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Simultaneous robust optimization of tuned mass damper and active control system

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***Abstract*—This paper presents a new design method for an active tuned-mass-damper (AMD) system. A tuned-mass-damper (TMD) is one of the common passive-control methods. A TMD reduces a response for resonance frequency. Recently, an AMD, which applies a control engineering for a TMD, is used to increase control performance. Usually, the TMD and the control system of an AMD are designed separately and some nonstructural member or nonlinear characteristics are ignored. This paper presents a simple method that designs both the TMD and the control system of the AMD simultaneously and considers the uncertainties of a structure. The numerical example uses a single-degree-of-freedom model with an AMD and several earthquake waves. The numerical example demonstrates that the presented method suppresses the displacement without increasing absolute acceleration.**

***Index Terms*—Active control, Robust control, Tuned mass damper (TMD), Active tuned mass damper (AMD), linear matrix inequality (LMI)**

I. Introduction

A tuned mass damper (TMD) has been widely used in civil and mechanical engineering to suppress the influence of disturbances. In civil engineering, high-rise buildings employ a TMD to protect a building from a strong wind or a large earthquake. For example, Taipei 101 [1], Tokyo sky tree [2], Chiba port tower [3], Shanghai tower [4], etc. Since the control performance of a TMD depends on its size of the mass, a large TMD is required to improve the control performance despite a large TMD is a burden on the structure. To solve this problem and to improve the control performance, an active structural control method is applied to a TMD, which is called an active TMD (AMD). An AMD has been employed in many buildings such as the Kyobashi-Seiwa building (Japan), the Yokohama landmark tower (Japan), TC Tower (Taiwan), Nanjing Communication Tower (China), etc. [5]. A TMD is usually designed by the fixed-point theory [6]. This method is for an undamped linear structure and designs an optimal TMD to minimize the response of a main structure.

On the other hand, to date, linear control strategies have been applied to design a control system of an AMD, for example, a linear quadratic regulator (LQR) [7-9], modal control [10], robust control [11-13].

In most cases, the TMD and the control system of an AMD are designed separately. Moreover, although a linear model is used to describe a building, it includes uncertainties because some nonstructural members or nonlinear characteristics are ignored. To present a simple method that designs a TMD and the control system of an AMD simultaneously and to deal with the uncertainties of the models, this paper applies a robust-control strategy via a linear matrix inequality (LMI) that considers perturbation of a mass, damping, and stiffness of the structure.

A numerical example demonstrates the validity of our method by using a single-degree-of-freedom (SDOF) structural model and several earthquake waves. The results show that the presented method suppresses the displacement of the main structure without increasing the absolute acceleration.

II. Structural model

The dynamics of a building with an AMD (Fig. 1) is

$$M_S\ddot{x}(t) + C_S\dot{x}(t) + K_S x(t) = -M_S E_d \ddot{x}_g(t) + E_u u(t). \quad (1)$$

where,

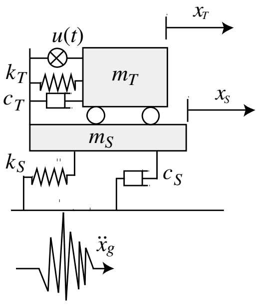


Fig. 1. Model of AMD with an SDOF structure

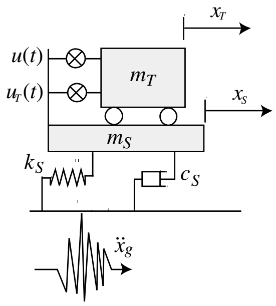


Fig. 2. AMD model with two feedback inputs

$$\begin{cases} x(t)=\begin{bmatrix} x_S(t) \\ x_T(t) \end{bmatrix}, & M_S=\begin{bmatrix} m_S & 0 \\ 0 & m_T \end{bmatrix} \\ K_S=\begin{bmatrix} k_S+k_T & -k_T \\ -k_T & -k_T \end{bmatrix}, & C_S=\begin{bmatrix} c_S+c_T & -c_T \\ -c_T & -c_T \end{bmatrix}, \\ E_u=\begin{bmatrix} 1 \\ -1 \end{bmatrix}, & E_d=\begin{bmatrix} -1 \\ -1 \end{bmatrix}, \end{cases}$$

