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Doctoral Thesis

Novel Methodologies of Modeling and Analyzing Conflict Resolution with Coarse Information

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Abstract

School of Environment and Society
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Doctor of Science

Novel Methodologies of Modeling and Analyzing Conflict Resolution with Coarse Information

by Yukiko KATO

This study addresses methodologies for conflict resolution. In particular, frameworks for finding solutions that avoid the worst-case scenario for all possible related parties when the information available for analysis is coarse is discussed, and new methods are proposed. As a basic framework, we employed Graph Model for Conflict Resolution (GMCR), a derivative of the non-cooperative game theory. We present methods that can be used to analyze fine and coarse information by integrating a framework that can coarsen the resolution of the analysis when the necessary information, such as the identification of parties involved and their preferences, is not sufficiently complete. In the standard method, analysis is not possible unless information about the elements required for analysis is available. In addition, when sufficient information is difficult to obtain, many procedures set up further assumptions about fuzziness and uncertainty. This study proposes new frameworks and analysis methods that allow us to obtain solutions within the range of coarseness when the information is coarse and replace it with more refined information when available.

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List of Abbreviations

AHP Analytic Hierarchy Process
CGMR Coalition General Metarationality
CNash Coalition Nash stability
CONR Convergence Rechability
CSMR Coalition Symmetric Metarationality
CSEQ Coalition Sequential Stability
DESR De-escalation Reachability
DIVR Divergence Reachability
DM Decision Maker
ELECTRE Elimination Choice Expressing Reality
ESR Escalation Reachability
EXCOMM Executive Committee of the National Security Council
GMCR Graph Model for Conflict Resolution
GMCR-DA GMCR for Disaster Aversion
GMCR-PR GMCR Incorporating Permissible Range
GMR General Metarationality
i.e. id est
LM Limited-Move stability
MCDM Multi Criteria Decision Making
NIS Negative Ideal Solution
NM Non-Myopic stability
PGMR GMR state in GMCR-PR
PIS Positive Ideal Solution
PNash Nash state in GMCR-PR
PR Permissible Range
PROMETHEE Preference Ranking Organization Method for Enrichment of Evaluations
PSEQ SEQ in GMCR-PR
PSMR SMR in GMCR-PR
PV Permissible Value
RG Rapoport and Guyer
SMR Symmetric Metarationality
SEQ Sequential Stability
s.t. such that
ST Stackelberg equilibrium
tGMR time GMR state for DM *i* against credible moves by others
when DM *i* takes only credible moves
tcGMR time credible GMR state for DM *i* against moves by others
tGMR-c time GMR state for DM *i* against credible moves by others
tcGMR-c time credible GMR state for DM *i* against credible moves by others
when DM *i* takes only credible moves
tNash time Nash state
tNash-c time Nash state for DM *i* against credible moves by others

TOPSIS Technique for Order of Preference by Similarity to Ideal Solution
tSMR time SMR state for DM *i* against moves by others
tcSMR time credible SMR state for DM *i* against moves by others
tSMR-c time SMR state for DM *i* against credible moves by others
tcSMR-c time credible SMR state for DM *i* against credible moves by others
tSEQ time SEQ state for DM *i* against moves by others
tcSEQ time credible SEQ state for DM *i* against moves by others
tSEQ-c time SEQ state for DM *i* against credible moves by others
tcSEQ-c time credible SEQ state for DM *i* against credible moves by others when DM *i* takes only credible moves
UI Unilateral Improvement
UM Unilateral Movement
ZF Zermelo–Fraenkel set theory

Chapter 1

Introduction

1.1 Objective of the Research

This study addresses a framework for conflict resolution. In particular, a framework for finding a solution that avoids the worst-case scenario for all possible related parties when the information available for analysis is coarse is discussed, and new methods are proposed. When the interests among decision-makers (DMs), such as individuals, corporations, and nations, may not coincide while they wish to change the situation for the better themselves, some information is needed for the analysis seeking a solution. Although all of the information necessary for an examination is rarely available sufficiently fine to be analyzed, standard analysis methods assume that the required information is available, not always usable for practical applications. In this paper, we present methods that can be used to analyze fine and coarse information by integrating a framework that can coarsen the resolution of the analysis when the necessary information, such as the identification of parties involved and their preferences, is not sufficiently complete.

Conflicts in this study are assumed to be situations involving severe crises that endanger a wide area's human life and health and the environment and economic stability, where experimentation is not feasible. For example, conflicts between countries with the potential for military disputes, the construction of critical monetary or trade systems, or cases involving severe environmental problems. Conflict resolution is about finding equilibrium in the interdependent activities of the parties involved in the conflict as they attempt to change the current state to a state that they each consider desirable. We examine frameworks for decision-making by reviewing the balance and stability of the state of conflict and providing mathematical definitions and frameworks. In international disputes with conflicting national interests, DMs try to gain new stability by exercising rational choices that maximize their own interests while threatening others. Deterrent stability will be established if both decision-makers do not want the conflict to escalate beyond initially expected. However, even in such a case, the gain of the decision-maker that exercises a more robust threat to the limit of outburst is greater. We propose frameworks for such kinds of conflicts in an environment with fine or coarse information density. In particular, we aim to provide a framework that can support decision-making to avoid the worst-case scenario in conflicts when only coarse-density information is available.

The basic framework employed in this research is Graph Model for Conflict Resolution (GMCR), derived from non-cooperative game. In normal form games, the conflict phase is depicted as a combination of strategies taken by DMs, whereas in GMCR, it is viewed as a "possible state." Furthermore, the stability concepts are specified based on the behavioral norm of how many steps ahead the decision-maker is predicting, considering the reactions of other decision-makers to the decision maker's actions. Therefore, it is possible to draw the state transition of conflicts flexibly.

1.2 Background and Underlying Concepts

1.2.1 Decision Theory

In order to manifest the position of the framework we adopt in this paper, we would like to discuss the overall picture of decision-making systems and clarify our ideas before the detailed theoretical study.

In decision making, we generally deal with the following essential elements: a set of "alternatives," a set of "states," a set of "consequences," a set of "maps from alternatives or states to consequences," and "a preference structure for consequences." Although there are some differences in the definitions and terminologies of each item depending on different methods, it is expected that these are the essential elements in the methods discussed in this paper. An "alternative" is a description of a situation that a decision-maker can choose for himself. In a conflict, elements of the situation that are left to the decisions of other decision-makers are not "alternative," and factors that the decision-maker has not yet identified but that may affect their situation, such as climate or global economic trends, are not "alternative." A "state" is an element of a situation that the DM cannot choose or change. It corresponds to the examples of factors that the DM cannot control in the "alternative" mentioned above. Generally, in game theory, analysis is conducted using probability distributions for these "states." However, in GMCR, which is adopted as the basic framework for this research, "state" is defined as a concept that integrates "alternative" and "state" in a narrow sense. The set of "consequences" is a description of the final state obtained by an "alternative" and a "state."

Decision-making requires information about the preference structure, which represents which possible "consequences" are preferred. The preferences must be complete and transitive. Preference is an expression of the utility function that DM possesses. There are two types of utility functions: ordinal functions, to deal only with the order of preferences, and cardinal utility functions, which also deal with the quantitative difference in utility. In the normal form game, ordinal real numbers are used, but in GMCR, which is adopted in this paper, only the preference relation is focused and real numbers are not required. If a preference satisfies completeness and transitivity, then the preference is weakly ordered. The rational DM then chooses actions that will be able to stabilize the state in a higher-order preference. In other words, rationality is the criterion for choosing such actions.

Constructing a framework for a decision-making system provides consistent descriptions and suggestions on the DM's activity. This study aims to support DMs make decisions through comprehensive suggestions based on descriptions and norms.

1.2.2 Coarse Decision Model

As mentioned at the beginning of this thesis, the conflicts envisioned for analysis in this study are between nations with the risk for military confrontations, the collapse of key currencies and trading systems, and disputes involving environmental destruction. It should be noted that the decision theory described in the previous subsection is about a general framework; there are constraints when dealing with *grave crises* due to their nature. Such *grave crises* have either never occurred or have occurred very rarely. Therefore, it is assumed that 1) the amount of information required for analysis by any analyst is considerably limited, and 2) the severity of the crisis makes empirical testing of the model impossible. While it is possible to retrospectively analyze the simultaneous terrorist attacks in 2001, it is impossible to empirically test a model that examines the conditions under which the events occur. If all the environmental dynamics are known, the optimal response to risk can be obtained by dynamic programming based on state transitions and the utility gained from these transitions in a given world. However, when the amount of information available is limited, and the worst-case scenario is known as catastrophic, the choice of model and the appropriateness of the information partitioning used in the model should be deliberately considered in light of the constraints. This study is based on the concept that adopting a coarse framework is rational and functional for decision-making to avoid the worst-case scenario in situations where the availability of information is limited. The assumed rationale that coarse decision-making systems are rational and efficient is presented in Remark 2.1.4 regarding *coarse criteria*.

1.2.3 Underlying Concepts in This Research

In a conflict, when all parties involved are unable to change the current situation to a more favorable one, the conflict is considered to be in equilibrium because the situation is not expected to change any further. There are various notions of stability that each decision-maker can establish, including Nash equilibrium. In addition to Nash equilibrium [69] [70], which is concerned with the stability of one move ahead, this research deals with General Metarationality (GMR) [33] and Sequential Stability (SEQ) [25] [27], which consider two moves ahead, and Symmetric Metarationality (SMR) [33], which considers three moves ahead. These stability concepts allow for more profound interpretations of the stability analysis results, depending on the degree to which the decision-maker has a norm of proactive behavior. The "state" that is each phase of the conflict is considered a particular situation that is commonly recognized as a different phase by all parties involved.

The state is generally considered to be the combination of strategies by the decision-makers in the game theory, whereas, in this research, the state is not determined by the combination of strategies that the decision-makers can take, but rather the "state of the world" in a broader sense, shared as common knowledge by decision-makers. Decision makers' preference for each state is expressed in a linear order, and cardinality is not involved in the description of the degree.

Dealing with coarse information, for example, assumes coping with the existence of unidentifiable parties or the lack of sufficient information about DM preferences. Regarding the former, a typical example is the "prisoner's dilemma." In a standard analysis, the only decision-makers in this conflict are the two prisoners. The interrogator decides their fate, but the interrogator is not considered as a relevant party. In this study, we propose a method that can describe only the prisoner's moves affected by the interrogator's policy without adding the interrogator as a DM in such a case. The fate of the prisoner is affected by the decision on confession and the severity of the sentence. The interrogator in charge may have the authority to make the decision, or the prosecutor's office may have the authority, but in the analysis, it is sufficient to describe only the effects on the prisoner. Suppose it turns out that bribes from the prisoner to the interrogator can facilitate a reduced sentence. In that case, we can add the interrogator to the conflict DMs and switch to a "finer-grained" analysis based on the available information on his preferences.

When the information about preferences is not sufficiently fine, we propose the concept of "permissible range" obtained by estimating the acceptable limit in the preference order and setting a threshold. The setting of permissible range by thresholds enables to treat DMs' preferences as binary information, "permissible or impermissible." In the stability analysis to find a solution to a conflict, the level of analysis depending on the coarseness of the information can be captured as follows: when only very coarse preference information is available and only "permissible (impermissible)" is known, when preference rankings are known for only the permissible range (only the permissible range), and when preference rankings are known for all states. Since preference information is captured on the set of the entire linear order, it can be analyzed in the same framework, even if one DM has fine preference information and the other DM has coarse preference information. Analysis with coarse information may be beneficial for primary analysis of conflicts at the point where information is scarce or at the end of conflicts when negotiations have converged.

1.3 Relevance to the Existing Research

1.3.1 Game Theory

The method of describing conflicts is based on the framework established by von Neumann [71] and Morgenstern [72], who established game theory as an effective means of rigorously investigating conflicts, and by J.F. Nash [69] [70], who found strategic equilibrium for it. Subsequent research has developed game theory in various directions, but the starting point is considered logical construction by von Neumann and Morgenstern. Since then, game theory has been actively studied mainly in economics, but also in biology, computer science, and political science.

Turning to the mathematical aspects, the research on finite games under perfect information by Ernst Zermelo, who published the ZF axiomatic system [30], the foundation of modern mathematics, at the beginning of the twentieth-century, and the research on early game theory by the French mathematician Borel [13] is also significant. This is because game theory is substantially the only framework that can be constructed in an axiomatic system to describe social events. In other words, the fundamental concept of this research is to describe conflict situations with a system of axiomatic characterizations of DMs by factors including their respective preferences, utility, and behavioral standard. Zermelo's "On an Application of Set Theory to the Theory of the Game of Chess" is regarded by the mathematical community as a landmark and is known as the first paper on game theory [81]. Zermelo's work presented that in a two-player zero-sum game with perfect information, he will always win if one player is in a winning position, no matter what strategy the other players adopts. His algorithm is a foundation of game theory, and it has also been used for applications in fields other than finite games.

The normal form game is formulated in terms of three tuples: the DM, the DM's strategy, and the DM's preference. DMs' preferences are generally described by the von Neumann expected utility function. However, we do not deal with quantitative utility in this study but employs static preference relations. Since we do not suppose the probability space, we also do not assume maximizing expected utility through a subjective probability distribution.

Furthermore, in this research, conflicts are regarded as state transitions based on DM's preference for a particular "state." The "state" is the equivalent notion as the shared "information partition" of the "state of the world" on DM's perception in game theory. A precise mathematical formulation of common knowledge was given by Lewis [58] and Aumann [6], who used the concept of information partition in a set-theoretic model. Aumann's model of common knowledge about the structure of a game made it possible to analyze mathematically the properties of events that are common knowledge among players. The basis of this model is a state set Ω that completely describes the world. Each state is viewed as a complete description of the world, including the information and beliefs of the decision-maker and his actions. One way to define the extent of the decision maker's knowledge of the occurring states is to specify an information function that maps every state $\omega \in \Omega$ to a non-empty subset $\mathcal{P}(\omega)$. When a state is ω, the DM knows only that the state is contained in the set $\mathcal{P}(\omega)$. In other words, the DM considers that the true state is one of the states in $\mathcal{P}(\omega)$ and not a state that is not in $\mathcal{P}(\omega)$. In this paper, we assume that the state depicted as this information partition is the common knowledge of the DMs involved in the conflict. As for the perspectives of information structure, equilibrium, and crisis avoidance, this study refers to the concept of global games, which has been studied in the field of economics. As defined by Carlsson and van Damme [17], a global game is a game of incomplete information in which players receive correlated signals about the basic state of the world. The most important practical example of global games is the study of crises in financial markets, such as currency crises and bubbles. There are also examples of applications to economic situations where payoffs show strategic complementarities, such as complementary investments, beauty contests, and political upheavals and revolutions. In game theory, it is known that multiple Nash equilibria can be established as a consequence of the rational actions of each DM. How can this be avoided if one of the equilibria is a serious crisis that is irreversible for the whole? Stephen Morris and Hyun Song Shin considered a stylized currency crisis model in which traders observe the relevant fundamentals with small noise, using a global game framework, and showed that this leads to the choice of only one equilibrium [66] . This result overrides the results of the complete information model, which features multiple equilibria. This study is founded on the concept of the state of the world suggested by these research results. GMCR adopted in this study is one of the solutions to the problem of incorporating into the model the question of what kind of vision of the world each DM has.

Each decision-maker is assumed to act rationally to move from the current situation to more favorable to him. However, suppose the scope of the analysis is not a one-time game, as in normal form games, but by several moves ahead. In that case, the DM does not always necessarily act rationally on every step because he believes that it is possible to pursue the possibility of reaching a better state at the cost of one step. Metagames and GMCRs are frameworks developed based on insights into how DMs perceive conflicts to overcome these limitations of analytical capabilities in the basic framework of game theory.

1.3.2 Metagame

Metagames, developed by Howard [33], improved by Kilgour, Hipel and Fang [54] [23], the founders of GMCR, is a non-quantitative reformulation of the mathematical aspects of game theory. The possible strategies taken by DMs are defined as combinations of options, and the states that result from each DM's preference for strategies are termed scenarios. The goal of each DM is to stabilize the situation in a scenario that is more favorable to him or her.

Suppose a DM is able to change his strategy while all other DMs do not. In that case, this is called a "unilateral move(UM)," and if he can move to a higher preference scenario, this is called a "unilateral improvement (UI)." If a DM has no unilateral improvement from a given scenario (including no unilateral transition), the scenario is “rationally stable” for the DM. In other cases, a DM’s unilateral improvement scenario may result in a further unilateral improvement by another DM, resulting in a shift to a scenario with a lower order of preference for the original DM. This is called “sanctioning.” If others do not sanction the unilateral improvement scenario, it will naturally shift to that scenario, and the original scenario will be “unstable.” On the other hand, if a country is sanctioned, it is forced to stay in the original scenario, making it "sequentially stable. If a scenario is "reasonably stable" or “sequentially stable” for a DM, then it is “stable” for that DM. A scenario in which all DMs are “stable” is called an “equilibrium solution.” The operation of searching for an equilibrium solution in a metagame is specifically called stability analysis. Howard defined two types of stability that can occur in conflicts, GMR and SMR, in addition to Nash, which had been the only stability concept that represented rationality. With these new definitions of stability, it became possible to describe the behavior of a DM with a norm that seeks to improve the situation not only one move ahead but two steps forward.

The idea of states and transitions of states and the concept of stability in metagame can be said to be the foundation of GMCR, which is the basic framework adopted in this paper. Howard’s definition of state transition and the new notion of stability is significant because this research aims to propose a structure for decision-making systems that can be intuitively grasped and manipulated by DMs.

1.3.3 GMCR

Fraser and Hipel simplified metagame analysis and laid the foundation for the GMCR. Later, Kilgour et al. continued this work and developed it into a comprehensive and detailed GMCR [23] [54]. Walker et al.[10] constructed four preferences and four stability concepts based on DM’s’ attitudes. In the stability concepts GMR [33] and SMR [33], only potential sanctions matter, not the DM’s own preferences for sanctioning. However, it is believed that a rational DM must not want to sacrifice himself in the process of sanctioning other DMs. Therefore, Hipel et al. newly defined and proposed the stability concept of SEQ [25] [27] . For a given DM, a state is defined to be SEQ if each UI from the starting state can be blocked in a credible manner. This means that the sanctioning DM provides UI only to try to prevent the opponent from improving.

In GMCR, DM transitions are illustrated as directed graphs. The vertices represent states, and the arcs connecting the states represent transitions between states that the DM has unilateral control over. The four stability concepts of Nash, GMR, SMR, and SEQ, preferences, and graphs made it possible to describe and analyze conflicts intuitively.

Subsequently, comprehensive studies incorporating coalition [40], attitude [11] [10], transition time [39], etc., have been carried out. The engineering approach is the mainstream in the research on DM preferences, with studies considering four aspects: fuzzy preference [1] [8], grey preference [55] [56], the strength of preference [31], and unknown preference [60] [61].

In this study, GMCR will be adopted as the basic framework for modeling and analysis. On the other hand, we will not take an approach in the direction of research requiring more detailed information about preferences. Still, we will examine methods to be able to perform analysis intuitively using less information to study real-world problems. We evaluate that the most significant advantage of GMCR is its simplicity and flexibility. Because, unlike traditional game theory, we can predict where the conflict will be resolved if we know only the preference relations and the DM's behavioral norms, i.e., how many moves ahead the DM is foreseeing. In the development process from game theory to metagames, describing the game in an ordinal manner was used to prioritize intuitiveness. Following this trend, we aim to ensure the usefulness as a decision-making tool in the real world by constructing a simple framework while describing it in an ordinal manner, rather than making it more complicated by conducting quantitative modeling and analysis of the main elements.

