actual bending stiffness of the pile EI_p and the area moment of inertia of a solid cylinder with
148 the pile diameter.

$$149 \quad E_p = \frac{EI_p}{\pi D_p^4 / 64} \quad (6)$$

150 The selected properties of the structure are listed in **Table 1**. Hereafter, the pile and tower
151 dimensions are normalized by the monopile diameter (D_p). The Poisson’s ratio of steel
152 material and soil materials are selected to be a constant value of 0.3 because the influence of
153 Poisson’s ratio change is relatively small. However, if it is necessary, a Poisson’s ratio factor
154 proposed by [Randolph \(1981\)](#) can be applied to this study.

155 **2.2 Study range**

156 Taking the currently installed pile dimensions and its expected increase in the future into
157 account, the probable range of the slenderness ratio (L_p/D_p) is between four and twenty. Six
158 slenderness ratios (L_p/D_p) 4, 6, 8, 10, 16 and 20 are selected as representative values. The
159 stiffness ratios (E_p/E_s) are set between 0.2e3 and 960e3, that accounts for the situations to
160 install monopiles into the softest soil ($E_s = 0.075$ MPa, non-consolidated soft clay) to the
161 stiffest soil ($E_s = 361$ MPa, soft rock). $E_p = 72.2$ GPa is selected in the reference model. The

162 ten different stiffness ratios selected for the analyses and the corresponding soil stiffness are
163 listed in **Table 2** with the soil numbering from one to ten.

164 **2.3 Boundary conditions**

165 Since the geometry and motion of a WTG structure are supposed to be symmetric against
166 a plane parallel to the applied horizontal loading, only the half of the structure and the soil
167 are modelled. The analysis domain is selected sufficiently large to avoid any boundary effects.
168 The depth of the soil is at least twice as large as the pile length and the distance from the pile
169 to the vertical side boundaries is twenty times larger than the pile diameter.

170 **2.4 Pile modelling**

171 Winkler approach is often used as a reference model of parametric studies or in the
172 detailed design phase of monopiles and transition pieces (Bhattacharya 2019). In the
173 approach, piles are modelled as a beam element without volume and the soil modelled as
174 numbers of discs in layers with a stiffness. An advantage of using this simplified model is that
175 the global pile deformation and internal forces (shear force and bending moment) can be
176 easily estimated. However, the pile diameter size effect shall be accounted for in this study
177 because the volume effect of the pile is not negligible due to the relatively small slenderness
178 ratio of monopiles. Hence, a hybrid model (**Fig. 3**) composed of a beam element in the middle
179 of a cylindrical solid element is employed to model a monopile taking the size effect of pile
180 diameter into account. The bending stiffness of the pile is shared by the beam element and
181 the solid element. The solid element is modelled to have the total weight of the monopile.

182 An important factor of the hybrid model is the ratio between the bending stiffness of the
183 beam element and the solid element. The larger the bending stiffness of the beam element is
184 to the solid element, the closer the mechanical behaviour of the hybrid model is to a single
185 beam. However, the bending stiffness of the solid element should be large enough to ensure

186 the relatively stiff property of the monopile with respect to the soil at the same time so that
187 pile-size effect is well represented. Zhang et al. (2000) made a parametric study to determine
188 the sharing ratio between the stiffness of the beam element to the solid element and selected
189 a ratio: nine to one for their simulation of the lateral pile loading test. The study compares the
190 load-displacement relation and Eigen-period of a pile-ground system for several ratios and
191 periods. Since the results with the ratio adopted by Zhang et al. (2000) look reasonable, the
192 ratio nine to one is adopted for modelling of monopiles in this study. The Young's modulus of
193 the solid element (7.22 GPa, 10% of the pile bending stiffness E_p) is still twenty times larger
194 than the maximum Young's modulus of the soil (E_s) in the study range 361 MPa.

195 To equivalently transmit the applied loadings to the solid element on the hybrid model, a
196 relatively rigid disc element with 50 mm of thickness is embedded at the top of the pile solid
197 element. The tower modelled as a beam element is connected with the beam element of the
198 pile at the seabed level.

199 **2.5 Finite element mesh**

200 3-node line elements are used for the tower beam element and the beam element of the
201 hybrid pile. 6-node plate elements discretize the shell element of the soil-pile interface. The
202 solid element of the hybrid pile and the soil are discretized by 10-node tetrahedral elements.
203 High mesh density is selected along the pile and the soil around it to ensure the accuracy of
204 the load-displacement analysis. The generated mesh in the analysis domain is shown in **Fig. 4**.
205 Parameters for the soil-structure interface element are selected as the same as the
206 surrounding soil and no slipping at the interface is accounted.

207

208 **3. FE static load-displacement analysis**

209 **3.1 Overview**

210 Sixty cases of FE static load-displacement analyses with combinations of the ten stiffness
 211 ratios and the six slenderness ratios are investigated in order to estimate the formulae of the
 212 non-dimensional stiffness (η_L , η_{LR} and η_R) as function of the stiffness ratio and slenderness
 213 ratio. Before the main analyses, the size effect of scaled piles with different diameters (1 m
 214 and 4 m) is investigated to confirm that the pile diameter in the FE analyses does not affect
 215 the non-dimensional stiffness values estimation. Hence, the scaled pile model diameter is set
 216 to the unit length 1m. A horizontal load H_1 in **Eqn. (7)** and a moment M_2 in **Eqn. (8)** are applied
 217 respectively at the seabed level and the lateral displacements u and rotations ϑ are obtained
 218 as the FE analysis results.

$$219 \begin{bmatrix} H_1 \\ 0 \end{bmatrix} = \begin{bmatrix} K_L & K_{LR} \\ K_{RL} & K_R \end{bmatrix} \begin{bmatrix} u_1 \\ \theta_1 \end{bmatrix} \quad (7)$$

$$220 \begin{bmatrix} 0 \\ M_2 \end{bmatrix} = \begin{bmatrix} K_L & K_{LR} \\ K_{RL} & K_R \end{bmatrix} \begin{bmatrix} u_2 \\ \theta_2 \end{bmatrix} \quad (8)$$

221 The four elements of the stiffness matrix are obtained solving the two matrices **Eqns. (7) and**
 222 **(8)**. As the coupling spring stiffness K_{LR} and K_{RL} are the same in the elastic domain, K_{LR} is
 223 used as the representative notation hereafter.