and, M_S is a mass matrix, C_S is a damping matrix, K_S is a stiffness matrix, E indicates a control input channel, $x_g(t)$ is a displacement of the ground, and $u(t)$ is a control input. The control law is

$$u(t)=K_P\left[x^T(t) \quad \dot{x}^T(t)\right]^T . \tag{2}$$

To apply a robust-control strategy to design both the feedback gain and the damping coefficient and the stiffness of the TMD simultaneously, we regroup (1) as follow:

$$M_S\ddot{x}(t)+C_0\ddot{x}(t)+K_0x(t)=-M_SE_d\ddot{x}_g(t)+E_uu(t)+E_Tu_T(t), \tag{3}$$

where

$$\begin{cases} u_T(t)=K_{PT}\left[x^T(t) \quad \dot{x}^T(t)\right]^T, \quad K_{PT}=\left[K_T \quad C_T\right], \\ K_0=\begin{bmatrix} k_S & 0 \\ 0 & 0 \end{bmatrix}, \qquad C_0=\begin{bmatrix} c_S & 0 \\ 0 & 0 \end{bmatrix}, \\ K_T=\begin{bmatrix} k_T & 0 \\ 0 & -k_T \end{bmatrix}, \quad C_T=\begin{bmatrix} c_T & 0 \\ 0 & -c_T \end{bmatrix} \\ E_T=\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \end{cases}$$

Equation (3) assumes that the restoring and damping force by the TMD has the same function as a state-feedback control input. Thus, (3) indicates that the structure has two feedback inputs (Fig. 2).

The state-space representation of (3) is

$$\dot{z}(t)=Az(t)+Bu(t)+B_Tu_T(t)+B_d\ddot{x}_g(t) \; , \tag{4}$$

where, $z(t)$ is a state of the control system,
A is a system matrix, which determines the dynamic characteristics, B is a input matrix for the active control, B_T is an input matrix for the damping and restoring force of the TMD, and B_d is a disturbance input matrix:

$$\begin{cases} z(t)=\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, & A=\begin{bmatrix} 0 & 1 \\ -M_S^{-1}K_S & -M_S^{-1}C_S \end{bmatrix} \\ B=\begin{bmatrix} 0 \\ -M_S^{-1}E_u \end{bmatrix}, & B_d=\begin{bmatrix} 0 \\ -M_S^{-1}E_d \end{bmatrix}, \\ B_T=\begin{bmatrix} 0 \\ -M_S^{-1}E_T \end{bmatrix}. \end{cases}$$

III. Robust control design

Let consider the following model:

$$\begin{cases} \dot{z}(t)=\left(A+\Delta A\right)z(t)+Bu(t)+B_Tu_T(t)+B_d\ddot{x}_g(t) \\ \psi(t)=C_\psi z(t) \end{cases} , \tag{5}$$

where ψ is a variable to be assessed the control performance and ΔA is an uncertainty of the system matrix, A, and is given by

$$\begin{cases} \Delta A=B_N\Delta(t)N, \\ \Delta A=\left[-\Delta_{MK}M_S^{-1}K_S \quad -\Delta_{MC}M_S^{-1}K_S\right], \quad B_N=\begin{bmatrix} 0 \\ I \end{bmatrix}. \end{cases} \tag{6}$$

In the above equation, Δ_{MK} and Δ_{MC} are the possible errors of $M_S^{-1}K_S$ and $M_S^{-1}C_S$. Substituting (6) into (5) yields

$$\begin{cases} \dot{z}(t)=\Delta Az(t)+Bu(t)+B_Tu_T(t)+B_dd(t)+B_Nw(t) \\ y(t)=Nz(t), \end{cases}$$

where $w(t)$ can be represented by using a scalar $\Delta_i \in R^{n\times n}\ (i=1,2,...s)$

$$\begin{cases} w(t)=\Delta(t)y(t), \quad \|\Delta(t)\|_2\le 1, \\ \Delta(t)=\mathrm{diag}(\Delta_1,\Delta_2,...\Delta_s) \end{cases}$$