1.3.4 Simple Game

Suppose a situation in which information about the DM's preferences is not sufficiently clear in the conflict or a situation in which the preferences of each DM become apparent later in the conflict and are either "permissible or impermissible." The situation can be represented by a "simple game [82] [86]."

A simple game can be considered a game in which an individual's utility is not yet tied to a possible outcome: a game with virtually no utility function [28]. It is a system in which each individual is given a series of strategies to choose from, and the outcome depends on the strategy selected by each individual. For example, suppose we formulate a committee in the framework of a simple game. In that case, we can define a decision-making body of n people as the set $N = \{1, ..., n\}$, and the decision rule can be described by specifying which subset of N ensures acceptance of the alternative. The strategy is "yes or no," and the outcome is either of $\{0, 1\}$. In other words, simple games are a particular type of cooperative games with characteristic functions. In GMCR Incorporating Permissible Range in this research, we propose a method of incorporating coarsened preference information into GMCR by representing it with the concept of simple games. We assume the decision rule is "unanimity," and interpret the "alternative" as the "state." Furthermore, we set a threshold on the DM's preference ranking to obtain the binary options; "permissible or impermissible." Then, a stability analysis is performed, with 1 being a permissible state and 0 being an impermissible state.

The notion of core, the solution for the decision in a committee, is defined using the notion of dominance relations for the set A of all possible alternatives. These concepts were developed in the theory of committee [74] and will be defined where each voter has only one opinion.

Examples of research on applying the core concept to the real world include Edgeworth's discussion of the stability of an allocation in the grand coalition [20] and Walras's general equilibrium [4]. The idea is that, under certain regularities, when applied to an economy with a sufficiently large set of agents, the predictions provided by different game-theoretic solution concepts tend to converge to a set of competitive equilibrium allocations. Moreover, Aumann [5] models the economy as an atomless measure space and presents the following core equivalence theorem. Suppose that the economy consists of a continuum of atomless agents. Then the core is. The core coincides with the set of competitive allocations. He proved that in the presence of uncountable infinite agents, Edgeworthian core [20] and Walrasian equilibrium allocations [4] are equivalent.

This study examines methodologies for finding solutions to severe crises that are difficult or impossible to conduct experiments. Therefore, we do not include detailed theoretical discussions of economics even though these issues concerning the core are fundamental to the decision-making mechanism in conflict. Nevertheless, since a deep insight into the mechanism of social equilibrium is essential for conflict resolution, we believe that the fundamentals of microeconomics, which allow for some empirical research, may contribute to our theory building.

1.3.5 Coarse Information and Categorization

The basic supposition of this thesis is that it is rational for DMs to employ a coarser framework when the amount of information is low, and this is especially true when making decisions that involve significant risks. There is a great deal of insight to be gained from literature in the fields of economics, finance, and psychology about the models and information partitioning that DMs adopt and their validity [2] [14] [63] [65] [83] [89]. Among them, *rational inattention* of DMs under limited information processing capacity, proposed by Sims [84] is remarkable. A priori, we know that it is impossible to solve the problem of temporal imprecision when considering the common knowledge that is the premise of the state of the world in decision-making. In this sense, it can be said to be reasonable to use a coarser granularity [21].

Since we aim to formulate, apply, and examine mathematical models, we will not discuss in detail the validity of logic for the model selection and information partitioning, which are prerequisites for the validatable models, while we clarify our position on the assumptions underlying our proposals on these issues; the theoretical ground for the relationship between the size of the utility/risk and the model and information to be analyzed are fundamental elements of decision making.

In this study, using one framework, GMCR, we propose different analysis methodologies according to the fineness of information required for decision-making in terms of information partitioning perspective. The base concept is *common knowledge* by Aumann [6]. Fundamental axioms of game theory are that DMs are oriented toward maximizing expected utility and that each DM has *common knowledge* of the situation. However, this basic premise is explicitly given even neither in Nash [69] nor in Luce-Raiffa [62]. Generally, in conflict analysis, *common knowledge* is usually grounded without specific definitions given as a premise. It is necessary to clarify the information structure of conflicts to deal with the refinement and coarsening of information about the components of conflicts.

Suppose the world is a set of states. Each state is understood as a complete specification of the relevant facts about the world; each element of the GMCR is seen as a partition of information about the world. For example, the fineness or coarseness of the preference information and the difference between the states obtained by combining options and the states involving other factors can also be explained by *information partitioning*.

We used the concept of *information partitioning* to clearly define the information structure in conflicts, to be able to deal with the fineness and coarseness, and integrated it into the GMCR.

Decision-making systems with *coarse information* are undoubtedly related to the problem of *bounded rationality*. However, in this paper, we do not discuss bounded rationality itself. Designating the DM's rationality as the starting point, we integrate the possible irrationality by developing frameworks with an axiomatic approach aiming for further refined methodology.

Namely, we adopt the GMCR as a plausible platform for coarse decision analysis.

1.3.6 Preference Uncertainty in GMCR

Table 1.1 summarizes the extension concepts in GMCR literature when only uncertain preference information is available.

TABLE 1.1: Methodologies for Preference Uncertainty in GMCR [32]

Type of Preferences	Summary
Unknown [60] [61]	When DM i's preference for two states x and y is unknown, it is denoted by xU_iy (see Definition 2.1.9.)
Fuzzy [1] [8] [9]	Fuzzy sets [96] represent the degree of preference of a DM for one state over another.
Grey [55] [56]	Grey numbers are used for preference elicitation in an uncertain, vague, and fuzzy environment.
Probabilistic [79]	The probability of a preferred state to a given DM is used for the degree of preference.
Combinations	Use a combination of the above methods and explicit preferences.

We can find in Table 1.1 that these methods require more categorization to handle unknown preferences and are computationally more complicated. To solve these frameworks to treat uncertainty, the matrix representation of GMCR has been developed, and more and more detailed frameworks are being studied as more complex calculations become possible. Meanwhile, as mentioned in the previous subsection 1.3.5, our methodologies give primary importance to the appropriateness of information partitioning for criteria in making decisions. It is considered to be more rational to use a coarser framework for analysis in situations where sufficient information is not available. The methodologies we propose in this thesis are grounded on an entirely different premise from the structurally intricate system-oriented methods.

1.4 Novelty of the Research

In the standard method, analysis is not possible unless information about the elements required for analysis is available. In addition, when sufficient information is difficult to obtain, many methods set up further assumptions about fuzziness and uncertainty. In this study, we propose a new framework and analysis methods that allow us to obtain solutions within the range of coarseness when the information is coarse, and to replace it with finer information when it is available.

As outlined in the previous section, a conflict resolution framework requires information on DMs, possible states, DM's state transition, and each DM's preferences for states. However, it is not easy to obtain all the information with the same fineness needed for analysis in an actual decision-making situation. It might often be difficult for a DM in a conflict to identify other DMs and their preferences, even if one can roughly predict the possible states. In GMCR, possible states are generated by combinations of options considered to be clarified by DMs. However, let's assume the perceptions in the real world. It may be that what is required for risky and critical decision-making is how to deal with "situations that DMs know they want to avoid, although it is not clear who will cause them and based on what preferences" in the first place. This paper proposes three new modeling and analysis methods to answer such questions, which are challenging to address with existing frameworks:

1) GMCR Incorporating Permissible Range, 2) Preference Order Setting for Disaster Aversion, 3) New Reachability by Unspecified External Causes.

In *"GMCR Incorporating Permissible Range"*, we integrate the concept of simple games into GMCR and present an analysis method for coarse preference information. It is a method to evaluate a state as either "permissible" or "impermissible" in the case of the coarsest information about DM's preferences, where the roughest information is known, but the exact preference order is not. Although GMCR evolved from non-cooperative games, it is reasonable to assume that in the real world, there is an implicit agreement to avoid such situations whenever possible in conflicts where the worst-case scenario is significantly severe to all parties and irreversible.

Suppose a serious contentious situation, such as conflicts that involve the sacrifice of human lives by military disputes or catastrophic environmental destruction. In such conflicts, DMs may pursue their interests but may not want to end up losing everything in the conflict. We consider the situation as transitions of states searching for equilibrium with a non-explicit consensus that ultimately avoids the worst. Based on this idea, an attempt to interpret the "core" of simple games in the context of the stability concept of GMCR will be made through the method of setting an permissible range in preference order.

This method enables us to deal with the following three cases: a) only permissible (impermissible) states are known, and all states can be expressed in binary terms: permissible or impermissible, b) within the range of permissible (impermissible) states, the preference order is clear, and the rest are recognized as impermissible (permissible), c) the preference order is all clear. When the information is coarse, method a) can be used, and when the information is fine, method c) can be used, and the analysis can be conducted within the same framework.

In *"Preference Order Setting for Disaster Aversion,"* we propose a method to formulate a preference order "far from the least favorable" when there is insufficient information to develop a preference order. This method can be said to be an inverse version of Option prioritization in GMCR. Option prioritization is one of the methods within the standard GMCR modeling method in which options are ranked and weighted to obtain a preference ranking for all states automatically. In conflicts where the worst-case scenario is grave, it may be prioritized to avoid the worst-case scenario rather than seek a better situation. This method describes the DM's preference for a state far from the worst in the most intuitive way possible. This paper will employ Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), a type of multivariate analysis method, to weight options according to their degree of "unfavorability" and conduct a stability analysis using the obtained preference rankings. In the standard GMCR method, we set the options that each DM can control, combine whether each option is executed or not (y/n), and then eliminate the infeasible states to obtain all states for analysis. In other words, the number of states may become very large unless the modeling is done in a focused way. Furthermore, it is necessary to set preferences for all states in a linear order for each DM, making the modeling and preference setting complicated. However, with *"Preference Order Setting for Disaster Aversion,"* only the degree of unfavorability of options is needed to analyze it, allowing decision making to avoid the worst-case scenario in a simple process.

In *"New Reachability by Unspecified External Causes,"* we assume a situation in which the DM is affected in a conflict. Still, the events that are the source of the effect are not subject to analysis as to which DM causes them and how. For example, as mentioned in Subsection 1.2.3, in the Prisoner's Dilemma, the interrogator does not constitute an element of the conflict in the standard conflict description method, even though the decision of the interrogator determines the fate of the prisoners. All the interrogator needs to know is the confession and the conclusion about the sentence that the interrogator has reached for the prisoner. In GMCR framework, it is possible to describe situations in which, for example, the interrogator changes his decision on sentence. In the standard analysis method, a state is a combination of DMs' options, so it is impossible to describe a state that contains elements other than DM's options. However, there are no restrictions by definition on how states can be formulated, and modeling that incorporates external factors that are not controllable by the DM, such as the interrogator in the prisoner's dilemma, is possible. Furthermore, even if the DM is not identifiable, such as the climate or the global economic situation, it is possible to address only the reachability as an effect of such factors.

In this method, if the influencing agent itself does not have a significant role in the conflict, or if information about it is difficult to obtain, it can be described by focusing only on the transition possibilities as an influence to be received and used for the primary analysis. If it is necessary to consider the preferences of the influencing agent, and if that information is available, the agent can be incorporated into the conflict framework as a DM and analyzed in the usual way. In other words, depending on the information available, the resolution of the analysis can be changed.

1.5 Structure of the Thesis

In this section, the purpose and background of this study, outline of novel methodologies, their relevance to previous studies, and their novelty are reviewed. In Chapter 2, all the definitions and frameworks used in this study, including game theory, GMCR, stability concepts and information refinement and coarsening, are comprehensively presented. We will provide mathematical definitions of the frameworks and each element in the frameworks: DM, preferences, states, state transitions, and stability concepts explained in the previous chapters.

In Chapter 3, we describe in detail the new framework proposed in this study; *"GMCR Incorporating Permissible Range,"* an analytical method incorporating the novel concept of permissible range into the basic framework of GMCR is outlined using examples. In Chapter 4 *"Preference Order Setting for Disaster Aversion,"* a method to obtain a "preference order far from the most undesirable" using TOPSIS is presented and discussed, comparing the results with those of stability analysis using the standard method of preference order. In Chapter 5 *"New Reachability by Unspecified External Factors,"* the new concept of change in state transition possibility due to external factors will be outlined using a case study.

Chapter 6 will evaluate the new frameworks proposed in this study and discuss future issues and possibilities for extension of research.

In each chapter we will apply the newly proposed methodologies described to conflicts in real society or cases presented in previous studies for a deeper analysis. As application cases to examine the proposed structure, we will employ Rapoport and Guyer's Taxonomy [78], a representative study that comprehensively scrutinizes 2×2 games. For conflicts in the real world, we will address the Cuban Missile Crisis, a conflict between the United States and the Soviet Union, and the Elmira Conflict, an environmental pollution dispute in Ontario, Canada. The Cuban Missile Crisis is one of the most severe conflicts in modern history, and yet it is instructive how the worst was averted after two weeks of negotiations. There have also been many studies in conflict resolution on the structure of all phases of the crisis, from its onset to its solution. The Elmira conflict is a case in which Hipel, Kilgour, and Fang, the developers of the GMCR framework, were directly involved as technical consultants [90], and thus is the most widely published case of research using GMCR analysis.

Chapter 2

Models and Definitions

2.1 Basic Elements of Conflict Analysis

In this study, we assume that DMs share knowledge of state space and act to maximize their own utility within that space. Here, knowledge does not necessarily imply that they know about the situation with certainty, but that they know about the feasibility of such a state when making decisions and that this knowledge is common. Each DM makes decisions based on this knowledge. The conflict resolution framework deals with interactive situations where two or more DMs make decisions and determine the final outcome in such world.

This chapter gives definitions of terms and analysis methods in the existing framework commonly used in conflict analysis. Our own proposed methods and detailed descriptions will be explained in the next chapter.

2.1.1 Decision-Maker

Definition 2.1.1 (Agent)**.** The factor that provides the movement that causes changes in conflicts is called *agent*.

Definition 2.1.2 (DM)**.** The agent that makes moves subjectively in the conflict framework is called *decision-maker (DM)*. A set of DM is generally represented as N.

2.1.2 Information

Definition 2.1.3 (Information)**.** Let Ω be a finite set of states, where each state is to be understood as a complete specification of the *relevant facts* about the world. The specification of the world is represented by *information*.

Definition 2.1.4 (Information Partition)**.** Let $\mathcal{P}_i$ represent the partition of Ω by agent i. We consider $\mathcal{P}_i$ as a function that maps from the states of the world to the cells of Ω. An information partition is a partition I of Ω (that is, a collection of subsets of Ω that (1) are pairwise disjoint and (2) whose union covers the entire set Ω); the elements of the partition are called *information sets*. For every $\omega \in \Omega$, $I(\omega)$ represents the *information set* that contains state ω. A partition of a set Ω is a set of non-empty subsets of Ω such that every element ω in Ω is in exactly one of these subsets.

Definition 2.1.5 (State of the world)**.** Ω represents the set of all possible states of the world; a typical element of Ω, e.g., ω, is a *state of the world*. In the framework of GMCR, we call the elements ω as *state*, or s, and a set of states as S.

Definition 2.1.6 (Knowledge Function)**.** If $\mathcal{E}$ is a set of states of the world, and the true state of the world is ω, and $\omega \in \mathcal{E}$, then not only is ω true, but so is $\mathcal{E}$. This understanding can be expressed by the following *knowledge function* equation K:

$$K_i(\mathcal{E}) = \{\omega \in \Omega \mid \mathcal{P}_i(\omega) \subseteq \mathcal{E}\}. \tag{2.1}$$

We can also define a function describing the event "*everyone in N knows*" as follows:

$$K_N(\mathcal{E}) = \{\omega \in \Omega \mid \bigcup_{i \in N} \mathcal{P}_i(\omega) \subseteq \mathcal{E}\}. \tag{2.2}$$

Definition 2.1.7 (Common Knowledge [6])**.** The information function for a set of states Ω is a function K that maps any state $\omega \in \Omega$ to a non-empty subset $K(\omega)$. The conflict resolution framework is valid when such an information set is shared as *common knowledge*; the set of states Ω and the information function K that constitute the *common knowledge* satisfy the two conditions in Theorem 2.1.1.

Theorem 2.1.1 (Aumann [6])**.** *Let $K(\omega)$ denote the smallest set containing ω that is known to all agents. $\mathcal{E}$ is* common knowledge *at ω if and only if $K(\omega)$ satisfies the following two conditions:*

$$\forall \omega \in \Omega, \ \ \omega \in K(\omega), \tag{2.3}$$

and

$$\omega' \in K(\omega), \text{ then } K(\omega') = K(\omega). \tag{2.4}$$

Example 2.1.1 (Prisoners' Dilemma)**.** Suppose the prisoner's dilemma situation. There are four states in the world of decision-making , namely $\mathcal{E}$, about the interrogation for the two prisoners. Then we can describe the conflicts as follows;
$N = \{1,2\}$, $\mathcal{E} = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, where ω_1: Both prisoners confess: both will be sentenced to the most severe punishment, ω_2: Prisoner 1 confesses, and Prisoner 2 does not confess: Prisoner 1, who confessed, will be acquitted because he cooperated with the investigation, while Prisoner 2 will be sentenced to a heavier sentence, ω_3: Prisoner 2 confesses, and Prisoner 1 does not confess : Prisoner 2 will be acquitted, and Prisoner 1 will be sentenced to a heavy sentence, ω_4: Neither of them confesses: they will both receive only a light sentence.

Remark. Associated with Definition 2.1.4, it is also valid to apply $\mathcal{P}_i$ to the set of elements of Ω. Here, for an event $\mathcal{E}$ (i.e., the set of states of the world on the decision making for the two prisoners), let $\mathcal{P}_i(\mathcal{E})$ denote the set of all states of the world that DM i thinks are possible if the true state is ω. Thus, $\mathcal{P}_i(\mathcal{E}) = \bigcup_{\omega \in \mathcal{E}} \mathcal{P}_i(\omega)$ This implies that someone *knows* something.

This means that if the true state is ω, then DM i may be uncertain about what the true state of the world is, but if everything he believes to be possible implies $\mathcal{E}$, then we can say DM i knows $\mathcal{E}$.

The framework of conflicts we address in this paper assumes the situation in Equation 2.2.

Remark (Refinements and Coarsening of Information)*.* Suppose there are four states of the world : $\Omega = \{1,2,3,4\}$, where $\mathcal{P}_1 = \{\{1,2\},\{3,4\}\}$. We can distinguish between more states if we can more information: $\mathcal{P}_1' = \{\{1\},\{2\},\{3\},\{4\}\}$. $\mathcal{P}_1'$ is *finer* than $\mathcal{P}_1$, and $\mathcal{P}_1'$ is a *refinement* of $\mathcal{P}_1$. In case of the partition $\mathcal{P}_1'' = \{\{1,2,3\},\{4\}\}$, $\mathcal{P}_1''$ is *coarsening* of $\mathcal{P}_1'$.

2.1.3 Preference

Definition 2.1.8 (Preference Relation)**.** Take a relation $\succsim$ on X where a relation on X is a subset of $X \times X$.