224 3.2 Non-dimensional spring stiffness

225 The SR spring stiffnesses (K_L , K_{LR} and K_R) acquired in the analysis are converted into the
 226 non-dimensional stiffnesses (η_L , η_{LR} and η_R) defined in the **Eqns. (2), (3) and (4)** which are
 227 plotted on double-logarithmic graphs. **Figure 5** shows the results of η_L as a function of
 228 stiffness ratio (E_p/E_s) with six different slenderness ratios (L_p/D_p). The plotted result lines
 229 overlap one another when the stiffness ratio (E_p/E_s) is relatively small. In this study, piles in
 230 this range are defined as long-flexible piles. Non-dimensional SR spring stiffness of the flexible
 231 piles is described by a power function of the stiffness ratio (E_p/E_s). On the other hand, there
 232 is a range where the spring stiffness converges into a constant value for each slenderness ratio

233 (L_p/D_p) . Piles in this range are categorized as perfectly rigid piles whose non-dimensional
234 stiffness is as a power function of the slenderness ratio. The results show that the non-
235 dimensional spring stiffness η_L of intermediate (short-rigid) piles is as a function of both the
236 stiffness ratio (E_p/E_s) and the slenderness ratio (L_p/D_p) . The same trend is observed on the
237 plots of η_{LR} and η_R .

238 **3.3 Deformations and internal forces diagrams**

239 To associate the behaviour of piles in soil with the observed SR spring stiffness trends, the
240 deformations and internal forces observed on the whole embedded piles are also investigated,
241 even though only the deformations at the seabed level are required for the determination of
242 the SR spring stiffness. Pile deflection diagrams of the hybrid model when a moment is applied
243 at the seabed level are shown in **Fig. 6** (lateral displacement diagram) and **Fig. 7** (rotation
244 diagram) for $L_p/D_p = 8$, as an example. The moment is applied in a clockwise direction.
245 Notation for the displacement to the right-hand side and for the rotation in counter-clockwise
246 are defined positive. To plot the results for different soil stiffness, the lateral displacements
247 are normalized to its maximum absolute value at the seabed level.

248 As for the lateral displacement diagram (**Fig.6**), the six cases with different soil stiffness can
249 be globally categorized into the two groups by their deformation patterns. The piles
250 embedded in Soil 1 and 2 seem to be categorized into flexible piles from the deflection
251 patterns. In contrast to Soil 1 case where there is no pile toe displacement, a small pile toe
252 displacement is observed in the pile in Soil 2. It tells that the definition of long-flexible piles in
253 this study does not necessarily mean that the pile length is large enough to show the same
254 behaviour of pile toe as an infinitely long pile. However, the influence of the slenderness ratio
255 (L_p/D_p) is insignificant. As for the piles in Soils 3, 4 and 5, the deflection diagrams and the
256 moment diagrams of these piles are very similar to that of the perfectly rigid pile in Soil 10,

257 despite that their SR spring stiffness values seem to be in the range of long-flexible or short-
258 rigid piles in **Fig. 5**.

259 As is the case with the lateral displacements, the internal forces (moment, shear force)
260 diagrams are very similar among the piles in Soils 3 ~ 10. On the other hand, the rotation
261 diagram (**Fig. 7**) of piles in Soil 1 ~ 5 (flexible to intermediate) show a remarkable difference
262 from one another. No rotational deflection is observed in Soil 10 when the pile is considered
263 as perfectly rigid.

264

265 **4. New formulae proposal**

266 **4.1 Overview**

267 The next step is the proposal of a new set of formulae for the SR spring stiffness estimation
268 by means of curve fittings on the double-logarithmic graphs with the stiffness ratio on the x-
269 axis and the non-dimensional stiffness η_L , η_{LR} and η_R on the y-axis. The common-logarithm
270 of the non-dimensional stiffness: $\log_{10}(\eta_L)$, $\log_{10}(-\eta_{LR})$, $\log_{10}(\eta_R)$ linearly increases as a
271 function of common-logarithm of the stiffness ratio: $X = \log_{10}(E_p/E_s)$ for flexible piles with
272 relatively small stiffness ratios (E_p/E_s) and it reaches a constant value for rigid piles with
273 relatively large stiffness ratios. The non-dimensional stiffness in the intermediate (short-rigid
274 pile) range depicts non-linear curves connecting the linear function and the constant function.

275 **4.2 Sigmoid**

276 To fit these “S-shaped curves” $f_i(X)$, the sigmoid curve function may well represent the
277 stiffness features. The most generalized sigmoid function is the Richards curve **Eqn. (9)** named
278 after F.J. Richard, who developed the simple logistic growth curve proposed by Pierre Verhulst
279 as a population growth model ([Richards 1959](#); [Bacaër 2011](#)).

280
$$f_i(X) = L_{-\infty} + \frac{L_{+\infty} - L_{-\infty}}{(1 + Ae^{-B(X-x_0)})^{1/C}} \quad (9)$$

281 The Richards curve has six parameters: $L_{+\infty}$, $L_{-\infty}$, x_0 , B , A and C . In this study, four
 282 parameters ($L_{+\infty}$, $L_{-\infty}$, x_0 and B) out of the six are defined as a function of the slenderness
 283 ratio. For the remaining two (A , C), a constant value $A = 1$ and $C = 100$ are selected, and
 284 eliminated from the parameter study.

285 4.3 Parameters

286 The parameters are determined by trial and error using the non-linear least square method
 287 on the Matlab curve fitting toolbox until most of the errors between the FE load-displacement
 288 results and the sigmoid function are within a relatively narrow band ($\pm 3.5\%$ for K_L , $\pm 4\%$ for
 289 K_{LR} and $\pm 7\%$ for K_R). Regarding K_{LR} and K_R , the accuracy of short-rigid piles is prioritized
 290 than that of slender piles because the relatively rigid range is the most important range for
 291 monopile designers. Errors up to 7% and 14% are observed in case of K_{LR} (L_p/D_p : 20) and K_R
 292 (L_p/D_p : 16 and 20), respectively. Each of the four parameters has a special characteristic on
 293 the data curves. These are determined one after another in the following order.