For the control system (8), let S is a scaling matrix with $\varepsilon_i \in R(i=1,2,...s)$

$$S=\mathrm{diag}(\varepsilon_1,\varepsilon_2,...\varepsilon_s). \tag{7}$$

Robust stability of the control system (5) with the scalar, S, is guaranteed by the following theorem:

Theorem 1. *The followings are equivalent*

1) The control system (5) is robustly stable

2) There exists a positive symmetrical matrix, X, and appropriate dimension matrix F_T and F such that

$$\begin{bmatrix} XA + A^T X + B_T F_T + F_T^T B_T + BF + F^T B & XN^T & B_N \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{8}$$

where

$$F = [f_1 \quad f_2 \quad f_3 \quad f_4], \quad F_T = \begin{bmatrix} f_{t1} & 0 & f_{t2} & 0 \\ 0 & f_{t1} & 0 & f_{t2} \end{bmatrix}.$$

(9)

Note that in the above equation (9), f_i is a tuning parameter for the feedback controller gain and f_{t1} and f_{t2} are the parameters to adjust k_{TMD} and c_{TMD}, respectively.

Proof. Using a Positive-real Lemma [14] for the control system (5) with $\ddot{x}_g(t) = 0$ yields the LMI (8), and it guarantees robust stability.

This completes the proof.

The controller gain K_P and K_{PT} are given by

$$K_P = FX^{-1}, \quad K_{PT} = \begin{bmatrix} k_T & 0 \\ 0 & c_T \end{bmatrix} = F_T X^{-1}.$$

IV. Numerical Verification

The nominal parameters of the building was m_S =5800 kg k_S= 25442 N/m, c_S = 728 Ns/m, and m_T = 290 kg. This paper assumed that the model includes 30% uncertainties for the stiffness and the damping coefficient, and k_S= 17809 N/m, c_S = 510 Ns/m were used in the numerical simulation. The natural period of the structure, ω, is given by

$$\omega = \sqrt{\frac{k_S}{m_S}}, \tag{9}$$

In this paper, the natural period of the model is about 3.6 s (0.28 Hz).

For the LMI (8), the parameters are selected to be

- $\Delta_{MK} = 0.3$, $\Delta_{MC} = 0.3$,
- $\varepsilon_i = 10^{-10.8}$,
- $X = \mathrm{diag}(X_1, X_1, X_2, X_2)$,
- $F_T = \begin{bmatrix} \alpha F_{T1} & 0 & \beta F_{T2} & 0 \\ 0 & \alpha F_{T1} & 0 & \beta F_{T2} \end{bmatrix}, \quad \alpha = 13, \quad \beta = 10^{-8}$,

  and
- $F = [\phi F_1 \quad \phi F_2 \quad \phi F_3 \quad \phi F_4], \quad \phi = 10^{-8}$.

For the above equations, α and β adjust the stiffness and the damping coefficient of the TMD, k_T, and c_T; and φ adjusts the controller gain.

This paper solves the LMI (8) with the following LMI that optimizes the H_∞ norm of the nominal model:

$$\begin{bmatrix} XA + A^T X + B_T F_T + F_T^T B_T + BF + F^T B & XC_\psi^{\ T} & B_d \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0.$$

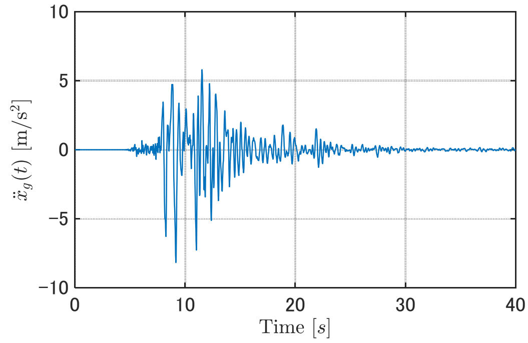
If there exists a positive symmetrical matrix, X (>0), the H_∞ norm of the transfer function that is from B_d to C_Ψ is less than 1.0 and it guarantees that robust stability by small gain theorem.