- A relation $\succsim$ is said to be *complete* if and only if, given any $x, y \in X$, $x \succsim y$ or $y \succsim x$.
  A relation $\succsim$ is said to be *transitive* if and only if, given any $x, y, z \in X$, $[x \succsim y$ and $y \succsim z] \Rightarrow x \succsim z$.
- A relation is a *preference relation* if and only if it is *complete* and *transitive*.
- Given any *preference relation* $\succsim$, we can define *strict preference* $\succ$ by $x \succ y \Leftrightarrow [x \succsim y$ and $y \not\succsim x]$, and the *indifference* $\sim$ by $x \sim y \Leftrightarrow [x \sim y$ and $y \sim x]$.

DMs have a *preference order* for possible states, and the concept that quantifies the *preference order* of a DM is called *utility*. We can say a utility function $u_i : X \to \mathbb{R}$ represents the *preference relation* $\succsim_i$ if for all outcomes x, y in X we have $u_i(x) \geq u_i(y) \leftrightarrow x \succsim_i y$.

Definition 2.1.9 (Unknown Preference [60])**.** A binary relation U_i represents the uncertainty in DM i's preference of which of the two states is preferable. The preference structure in the graph model can be extended to a triplet with U_i. U_i is *symmetric* and $\{\succsim_i, \sim_i, U_i\}$ must be strongly *complete*.

Remark (Preference Order Elicitation)*.* As for preference ordering, if no particular method is mentioned, it is based on the *direct evaluation* by the analyst who built the case setting. For example, the preference order in the base-case analysis of the Elmira environmental dispute was derived by the GMCR developers based on information obtained through direct interviews with stakeholders of the conflict. For the Cuban Missile Crisis, the author of this paper formulated through a literature review. In addition to direct evaluation, this thesis discusses *option prioritization* 2.1.7 and *TOPSIS* 4 as methods for eliciting preference rankings with less information.

2.1.4 Coarse Criteria

The coarser the criteria (fewer categories for each criterion), the lower the decision-making cost, even though the DM has to use more criteria [63]. The maximum number of alternative distinctions that can be generated considering the number of categories for each criterion is equal to the product of the number of categories for the criterion deployed. Theoretically, it can be said that there is a trade-off between categories and criteria when considering an efficient decision-making function with a limited amount of information. Obviously, in the analysis aimed at a solution that avoids the worst-case scenario, which is the subject of this study, a more reasonable solution can be obtained by reducing the number of criteria.

Remark (**Efficiency of Coarser Criteria** [63])*.* Consider a set of criteria $\mathcal{C}$ with the choice function c, denoted by $(\mathcal{C}, c)$. If $x, y \in X$ are contained in the same C_i-category for each $\mathcal{C}_i \in C$ there is a choice class of c that contains x and y.

1. The pair $\mathcal{C}_i \in C$ is maximally discriminates when the number of alternative choices of c equals $min[\prod_i^N = e(C_i), |X|])$, where e indicates the number of categories in the criterion.

   If $\mathcal{C}_i \in C$ is efficient, then $\mathcal{C}_i \in C$ maximally discriminates.

2. Consider the set $\mathcal{X}$ of domains contains X with m choices for all $m > 1$. In this case, the following two statements are equivalent.
   (a) Any efficient $\mathcal{C}$ with $\mathcal{X}$ as its domain contains only the binary criteria.
   (b) For all integers $e > 2$, we have $\kappa(e) > \kappa(2)\lceil log2 \rceil e$, where κ denotes the costs of criterion $\mathcal{C}$.

A sufficiently small number of decision-making criteria is more efficient, and the smallest partition is binary.

2.1.5 Rationality

Definition 2.1.10 (Rationality)**.** DM i is *rational* at state ω if, given the DM's *preference order* , there is no other states ω' that yields a higher expected utility for DM i than at ω.

In this study, when interstate conflict is the subject of analysis unless otherwise noted, the following assumptions on a phenomenon X are made based on the Rational Actor Model by Allison and Zelikow [3]: *"state" in the following definitions refers to a "nation state."*

Assumption 1 (Rationality of a State)**.**

- *X is the action of a state.*
- *The state is a unified actor.*
- *The state has a coherent utility function.*
- *The state acts in relation to threats and opportunities.*
- *The state's action is value-maximizing(or expected value-maximizing.)*

Remark (Bounded Rationality). In this study, if there is not enough information to make a reasonable preference setting, it is said to be rational to be inattention concerning the information that is not available *rational inattention* [84]. Even within the range of information that is available, it is rational to make efficient criteria that are sufficient enough and minimal to discriminate for decision making [63].

2.1.6 Conflict

A *conflict* is an interaction involving two or more independent DMs, each of whom makes choices that together determine how the *conflict* evolves, and has *preferences* over the outcome i.e., *resolution* of the *conflict*. *Conflict resolution* attempts to provide interpretations of equilibria that can be found by describing a conflict structure with a mathematical model and bringing it closer to the actual resolution of the conflict in the real world.

2.1.7 Option

Definition 2.1.11 (Option [90])**.** The actions that the DM can control are called *options*. The set of all options in a conflict is $O = \bigcup_{i \in N} O_i$, where i indicates which DM controls the options. Let O_i denote the *option set* of DM i for $i \in N$ for which $o_{ij} \in O_i$. An option selection for DM i is a mapping $g : O_i \to \{0, 1\}$, such that for $j = 1, 2, \ldots, h_i$, where o_{ij} is DM i's jth option.

$$g(o_{ij}) = \begin{cases} 1 \text{ if DM } i \text{ selects option } o_{ij}, \\ 0 \text{ otherwise.} \end{cases} \tag{2.5}$$

Definition 2.1.12 (State in Option Form [90]). Let $O = \bigcup_{i \in N} O_i$ be the set of all options for $o_{ij} \in O_i, i = 1, 2, \ldots, n$. A *state* is a mapping $f : O \to \{0, 1\}$, such that for $i = 1, 2, \ldots, n$.

$$f(o_{ij}) = \begin{cases} 1 \text{ if DM } i \text{ selects option } o_{ij}, \\ 0 \text{ otherwise.} \end{cases} \tag{2.6}$$

Let $|O|$ represent the total number of options available to the DMs. A state can be treated as a $|O|$-dimensional column vector with 0 or 1 as an element. $g^s(O_i)$ denotes DM i's option selection corresponding to state s for $i = 1, 2, \ldots, n$ and is a $|O_i|$-dimensional column vector whose elements are

$$g^s(o_{ij}) = \begin{cases} 1 \text{ if DM } i \text{ selects option } o_{ij} \text{ in state } s, \\ 0 \text{ otherwise.} \end{cases} \tag{2.7}$$

Example 2.1.1(Prisoners' Dilemma) can be illustrated as in Table 2.1 , in which "Y" indicates that an option is selected by the DM, while "N" means that the option is not selected.

TABLE 2.1: Prisoners' Dilemma

		States			
DM	**Option**	s_1	s_2	s_3	s_4
DM1	Not Confess	Y	Y	N	N
DM2	Not Confess	Y	N	Y	N

Remark (Option Form as Binary Information). the *option form* or *binary form* as shown in Table 2.1, which expresses conflicts based on the selection of options, was initially proposed by Howard [33]. However, the *option form* is considered one of the methods of describing a state in the framework of GMCR, which is the underlying conceptual foundation [23].

Remark (Option Prioritization). It is possible to derive the preference ranking in GMCR from the combination of options that can be directly controlled by the DMs. A method called *option prioritization* that satisfies first-order predicate logic was developed to conduct preference ranking with a simpler and more logical procedure [90]. Taking the prisoner's dilemma as an example again, we present the process of setting the preference order using the method of *option prioritization* as depicted in Figure 2.1.

Figure 2.1 shows the preference ranking process for DM1. Suppose we set "DM1 does not confesses" as the option 1, and "DM2 does not confesses" as the option 2. For each of these options, assume the combinations such that, non-conditional: negation, conjunction, disjunction, and conditional: if, or iff. The most desirable situation for DM1 is that DM2 selects the option "Not confess"(the option 2) in the first round, then DM1 selects the option 1. Once the options have been structured, the next step is to score each option using the coefficients obtained in the following method:

Let $K = \{O^1, O^2, \ldots O^l, \ldots, O^k\}$ be the set of option statements listed by priority for a DM, where O^l represents the l^{th} option statement. In a state $s \in S$, an option statement O^l is true or false.

$$\Phi_l(s) = \begin{cases} 2^{k-l} \text{ if } O_l(s) = 1, \\ 0 \text{ otherwise.} \end{cases} \tag{2.8}$$

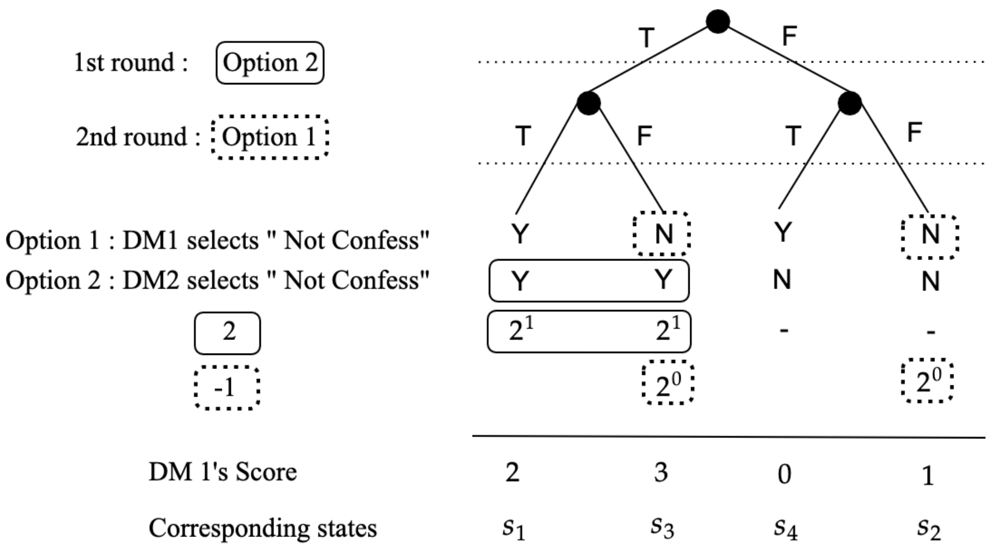

FIGURE 2.1: Option Prioritization of DM1 in Prisoner's Dilemma

The total score of all the option statements K in state s can be expressed as follows:

$$\Phi(s) = \sum_{l=1}^{k} \Phi_l(s). \tag{2.9}$$

With the sum of these scores, the preference ranking of each state for DM1 can be automatically obtained: $s_3(NY) \succ_1 s_1(YY) \succ_1 s_4(NN) \succ_1 s_2(YN)$.

2.1.8 Strategy

A *strategy* is a situation in which a DM decides which of their options to adopt or not to adopt in a normal form game.

2.1.9 State

A *state* is an outcome of each DM's decision on which strategy to adopt.

Remark (Option, strategy, state). In Table 2.1 and Figure 2.1, "Not confess" is the option, Y/N of each DM selecting or not selecting the option is the strategy, and the strategy outcomes are the states s_1 to s_4.

2.1.10 State Transition

Definition 2.1.13 (Reachability)**.** State ω' is *reachable* from state ω if there exist transitions by DMs, $\omega' \in \mathcal{P}_k(\dots(\mathcal{P}_2(\mathcal{P}_1(\omega))))$.

Remark. In GMCR, we express the sequence of the states by DMs as *reachability* or *state transition*. State transition in GMCR do not imply chronological or causal relationships as in the extensive form game but are conceptual move concepts.

Definition 2.1.14 (State Transition as Common Knowledge - Lewis[58])**.** Event $\mathcal{E}$ is *common knowledge* at state ω if for every $n \in \{1, 2, \dots\}$ and every transition $(i_1, i_2, ...i_n)$, we have $\mathcal{P}_{i_n}(\dots(\mathcal{P}_{i_2}(\mathcal{P}_{i_1}(\omega))))$.

2.2 Frameworks

2.2.1 Normal Form Game

In this section, we give definitions of the basic concepts of normal form game that has common theoretical foundation with GMCR.

Where N is finite and, for every $i \in N$, T_i is finite, and $u_i : T_i \times T_{N\backslash\{i\}} \rightarrow \mathbb{R}$. A set of triple represents the normal form game: DMs (N), the nonempty set of strategies each DM i adopts (T_i), and the utility of DM i adopting each strategy (u_i).

Definition 2.2.1 (Normal Form Game [90]).

$$G = (N, \{T_i\}_{i \in N}, \{u_i\}_{i \in N}). \tag{2.10}$$

The normal form game representation of Example 2.1.1 (Prisoners' Dilemma) is shown in Table 2.2, where

TABLE 2.2: Prisoners' Dilemma in Normal Form Game

		DM2	
		Not Confess	Confess
DM1	Not Confess	3,3	1,4
	Confess	4,1	2,2

$$\begin{aligned}
&N = \{DM1, DM2\}, T_i\{\text{Not Confess}\,(NC)\,, \text{Confess}\,(C)\}\\
&s_1 = (NC, NC), s_2 = (NC, C), s_3 = (C, NC), s_4 = (C, C),\\
&u_1(s_1) = 3, u_1(s_2) = 1, u_1(s_3) = 4, u_1(s_4) = 2,\\
&u_2(s_1) = 3, u_2(s_2) = 4, u_2(s_3) = 1, u_2(s_4) = 2.
\end{aligned}$$

The normal form game can be expressed in option form as in Definition 2.2.2, , where $N = \{1, 2, \ldots, n\}$ is an nonempty set of DMs, O_i is the nonempty option set of DM i for each DM $i \in N$, $S = \{s_1, s_2, \ldots, s_m\}$ is a nonempty set of feasible states, and DM i's preference over the states.

Definition 2.2.2 (Normal Form Game in Option Form [90]).

$$G = (N, \{O_i\}_{i \in N}, S, \{\succsim\}_{i \in N}). \tag{2.11}$$

In option form, the set of *unilateral moves* from state $s \in S$ for DM i is defined as in Definition 2.2.3, where number of strategies for DM i in T_i is indicated as h_i thus $1 \leq l \leq h_i$ and $1 \leq k \leq h_j$:

Definition 2.2.3 (Unilateral Moves in Option Form[90]).

$$R_i(s) = \{q \in S : g^q(o_{il}) \neq g^s(o_{il}) \text{ for } o_{il} \in O_i \text{ and } g^q(o_{jk}) = g^s(o_{jk})\ , \forall j \in N \backslash \{i\}\}. \tag{2.12}$$

In option form, the set of *unilateral improvement* from state $s \in S$ for DM i is defined as in Definition 2.2.4.

Definition 2.2.4 (Unilateral Improvement in Option Form).

$$R_i^+(s) = \{q \in R_i(s) \text{ and } q \succ_i s\}. \tag{2.13}$$

Example 2.2.1. We present *unilateral move* and *unilateral improvement* in the prisoner's dilemma illustrated in Table 2.1 and Table 2.2. Suppose a state change occurs in s_1. DM1 can move to s_3, where DM2 has the same strategy choice as its strategy in s_1, Y, by changing its own strategy from Y to N (*unilateral move*). Comparing the utility for DM1 for s_1 and s_3, we find that $s_1 \succ s_3$. Therefore, for DM1, moving from s_1 to s_3 is a *unilateral improvement*. The *unilateral improvements* of the two DMs can be described as follows:

$$R_1^+(s_1) = \{3\}, R_1^+(s_2) = \emptyset, R_1^+(s_3) = \{4\}, R_1^+(s_4) = \emptyset,$$

$$R_2^+(s_1) = \emptyset, R_2^+(s_2) = \emptyset, R_2^+(s_3) = \{3\}, R_2^+(s_4) = \{4\}.$$

Definition 2.2.5 (Nash Equilibrium in Normal Form Game)**.** A list of strategies, $(t_i^*, t_{N\setminus\{i\}}^*) \in T_i \times T_{N\setminus\{i\}}$, is a Nash equilibrium if and only if for all DM i Equation 2.14 holds:

$$u_i(t_i^*, t_{N\setminus\{i\}}^*) \geq u_i(t_i^*, t_{N\setminus\{i\}}^*), \forall t_i \in T_i. \tag{2.14}$$

Definition 2.2.6 (Pareto Optimal in Normal Form Game)**.**

$t^* \in T$ is *Weakly Pareto optimal* if

$$\nexists t^{**} \in T, s.t. \forall i, u_i(t^{**}) > u_i(t^*). \tag{2.15}$$

$t^* \in T$ is *Strongly Pareto optimal* if

$$\nexists t^{**} \in T, s.t. ((\forall i, u_i(t^{**}) \geq u_i(t^*)), (\exists j, u_j(t^{**}) > u_j(t^*))). \tag{2.16}$$

2.2.2 GMCR

In GMCR, evolution of conflict is described as a sequence of state transitions. The possible states of a conflict are provided as a set of graph vertices, and the possible state transitions of a conflict are represented by a set of graph arcs. Furthermore, GMCR requires information about the DMs involved in the conflict and their relative preferences regarding the possible states of the conflict.

A conflict is represented by four tuples: DMs (N), set of feasible states (S), the graph of DMs (A_i), and the preferences of each DM $(\succsim_i)$, where set of all DMs $N : |N| \geqslant 2$, set of all states $S : |S| \geqslant 2$, and preference of DM i satisfies reflectiveness, completeness and transitivity :

$s \succsim_i s'$: s is equally or more preferred to s' by DM i;

$s \succ_i s'$: s is strictly preferred to s' by DM i;

$s \sim_i s'$: s is equally preferred to s' by DM i.

Definition 2.2.7 (Graph Model [90])**.**

$$G = (N, S, (A_i)_{i \in N}, (\succsim_i)_{i \in N}). \tag{2.17}$$

Definition 2.2.8 (Directed Graphs of DM i)**.**

$$G_i : (S, A_i). \tag{2.18}$$

Example 2.2.2 (Graph of Prisoners' Dilemma)**.** Example 2.1.1(Prisoners' Dilemma), whose summary of option statement is provided in Table 2.1, can be presented in graphs as in Figure 2.1. The graph on the left is A_1, and the graph on the right is A_2, respectively. The arcs show transitions that each DM can control by itself.

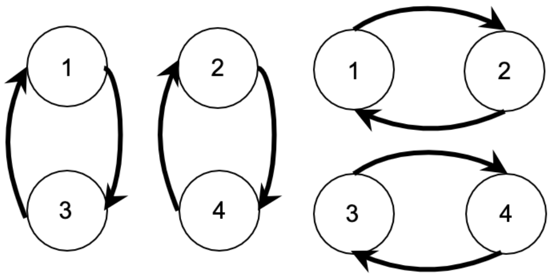

FIGURE 2.2: Graph Model of Prisoners' Dilemma

The four arcs in Figure 2.2 show the transitions of DMs, all of which are bidirectional. This means that there are no impossible or *irreversible* transitions in the state transitions that the DMs can control themselves. The prisoner's dilemma can be described in the framework of GMCR as follows:

$$N = \{1, 2\}, S = \{s_1, s_2, s_3, s_4\},$$
$$A_1 = \{(s_1, s_3), (s_3, s_1), (s_2, s_4), (s_4, s_2)\}, A_2 = \{(s_1, s_2), (s_2, s_1), (s_3, s_4), (s_4, s_3)\},$$
$$s_3 \succ_1 s_1 \succ_1 s_4 \succ_1 s_2, \ \ s_2 \succ_2 s_1 \succ_2 s_4 \succ_2 s_3.$$

Suppose we adopt the interpretation rule that prisoners cannot retract their confessions once they have made them. The state transitions of DMs change as follows, and the corresponding arcs disappear as in Figure 2.3 .