- 294 1. $L_{+\infty}(L_p/D_p)$: Upper asymptote
- 295 2. $x_0(L_p/D_p)$: Inflection point
- 296 3. $L_{-\infty}(L_p/D_p)$: Lower asymptote
- 297 4. $B(L_p/D_p)$: Sharpness of the curve

298 4.4 Proposal and evaluation

299 Twelve closed-form equations to determine the four parameters ($L_{+\infty}$, $L_{-\infty}$, x_0 and B) for
 300 each spring stiffness ratio (η_L , $-\eta_{LR}$ and η_R) are proposed and listed in **Table 3**. The proposed
 301 sigmoid curves (solid lines), the SR spring stiffness values estimated from the FE analysis, the
 302 formulae of [Gazetas \(1984\)](#) (dashed lines) for flexible piles and the formulae of [Higgins et al.](#)

303 (2013) (dotted lines) for rigid piles are plotted on the same graphs (Figs. 8 ~ 10) for comparison.
304 The sigmoid curves match very well with the formulae of Gazetas (1984) in the flexible pile
305 range and with the formulae of Higgins et al. (2013) in the perfectly rigid pile range.

306

307 5. First natural frequency estimation

308 5.1 Overview

309 According to the previous studies such as Zaijier (2006) and Arany (2016), the effect of the
310 soil-pile interaction is often too large to be ignored in the natural frequency estimation of
311 WTG structures. The actual natural frequencies f_n accounting for the foundation stiffness are
312 lower than the fixed-base frequencies f_{FB} assuming cantilever beam with a rigid boundary
313 condition at seabed level. Free vibration study in an analytical method is performed on a
314 simplified tower structure supported by the SR spring system, and the first natural frequency
315 of the structure f_1 is estimated based on the Euler-Bernoulli beam theory applicable to flexible
316 structures. Arany et al. (2015) report that the Euler-Bernoulli beam model is accurate enough
317 to calculate the first natural frequency of a WTG tower because the effect of shear
318 deformation is not significant.

319 5.2 Equation of motion

320 The first natural frequency of the structure is estimated by a free vibration on an Euler-
321 Bernoulli beam supported by the SR springs. The equation of motion for an Euler-Bernoulli
322 beam is written as the following Eqn. (10).

$$323 M\ddot{u} + Ku = 0 \quad (10)$$

324 As the tower has a distributed mass along the structure, calculation of this matrix equation is
325 computationally expensive. For further simplification of the model to reduce the number of
326 the degree of freedom, an effective mass (M_{eff}) at the top of the tower is introduced. The

327 effective mass is defined by equating the kinematic energy of the cantilever beam with the
 328 distributed mass and that of a cantilever beam with an effective mass (M_{eff}) at the top of the
 329 beam. Following the calculation documented in Gürgöze (2005), M_{eff} is expressed as a
 330 function of the mass of the tower m_t and the mass of the nacelle m_{RNA} as presented in **Eqn.**
 331 **(11)**.

$$332 \quad M_{eff} = \frac{33}{140} m_t + m_{RNA} \quad (11)$$

333 This simplified model has three degrees of freedom. The first nodal displacement u is the
 334 relative displacement between the tower top and the seabed in the horizontal direction. The
 335 second nodal displacement u_b and the third nodal rotation θ_b is at the seabed in the lateral
 336 direction and the rotational direction, respectively. The absolute nodal displacement vector
 337 in the equation of motion (**Eqn. (10)**) is written in **Eqn. (12)**. This method is used by [Chowdhury](#)
 338 [\(2016\)](#).

$$339 \quad \mathbb{u} = \begin{bmatrix} u + u_b + \theta_b L_t \\ u_b \\ \theta_b \end{bmatrix} (12)$$

340 **5.3 First natural frequency**

341 Supposing that the nodal displacement vector has a harmonic solution, the equation of
 342 motion (10) is transformed into **Eqn. (13)**.

$$343 \quad (\mathbb{K} - \omega^2 \mathbb{M}) \mathbb{u} = \mathbb{0} \quad (13)$$

344 The mass matrix **Eqn. (14)** is defined only with the effective mass when the mass effect of the
 345 pile is very small.

$$346 \quad \mathbb{M} = \begin{bmatrix} M_{eff} & & \\ & 0 & \\ & & 0 \end{bmatrix} (14)$$

347 The stiffness matrix **Eqn. (15)** can be obtained by applying either the force method or the
 348 displacement method to the system.

$$349 \quad \mathbb{K} = \begin{bmatrix} K_t & -K_t & -K_t L_t \\ -K_t & K_t + K_L & K_t L_t + K_{LR} \\ -K_t L_t & K_t L_t + K_{RL} & K_t L_t^2 + K_R \end{bmatrix} \quad (15)$$

350 The stiffness matrix elements are described with the combinations of tower lateral stiffness
 351 for the first mode K_t : **Eqn. (16)**, the tower height L_t and the SR spring stiffness (K_L , K_{LR} and
 352 K_R).

$$353 \quad K_t = \frac{3EI_t}{L_t^3} \quad (16)$$

354 To obtain a unique solution from Eq. (13), the determinant of the matrix ($\mathbb{K} - \omega^2 \mathbb{M}$)
 355 should be zero. Introducing the following non-dimensional stiffness normalized by the tower
 356 stiffness and the tower height (ζ_L^* : **Eqn. (17)**, ζ_{LR}^* : **Eqn. (18)**, ζ_R^* : **Eqn. (19)**) and the fixed-base
 357 angular velocity of the first mode ($\omega_{FB,1}$: **Eqn. (20)**), the determinant of the matrix
 358 ($\mathbb{K} - \omega^2 \mathbb{M}$) is calculated.

$$359 \quad \zeta_L^* = \frac{K_L}{K_t} \quad (17)$$

$$360 \quad \zeta_{LR}^* = \frac{K_{LR}}{K_t L_t} \quad (18)$$

$$361 \quad \zeta_R^* = \frac{K_R}{K_t L_t^2} \quad (19)$$

$$362 \quad \omega_{FB,1} = \sqrt{\frac{K_t}{M_{eff}}} \quad (20)$$

363 The ratio of the natural frequency of the structure to the fixed-base frequency for the first
 364 mode ($f_1/f_{FB,1}$) is estimated in the following **Eqn. (21)** with the soil-pile-tower interaction
 365 factor χ : **Eqn. (22)**.

$$366 \quad \frac{f_1}{f_{FB,1}} = \frac{\omega_1}{\omega_{FB,1}} = \sqrt{1 - \frac{1}{1+\chi}} \quad (21)$$

$$367 \quad \chi = \frac{\zeta_L^* \zeta_R^* - \zeta_{LR}^{*2}}{\zeta_L^* - 2\zeta_{LR}^* + \zeta_R^*} = \frac{1}{K_t} \cdot \frac{K_L \cdot K_R - K_{LR}^2}{K_L L_t^2 - 2K_{LR} L_t + K_R} \quad (22)$$

368 The factor χ is the same for the proportionally enlarged structure models because it only
369 depends on the non-dimensional spring stiffness normalized by the tower stiffness.