To demonstrate our method two earthquakes were used:

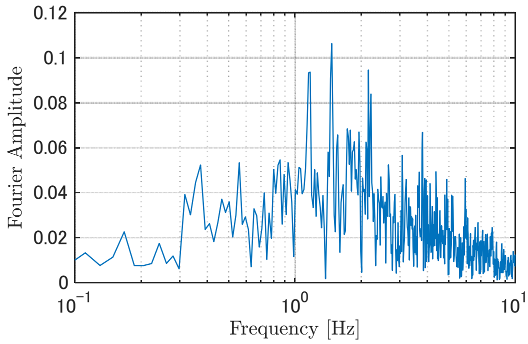
・Kobe wave (Great Hanshin earthquake, 1995)

・El Centro wave (El Centro earthquake, 1940).

The accelerogram and the Fourier amplitude of them are shown as follows.

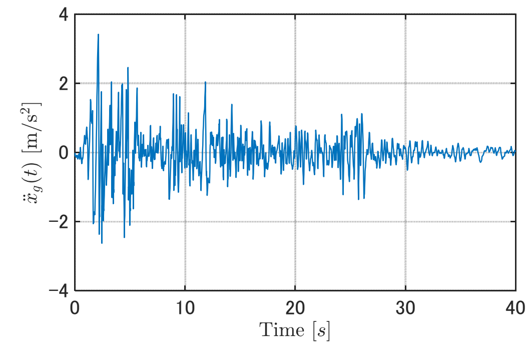


(a) Accelerogram

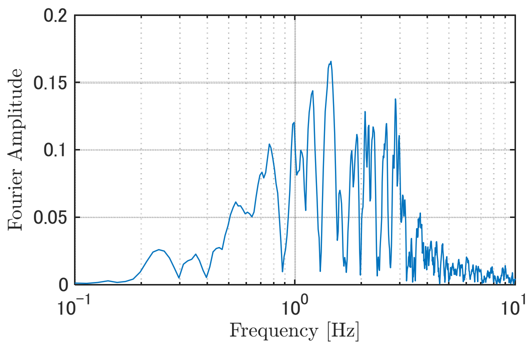


(b) Fourier amplitude

Fig. 3. Kobe wave



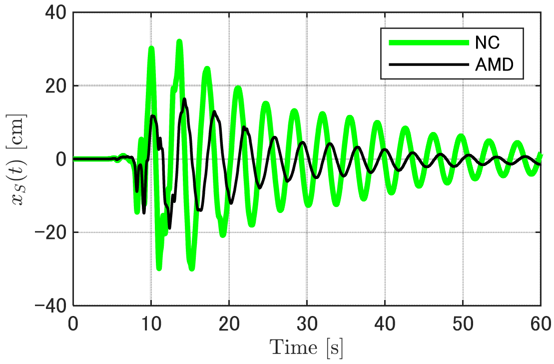
(a) Accelerogram



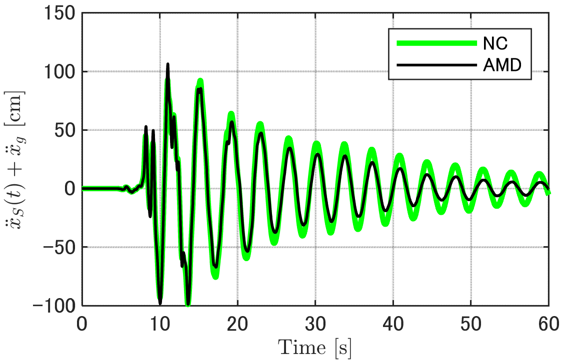
(b) Fourier amplitude

Fig. 4. El Centro wave

Simulation results for the waves are shown in Figs. 5 and 6.