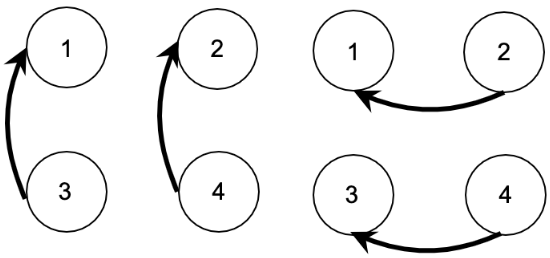

FIGURE 2.3: Irreversible Graph Model of Prisoners' Dilemma

If we denote the state transitions depicted in Figure 2.3 as A'_1 and A'_2, we get the following:

$$A'_1 = \{(s_3, s_1), (s_4, s_2)\}, \ \ A'_2 = \{(s_2, s_1), (s_4, s_3)\}.$$

2.2.3 Transform between Normal Form Game and GMCR

Since GMCR has a history of deriving from non-cooperative games, it is possible to transform from the normal form game to the GMCR framework: the set of DMs (N) remains the same, the set of all strategies of DM i, (T_i) corresponds to the set of states, and the set of preferences over possible outcomes of selection of strategy ($\succsim_i$) is compatible with the set of preferences over states. Nonetheless, the concept of state transitions is found only in the GMCR, and the following situations cannot be described in the normal form game: 1) *common move* [23]: one or more DMs perform unilateral moves, resulting in a change from one identical state to another identical state, 2) *irreversible move* [23]: Once a DM has transitioned from one state to another, it cannot return to its original state, 3) *no move*: no state transition occurs since any DM has a possible unilateral move from one state to another.

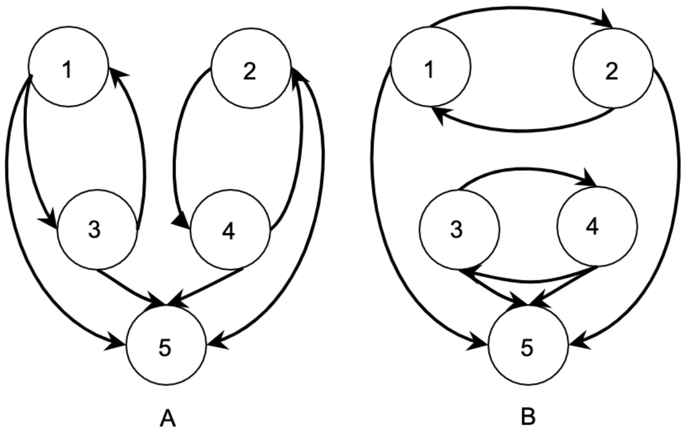

FIGURE 2.4: Superpower Nuclear Confrontation[23]

Figure 2.4 shows a model for nuclear confrontation among two superpowers A and B. It is assumed that each DM has options of using or not using conventional or nuclear weapons. The five states each indicate the following. s_1: Both DMs are at peace without using any weapons, s_2: B uses conventional weapons, s_3: A uses conventional weapons, s_4: Both use conventional weapons, s_5: Both use nuclear weapons. In the graph of A, the state transition from $s_1, s_2, s_3, s4$ to s_5 is shown to be irreversible (*irreversible move*). In both graphs, the transitions from $s_1, s_2, s_3, s4$ to s_5 are UMs for both DMs (*common move*), and from s_5, there are no UMs for either DM, which indicates that there will be no further state changes (*no move*). Since these moves cannot be described in the normal form game, it can be understood that transforming a conflict described in the normal form game to the GMCR is possible, but not vice versa.

2.3 Stability Analysis

2.3.1 GMCR Stability Analysis

The following *reachable lists* are needed to analyze a decision-making situation using GMCR.

Definition 2.3.1 (Reachable Lists)**.** DM i's reachable list from $s \in S$ are subsets of S as follows:

i. DM i's *reachable list* from s to s' by unilateral moves:
$R_i(s) = \{s' \in S \mid (s, s') \in A_i\}$, $(s, s') \in A_i$ denotes the reachability; DM i can reach from s.

ii. DM i's reachable list from s by unilateral improvements:
$R_i^+(s) = \{s' \in R_i(s) \mid s' \succ_i s\}$.

iii. DM i's list at s by equally or less preferred moves:
$\phi_i^{\simeq}(s) = \{s' \in S \mid s \succsim_i s'\}$.

iv. $R_{N\setminus\{i\}}(s)$ is defined as the set of all states which can be achieved by the sequences of unilateral moves of DMs other than DM i.

v. $R^{+}_{N\setminus\{i\}}(s)$ is defined as the set of all states which can be achieved by the sequences of unilateral improvements of DMs other than DM i.

vi. DM i's reachable list at s to s' with unknown preference.
$R^{U}_{i} = \{s' \in S \mid s\ U_i\ s'\}$.

Example 2.3.1 (Reachable Lists of Prisoners' Dilemma in GMCR)**.** Reachability lists of the two DMs in the prisoner's dilemma can be presented as follows:

$$R_1(s_1) = \{s_3\}, R_1(s_2) = \{s_4\}, R_1(s_3) = \{s_1\}, R_1(s_4) = \{s_2\},$$

$$R_2(s_1) = \{s_2\}, R_2(s_2) = \{s_2\}, R_2(s_3) = \{s_4\}, R_2(s_4) = \{s_3\}.$$

Definition 2.3.2 (States and Preference Structure)**.** The set of state S can be partitioned into subsets according to relative preference defined in Definition 2.2.7 as follows:

$\Phi^{+}_{i}(s) = \{q : q \succ_i s\}$, the states strictly preferred to state s by DM i,
$\Phi^{=}_{i}(s) = \{q : q \sim_i s\}$, the states indifferent to state s by DM i,
$\Phi^{-}_{i}(s) = \{q : s \succ_i q\}$, the states strictly less preferred to state s by DM i,
$\Phi^{U}_{i}(s) = \{q : s\ U_i\ q\}$, the states DM i' s preference is uncertain.

Relations among the subset of S and reachable lists other than uncertain preference are depicted as in Figure 2.5 [90].

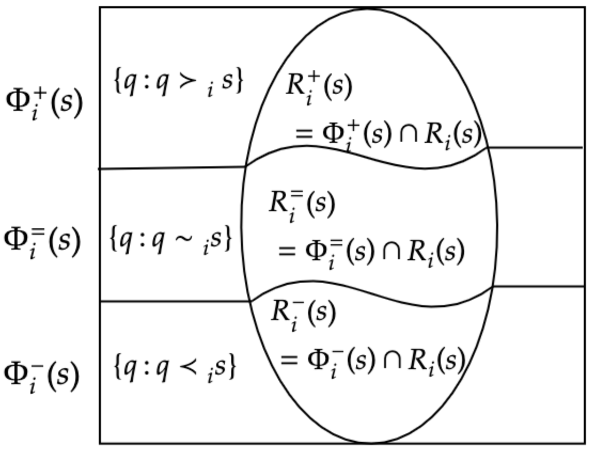

FIGURE 2.5: Structure of States and Reachable Lists [90]

In GMCR, there exists four types of the concept of stability, Nash, GMR, SMR and SEQ, where the situation reaches an equilibrium state, with no possibility for future state transition for each DM. The concepts of stability are based on the situation of the list of unilateral improvements of DM i from state s in Definition 2.3.1 and defined as in Definition 2.3.3, 2.3.4, 2.3.5, and 2.3.6.

When DM i has no unilateral improvements from state s, there are no further state transitions, thereby establishing stability.

Definition 2.3.3 (Nash [69] [70])**.** For $i \in N$, state $s \in S$ is Nash stable for DM i, denoted by $s \in S_i^{Nash}$, if and only if

$$R_i^+(s) = \emptyset. \tag{2.19}$$

When DM i cannot reach a state more favorable than state s through any of its unilateral improvements because of the other DMs' response, there are no further state transitions, thereby establishing stability.

Definition 2.3.4 (GMR [33])**.** For $i \in N$, state $s \in S$ is GMR stable for DM i, denoted by $s \in S_i^{GMR}$, if and only if

$$\forall s' \in R_i^+(s),\ R_{N\setminus\{i\}}(s) \cap \phi_i^{\simeq}(s) \neq \emptyset. \tag{2.20}$$

When a state occurs where for any of DM i's unilateral improvements, the response of another DM would cause a state less favorable than state s , and further, regardless of how DM i responds to the other DMs' response, a state more favorable than state s cannot occur, there are no further state transitions, thereby establishing stability.

Definition 2.3.5 (SMR [33])**.** For $i \in N$, state $s \in S$ is SMR stable for DM i, denoted by $s \in S_i^{SMR}$, if and only if

$$\forall s' \in R_i^+(s), \exists s'' \in R_{N\setminus\{i\}}(s) \cap \phi_i^{\simeq}(s),\ R_i(s'') \subseteq \phi_i^{\simeq}(s). \tag{2.21}$$

When DM i has at least one unilateral improvement from state s, regardless of DM i's unilateral movement, the state that would occur due to other DM's unilateral improvement, and there are no further transitions, thereby establishing stability.

Definition 2.3.6 (SEQ [25] [27])**.** For $i \in N$, state $s \in S$ is SEQ stable for DM i, denoted by $s \in S_i^{SEQ}$, if and only if

$$\forall s' \in R_i^+(s),\ R_{N\setminus\{i\}}^+(s') \cap \phi_i^{\simeq}(s) \neq \emptyset. \tag{2.22}$$

Remark (Credible Move)*.* SEQ is similar to GMR in state transitions but includes only *credible* sanctions. A *credible* action is caused only by a *unilateral improvement* defined in 2.2.4.

In addition to rational solution concepts, Pareto optimality is also considered for efficiency of a conflict. Pareto optimality in GMCR is described as follows:

Definition 2.3.7 (Pareto Optimal in GMCR)**.**

For $i \in N$, state $s \in S$ is *Weakly Pareto optimal* for DM i, denoted by $s \in S^{wPareto}$ if and only if ,

$$\neg[\exists s' \in S, \forall i \in N, s' \succ_i s]. \tag{2.23}$$

For $i \in N$, state $s \in S$ is *Strongly Pareto optimal* for DM i, denoted by $s \in S^{sPareto}$ if and only if ,

$$\neg[\exists s' \in S, [[\forall i \in N, s' \succsim_i s] \wedge [\exists j \in N, s' \succ_j s]]]. \tag{2.24}$$

Definition 2.3.8 (Worst-Case Scenario)**.** For $i \in N$, state $s \in S$ is identified *worst-case scenario* or *disaster* if one of the following cases pertaining to Nash stability and Pareto efficiency is applicable.

1. A Nash equilibrium is established in the least preferred state to DM i, $s \in S^{Nash}$, $s = \{\min \succsim_i\}$, where $\min \succsim_i$ denotes the DM i's least preferred state.

2. The least preferred state to all DMs: $\forall i \in N, s = \{\min \succsim_i\}$.

3. A Nash equilibrium is established in a state which is not Pareto efficient.: $s \in S^{Nash}, s \notin S^{Pareto}$.

   Similarly, the worst-case scenario in DM i's option selection (2.1.11, 2.1.12) is defined as follows:

4. $s \in S^{Nash}$, DM i's least preferred option selection is done at s.

Remark (Examples of Applicable Cases: Worst-Case Scenario).

- Definition 2.3.8- 1 : s_9 for **M** and **L** in Elmira Conflict (2.5).
- Definition 2.3.8- 2 : s_4 in chicken game (3.2.2).
- Definition 2.3.8- 3 : s_9 in Elmira Conflict (2.5), s_9 in Cuban Missile Crisis (3.12), s_4 in prisoner's dilemma (5.1).
- Definition 2.3.8- 4 : s_9 for the United States in Cuban Missile Crisis (3.12), s_9 for **M** and **L** in Elmira Conflict (2.5).

Extended Stability Concepts

In this study, four types of stability concepts, namely Nash, GMR, SMR, and SEQ, are employed in GMCR rationality analysis. Other notions of conflict resolution in GMCR include *Limited-Move Stability of horizon h* (L_h) [53] based on the notion of *anticipation* [53], and *Non-Myopic Stability (NM)* [16] [52] without the bound of *horizon h*, and also *Stackelberg equilibrium (ST)* [7] [23] [44] [57] [91], in which there is a leader in the conflict. In some cases, the analysis is extended to a total of seven stability concepts.

A DM who foresees a sequence of length at most h is said to be a DM with horizon h. Knowledge of DM i 's anticipation vector allows prediction of subsequent moves at every status quo state, then L_h holds as described in Remark below.

Remark (Limited-Move Stability (L_h)). A state $s^* \in S$ is *limited-move*, horizon h, stable L_h for DM $i \in N$ iff $V_h(i, s^*) = s^*$, where

- $V(i, s^*)$: anticipated final state of conflict beginning at state s^* with initial move made by DM i.
- State $s^* \in S$ is stable for DM i iff $V(i, s^*) = s^*$.
- State $s^* \in S$ is equilibrium for the conflict iff $V(i, s^*) = s^*$ for all $i \in N$.

Remark (Non-Myopic Stability (NM)). A state $s^* \in S$ is *Non-Myopic Stable (NM)*, for DM i iff there is a positive integer k' st, $V_k(i, s^*) = s^*$ for all $k \geq k'$.

Some studies incorporate Stackelberg equilibrium into GMCR. The assumption is that there is a leader with bargaining power in the conflict and followers who follow the leader and that the leader's actions are irreversible. This stability concept is considered to be applied to situations in which one of the players is the leader in a chicken game structure, as one of the most representative cases.

Remark (Stackelberg Stability (ST)). Let $i \in N$. State $s \in S$ is *Stackelberg stable* for i as leader iff s is L_2 stable for DM i and Nash stable for DM j.

The interrelationships between Nash, GMR, SMR, SEQ, LM, and ST in two DMs conflicts have been clarified by Fang, Hipel and Kilgour [23]. ST is naturally the smaller set among all the stability sets.

2.3.2 Coalition Stability Analysis

For a situation in which DMs may cooperate to achieve mutually beneficial gains within coalition, we can provide four stability concepts initially defined by Inohara and Hipel [40]: *coalition Nash stability* (CNash), *coalition general metarationality* (CGMR), *coalition symmetric metarationality* (CSMR) and *coalition sequential stability* (CSEQ). Given a coalition $H \subseteq N$ and $s' \in S$, a state s' is a *coalitional improvement* of coalition H from state s if and only if coalitional state s' is included in the reachable list $R_H(s)$ of coalitions from state s and each DM i of coalition H strictly prefers state s' to state s.

Definition 2.3.9 (Reachable List of Coalition)**.** Coalition H's reachable list from $s \in S$ are subsets of S as follows:

i. Coalition H's list from s by unilateral moves:
$R_H(s) = \{s' \in S \mid (s, s') \in A_H\}$, $(s, s') \in A_H$ denotes the reachability; coalition H can reach from s to s'.

ii. Coalition H's list from s by unilateral improvements:
$R_H^{++}(s) = \{s' \in R_H(s) \mid \forall i \in H, s \succ_i s'\}$.

iii. Coalition H's list at s by equal or less preference:
$\phi_H^{\simeq}(s) = \{s' \in S \mid \exists i \in H, s' \precsim_i s\}$.

iv. $R_{\mathbb{P}(N \setminus H)}(s)$ is defined as the set of all states which can be achieved by the sequences of unilateral moves of coalitions other than coalition H.

v. $R^{++}_{\mathbb{P}(N \setminus H)}(s)$ is defined as the set of all states which can be achieved by the sequences of unilateral improvements of coalitions other than coalition H.

When coalition H has no unilateral improvements from state s, there are no further state transitions, thereby establishing stability.

Definition 2.3.10 (CNash)**.** For $H \subseteq N$, state $s \in S$ is CNash stable for DM i, denoted by $s \in S_H^{CNash}$, if and only if

$$R_H^{++}(s) = \emptyset. \tag{2.25}$$

When coalition H cannot reach a state more favorable than state s through any of its unilateral improvements because of the other coalition's response, there are no further state transitions, thereby establishing stability.

Definition 2.3.11 (CGMR)**.** For $H \subseteq N$, state $s \in S$ is CGMR stable for coalition H, denoted by $s \in S_H^{CGMR}$, if and only if

$$\forall s' \in R_H^{++}(s), R_{\mathbb{P}(N \setminus H)}(s) \cap \phi_H^{\simeq}(s) \neq \emptyset. \tag{2.26}$$

When a state occurs where for any of coalition H's unilateral improvements, the response of another coalitions would cause a state less favorable than state s , and further, regardless of how coalition H responds to the other coalitions response, a state more favorable than state s cannot occur, there are no further state transitions, thereby establishing stability.

Definition 2.3.12 (CSMR)**.** For $H \subseteq N$, state $s \in S$ is CSMR stable for coalition H, denoted by $s \in S_H^{CSMR}$, if and only if

$$\forall s' \in R_H^{++}(s), \exists s'' \in R_{\mathbb{P}(N\setminus H)}(s) \cap \phi_{\widetilde{H}}^{\simeq}(s),\ R_H(s'') \subseteq \phi_{\widetilde{H}}^{\simeq}(s). \tag{2.27}$$

When coalition H has at least one unilateral improvement from state s, regardless of coalition H's unilateral movement, the state that would occur due to the other coalitions' response by unilateral improvement would not be more preferable than the current state, and there are no further transitions, thereby establishing stability.

Definition 2.3.13 (CSEQ)**.** For $H \subseteq N$, state $s \in S$ is CSEQ stable for coalition H, denoted by $s \in S_H^{CSEQ}$, if and only if

$$\forall s' \in R_H^{++}(s),\ R_{\mathbb{P}(N\setminus\{H\})}^{++}(s') \cap \phi_{\widetilde{H}}^{\simeq}(s) \neq \emptyset. \tag{2.28}$$

Example 2.3.2 (Elmira Conflict)**.** The Elmira conflict is an environmental contamination dispute in Ontario, Canada; numerous studies have been already conducted using the GMCR . Three DMs are involved in the conflict: the Ministry of Environment (**M**), Uniroyal (**U**), and the local government (**L**). **M** discovered contamination and issued a control order to **U** that included decontamination operation by **U**. They would like to exercise their authority efficiently. **U** owns questionable chemical plants and intends to exercise its right to object, aiming to lift or relax the control order. **L** represents diverse interest groups and intends to protect the residents and the local industrial base.

The options for each DM are as follows. **M**: can irreversibly modifying the control order; **U**: may continue to delay the objection process, irreversibly accept the control order, or abandon irreversibly chemical plants; **L**: may argue for the application of the original control order. Table 2.3 summarizes all the feasible states based on the DM's options.