370

371 **6. FE free vibration analysis**

372 **6.1 Overview**

373 As the factor χ does not change, even if the structure size proportionally enlarged, the pile
374 and tower model with $D_p = 1$ m is used in the FE free vibration analysis to validate the natural
375 frequencies estimated in the theoretical calculation. In this study, m_{RNA} is defined as zero and
376 m_t is calculated from the density of steel and the volume of the tower. A Newmark β value of
377 one-fourth is selected to ensure the calculation stability and at least five structural vibration
378 waves are investigated after releasing a horizontal loading at the tower head. Even though the
379 material damping ratios of the soil, pile and tower are set to zero, an unknown damping ratio
380 ξ , which is defined by the angular velocity of the first mode ω_1 and the angular velocity with
381 damping ω_D (**Eqn. (23)**), is observed in the results.

$$382 \quad \omega_D = \omega_1 \sqrt{1 - \xi^2} \quad (23)$$

383 **6.2 Natural frequency validation**

384 Eliminating the effect of the damping, the first natural frequencies determined by the FE
385 free vibration analysis (open circles) are compared with the theoretical estimation (solid lines)
386 in **Fig. 11**. Even though the “Inverse-S shaped” curves are well depicted within the theoretical
387 model using the proposed sigmoid functions, the frequency values are overestimated in the
388 cases with small slenderness ratios $L_p/D_p = 4$ and 8. This kind of overestimation of the natural
389 frequency with the SR spring system is also reported in previous studies, for example by [Okada](#)
390 [et al. \(2007\)](#).

391 To investigate a cause of the natural frequency overestimation, a parameter study is
392 conducted. From a free vibration analysis for an extreme case with the pile and soil mass,
393 which is nearly zero, it is confirmed that the effect of the embedded pile mass is negligible in
394 the first natural frequency estimation and not a reason of the overestimation. The
395 overestimation of the natural frequency cannot be explained by the effective mass M_{eff} nor
396 the tower length L_t because the first natural frequency estimation is very accurate in cases
397 with the relatively stiff soil materials ($E_p/E_s=0.2e3$).

398 Hence, the only reason which can explain the natural frequency overestimation is the
399 parameters of the SR spring stiffness. As seen in **Fig. 11**, the SR spring method result curve
400 matches perfectly if a dynamic pile effective stiffness $E_{p_dynamic}$, which is 1.75 times smaller
401 than static pile effective stiffness E_p , is selected to determine the SR spring stiffness in the
402 case of $L_p/D_p = 4$. On the other hand, 2.25 times smaller E_p should be selected as $E_{p_dynamic}$
403 in case of $L_p/D_p = 8$. The dynamic stiffness of the pile to be used in the SR dynamic analysis
404 differs from the static pile stiffness. The statement is supported by the research of [Novak et](#)
405 [al. \(1977\)](#). According to his research group, the modification of pile stiffness and consideration
406 of damping generation are required in dynamic models due to the energy radiation and
407 dissipation caused by the soil-pile interaction. The modification is proposed to be a non-linear
408 change with respect to the soil and pile properties. It is verified that the first natural frequency
409 can be estimated with a high accuracy taking the dynamic pile stiffness effect into account.
410 However, further study is required to obtain the ratio between the static pile effective
411 stiffness E_p and the dynamic pile effective stiffness $E_{p_dynamic}$.

412

413 **7. Contribution of new formulae**

414 **7.1 Overview**

415 The accuracy of the newly proposed formulae is evaluated and justified in this section. The
416 SR spring stiffness, deformations at the seabed level and the first natural frequencies are
417 estimated with the bilinear curves (dashed lines in **Fig. 12**) with an abrupt transition from long-
418 flexible to perfectly rigid behaviour and the newly proposed sigmoid curves (solid lines in **Fig.**
419 **12**). In the following subsections, the results of the two different formulae for three
420 representative slenderness ratios ($L_p/D_p = 4, 8, 20$) are compared. At the end of this section,
421 how to use the new set of formulae in practice is presented.

422 **7.2 SR spring stiffness**

423 The bilinear curves are defined by extrapolating the long-flexible pile linear function and
424 the rigid pile constant function in the double-logarithmic graphs. The limitation of the
425 previously proposed equations is that the behaviour of short-rigid piles is not described in the
426 equations. As seen in **Fig. 12**, the SR spring stiffness is overestimated in the short-rigid pile
427 cases compared to the sigmoid formulae. The errors due to the negligence of the short-rigid
428 pile effects are defined by the following error function (**Eqn. (24)**).

$$429 \quad Error = \left(\frac{X(\text{bilinear})}{X(\text{sigmoid})} - 1 \right) \times 100 [\%] \quad (24)$$

430 X is substituted by η , u , θ or f_1 to calculate the corresponding errors. The maximum errors
431 in the SR spring stiffness estimations among the three slenderness ratios are 16.7% (K_L),
432 28.9% (K_{LR}) and 52.6% (K_R) and are located at the intersections of the bilinear curves for the
433 slenderness ratio $L_p/D_p = 4$. The smaller the slenderness ratio is, the larger errors are
434 observed. This means that contribution of the new formulae is large for short-rigid piles such
435 as monopiles. These SR estimation errors cause errors in the estimation of the deformations
436 and the first natural frequency.