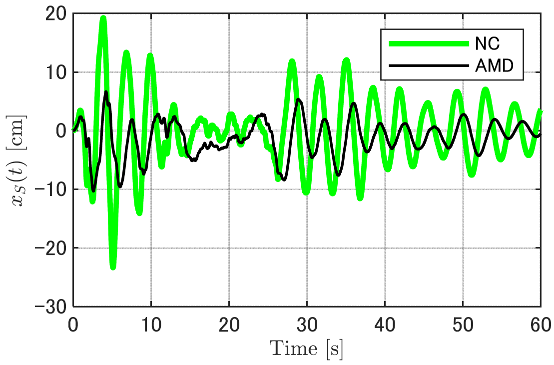


(a) Displacement

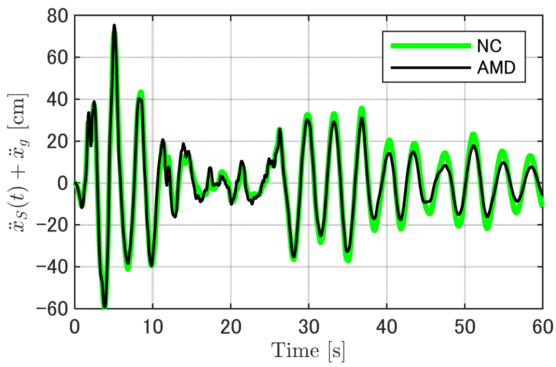


(b) Absolute acceleration

Fig. 5. Simulation results for Kobe wave.



(a) Displacement



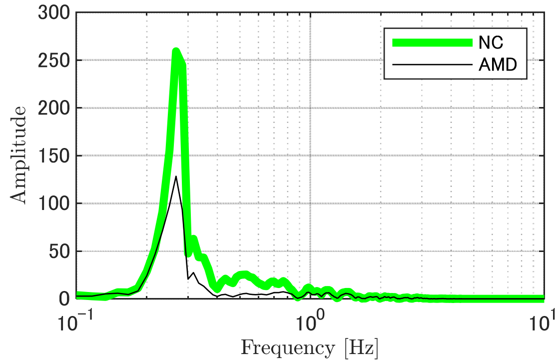
(b) Absolute acceleration

Fig. 6. Simulation results for El Centro wave.

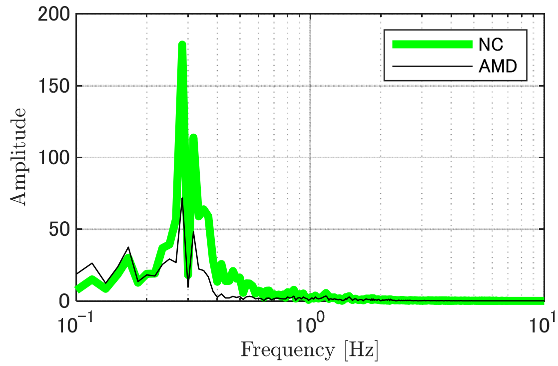
The simulation results show that the presented method suppresses the displacement and the absolute acceleration with structural uncertainties, $\Delta_{MK} = 0.3$ and $\Delta_{MC} = 0.3$.

There is a tradeoff between the displacement and the absolute acceleration of a structure. However, the presented method suppressed both the displacement and the absolute acceleration.

Figures 7 show that the results of fast Fourier transform (FFT) of Figs. 5 and 6.



(a) FFT of displacement for Kobe wave



(a) FFT of displacement for El Centro wave

Fig. 7 Results of FFT for (a) Kobe wave and (b) El Centro wave

As we mentioned before that the natural period of the structure is 0.28 Hz and the dominant frequencies of the results of FFT (Fig. 7) for the earthquakes are 0.28 Hz. The FFT results of AMD for them show that the AMD suppresses the amplitude of the natural frequency component appropriately.

V. Conclusion

This paper presented a new design method for an active tuned mass damper (AMD) based on a robust control strategy in the form of a linear matrix inequality (LMI). An AMD consists of a tuned mass damper (TMD) and a control system, and they are designed separately. To simplify the design proces this paper presented a new method that designs a TMD and a control system simultaneously for an AMD. Moreover, this paper considers the uncertainties of a model because usually, a linear model includes uncertainties. This paper clarified the following points:

1) Since the restoring and damping force of a TMD is the same function as feedback control, a feedback control theory can be applied to design the stiffness and the damping coefficient of a TMD.
2) A TMD and a control system for an AMD can be designed simultaneously using our method and it simplifies the design process of an AMD system.
3) The numerical example shows that an AMD system designed by our method suppresses the displacement without increasing the absolute acceleration of the model.
4) Moreover, the presented method decreases the response while the structural model includes errors.
5) The results of FFT for the simulation showed that the presented method suppresses the component for the resonance frequency.

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