TABLE 2.3: Elmira Conflict - Options and States

		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
M	Modify	N	Y	N	Y	N	Y	N	Y	-
U	Delay	Y	Y	N	N	Y	Y	N	N	-
	Accept	N	N	Y	Y	N	N	Y	Y	-
	Abandon	N	N	N	N	N	N	N	N	Y
L	Insist	N	N	N	N	Y	Y	Y	Y	-

The preference orders of the three DMs are illustrated in Table 2.4 and the reachability lists in following, where the numbers indicate the state number, in both Table 2.4 and in the reachability lists.

$$A_M = \{(s_1, s_2), (s_3, s_4), (s_5, s_6), (s_7, s_8)\},$$

$$A_U = \{(s_1, s_3), (s_1, s_9), (s_2, s_4), (s_2, s_9), (s_3, s_9), (s_4, s_9), (s_5, s_7), (s_5, s_9), (s_6, s_8), (s_6, s_9), (s_7, s_9), (s_8, s_9)\},$$

$$A_L = \{(s_1, s_5), (s_2, s_6), (s_3, s_7), (s_4, s_8), (s_5, s_1), (s_6, s_2), (s_7, s_3), (s_8, s_4)\}.$$

The results of the stability analysis based on the preference information and reachability of each DM are shown in Table 2.5. The result reflects the situation where the **L** insists on enforcing the original control order, whereas **U** avoids the

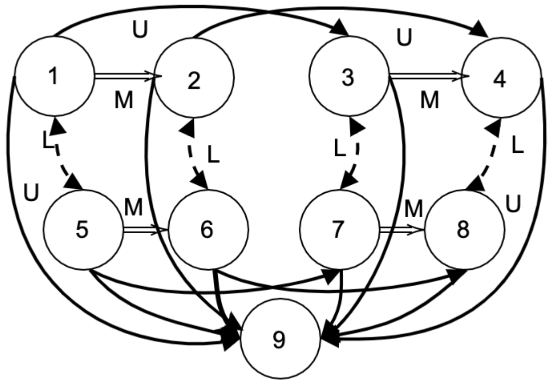

FIGURE 2.6: Graph Model of Elmira conflict

costs associated with complying with the control order and aims to stall the discussions until the order is modified in a direction that is convenient for the company.

TABLE 2.4: Elmira Conflict - Preference Order

most preferred								least preferred
M	s_7	s_3	s_4	s_8	s_5	s_1	s_2	s_6	s_9
U	s_1	s_4	s_8	s_5	s_9	s_3	s_7	s_2	s_6
L	s_7	s_3	s_5	s_1	s_8	s_6	s_4	s_2	s_9

TABLE 2.5: Elmira Conflict - Stability Analysis

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
Nash					✓			✓	✓
GMR	✓			✓	✓			✓	✓
SMR	✓			✓	✓			✓	✓
SEQ					✓			✓	✓
CNash								✓	✓
CGMR	✓			✓				✓	✓
CSMR	✓			✓				✓	✓
CSEQ								✓	✓
Pareto	✓		✓	✓	✓		✓	✓	

GME is established in s_5 and s_8 while coalitional stability concepts CNash, CGMR, CSMR, CSEQ hold only in s_8. Ex facto, **M** and **U** made transitions in coalition, and the conflict was solved in s_8.

2.3.3 State Transition Time Analysis

The framework of GMCR considering state transition time was presented by Inohara [39], who gave definitions of stability concepts that are established by considering that if the relative speed of time required for DM i to transition from state s to state s' is faster than the transition speed of other DMs, then the opponent's move may be sanctioned. Fourteen new stability concepts incorporating the transition time and the credibility of DMs behavior based on Nash, GMR, SMR, and SEQ have been provided. To define new solution concepts, we need *time reachable lists* and *time unilateral improvement lists* , which can be analogously obtained from the standard approach. The GMCR incorporating the state transition time is formulated as follows:

$$G = (N, S, (A_i)_{i\in N}, (\succsim_i)_{i\in N}, (tt_i(s,s'))_{i\in N, s\in S, s'\in R_i(s)}) \\ \text{where } tt_i(s,s') \text{ is positive real number.} \tag{2.29}$$

Definition 2.3.14 (Time Reachable List)**.** DM i's time reachable list from $s \in S$ are as follows:

i. DM i's *time reachable list*: For $i \in N, s \in S$, and $T \subset N\backslash\{i\}$, from s against the move by T:
$tR_{i,T}(s) = \{s' \in R_i(s) \mid \forall j \in T, \forall s'' \in R_j(s), tt_j(s,s'') \geq tt_i(s,s')\}$.

ii. DM i's reachable list from s against the credible move by T:
$tcR_{i,T}(s) = \{s' \in R_i(s) \mid \forall j \in T, \forall s'' \in R_j^+(s), tt_j(s,s'') \geq tt_i(s,s')\}$.

iii. DM i's time improvement list from s against the moves by T:
$tR^+_{i,T}(s) = tR_{i,T}(s) \cap R_i^+(s)$.

iv. DM i's time improvement list from s against the credible moves by T:
$tcR^+_{i,T}(s) = tcR_{i,T}(s) \cap R_i^+(s)$.

Using the lists in Definition 2.3.14, we can provide the definitions of fourteen stability concepts: tNash, tNash-c, tGMR, tcGMR, tGMR-c, tcGMR-c, tSMR, tcSMR, tSMR-c, tcSMR-c , tSEQ, tcSEQ, tSEQ-c, tcSEQ-c.

Definition 2.3.15 (tNash)**.** s is a time Nash state (tNash) for DM i against moves by others, denoted by S_i^{tNash}, if and only if

$$tR^+_{i,N\backslash\{i\}}(s) = \emptyset. \tag{2.30}$$

Definition 2.3.16 (tNash-c)**.** s is a time Nash state for DM i against credible moves by others, denoted by $S_i^{tNash-c}$, if and only if

$$tcR^+_{i,N\backslash\{i\}}(s) = \emptyset. \tag{2.31}$$

Definition 2.3.17 (tGMR)**.** s is a time GMR state for DM i against moves by others when DMi takes only credible moves , denoted by S_i^{tGMR}, if and only if

$$\forall s' \in tR^+_{i,N\backslash\{i\}}(s),\ tR_{N\backslash\{i\},\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset. \tag{2.32}$$

Definition 2.3.18 (tcGMR)**.** s is a time credible GMR state for DM i against moves by others, denoted by S_i^{tcGMR}, if and only if

$$\forall s' \in tR^+_{i,N\backslash\{i\}}(s),\ tcR_{N\backslash\{i\},\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset. \tag{2.33}$$

Definition 2.3.19 (tGMR-c)**.** s is a time GMR state for DM i against credible moves by others, denoted by S_i^{tGMR-c}, if and only if

$$\forall s' \in tcR^+_{i,N\backslash\{i\}}(s), tR_{N\backslash\{i\},\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset. \tag{2.34}$$

Definition 2.3.20 (tcGMR-c). s is a time GMR state for DM i against credible moves by others when DM i takes only credible moves(tcGMR-c), denoted by $S_i^{tcGMR-c}$, if and only if

$$\forall s' \in tcR^+_{i,N\setminus\{i\}}(s), tcR_{N\setminus\{i\},\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset. \tag{2.35}$$

Definition 2.3.21 (tSMR). s is a time SMR state for DM i against moves by others, denoted by S_i^{tSMR}, if and only if

$$\begin{aligned}&\forall s' \in tR^+_{i,N\setminus\{i\}}(s), \exists s'' \in tR_{i,N\setminus\{i\}}(s') \cap \phi_i^{\simeq}(s),\\ &s.t.\ s'' \in \phi_i^{\simeq}(s), \forall s' \in tR_{i,N\setminus\{i\}}(s'').\end{aligned} \tag{2.36}$$

Definition 2.3.22 (tcSMR). s is a time credible SMR state for DM i against moves by others, denoted by S_i^{tcSMR}, if and only if

$$\begin{aligned}&\forall s' \in tR^+_{i,N\setminus\{i\}}(s), \exists s'' \in tcR_{i,N\setminus\{i\}}(s') \cap \phi_i^{\simeq}(s),\\ &s.t.\ s'' \in \phi_i^{\simeq}(s), \forall s' \in tR_{i,N\setminus\{i\}}(s'').\end{aligned} \tag{2.37}$$

Definition 2.3.23 (tSMR-c). s is a time SMR state for DM i against credible moves by others, denoted by S_i^{tSMR-c}, if and only if

$$\begin{aligned}&\forall s' \in tcR^+_{i,N\setminus\{i\}}(s), \exists s'' \in tR_{i,N\setminus\{i\}}(s') \cap \phi_i^{\simeq}(s),\\ &s.t.\ s'' \in \phi_i^{\simeq}(s), \forall s' \in tcR_{i,N\setminus\{i\}}(s'').\end{aligned} \tag{2.38}$$

Definition 2.3.24 (tcSMR-c). s is a time credible SMR state for DM i against credible moves by others, denoted by $S_i^{tcSMR-c}$, if and only if

$$\begin{aligned}&\forall s' \in tcR^+_{i,N\setminus\{i\}}(s), \exists s'' \in tcR_{i,N\setminus\{i\}}(s') \cap \phi_i^{\simeq}(s),\\ &s.t.\ s'' \in \phi_i^{\simeq}(s), \forall s' \in tcR_{i,N\setminus\{i\}}(s'').\end{aligned} \tag{2.39}$$

Definition 2.3.25 (tSEQ). s is a time SEQ state for DM i against moves by others, denoted by S_i^{tSEQ}, if and only if

$$\forall s' \in tR^+_{i,N\setminus\{i\}}(s), tR^+_{i,N\setminus\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset, \tag{2.40}$$

Definition 2.3.26 (tcSEQ). s is a time credible SEQ state for DM i against moves by others, denoted by S_i^{tcSEQ}, if and only if

$$\forall s' \in tR^+_{i,N\setminus\{i\}}(s), tcR^+_{i,N\setminus\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset. \tag{2.41}$$

Definition 2.3.27 (tSEQ-c). s is a time SEQ state for DM i against credible moves by others, denoted by S_i^{tSEQ-c}, if and only if

$$\forall s' \in tcR^+_{i,N\setminus\{i\}}(s), tR^+_{i,N\setminus\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset. \tag{2.42}$$

Definition 2.3.28 (tcSEQ-c). s is a time credible SEQ state for DM i against credible moves by others, denoted by $S_i^{tcSEQ-c}$, if and only if

$$\forall s' \in tcR^+_{i,N\setminus\{i\}}(s), tcR^+_{i,N\setminus\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset. \tag{2.43}$$

2.3.4 Interrelationships of Stability Concepts

In this subsection, the theoretical interrelationships between stability concepts are presented. The Theorem 2.3.1 and 2.3.2 were developed directly from the definitions of Nash, GMR, SMR, and SEQ [22] [75], and coalitional stability concepts [40]. The interrelationships among the eight stability concepts, Nash, GMR, SMR, SEQ, CNash, CGMR, CSMR, and CSEQ, are depicted in Figure 2.7, 2.8 and 2.9. For the interrelationship of the GMCR stability concepts incorporating the transition time, developed and proved by Inohara [39], are presented in Propositions 2.3.5, 2.3.6, 2.3.7 , 2.3.8, 2.3.9, and 2.3.10. Furthermore, the interrelationships among the t-GMCR concepts are depicted in Table 2.10.

Theorem 2.3.1 (Fang et al.1989 [22]). For $i \in N$ and $s \in S$, if $s \in S_i^{Nash}$, then $s \in S_i^{SMR}$; if $s \in S_i^{SMR}$, then $s \in S_i^{GMR}$.

Theorem 2.3.2 (Fang et al.1989 [22]). For $i \in N$,and $s \in S$, if $s \in S_i^{Nash}$, then $s \in S_i^{SEQ}$; if $s \in S_i^{SEQ}$, $s \in S_i^{GMR}$.

The theorems for coaltional stability concepts are provided based on Theorem 2.3.1 and 2.3.2.

Theorem 2.3.3. For coalition $H \subset N$ and $S \in S$, if $s \in S_H^{CNash}$, then $s \in S_H^{CSMR}$; if $s \in S_H^{CSMR}$, then $s \in S_H^{CGMR}$.

Theorem 2.3.4. For coalition $H \subset N$ and $s \in S$, if $s \in S_H^{CNash}$, then $s \in S_H^{CSEQ}$; *if* $s \in S_H^{CSEQ}$, then $s \in S_H^{CGMR}$.

Proposition 2.3.5 (Inohara2016 [39]). For $i \in N$, $S_i^{Nash} \subseteq S_i^{tNash-c} \subseteq S_i^{tNash}$.

Proposition 2.3.6 (Inohara2016 [39]). For $i \in N$, $S_i^{tGMR} \subseteq S_i^{tcGMR}; S_i^{tGMR-c} \subseteq S_i^{tcGMR-c}$.

Proposition 2.3.7 (Inohara2016 [39]). For $i \in N$, $S_i^{tSMR} \subseteq S_i^{tcSMR}; S_i^{tSMR-c} \subseteq S_i^{tcSMR-c}$.

Proposition 2.3.8 (Inohara2016 [39]). For $i \in N$, $S_i^{tSEQ} \subseteq S_i^{tcSEQ}; S_i^{tSEQ-c} \subseteq S_i^{tcSEQ-c}$.

Proposition 2.3.9 (Inohara2016 [39]). For $i \in N$:

i $S_i^{tNash} \subseteq S_i^{tSMR}, S_i^{tSMR} \subseteq S_i^{tGMR}, S_i^{tNash} \subseteq S_i^{tSEQ}, S_i^{tSEQ} \subseteq S_i^{tGMR}$.

ii $S_i^{tNash} \subseteq S_i^{tcSMR}, S_i^{tcSMR} \subseteq S_i^{tcGMR}, S_i^{tNash} \subseteq S_i^{tcSEQ}, S_i^{tcSEQ} \subseteq S_i^{tcGMR}$.

iii $S_i^{tNash-c} \subseteq S_i^{tSMR-c}, S_i^{tSMR-c} \subseteq S_i^{tGMR-c}, S_i^{tNash-c} \subseteq S_i^{tSEQ-c}, S_i^{tSEQ-c} \subseteq S_i^{tGMR-c}$.

iv $S_i^{tNash-c} \subseteq S_i^{tcSMR-c}, S_i^{tcSMR-c} \subseteq S_i^{tcGMR-c}, S_i^{tNash-c} \subseteq S_i^{tcSEQ-c}, S_i^{tcSEQ-c} \subseteq S_i^{tcGMR-c}$.

Proposition 2.3.10 (Inohara2016 [39]). If all $tt_i(s, s')$s are the same for all $i \in N$, all $s \in S$ and $s' \in R_i(s)$ then for $i \in N$:

i $S_i^{Nash} = S_i^{tNash-c} = S_i^{tNash}$.

ii $S_i^{GMR} = S_i^{tGMR} = S_i^{tcGMR} = S_i^{tGMR-c} = S_i^{tcGMR-c}$.

iii $S_i^{SMR} = S_i^{tSMR} = S_i^{tcSMR} = S_i^{tSMR-c} = S_i^{tcSMR-c}$.

iv $S_i^{SEQ} = S_i^{tSEQ} = S_i^{tcSEQ} = S_i^{tSEQ-c} = S_i^{tcSEQ-c}$.

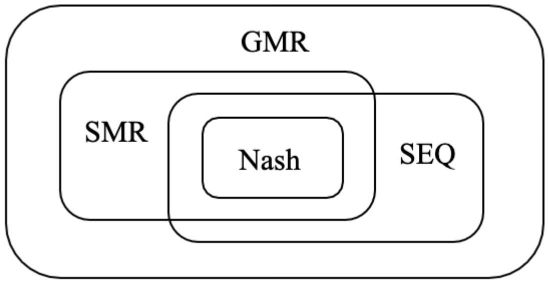

FIGURE 2.7: Interrelationship of Stability Concepts [22] [75]

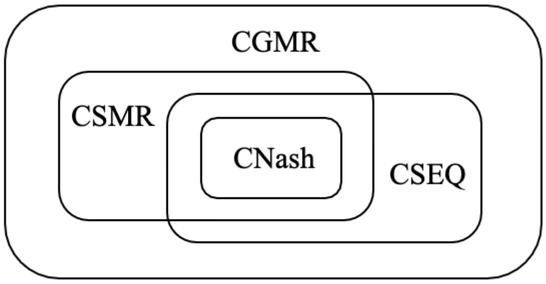

FIGURE 2.8: Interrelationship of Coaltional Stability Concepts [40]

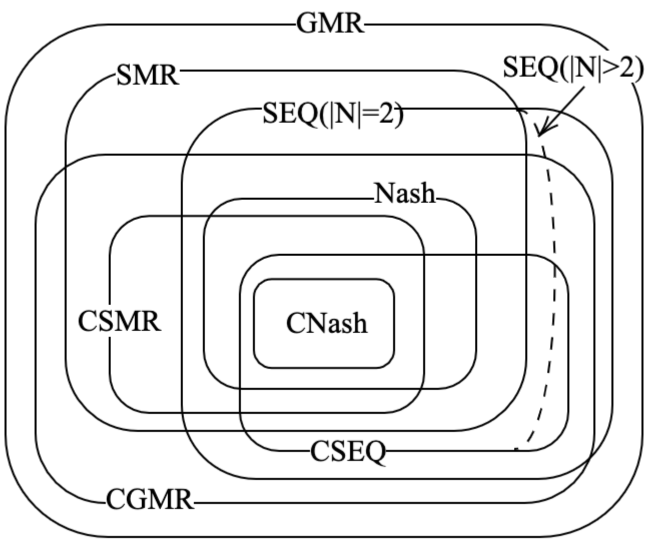

FIGURE 2.9: Interrelationship of Non-coalitional and Coaltional Stability Concepts [40]

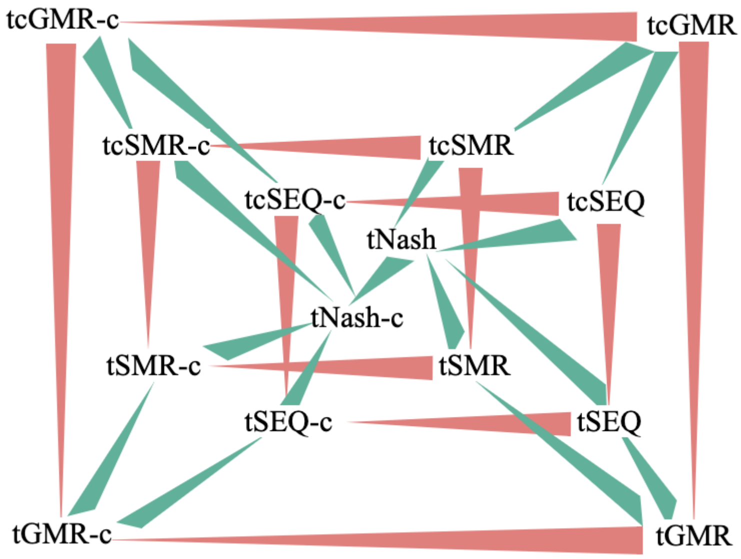

FIGURE 2.10: Interrelationship of t-GMCR Stability Concepts [39]

Chapter 3

GMCR Incorporating Permissible Range

Conflict resolution analysis requires information regarding the DM's value perception of the utility on the state of affairs that may arise in the conflict because conflict resolution is a process that can rationally or efficiently adjust the DMs' utility preferences. It is difficult to obtain information pertaining to DM preferences on possible states of affairs in real-world conflicts in many circumstances. However, it may be easier to identify "unacceptable situations" for some DMs. In this section, we propose an analysis method that incorporates the permissible range (PR) in DM's preferences, **when DM's binary choice between permissible and impermissible is known for feasible states** [47]. Using GMCR as the basic framework, we developed a framework with thresholds for the DMs' preference order over states. By introducing the new concept of "setting PR with thresholds for preference order in GMCR," it is possible to analyze cases in which the DM's preference information for all states is not available or only vague intentions are available. Because the DM's preference order is classified into only two sets, i.e., "permissible or impermissible," and the linear order of preference within the sets is no longer important in the proposed framework. Alternatively, instead of conducting the analysis using only binary information from the beginning, the available preference order information can be reserved and the analysis with more detailed information completed when the information for all states becomes clear. When a conflict occurs between two DMs, a preliminary analysis can be conducted using the preference information for all states of a DM and the information on PR known at the time for the other DM. In other words, the proposed method allows the analysts to switch the "resolution of the information" each time, depending on the fineness level of the information available. Hence, the proposed analysis method can enhance the applicability and usefulness of GMCR analysis .