437 **7.3 Deformations at seabed level**

438 The lateral displacement and the rotation intensity is highly depending on the applied static
439 loading at the seabed level. A total (static and dynamic) lateral loading-moment combination
440 under the yearly mean wind speed 20 m/s is estimated to be $(H, M) = (1.79 \text{ MN}, 121.5 \text{ MNm})$
441 according to [Arany et al. \(2014\)](#). These loadings are selected as the largest service state load
442 in this study. Due to the overestimation of the SR spring stiffness, the lateral displacement
443 and the rotation at the seabed level tend to be underestimated in the simple bilinear
444 estimation as seen in **Fig. 13**. The error values hit 41.3% ($4.2e-2$ [m]: sigmoid and $2.5e-2$ [m]:
445 bilinear) for the lateral displacement and 46.9% ($4.9e-3$ [rad]: sigmoid and $2.6e-3$ [rad]:
446 bilinear) for the rotation at $E_p/E_s = 2.5e3$ in the case of $L_p/D_p = 4$.

447 According to [DNVGL-ST-0126 \(2018\)](#), the rotational allowance due to the excitation
448 loadings is 0.25° , which is equal to $4.4e-3$ [rad]. Selecting $L_p/D_p = 4$ as an example, the
449 effective pile stiffness E_p is supposed to be 1.5 times overestimated by the bilinear function
450 ($E_p/E_s = 3.3e3$) than the sigmoid function ($E_p/E_s = 2.2e3$) when the soil stiffness is the same.
451 The difference might be significant in the design phase. The newly proposed sigmoid formulae
452 should be used instead of the bilinear formulae to select sufficient pile dimensions to fulfil the
453 design requirements.

454 **7.4 First natural frequency**

455 As demonstrated in Section 6, the ratio of the first natural frequency of the structure to the
456 fixed-base frequency ($f_1/f_{FB,1}$) is determined by the soil-pile-tower factor χ depending on the
457 non-dimensional spring stiffness normalized by the tower stiffness. The ratio calculated by the
458 sigmoid formulae and the bilinear formulae is compared in **Fig. 14**. The maximum errors of
459 the f_1/f_{FB} are 7.3% ($L_p/D_p = 4, E_p/E_s = 2.5e3$), 10.3% ($L_p/D_p = 8, E_p/E_s = 3.2e4$) and 5.0%
460 ($L_p/D_p = 20, E_p/E_s = 9.3e4$).

461 When the structure fixed-base frequency $f_{FB,1}$ is 0.2 ~ 0.8 Hz, the 10% error of the first
462 natural frequency f_1 can be equal to 0.01 ~ 0.05 Hz. This error range might have a significant
463 influence in some projects in which the first natural frequency of structures should fall within
464 a very small range between rotor frequency: 1P (0.08 ~ 0.22 Hz) and the blades passing
465 frequency: 3P (0.25 ~ 0.65 Hz) (Data source: Arany et al. 2016). In the case of a turbine
466 generation with a capacity of 8 MW or larger, the frequency band between 1P and 3P is only
467 0.03 Hz (0.22 ~ 0.25 Hz).

468 7.5 How to use the formulae

469 This subsection aims to explain how the newly developed formulae can be practically used. If
470 the sequence is once understood and traced in a simple calculation tool such as Microsoft
471 Excel, it will take only a few minutes to calculate (i) SR spring values (K_L , K_{LR} and K_R), (ii)
472 displacement and rotation at seabed level and (iii) first natural frequency of the structure.

473 First of all, the effective pile stiffness E_p , soil stiffness E_s , embedded pile length L_p and pile
474 diameter D_p shall be defined. Secondly, four parameters ($L_{+\infty}$, $L_{-\infty}$, x_0 and B) shall be
475 calculated with **Table 3** by substituting the parameter L_p/D_p with an actual value. The non-
476 dimensional stiffness (η_L , $-\eta_{LR}$ and η_R) can be obtained by substituting the parameter E_p/E_s
477 and the four parameters ($L_{+\infty}$, $L_{-\infty}$, x_0 and B) of **Eqn. (9)**. The remaining two parameters (A
478 and C) excluded from the parameter study are one and a hundred, respectively. Or
479 alternatively, η_L , $-\eta_{LR}$ and η_R can be roughly estimated using **Fig. 8**, **Fig. 9** and **Fig. 10**. Note
480 that η_{LR} is negative. K_L , K_{LR} and K_R shall be calculated using **Eqn. (2)**, **(3)** and **(5)**.

481 **Eqn. (1)** enables to obtain the lateral displacement and rotation at the seabed level. As for the
482 first natural frequency of the structure, please follow the steps written in Section 5.2 and 5.3.

483 The following properties of the tower: mass of tower m_t , mass of nacelle m_{RNA} , diameter of
484 tower D_t , thickness of tower t_t and height of tower L_t are additionally required to estimate the
485 first natural frequency of the structure. With the parameters, the effective mass M_{eff} and K_t
486 shall be defined with the **Eqn. (11)** and **Eqn. (16)**. Assuming that the tower is perfectly fixed to
487 the ground, the fixed-base angular velocity ($\omega_{FB,1}$) or fixed-base frequency ($f_{FB,1}$) is obtained
488 as a function of M_{eff} and K_t . The stiffness ratios between the Sway-Rocking (SR) spring
489 stiffness and the tower stiffness (ζ_L^* , ζ_{LR}^* , ζ_R^*) can be defined by **Eqns. (17), (18) and (19)**. The
490 ratio between the natural frequency of the structure and the fixed-base frequency for the first
491 mode ($f_1/f_{FB,1}$) can be simply determined with the soil-pile-tower interaction factor χ
492 calculated by **Eqn. (22)**.

493

494 **8. Conclusions**

495 A new set of formulae of the Sway-Rocking (SR) extend the application of the SR spring
496 system, which was limited to long-flexible and perfectly rigid foundations, to 'short-rigid' piles.
497 The new SR spring stiffness formulae is a function of the stiffness ratio (E_p/E_s) and
498 slenderness ratio (L_p/D_p). The study range of the soil and pile properties are chosen such that
499 it simulates currently used offshore wind monopile foundations. The methodology to estimate
500 the displacement at seabed level and the first frequency with the SR spring system is also
501 demonstrated in this paper.