Before we get into the discussion of PR, we review the impact of DM's preferences order on conflict analysis in the following subsection. Then we give definitions of the main concepts of GMCR incorporating PR. As an example, we analyzed and classified the changes in the stability of the chicken game when the DMs changed their acceptable range by setting a threshold for each DM in the GMCR framework. Subsequently, we attempted to generalize the framework using PR by limiting the scope of analysis to 2×2 games with no dominant strategy. Furthermore, we demonstrated application cases for the Elmira conflict and Cuban Missile Crisis.

3.1 Outline of the New Method

3.1.1 Background

The elements in the GMCR framework are sets of information about DM, state, DM's graph, and DM's preferences. In a conflict, a DM attempts to change the current state by seeking a more favorable state, weighing the utility of each possible state for him/her. In this process, DMs use their own preferences to guide their actions; thus, preferences are the most influential factor in solving conflicts. Using different preference orders, we show the sensitivity analysis results for the Elmira conflict we have already presented in Example 2.3.2.

Example 3.1.1 (Elmira Conflict with Different Preference Order)**.** Information for baseline analysis is given as follows: $N = \{M, U, L\}, S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9\}$,
$A_M = \{(s_1, s_2), (s_3, s_4), (s_5, s_6), (s_7, s_8)\}$,
$A_U = \{(s_1, s_3), (s_1, s_9), (s_2, s_4), (s_2, s_9), (s_3, s_9), (s_4, s_9), (s_5, s_7), (s_5, s_9), (s_6, s_8), (s_6, s_9), (s_7, s_9), (s_8, s_9)\}$,
$A_L = \{(s_1, s_5), (s_2, s_6), (s_3, s_7), (s_4, s_8), (s_5, s_1), (s_6, s_2), (s_7, s_3), (s_8, s_4)\}$,
$\succsim_M$: $s_7 \succ s_3 \succ s_4 \succ s_8 \succ s_5 \succ s_1 \succ s_2 \succ s_6 \succ s_9$,
$\succsim_U$: $s_1 \succ s_4 \succ s_8 \succ s_5 \succ s_9 \succ s_3 \succ s_7 \succ s_2 \succ s_6$,
$\succsim_L$: $s_7 \succ s_3 \succ s_5 \succ s_1 \succ s_8 \succ s_6 \succ s_4 \succ s_2 \succ s_9$.
Suppose that the preference orders of **M** (Ministry of the Environment) and **L** (local government) were precisely the same;
$\succsim_M$: $s_7 \succ s_3 \succ s_4 \succ s_8 \succ s_5 \succ s_1 \succ s_2 \succ s_6 \succ s_9$,
$\succsim_L$: $s_7 \succ s_3 \succ s_4 \succ s_8 \succ s_5 \succ s_1 \succ s_2 \succ s_6 \succ s_9$.

The result of the sensibility analysis with the two preference orders for **L** is given in Table 3.1. In the table, a checkmark indicates an equilibrium that is established in both the original and the new preference order, D indicates an equilibrium that disappears due to the adoption of the new preference order, and E indicates an equilibrium that is newly established due to the adoption of the new preference order, respectively. The stability analysis shows that there is a significant change in equilibrium in s_4 and s_8. In this conflict, **M** and **L** are assumed to have similar preferences due to their public nature. Still, suppose we consider a situation where **M**'s preference is information that can be collected in detail directly from the people involved, and the information about **L** is conjecture. In that case, we can see that the conjecture about **L**'s preference significantly impacts the outcome, that is more favorable to **U**, who ranks s_4 as the second. It is evident that this difference in the stability analysis results has significant implications for the interpretation of conflicts, and it suggests a problem in the analysis using inferential preference ranking.

The proposed GMCR incorporating PR(GMCR-PR) aims to provide more firm decision-making information by preventing fatal deviations in the analysis of inferential information when the linear preference order information for all possible states is not available, but "permissible" or "impermissible" states for the DM are known.

3.1.2 Theoretical Novelty

As already mentioned in Subsection 1.3.4, previous studies for dealing with the notion of "permissible range," are mainly based on the simple game, and some achievements consider consensus building, within the the committee framework [34] [92] [94].

TABLE 3.1: Elmira Conflict - Sensitivity Analysis

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
Nash				E	✓			D	✓
GMR	✓			✓	✓			✓	✓
SMR	✓			✓	✓			✓	✓
SEQ				E	✓			D	✓
CNash				E				D	✓
CGMR	✓			✓				D	✓
CSMR	✓			✓				D	✓
CSEQ				E				D	✓
Pareto	✓		✓	✓	D		✓	D	

Works that integrated "permissible range" with GMCR were developed by Inohara [37] [35], but all of these are still based on the committee. In other words, it can be said that all of these previous studies were based on the framework of committee, which is a derivative of a cooperative game, and was originally based on a framework for reaching a consensus by setting some decision rules.

On the other hand, this study proposes that even in conflicts with non-cooperative conflict structures, the concept of permissible range can be incorporated into the framework of conflict resolution by considering that there exists an implicit rule that conflicts shall converge in such a direction if the incentives to avoid the worst-case scenario are activated.

Thus, this research is the first to propose a framework that directly sets the permissible range to preference information in the GMCR framework and calculates stable solutions.

3.1.3 Information Partition and Coarseness in GMCR-PR

Here, states of conflicts and reachability of DMs, the combination of states and preferences for which the linear preference of all DMs for all states is clear, are given; we call the situation "*finest*." In contrast, the case where information partition of preferences occurred is called "*coarser*." In GMCR-PR, the case where the partitioning is the coarsest, i.e., binary, is treated. The basic concept is to partition within $s \in S$ based on the known criteria: "permissible and impermissible." This is the coarsest criteria framework and is useful when we do not know detailed preferences but know only what is permissible or impermissible about feasible states.

3.2 Permissible Range

In this section, we discuss the configuration of PRs in DMs' preference orders. First, we define reachability and stability concepts for GMCR-PR.

3.2.1 Definitions of GMCR-PR

Definition 3.2.1 (Permissible Range (PR)). We denote DM i's *permissible range* by P_i^n, the fact that, in a conflict, DM i allows up to the n^{th} most preferred state among the feasible states.

GMCR-PR is defined by the following equation:

Definition 3.2.2 (GMCR with Permissible Range (GMCR-PR))**.**

$$G = (N, S, (A_i)_{i \in N}, (\succsim_i)_{i \in N}, P_i^n). \tag{3.1}$$

Preferences for states included in DM i's PR, P_i^n, can be binarized as $P_i^{m \geq n} = 1$ and $P_i^{m<n} = 0$, where $n, m \leq |S|$, to represent *permissible value (PV)* , denoting permissible and impermissible for DM i, respectively:

Definition 3.2.3 (Permissible Value (PV))**.**

$$PV = \begin{cases} 1, \text{if } P_i^{m \geq n}, \\ 0, \text{otherwise.} \end{cases} \tag{3.2}$$

Remark. Setting a threshold on the preference order over states can be generalized as obtaining a mapping of a subset of the feasible state set S, determined by the permissible function χ_P. For a subset of the state set S, let χ_X be the characteristic function $f : S \to 2 = \{0, 1\}$ determined by the equation 3.2. For subsets X and Y of S, $X \subset Y$ and $\chi_X \subseteq \chi_Y$ are *equivalent*. Let $\overline{P}$ be a condition on the element of S, the characteristic function of $X = \{s \in S \mid \overline{P}\}$:

$$v_{\overline{P}}(s) = \begin{cases} 1, \text{if } s \text{ satisfies } \overline{P}, \\ 0, \text{otherwise,} \end{cases} \tag{3.3}$$

is the function $v_{\overline{P}} : S \to 2 = \{0, 1\}$ determined by $\overline{P}$. The characteristic function χ_P of a subset $X \subset S$ is a truth-value function of the condition for $s \in S$, where $X = \{s \in S \mid \chi_X(s) = 1\}$, representing permissivity on an arbitrary element(a state) of S. Assume $v : S \to \{0, 1\}$ is a function, v is a characteristic function of $X = \{s \in S \mid v(s) = 1\}$.

Example 3.2.1 (Coarse and Fine Preference Information)**.** Partition of preference information can be considered as we have provided a definition in 2.1.2. Suppose the preference order for DM i in a four states conflict: $S = \{s_1, s_2, s_3, s_4\}$, where DM a's preference order and PR are given as $s_1 \succ s_2 \succ s_3 \succ s_4$ and P_a^2, respectively.

The information partitioning for DM a's preference order when we set PR from P_a^0 to P_a^4 is summarized as follows. $\mathcal{P}_a^n$ represents information partition when PR is set at n^{th} most preferred states for DM a, and $\overline{P}_a^n$ denotes the elements included in DM a's PR when threshold is set at n^{th} most preferred states:

$$P_a^0, \mathcal{P}_a^0 = \{\ s_1, s_2, s_3, s_4\}, \overline{P}_a^0 = \emptyset,$$

$$P_a^1, \mathcal{P}_a^1 = \{\{s_1\}, \{s_2, s_3, s_4\}\}, \overline{P}_a^1 = \{s_1\},$$

$$P_a^2, \mathcal{P}_a^2 = \{\{s_1, s_2\}, \{s_3, s_4\}\}, \overline{P}_a^2 = \{s_1, s_2\},$$

$$P_a^3, \mathcal{P}_a^3 = \{\{s_1, s_2, s_3\}, \{s_4\}\}, \overline{P}_a^3 = \{s_1, s_2, s_3\},$$

$$P_a^4, \mathcal{P}_a^4 = \{\{s_1, s_2, s_3, s_4\}\}, \overline{P}_a^4 = \{s_1, s_2, s_3, s_4\}.$$

Figure 3.1 illustrates the mappings of coarsening and refinement of preference information. The leftmost figure represents the preference with the finest information, and the center represents the $\mathcal{P}^2$ situation. The rightmost figure shows the

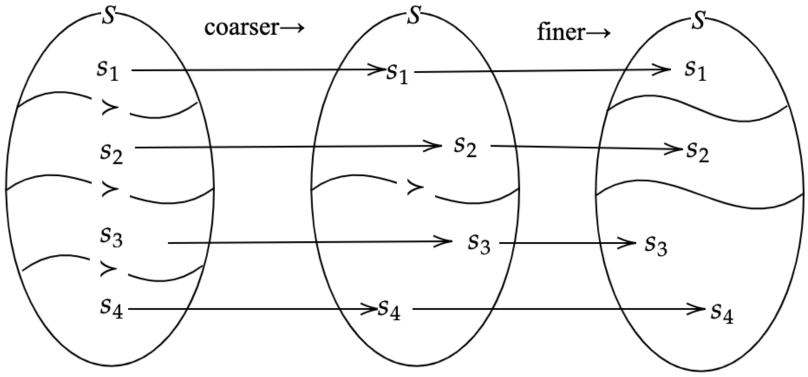

FIGURE 3.1: Coarsening and Refinement of Preference Information

information partitioning when detailed information about the elements included in $\overline{P}^2$ is available in the $\mathcal{P}^2$ situation.

GMCR incorporating permissible range, GMCR-PR, is represented by five tuples: DMs (N), set of feasible states (S), the graph of DM i (A_i), the preferences of each DM i ($\succsim_i$), and DM i's permissible range over their preferences (P_i).

Definition 3.2.4 (GMCR Incorporating Permissible Range)**.**

$$G = (N, S, (A_i)_{i \in N}, (\succsim_i)_{i \in N}, (P_i)_{i \in N}). \tag{3.4}$$

Definition 3.2.5 (Permissible Reachable Lists)**.** DM i's *permissible reachable list* from $s \in S$ are subsets of S as follows:

i. DM i's permissible reachable list from s to s' by unilateral moves:

$$PR_i(s) = \{s' \in S \mid (s, s') \in A_i\}. \tag{3.5}$$

$(s, s') \in A_i$ denotes the permissible reachability; DM i can reach from s to s'.

ii. DM i's permissible reachable list from s to s' by unilateral improvements:

$$PR_i^+(s) = \{s' \in PR_i(s) \mid PV(s') = 1, \text{ and } PV(s) = 0\}. \tag{3.6}$$

iii. DM i's permissible list about s and s' equally or less preferred :

$$P\phi_i^{\simeq}(s) = \{s' \in S \mid PV(s) \geq PV(s')\}. \tag{3.7}$$

iv. $PR_{N \setminus \{i\}}(s)$ is defined as the set of all states which can be achieved by the sequences of permissible unilateral moves of DMs other than DM i.

v. $PR^+_{N \setminus \{i\}}(s)$ is defined as the set of all states which can be achieved by the sequences of permissible unilateral improvements of DMs other than DM i.

When DM i has no unilateral improvements from state s, there are no further state transitions, thereby establishing stability.

Definition 3.2.6 (PNash)**.** For $i \in N$, state $s \in S$ is PNash stable for DM i, denoted by $s \in S_i^{PNash}$, if and only if

$$PR_i^+(s) = \emptyset. \tag{3.8}$$

When DM i cannot reach a state more favorable than state s through any of its unilateral improvements because of the other DM's response, there are no further state transitions, thereby establishing stability.

Definition 3.2.7 (PGMR). For $i \in N$, state $s \in S$ is PGMR stable for DM i, denoted by $s \in S_i^{PGMR}$, if and only if

$$\forall s' \in PR_i^+(s),\ PR_{N \setminus \{i\}}(s') \cap P\phi_i^{\simeq}(s) \neq \emptyset. \tag{3.9}$$

When a state occurs where for any of DM i's unilateral improvements, the response of another DM would cause a state less favorable than state s , and further, regardless of how DM i responds to the other DM's response, a state more favorable than state s cannot occur, there are no further state transitions, thereby establishing stability.

Definition 3.2.8 (PSMR). For $i \in N$, state $s \in S$ is PSMR stable for DM i, denoted by $s \in S_i^{PSMR}$, if and only if

$$\forall s' \in PR_i^+(s), \exists s'' \in PR_{N \setminus \{i\}}(s') \cap P\phi_i^{\simeq}(s),\ PR_i(s'') \subseteq P\phi_i^{\simeq}(s). \tag{3.10}$$

When DM i has at least one unilateral improvement from state s, regardless of DM i's unilateral movement, the state that would occur due to another DM's choice would not be more preferable than the current state, and there are no further transitions, thereby establishing stability.

Definition 3.2.9 (PSEQ). For $i \in N$, state $s \in S$ is PSEQ stable for DM i, denoted by $s \in S_i^{PSEQ}$, if and only if

$$\forall s' \in PR_i^+(s),\ PR_{N \setminus \{i\}}^+(s') \cap P\phi_i^{\simeq}(s) \neq \emptyset. \tag{3.11}$$

We analyze the chicken game as an example case of 2×2 conflicts and review the changes in stability due to changes in the PR of DMs. The chicken game can be described in terms of the GMCR as follows:

Example 3.2.2 (Chicken Game).
$(N, S, (A_i)_{i \in N}, (\succsim_i)_{i \in N}), N = \{1, 2\}, S = \{s_1, s_2, s_3, s_4\}$
$A_1 = \{(s_1, s_3), (s_3, s_1), (s_2, s_4), (s_4, s_2)\}$,
$A_2 = \{(s_1, s_2), (s_2, s_1), (s_3, s_4), (s_4, s_3)\}$,
DM_1's preference order $\succsim_1$: $s_3 \succ s_1 \succ s_2 \succ s_4$,
DM_2's preference order $\succsim_2$: $s_2 \succ s_1 \succ s_3 \succ s_4$, $S^{Nash} = \{s_2, s_3\}$.

We set the thresholds in the DM's preference order, and identify the conflict in the framework of the GMCR while incorporating the PR. The thresholds can be set in five levels for the four states in the chicken game; P^0 does not allow all states, P^1 allows only the most favorable state, P^2 allows the most and the second-most favorable state, P^3 allows the most, the second, and the third-most favorable state, and P^4 allows all states. Assuming that DM2's threshold is P^2, we have the following, $\succsim_2$: $s_2 \sim s_1 \succ s_3 \sim s_4$.

Table 3.2 shows the result of stability analysis when both DMs have a PR, P^3. The checkmarks indicate the stability established in both the standard analysis method and the analysis methods with PR P^3. E indicates the stability that can be newly established by setting the PR. As shown in the table, by setting the PR, a new equilibrium emerges in s_1. s_1 was initially a state in which Pareto optimum was established, but only GMR and SMR pertaining to rationality in the standard analysis were identified. It can be interpreted that if the two DMs know what each other's permissibility is in the chicken game, then the conflict may be settled with a solution at s_1. Table 3.3 summarizes the stability due to changes in the permissibility of DMs in the chicken game. The numbers (1,2,3,4) in the table represent the state numbers where the equilibrium (PNash, PGMR, PSMR, PSEQ, and Pareto) hold. In addition to the binary preferences by $P^0 - P^4$ thresholds already shown, the original four levels of preferences are shown as $\mathcal{O}$. The preference combination $P_1^3 - P_2^3$ in Table 3.2 corresponds to Table 3.3. Moreover, it was observed that all equilibria and Pareto optimality were established at s_1, s_2, and s_3 when $P_1^3 - P_2^0$, $P_1^3 - P_2^3$, and $P_1^3 - P_2^4$, respectively.

TABLE 3.2: Stability Analysis- Chicken Game

		s_1	s_2	s_3	s_4
	DM1	E	✓	✓	
Nash	DM2	E	✓	✓	
	Eq	E	✓	✓	
	DM1	✓	✓	✓	
GMR	DM2	✓	✓	✓	
	Eq	✓	✓	✓	
	DM1	✓	✓	✓	
SMR	DM2	✓	✓	✓	
	Eq	✓	✓	✓	
	DM1	E	✓	✓	
SEQ	DM2	E	✓	✓	
	Eq	E	✓	✓	

3.2.2 Interrelationship of Stability Concept: GMCR and GMCR-PR

The interrelationships between stability concepts in GMCR and GMCR-PR are given as follows:

Proposition 3.2.1. For $i \in N$ and $s \in S$, $s \in S_i^{Nash}$, $s \in S_i^{PNash}$.

Proof. if $s \in S_i^{Nash}$, $R_i^+(s) = \emptyset$. $R_i^+(s) = \emptyset$, then $PR_i^+(s) = \emptyset$: if DM i has no unilateral improvement with its finest preference set, DM i has no unilateral improvement with its coarser preference set defining PR. Thus $s \in S_i^{Nash}$, $s \in S_i^{PNash}$. □

Proposition 3.2.2. For $i \in N$ and $s \in S$, if $s \in S_i^{GMR}$, then $s \in S_i^{PGMR}$.

Proof. From Theorem 2.3.1, 2.3.2, and Proposition 3.2.1, if $s \in S_i^{GMR}$, then $s \in S_i^{PGMR}$ □

Proposition 3.2.3. For $i \in N$ and $s \in S$, if $s \in S_i^{SMR}$, then $s \in S_i^{PSMR}$.

Proposition 3.2.4. For $i \in N$ and if $s \in S_i^{Nash}$, then $s \in S_i^{SEQ}$; if $s \in S_i^{SEQ}$, then $s \in S_i^{PSEQ}$.

Proposition 3.2.3 can be proved analogously with the proof for Proposition 3.2.2.

Proposition 3.2.5. For $i \in N$ and $s \in S$, if $s \in S_i^{PNash}$, then $s \in S_i^{PSMR}$; if $s \in S_i^{PSMR}$, then $s \in S_i^{PGMR}$.