502 Improvement of the SR spring stiffness accuracy is maximum of 16.7% (K_L), 28.9% (K_{LR})
503 and 52.6% (K_R) with the slenderness ratio $L_p/D_p = 4$. Along with the improvement of the SR
504 spring stiffness estimation, accuracy of the deformation and natural frequency are also
505 improved. In an example case of a pile with a stiffness ratio $E_p/E_s = 2.5e3$, slenderness ratio
506 $L_p/D_p = 4$, horizontal loading $H = 1.79$ MN and moment $M = 121.5$ MNm, the expected

507 deformation differs maximum by 41.3 % for the lateral displacement and by 46.9% for the
508 rotation. The ratio of the first natural frequency to the fixed-base frequency $f_1/f_{FB,1}$ can be
509 estimated 10.3 % more accurately with the new formulae than the existing bilinear formulae
510 ($E_p/E_s = 3.2e4$ and $L_p/D_p = 8$). The first natural frequency considering the soil-pile
511 interaction can be well estimated by this method. However, the accuracy of the estimation
512 may be improved by introducing the “dynamic” pile effective stiffness $E_{p_dynamic}$ instead of
513 the “static” pile stiffness E_p .

514

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520

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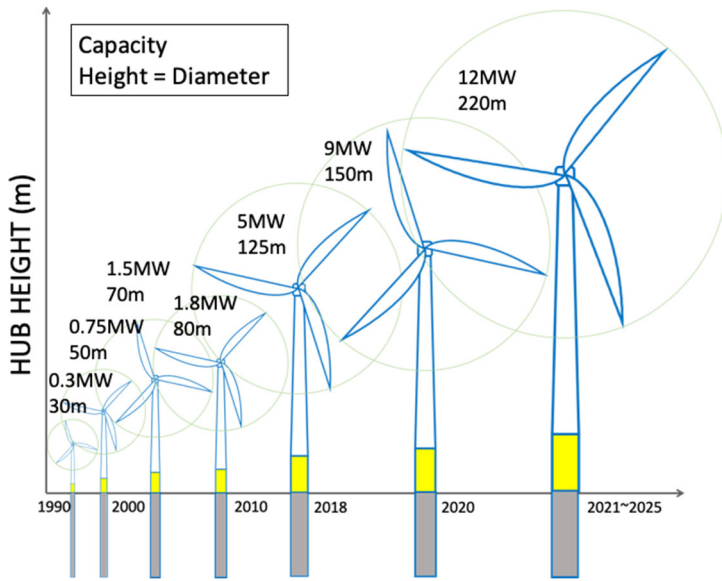
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Nomenclature

H	Lateral force at the seabed [N]	E_s	Young's modulus of soil [Pa]	m_t	Total mass of tower [kg]
M	Moment at the seabed [Nm]	E_p	Effective pile stiffness [Pa]	m_{RNA}	Mass of RNA [kg]
u	Lateral displacement [m]	ν	Poisson's ratio [-]	ω_1	Angular velocity (1 st mode) [rad/s]
θ	Rotational displacement [rad]	L_p	Embedded length of pile [m]	ω_{FB}	Fixed-base angular velocity [rad/s]
K_L	Lateral spring stiffness [N/m]	D_p	Diameter of pile [m]	f_1	Natural frequency (1 st mode) [Hz]
K_{LR}	Coupling spring 1 stiffness [N/rad]	t_p	Wall thickness of pile [m]	f_{FB}	Fixed-base natural frequency [Hz]
K_{RL}	Coupling spring 2 stiffness [Nm/m]	EI_p	Bending stiffness of pile [Nm ²]	ζ_L	K_L normalized by K_t and L_t [-]
K_R	Rotational spring stiffness [Nm/rad]	L_t	Tower height [m]	ζ_{LR}	K_{LR} normalized by K_t and L_t [-]
η_L	K_L normalized by E_s and D_p [-]	D_t	Diameter of tower [m]	ζ_R	K_R normalized by K_t and L_t [-]
η_{LR}	K_{LR} normalized by E_s and D_p [-]	t_t	Wall thickness of tower [m]	χ	Soil-pile-tower interaction factor [-]
η_{RL}	K_{RL} normalized by E_s and D_p [-]	EI_t	Bending stiffness of tower [Nm ²]	ω_D	Angular velocity with damping [rad/s]
η_R	K_R normalized by E_s and D_p [-]	K_t	Tower stiffness in lateral way [N/m]	ξ	Damping ratio [-]
		M_{eff}	Effective mass at the top of tower [kg]	$E_{p_dynamic}$	Dynamic pile effective stiffness [Pa]

600 **Figures**

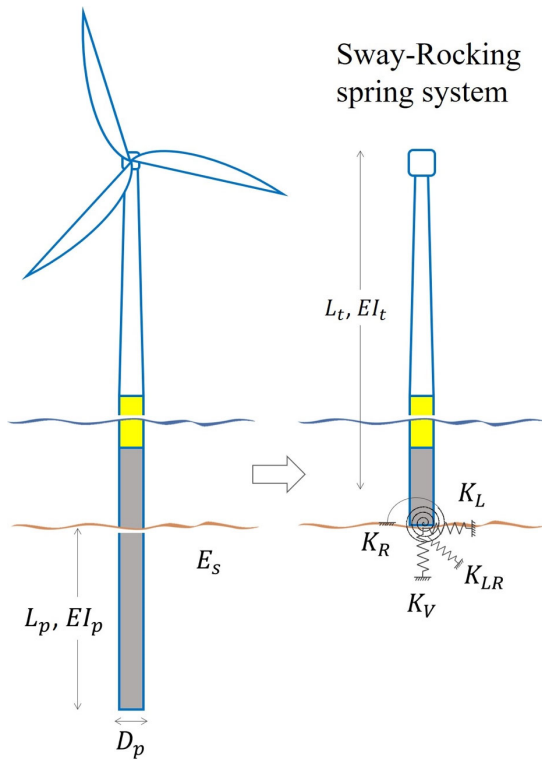


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602 Fig. 1. Growth in size of commercial turbines

603 (Data source: Wind-Europe 2017; 4C Offshore 2018; and IRENA 2019)

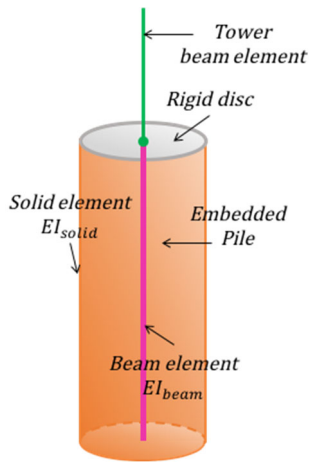
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606 Fig. 2. Sway-Rocking spring system

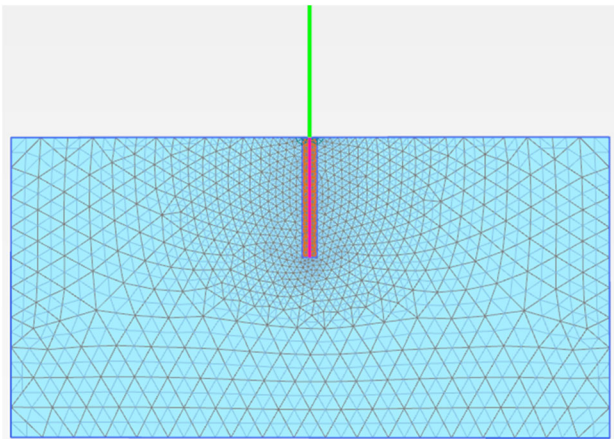
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609 Fig. 3. Hybrid pile model

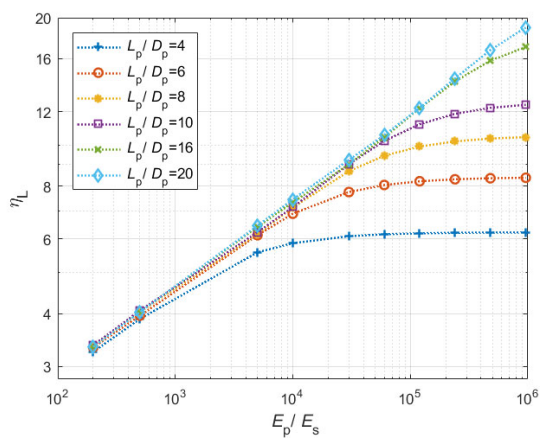
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612 Fig. 4. Generated mesh in the analysis domain ($L_p/D_p = 8$)

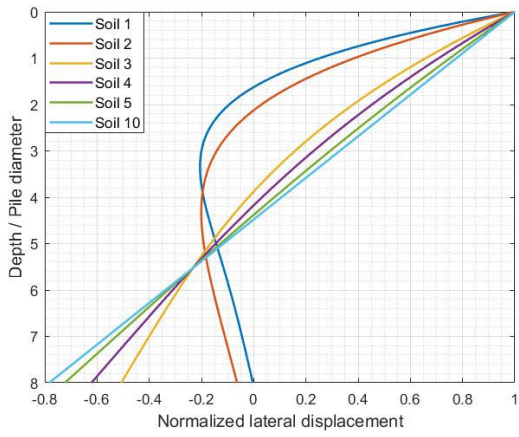
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615 Fig. 5. Non-dimensional stiffness (η_L), FE static load-displacement result

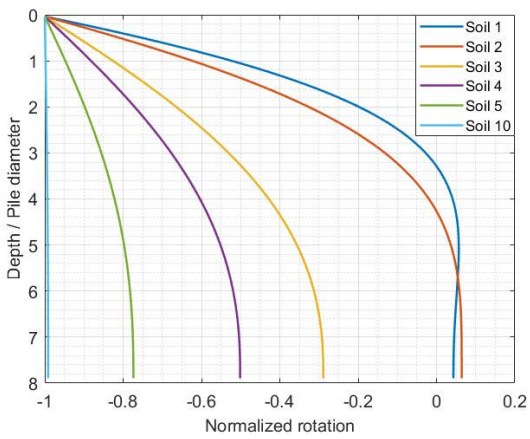
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618 Fig. 6. Lateral displacement diagram (M) where $L_p/D_p = 8$

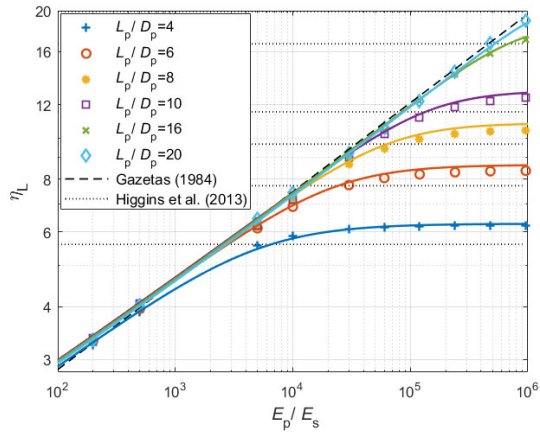
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621 Fig. 7. Rotation diagram (M) where $L_p/D_p = 8$

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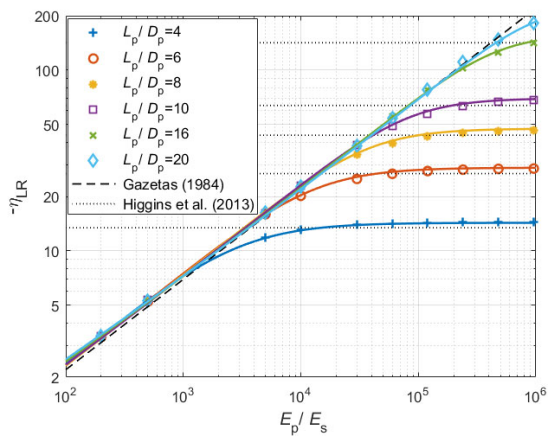


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624 Fig. 8. Non-dimensional stiffness (η_L), FE static load-displacement analysis vs. Sigmoid

625 curves and curves appear in literature

626

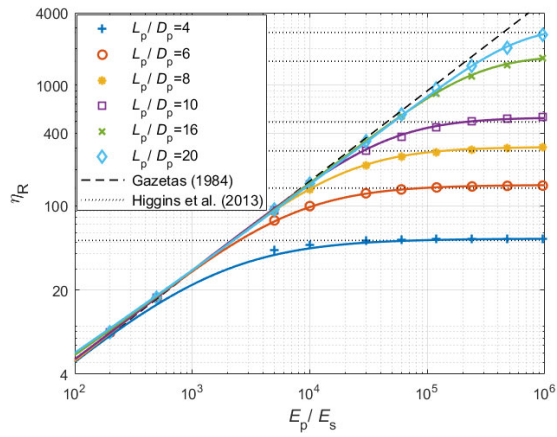


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628 Fig. 9. Non-dimensional stiffness ($-\eta_{LR}$), FE static load-displacement analysis vs. Sigmoid