TABLE 3.3: Stability Analysis - Chicken Game with PR

DM1	DM2	(P)Nash	(P)GMR	(P)SMR	(P)SEQ	Pareto
$\mathcal{O}$	$\mathcal{O}$	2,3	1,2,3	1,2,3	2,3	1,2,3
$\mathcal{O}$	P_2^0	2,3	1,2,3	1,2,3	2,3	3
$\mathcal{O}$	P_2^1	2,3	1,2,3	1,2,3	2,3	2,3
$\mathcal{O}$	P_2^2	2,3	1,2,3	1,2,3	2,3	1,3
$\mathcal{O}$	P_2^3	2,3	1,2,3	1,2,3	2,3	3
$\mathcal{O}$	P_2^4	2,3	1,2,3	1,2,3	2,3	3
P_1^0	P_2^0	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4
P_1^1	P_2^0	2,3,4	1,2,3,4	1,2,3,4	2,3,4	3
P_1^1	P_2^1	2,3,4	1,2,3,4	1,2,3,4	2,3,4	2,3
P_1^2	P_2^0	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,3
P_1^2	P_2^1	2,3,4	1,2,3,4	1,2,3,4	2,3,4	1,2,3
P_1^2	P_2^2	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1
P_1^3	P_2^0	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
P_1^3	P_2^1	2,3	1,2,3	1,2,3	2,3	2
P_1^3	P_2^2	1,2,3	1,2,3	1,2,3	1,2,3	1,2
P_1^3	P_2^3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
P_1^4	P_2^0	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4
P_1^4	P_2^1	2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	2
P_1^4	P_2^2	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2
P_1^4	P_2^3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
P_1^4	P_2^4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4

* The numbers 1, 2, 3, 4 in the columns (P)Nash, (P)GMR, (P)SEQ and Pareto represent the state numbers s_1, s_2, s_3, s_4, respectively.

Proof. From Theorem 2.3.1 and 2.3.2, Proposition 3.2.1, and 3.2.3, if $s \in S_i^{PNash}$, then $s \in S_i^{PSMR}$; if $s \in S_i^{PSMR}$, then $s \in S_i^{PGMR}$. □

Proposition 3.2.6. For $i \in N$ and $s \in S$, if $s \in S_i^{PNash}$, then $s \in S_i^{PSEQ}$; if $s \in S_i^{PSEQ}$, $s \in S_i^{PGMR}$.

Proposition 3.2.7. For $i \in N$ and $s \in S$, $S_i^{Nash} \subseteq S_i^{PNash}, S_i^{GMR} \subseteq S_i^{PGMR}, S_i^{SMR} \subseteq S_i^{SGMR}$.

Proposition 3.2.6 and 3.2.7 can be proved analogously with the proof for Proposition 3.2.5.

In the following section, we attempted to generalize the GMCR-PR stability analysis by restricting the scope of the analysis to 2×2 games in which both DMs have no dominant strategy.

3.3 Conflicts with No Dominant Strategy

In the taxonomy by Rapoport and Guyer [78] (RG), 2×2 games are classified into three classes, Class I : games in which both DMs have a dominant strategy; Class II : games in which only one DM has a dominant strategy; and Class III : games in which neither DM has a dominant strategy. The chicken game discussed in the previous subsection was classified as Class III. In Classes I and II, where either DM has a dominant strategy, a Nash equilibrium is established, and each conflict converges to a single equilibrium. In Class III presented in Table 3.4, where either no dominant strategy and no equilibrium (N) or two equilibria (W) exist, a set of conflicts that are difficult to resolve exists, which is denominated by RG as "pre-emption games," such as the chicken game, or "cycle games." We introduce the concept of permissibility in this class of conflict and analyze the equilibrium. Class III contained twenty one games from game numbers 58 to 78 in RG , where "Stag Hunt (61)," "Luke and Matthew (64)," "Chicken Game (66)," and "Battle of Sexes (68)" are classified.(In the Table 3.4, the number indicated in each state represents the DM's preference for the state by ordinal number (the bigger number for the more preferred state.)

TABLE 3.4: Games with No Dominant Strategy

Game No.	DM1				DM2				Nash	Note
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4		
58	4	2	1	3	4	3	1	2	W	
59	4	2	1	3	4	2	1	3	W	
60	4	2	1	3	4	1	2	3	W	
61	4	1	3	2	4	3	1	2	W	Stag Hunt
62	4	1	3	2	4	2	1	3	W	
63	4	1	2	3	4	2	1	3	W	
64	3	2	1	4	4	1	2	3	W	Luke and Matthew
65	2	3	1	4	4	1	2	3	W	
66	3	2	4	1	3	4	2	1	W	Chicken Game
67	2	3	4	1	3	4	2	1	W	
68	2	3	4	1	2	4	3	1	W	Battle of Sexes
69	2	4	3	1	2	3	4	1	W	
70	3	2	4	1	4	1	2	3	N	
71	3	2	4	1	3	1	2	4	N	
72	3	2	4	1	2	1	3	4	N	
73	2	4	3	1	4	1	2	3	N	
74	2	3	4	1	4	1	2	3	N	
75	2	4	3	1	3	1	2	4	N	
76	2	3	4	1	3	1	2	4	N	
77	2	4	3	1	2	1	3	4	N	
78	2	3	4	1	2	1	3	4	N	

The number indicated in each state column represents the DM's preference for the state by ordinal number.

We analyzed the stability of these twenty one games, assuming that each of the two DMs has a five level PR from P^0 to P^4: for $21 \times 5 \times 5 = 525$ cases, respectively. Among them, three games exist in which the preferences of the two DMs are symmetrical, including the chicken game, but they are not excluded. Let the class III games in RG be the games with two Nash equilibria as category W (No.58-No.69 in Table 3.4 , indicated as "W") and the games without Nash equilibria as category N(No.70-No.78 in Table 3.4 , indicated as "N"). Introduced PR for each of the twenty-one games and conducted stability analysis, we found Propositions 3.3.1 and 3.3.2 hold.

Proposition 3.3.1. In conflicts included in category N, at least one or more Nash equilibrium is established by introducing PR.

Proof. The stability analysis results are provided in Appendix B.1 . □

Proposition 3.3.2. S^{PNash}, S^{PGMR}, S^{PSMR}, and S^{PSEQ} for each game in category W are unchanged in all combinations of the two DMs' PR.

Proof. It is obvious from Proposition 3.2.7. □

Table 3.3 in the previous section shows the stability analysis results for the chicken game included in category N for all combinations of the two DM's PRs. We can see that the results in Table 3.3 are also consistent with Proposition 3.3.2.

Proposition 3.3.3. S^{PGMR} and S^{PSMR} for each game in category N are unchanged in all combinations of the two DMs' PRs.

Proof. The stability analysis results provided in Appendix B.1 are precisely the proofs of the proposition. □

TABLE 3.5: Stability Analysis - Game No.78 with PR

DM1	DM2	(P)Nash	(P)GMR	(P)SMR	(P)SEQ	Pareto
$\emptyset$	$\emptyset$	$\emptyset$	1,3	3	1,3	3,4
P_1^0	P_2^0	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4
P_1^0	P_2^1	1,2,4	1,2,3,4	1,2,3,4	1,2,4	4
P_1^0	P_2^2	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	3,4
P_1^0	P_2^3	1,3,4	1,3,4	1,3,4	1,3,4	1,3,4
P_1^0	P_2^4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4
P_1^1	P_2^0	2,3,4	1,2,3,4	1,2,3,4	2,3,4	3
P_1^1	P_2^1	2,4	1,2,3,4	1,2,3,4	2,3,4	3,4
P_1^1	P_2^2	2,3,4	1,2,3,4	1,2,3,4	2,3,4	3
P_1^1	P_2^3	3,4	1,3,4	1,3,4	3,4	3
P_1^1	P_2^4	2,3,4	1,2,3,4	1,2,3,4	2,3,4	3
P_1^2	P_2^0	2,3	1,2,3,4	2,3	2,3	2,3
P_1^2	P_2^1	2	1,2,3,4	2,3	2,3	2
P_1^2	P_2^2	2,3	1,2,3,4	2,3	2,3	3
P_1^2	P_2^3	3	1,3,4	3	3	3
P_1^2	P_2^4	2,3	1,2,3,4	2,3	2,3	2,3
P_1^3	P_2^0	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
P_1^3	P_2^1	1,2	1,2,3	1,2,3	1,2,3	1,2,3,4
P_1^3	P_2^2	1,2,3	1,2,3	1,2,3	1,2,3	3
P_1^3	P_2^3	1,3	1,3	1,3	1,3	1,3
P_1^3	P_2^4	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
P_1^4	P_2^0	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4
P_1^4	P_2^1	1,2,4	1,2,3,4	1,2,3,4	1,2,4	4
P_1^4	P_2^2	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	3,4
P_1^4	P_2^3	1,3,4	1,3,4	1,3,4	1,3,4	1,3,4
P_1^4	P_2^4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4

* The numbers 1, 2, 3, 4 in the columns (P)Nash, (P)GMR, (P)SEQ and Pareto represent the state numbers s_1, s_2, s_3, s_4, respectively.

Our analysis shows that two or more Nash equilibria were established, and all of them were Pareto optimal in all twenty-one games, when the PRs of DM1 and DM2 were $P_1^0 - P_2^0$, $P_1^0 - P_2^3$, $P_1^3 - P_2^0$, $P_1^3 - P_2^3$, $P_1^3 - P_2^4$, $P_1^4 - P_2^3$, and $P_1^4 - P_2^4$. In particular, the cases of $P_1^3 - P_2^3$, $P_1^3 - P_2^4$ and $P_1^4 - P_2^3$ are important when we exclude the case where it trivially holds that equilibrium is established in all states.

Hence, in 2×2 games in which two DMs have no dominant strategy, any conflict can be resolved when both DMs set their threshold as P^3; "accept all states except the least favorable one."

We reviewed Battle of Sexes : game number 68 in RG, as an example of another Class III game. Using the utility information provided in RG, the conflict can be described as follows:

Example 3.3.1 (Battle of Sexes). $(N, S, (A_i)_{i \in N}, (\succsim_i)_{i \in N}), N = \{1, 2\}, S = \{s_1, s_2, s_3, s_4\}$ $A_1 = \{(s_1, s_3), (s_3, s_1), (s_2, s_4), (s_4, s_2)\}, A_2 = \{(s_1, s_2), (s_2, s_1), (s_3, s_4), (s_4, s_3)\}, DM_1$'s preference order $\succsim_1$: $s_3 \succ s_2 \succ s_1 \succ s_4$, DM_2's preference order $\succsim_2$: $s_2 \succ s_3 \succ s_1 \succ s_4$, Nash equilibrium is established in s_2 and s_3.

The fact that the PR of two DMs' is P^3 indicates that both DMs can tolerate any state other than s_4; therefore, it is evident that s_1 becomes an equilibrium.

Proposition 3.3.4 (Correspondence of Nash and Pareto in 2×2 Game without Dominant Strategy). For $i \in N, s \in S$ in 2×2 Game without dominant strategy provided by RG [78], if $s \in S_i^{Nash}$ then $s \in S^{Pareto}$, if and only if $\overline{P}_i^n, s.t. n \geq 3$.

Proof. The proof is given by Definition of S^{Pareto} in 2.3.7 and the stability analysis results provided in Appendix B.1 □

Remark. Propositions 3.2.4, 3.2.5, and 3.2.6 are only valid for games in category W in Table 3.4, i.e., games with two Nash in the original preferences. In other words, Propositions 3.2.4, 3.2.5, and 3.2.6 do not hold for games that do not have Nash in the original fine preference information. As for Proposition 3.2.7, S_i^{PNash} is only applicable to category W games.

GMCR-PR analysis results of game number 78, as another example of the category N case, are shown in Table 3.5 ; the results show that Propositions 3.2.4, 3.2.5, and 3.2.6, where Nash in the state s is a necessary condition, do not hold, and only Propositions 3.2.2 and 3.2.3 hold.

In this subsection, we proposed setting a threshold in the preference order to define the PR and demonstrated the manner in which it can be introduced to twenty-one model games that are difficult to resolve to identify equilibrium in the conflict by relaxing the threshold. In the next subsection, we focus on Elmira Conflict and Cuban Missile Crisis for the further development of our framework, by applying the PR to the conflicts; subsequently, we discuss the results of the analysis.

3.4 Elmira Conflict - Analysis with Permissible Range

We applied the PR for each DM in an Elmira conflict and analyzed the stability for the following three threshold cases set based on DM options, as provided in the previous section 2.3.

Worst-Case Scenario in Elmira Conflict

As shown in the stability analysis 2.5, the worst-case scenario in the Elmira conflict is s_9: **U** abandons the plant, where all equilibria are established but not efficient. (Definition 2.3.8:1 and 3) The analysis explores the equilibrium possibilities in other states by setting PR.

3.4.1 Case I: P_M^6, P_U^9, P_L^6

M does not modify the control order, **U** accepts only the delay, **L** insists on the issued order. Based on the information shown in Tables 2.3 and 2.4, the PR for **M**, **U**, and **L** can be derived as P_M^6, P_U^9 and P_L^6, respectively. Table 3.6 shows the results of the stability analysis under these PR conditions. The binary information provided as $\{1,0\}$ for each DM is permissible or impermissible when the PR of DM i' is provided by P_i^n, $P_i^{m \geq n} = 1$, $P_i^{m<n} = 0$, according to the equation 3.2. In s_3 and s_7, where equilibrium was not established in the original conflicts but was Pareto optimal, strong stability including Nash, was established in the analysis with the PR. This is exactly a result that reflects the PRs of **M** and **U**. Since the PR of **U** is P_U^9, where all states are acceptable, such a method is useful as a primary analysis in a situation where only information pertaining to **M** and **L**'s overall intentions or prioritized options is known.

TABLE 3.6: Elmira Conflict - Stability Analysis: Case I

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
M	1	0	1	1	1	0	1	1	0
U	1	1	1	1	1	1	1	1	1
L	1	0	1	0	1	1	1	1	0
Nash	✓		✓		✓	✓	✓	✓	✓
GMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
SMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
SEQ	✓		✓		✓	✓	✓	✓	✓
Pareto	✓		✓		✓		✓	✓	

3.4.2 Case II: P_M^8, P_U^9, P_L^6

M accepts the modification of the control order, **U** accepts only the delay, and **L** insists on the issued order. The PRs of **M**, **U**, and **L** are P_M^8, P_U^9 and P_L^6, respectively. The stability analysis for Case II is presented in Table 3.7. By loosening the PR of **M**, the Pareto optimum was newly established in s_6, where **U** continues to stall and **L** adheres to enforce the issued control order, whereas **M** accepts the modification of the control order.

3.4.3 Case III: P_M^2, P_U^7, P_L^5

M does not accept the control order modification, **U** accepts the order without delay, and **L** does not insist on the issued order if **U** accepts any order without delay. The PRs of **M**, **U**, and **L** are P_M^2, P_U^7 and P_L^5, respectively. In this case, **L** accepts the states in which **M** modifies the order, provided that **U** accepts the order without stalling. As shown in Table 3.8, in Case III, the Pareto optimum holds at s_3 and s_7 , reflecting the narrower PR of **M**.

We discovered that the introduction of the PR enables us to conduct a meaningful analysis of the Elmira conflict with three DMs and nine states, even when the DM preferences for all states are not known. In the next subsection, we present a case study of its application to the Cuban Missile Crisis.

TABLE 3.7: Elmira Conflict - Stability Analysis: Case II

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
M	1	1	1	1	1	1	1	1	0
U	1	1	1	1	1	1	1	1	1
L	1	0	1	0	1	1	1	1	0
PNash	✓		✓		✓	✓	✓	✓	✓
PGMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
PSMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
PSEQ	✓		✓		✓	✓	✓	✓	✓
Pareto	✓		✓		✓	✓	✓	✓	

TABLE 3.8: Elmira Conflict - Stability Analysis: Case III

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
M	0	0	1	0	0	0	1	0	0
U	1	0	1	1	1	0	1	1	1
L	1	0	1	0	1	0	1	1	0
PNash	✓		✓		✓		✓	✓	✓
PGMR	✓		✓	✓	✓		✓	✓	✓
PSMR	✓		✓	✓	✓		✓	✓	✓
PSEQ	✓		✓		✓		✓	✓	✓
Pareto			✓				✓		

3.5 Cuban Missile Crisis

3.5.1 The Crisis

The Cuban Missile Crisis was a violent confrontation between the United States and the Soviet Union over constructing a Soviet missile base in Cuba in 1962.

Due to the escalating actions of both countries, the scale and area of the conflict quickly could expand globally, and there was a danger that it would develop into an all-out nuclear war. Still, direct dialogue between the two countries' leaders led to the Soviet Union's removal of its missiles.

On October 14, 1962, the U.S. Air Force discovered a Soviet missile base in Cuba. The United States recognized that the main reason for this deployment was for the Soviet Union to demonstrate its influence over Third World countries and its ability to strike first against the United States. President Kennedy immediately convened the EXCOMM (Executive Committee of the National Security Council) to discuss with a group of experts, and the result was that the United States could only take the following three strategies; (1) no offensive action, (2) full-scale airstrikes, and (3) a naval blockade. The Soviet response was predicted to be as follows; (1) not remove missiles from Cuba, (2) remove missiles, (3) escalate the conflict and attack the United States. Based on the results of this study, the United States decided on (3) the naval blockade and began formal action on October 17, 1962.

This confrontation between the United States and the Soviet Union was considered as a "risk-taking competition [80] [88]." It is often said that the situation was a game of chicken. The Cuban Missile Crisis can be described in normal form game as shown in Table 3.9. However, as a practical matter, if you were an analyst in the United States or the Soviet Union, you would not be able to propose an optimal strategy to the leader only with this state of information. Nash equilibrium [69] exists in two states, (4,2) and (2,4), and victory requires concessions from the opponent. Therefore, it is necessary to formulate some action in the state before the game, such as extracting concessions from the opponent.

Since the Soviet Union's deployment of missiles could be considered a retaliatory measure against the United States for its deployment of missiles in Turkey, the actual settlement may be regarded as a "no-winner conflict with compromise;" there are many discussions from the perspective of conflict analysis about the gap of actual consequences and game- theoretic considerations.

President J.F. Kennedy declared a naval blockade and demanded that the Soviet Union remove its missiles in a televised broadcast. As a result, the Soviet Union was forced to choose "concessions" as a strategy, and as a result, the situation in (3,3) was reached. According to Schelling [80], "the first player to make a move can force the opponent to concede." However, the normal form game description method cannot describe the flow of actions considering the opponent's moves or the transition from the initial state. Hence, the actual historical facts did not unfold as Schelling's theory. In this study, the state transitions of conflicts are scrutinized by using GMCR. Moreover, evolvement and convergence of the conflict are analyzed by using GMCR-PR.

TABLE 3.9: Cuban Missile Crisis in Normal Form Game

		USSR	
		Concede	Attack
US	Concede	3,3	2,4
	Attack	4,2	1,1

The conflict's basic structure is often regarded as a game of chicken. However, as already mentioned, analysis based on the game theory framework only shows two Nash equilibria, which is not useful information for optimal decision-making for both DMs. The analysis suggests that the optimal strategy is to attack first, which is a sufficient threat to the other side, and then block the other side's attack.