629 curves and curves appear in literature

630

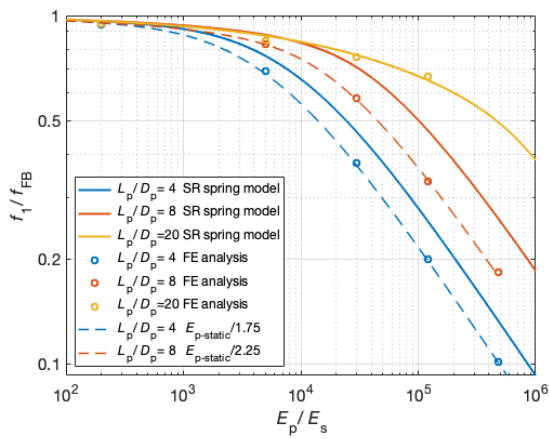


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632 Fig. 10. Non-dimensional stiffness (η_R), FE static load-displacement analysis vs. Sigmoid

633 curves and curves appear in literature

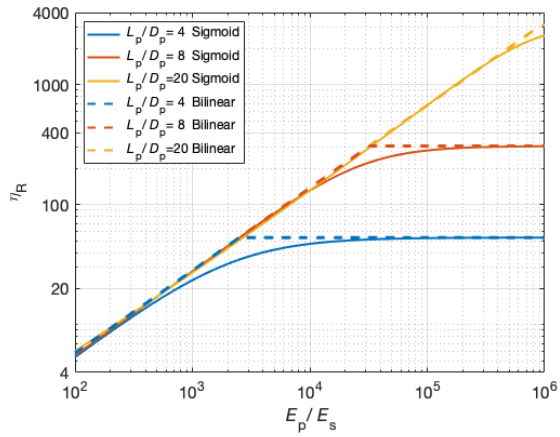
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636 Fig. 11. Comparison of f_1/f_{FB} , Theoretical estimation vs. FE free vibration analysis

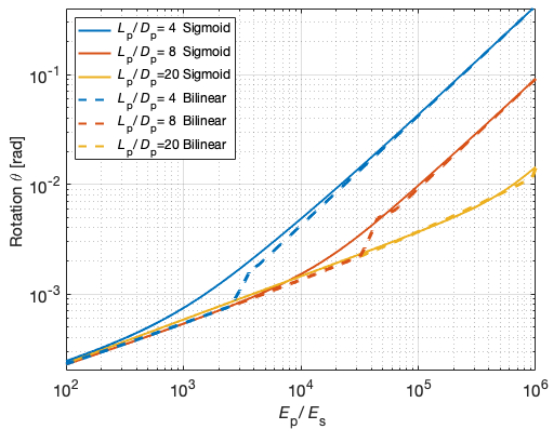
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639 Fig. 12. Non-dimensional SR spring stiffness (η_L), Sigmoid curves vs. Bilinear curves

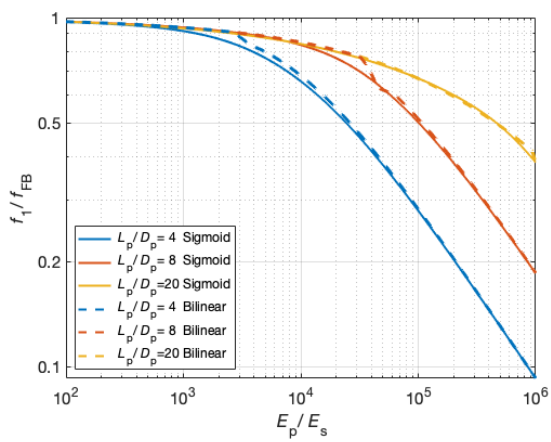
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642 Fig. 13. Estimation of rotation, Sigmoid curves vs. Bilinear curves

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645 Fig. 14. Estimation of the first natural frequency, Sigmoid curves vs. Bilinear curves

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647 **Tables**

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649 Table 1. Fixed properties of the tower and monopile of the reference model

Tower Diameter	Tower wall thickness	Tower height	Pile Diameter	Pile wall thickness	Steel unit weight	Young's modulus of steel	Effective Young's modulus of piles
D_t	t_t	L_t	D_p	t_p	γ_{steel}	E_{steel}	E_p
0.75 m	0.03 m	20 m	1.00 m	0.05 m	78 kN/m ³	210 GPa	72.2 GPa

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651 Table 2. Ten selected stiffness ratios and corresponding soil stiffness

	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5	Soil 6	Soil 7	Soil 8	Soil 9	Soil 10
E_p/E_s [-]	0.2e3	0.5e3	5.0e3	10e3	30e3	60e3	120e3	240e3	480e3	960e3
E_s [MPa]	361	144	14.4	7.22	2.40	1.20	0.60	0.30	0.15	0.075

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653 Table 3. List of closed-form equations for the four parameters

Parameter $L_{+\infty}$	$L_{+\infty}(\eta_L) = \log_{10} \left(2.1 \cdot (L_p/D_p)^{0.79} \right)$
	$L_{+\infty}(-\eta_{LR}) = \log_{10} \left(1.3 \cdot (L_p/D_p)^{1.73} \right)$
	$L_{+\infty}(\eta_R) = \log_{10} \left(1.6 \cdot (L_p/D_p)^{2.53} \right)$
Parameter $L_{-\infty}$	$L_{-\infty}(\eta_L) = -13 \cdot (L_p/D_p)^{-0.35}$
	$L_{-\infty}(-\eta_{LR}) = -32 \cdot (L_p/D_p)^{-0.35}$
	$L_{-\infty}(\eta_R) = -89 \cdot (L_p/D_p)^{-0.55}$
Parameter x_0	$x_0(\eta_L) = 3.68 \cdot \log_{10}(L_p/D_p) + 1.47$
	$x_0(-\eta_{LR}) = 3.68 \cdot \log_{10}(L_p/D_p) + 1.22$
	$x_0(\eta_R) = 3.68 \cdot \log_{10}(L_p/D_p) + 0.97$
Parameter B	$B(\eta_L) = 1.5 \cdot (L_p/D_p)^{0.31}$
	$B(-\eta_{LR}) = 1.9 \cdot (L_p/D_p)^{0.24}$
	$B(\eta_R) = 1.1 \cdot (L_p/D_p)^{0.41}$

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