Assuming that the possible options of the two DMs are ; U.S.: 1) Airstrike 2) naval blockade; and USSR: 1) withdraw the missiles and 2) escalate, the feasible states, the preference orders on the states of the two DMs, and the graph are shown in Table 3.10, Table 3.11, and Figure 3.2, where the solid and the dotted lines represent the state transitions for the United States and the Soviet Union, respectively. The results of the stability analysis conducted based on this information are presented in Table 3.12. Equilibrium was established in four states: status quo in s_1, "US: air strike, USSR: withdraw" in s_5, "US: naval blockade, USSR: withdraw" in s_6, and "US: naval blockade, USSR: escalate" in s_9, while Pareto optimal holds in s_1 and s_6 . Next, we apply GMCR-PR to the conflict.

TABLE 3.10: Cuban Missile Crisis - Options and States

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
U.S. airstrike	N	Y	N	Y	Y	N	Y	Y	N	Y
U.S blockade	N	N	Y	Y	N	Y	Y	N	Y	Y
USSR withdraw	N	N	N	N	Y	Y	Y	N	N	N
USSR escalate	N	N	N	N	N	N	N	Y	Y	Y

TABLE 3.11: Cuban Missile Crisis - Preference Order

	most preferred									least preferred
U.S.	s_6	s_5	s_7	s_1	s_3	s_2	s_4	s_9	s_8	s_{10}
USSR	s_1	s_3	s_9	s_2	s_8	s_6	s_5	s_4	s_7	s_{10}

3.5.2 Worst-Case Scenario in Cuban Missile Crisis

The stability analysis 3.12 indicates the strong equilibria hold in s_9 (US: Blockade, USSR: Escalate) which is consisted of the United States' least preferred option, "USSR: Escalate", and is not efficient. (Definition 2.3.8: 3 and 4) The analysis seeks de-escalation settlement.

3.5.3 Analysis with PR

In the baseline analysis, both rational equilibrium and Pareto optimalities were established at s_1 and s_6. This strong stability suggests that the situation may be stalemated at s_1 i.e., the status quo. Assuming that the United States will not tolerate the stalemate at the status quo and that the Soviet Union will not tolerate the United States conducting both a naval blockade and air strikes, then the two DMs' PRs are P^3_{US} and P^7_{USSR}, respectively. In fact, the United States, with the solid support of its allies, established a seven-step military plan that called for airstrikes if the Soviet Union did not agree to remove its missiles even after implementing a naval blockade. For the Soviet Union, it was presumed that they intended to avoid a situation in which the United States would launch airstrikes and naval blockades, where the conflicts were not localized but escalated [51] [67].

Table 3.13 shows the results of the stability analysis under the assumption of P^3-P^7. While equilibrium is established in more states than in the original analysis, Pareto disappears in the status quo s_1, and a new equilibrium is established in s_5, where the United States conducts airstrikes, and the Soviet Union retreats. These analysis results can be interpreted as reflection of the PR set by the United States. hence, this analytical method is considered to be valid assuming that the bottom line of the Soviet Union had known immediately after the deployment of the Soviet Union's missiles in Cuba.

3.6 Nash Stability and Efficiency for Permissible Range

Based on the study results in Sections 3.3, 3.4, and 3.5, we present propositions for Nash Stability and Efficiency in conflict analysis incorporating PR. The propositions are prepared separately for cases where at least one state commonly permissible to all DMs exists, in Subsection 3.6.1, and cases where it does not exist, in Subsection 3.6.2.

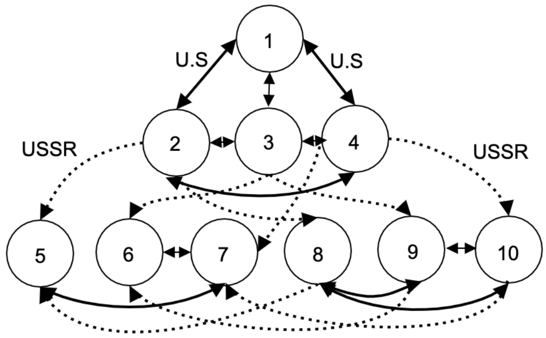

FIGURE 3.2: Graph Model of Cuban Missile Crisis

TABLE 3.12: Cuban Missile Crisis - Stability Analysis

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
Nash	✓				✓	✓			✓	
GMR	✓				✓	✓			✓	
SMR	✓				✓	✓			✓	
SEQ	✓				✓	✓			✓	
CNash	✓					✓			✓	
CGMR	✓					✓			✓	
CSMR	✓					✓			✓	
CSEQ	✓					✓			✓	
Pareto	✓					✓				

Consider a graph model of a conflict with permissible range: $(N, S, (A_i)_{i\in N}, (\succsim_i)_{i\in N}, (P_i^n))$. Here, for $i \in N$, P_i denotes the set of all permissible states for DM i, that is, $\overline{P}_i^n$. Therefore, if $s \in S$ is permissible for DM i, then it is denoted by $s \in P_i$, and otherwise, $s \notin P_i$.

3.6.1 Case with $\cap_{i\in N} P_i \neq \emptyset$

First, we consider the case with $\cap_{i\in N} P_i \neq \emptyset$, that is, there exists at least one state which is commonly permissible for all DMs. We have the following propositions:

Nash Stability

Proposition 3.6.1. State $s \in \cap_{i\in N} P_i$ is Nash equilibrium.

Proof. For $i \in N$, we have $R_i^+(s) = \emptyset$, because for all $s' \in S$, $s \succsim_i s'$. □

Proposition 3.6.2. Consider state $s' \notin \cap_{i\in N} P_i$. For $j \in N$, if $s' \in P_j$, then s' is Nash stable for DM j.

Proof. s' is Nash stable for DM j, because for all $s'' \in S$, $s' \succsim_j s''$. □

Proposition 3.6.3. Consider state $'s \notin \cap_{i\in N} P_i$. For $k \in N$, if $s' \notin P_j$, then s' is Nash stable for DM k if $R_k(s') \cap P_k = \emptyset$, and not if $R_k(s') \cap P_k \neq \emptyset$.

Proof. s' is Nash stable for DM k if $R_k(s') \cap P_k = \emptyset$, because we have $R_k^+(s') = \emptyset$ from $s' \notin P_k$ and for all $s'' \in R_k(s')$, $s'' \notin P_k$. s' is not Nash stable for DM k if $R_k(s') \cap P_k \neq \emptyset$, because we have $R_k^+(s') \neq \emptyset$ from $s' \notin P_k$ and there exists $s'' \in P_k(s')$ such that $s'' \in P_k$, which implies $s'' \succ_k s'$. □

Worst Case Nash Stability

For the special cases that for each DM, all states other than the DM's least preferred one are permissible for the DM, we have Corollary 3.6.3.1 of Proposition 3.6.3.

Corollary 3.6.3.1 (Corollary of Proposition 3.6.3)**.**
Consider the cases that $P_i = S \backslash \{\min \succsim_i\}$ for $i \in N$, where $\min \succsim_i$ denotes the DM i's least preferred state. $s' = \min \succsim_i$ is Nash stable for DM i if $R_i(s') = \emptyset$, and NOT if $R_i(s') \neq \emptyset$.

Proof. If $R_i(s') = \emptyset$, then we always have $R_i^+(s') = \emptyset$, which means that s' is Nash stable for DM i. If $R_i(s') \neq \emptyset$, then we have that $R_i(s') \cap P_i \neq \emptyset$, because $R_i(s') \subseteq S \backslash \{s'\} = S \backslash \{\min \succsim_i\} = P_i$. By using the result of Proposition 3.6.3, we have that s' is not Nash stable for DM i. □

Efficiency

The followings are propositions on the efficiency of states under the condition of $\cap_{i \in N} P_i \neq \emptyset$
Proposition 3.6.4. State $s \in \cap_{i \in N} P_i$ is weakly and strongly efficient.

Proof. In this case, for all $i \in N$ and all $s' \in S$, $s \succsim_i s'$. Therefore, $s' \succ_i s$ cannot be satisfied for any $i \in N$ and any $s' \in S$, which implies that s is weakly and strongly efficient. □

Proposition 3.6.5. Consider state $s' \notin \cap_{i \in N} P_i$. For $j \in N$, if $s' \in P_j$ (which implies that $s' \notin P_k$ for some $k \in N$), then s' is weakly efficient and NOT strongly efficient.

Proof. In this case, for all $i \in N$, $s \succsim_i s'$ and $s \succ_k s'$, because $s \in \cap_{i \in N} P_i$ and $s' \notin P_k$. This implies that s' is not strongly efficient. There is no $s'' \in S$ such that $s'' \succ s'$ for all $i \in N$, because $s' \in P_j$. This implies that s' is weakly efficient. □

Proposition 3.6.6. Consider state $s' \notin \cap_{i \in N} P_i$. If $s' \notin P_i$ for all $i \in N$, then s' is neither weakly efficient nor strongly efficient.

TABLE 3.13: Cuban Missile Crisis - Stability Analysis P^3-P^7

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
U.S.	0	0	0	0	1	1	1	0	0	0
USSR	1	1	1	0	1	1	0	1	1	0
PNash	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
PGMR	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
PSMR	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
PSEQ	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Pareto					✓	✓				

Proof. In this case, for all $i \in N$, $s \succ_i s'$, because for all $i \in N$, $s \in P_i$ and for all $i \in N$, $s' \notin P_i$. □

Worst Case Efficiency

For the special cases that for each DM, all states other than the DM's least preferred one are permissible for the DM, we have Corollary 3.6.6.1 of Proposition 3.6.5 and Proposition 3.6.6.

Corollary 3.6.6.1 (Corollary of Proposition 3.6.5 and Proposition 3.6.6).
Consider the cases that $P_i = S \backslash \{\min \succsim_i\}$ for all $i \in N$, where $\min \succsim_i$ denotes the DM i's least preferred state. $\min \succsim_i$ is weakly efficient and not strongly efficient, if $\min \succsim_i \neq \min \succsim_j$ for some i and $j \in N$. $\min \succsim_i$ is neither weakly efficient nor strongly efficient, if $\min \succsim_i = \min \succsim_j$ for all i and $j \in N$.

Proof. In the case that $\min \succsim_i \neq \min \succsim_j$ for some i and $j \in N$, $s' = \min \succsim_i \notin P_i$ and $s' \in P_j$. Then, by applying Proposition 3.6.5, we have that $\min \succsim_i$ is weakly efficient and not strongly efficient.

In the case that $\min \succsim_i = \min \succsim_j$ for all i and $j \in N$, $s' = \min \succsim_i \notin P_i$ for all $i \in N$. Then, by applying Proposition 3.6.6, we have that $\min \succsim_i$ is neither weakly efficient nor strongly efficient. □

3.6.2 Case with $\cap_{i \in N} P_i = \emptyset$

Next, let us consider the case with $\cap_{i \in N} P_i = \emptyset$, that is, there is no state which is commonly permissible for all DMs. We have the following propositions:

Nash Stability

Proposition 3.6.7. Consider state $s' \notin \cap_{i \in N} P_i$. For $j \in N$, if $s' \in P_j$, then s' is Nash stable for DM j.

Proof. s' is Nash stable for DM j, because for all $s'' \in S$, $s' \succsim_j s''$. □

Proposition 3.6.8. Consider state $s' \notin \cap_{i \in N} P_i$. For $k \in N$, if $s' \notin P_j$, then s' is Nash stable for DM k if $R_k(s') \cap P_k = \emptyset$, and not if $R_k(s') \cap P_k \neq \emptyset$.

Proof. s' is Nash stable for DM k if $R_k(s') \cap P_k = \emptyset$ s' is Nash stable for DM k if $R_k(s') \cap P_k = \emptyset$, because we have $R_k^+(s') = \emptyset$ from $s' \notin P_k$ and for all $s'' \in R_k(s')$, $s'' \notin P_k$. s' is not Nash stable for DM k if $R_k(s') \cap P_k \neq \emptyset$, because we have $R_k^+(s') \neq \emptyset$ from $s' \notin P_k$ and there exists $s'' \in P_k(s')$ such that $s'' \in P_k$, which implies $s'' \succ_k s'$. □

For the special cases that for each DM, all states other than the DM's least preferred one are permissible for the DM, we have Corollary 3.6.8.1 of Proposition 3.6.8.

Corollary 3.6.8.1 (Corollary of Proposition 3.6.8).
Consider the cases that $P_i = S \backslash \{\min \succsim_i\}$ for all $i \in N$, where $\min \succsim_i$ denotes the DM i's least preferred state. $s' = \min \succsim_i$ is Nash stable for DM i if $R_i(s') = \emptyset$, and not if $R_i(s') \neq \emptyset$.

Proof. If $R_i(s') = \emptyset$, then we always have $R_i^+(s') = \emptyset$, which means that s' is Nash stable for DM i. If $R_i(s') \neq \emptyset$, then we have that $R_i(s') \cap P_i \neq \emptyset$, because $R_i(s') \subseteq S \backslash \{\min \succsim_i\} = P_i$. By using the result of Proposition 3.6.8, we have that s' is not Nash stable for DM i. □

Efficiency

The followings are propositions on the efficiency of states under the condition of $\cap_{i \in N} P_i = \emptyset$

Proposition 3.6.9. Consider state $s' \notin \cap_{i \in N} P_i$. For $j \in N$, if $s' \in P_j$ (which implies that $s' \notin P_k$ for some $k \in N$), then s' is weakly efficient.

Proof. There is no $s'' \in S$ such that $s'' \succ_i s'$ for all $i \in N$, because $s' \in P_j$. Thus, s' is weakly efficient. □

Proposition 3.6.10. Consider state $s' \notin \cap_{i \in N} P_i$. Assume that $N = \{j, k\}$, that is $|N| = 2$. Then, for $j \in N$, if $s' \in P_j$ (which implies that $s' \notin P_k$ for the other $k \in N$), then s' is strongly efficient.

Proof. Assume that there exists $s'' \in S$ such that $s'' \succsim_j s'$ and $s'' \succsim_k s'$, and that $s'' \succ_j s'$ or $s'' \succ_k s'$. Because $s' \in P_j$, it is impossible that $s'' \succ_j s'$. This implies that $s'' \succ_k s'$. Then, we need to have that $s'' \in P_j$ and $s'' \in P_k$, which contradicts with the condition of $\cap_{i \in N} P_i = \emptyset$. Thus, we have that s' is strongly efficient. □

With respect to strong efficiency of state s' under the conditions of $\cap_{i \in N} P_i = \emptyset$, $s' \in P_j$ for some $j \in N$, $s' \notin P_k$ for some $k \in N$, and $|N| \geq 3$, see the next example. We see that s' is or is not strongly efficient depending on $(P_i)_{i \in N}$ in 3.6.1.

Example 3.6.1.

1. Case 1:

   Let $N = \{1, 2, 3\}$, $S = \{s_1, s_2, s_3\}$, and $P_1 = \{s_1, s_2\}$; $P_2 = \{s_2\}$; $P_3 = \{s_3, s_1\}$. In this case, $\cap_{i \in N} P_i = \emptyset$, and $s_1 \in P_1$; $s_1 \notin P_2$; $s_1 \notin P_3$. We see that s_1 is strongly efficient, because $s_1 \succ_3 s_2$ and $s_1 \succ_1 s_3$.

2. Case 2:

   Let $N = \{1, 2, 3\}$, $S = \{s_1, s_2, s_3\}$, and $P_1 = \{s_1, s_2\}$; $P_2 = \{s_2\}$; $P_3 = \{s_3\}$. In this case, $\cap_{i \in N} P_i = \emptyset$, and $s_1 \in P_1$; $s_1 \notin P_2$; $s_1 \notin P_3$. We see that s_1 is not strongly efficient, because $s_2 \succsim_1 s_1$; $s_2 \succ_2 s_1$; $s_2 \succsim_3 s_1$.

Proposition 3.6.11. Consider state $s' \notin \cap_{i \in N} P_i$. If $s' \notin P_i$ for all $i \in N$, then s' is weakly efficient and not strongly efficient.

Proof. Assume that there exists $s'' \in S$ such that for all $i \in N$, $s'' \succ_i s'$. Then, we need to have that for all $i \in N$, $s'' \in P_i$, which contradicts with the condition of $\cap_{i \in N} P_i = \emptyset$. Thus, we have that s' is weakly efficient.

Because we assume that $P_j \neq \emptyset$ for all $j \in N$, we can take $s'' \in P_j$. Then, it is satisfied that $s'' \succ_j s'$ and $s'' \succsim_i s'$ for all $i \in N$, because $s' \notin P_i$ for all $i \in N$. Therefore, s' is not strongly efficient. □

Worst Case Efficiency

Corollary 3.6.11.1 (Corollary of Prop. 3.6.9 and Prop. 3.6.10). *Consider the cases that* $P_i = S\backslash\{\min \succsim_i\}$ *for all* $i \in N$*, where* $\min \succsim_i$ *denotes the DM* i*'s least preferred state. Then, we have that* $\min \succsim_i$ *is weakly efficient. We also have that* $\min \succsim_i$ *is strongly efficient, if* $N = \{1, 2\}$.

Proof. Under the conditions of $\cap_{i\in N} P_i = \emptyset$ and $P_i = S\backslash\{\min \succsim_i\}$ for all $i \in N$, we have that $S = \{\min \succsim_i \mid i \in N\}$, because if not, $x \in S\backslash\{\min \succsim_i \mid i \in N\}$ satisfies that $x \in \cap_{i\in N} P_i$, which contradicts with the condition of $\cap_{i\in N} P_i = \emptyset$. $S = \{\min \succsim_i \mid i \in N\}$ implies the results by using Prop. 3.6.9 and Prop. 3.6.10. □

For strong efficiency in the cases with $P_i = S\backslash\{\min \succsim_i\}$ for all $i \in N$ and $|N| \geq 3$, we have the following proposition:

Proposition 3.6.12. Consider the cases that $P_i = S\backslash\{\min \succsim_i\}$ for all $i \in N$, where $\min \succsim_i$ denotes the DM i's least preferred state. Then, we have that $\min \succsim_i$ is strongly efficient, if $|N| \geq 3$.

Proof. Under the conditions of $\cap_{i\in N} P_i = \emptyset$ and $P_i = S\backslash\{\min \succsim_i\}$ for all $i \in N$, we have that $S = \{\min \succsim_i \mid i \in N\}$, because if not, $x \in S\backslash\{\min \succsim_i \mid i \in N\}$ satisfies that $x \in \cap_{i\in N} P_i$, which contradicts with the condition of $\cap_{i\in N} P_i = \emptyset$.

For all $s'' \in S = \{\min \succsim_i \mid i \in N\}$, there exists $i \in N$, such that $s'' = \min \succsim_i$, which implies that $s' \succ_i s''$. □

3.6.3 Summary of Results - Nash and Efficiency in PR Analysis

Table 3.14 summarizes the results for general cases in Subsection 3.6.1 and Subsection 3.6.2.

TABLE 3.14: Interrelationships between Nash Stability and Efficiencies

	$s \in S$: $s \in \cap_{i\in N} P_i$	$s' \in S$: $s' \in P_j$ and $s' \notin P_k$	$s' \in S$: $\forall k \in N, s' \notin P_k$
If $\cap_{i\in N} P_i \neq \emptyset$:	Nash for all $i \in N$ (Prop. 3.6.1)	Nash for j (Prop. 3.6.2)	—
		Nash for k depending on $R_k(s')$ and P_k (Prop. 3.6.3)	
	w.eff. (Prop. 3.6.4)	w.eff. (Prop. 3.6.5)	NOT w.eff. (Prop. 3.6.6)
	s.eff. (Prop. 3.6.4)	NOT s.eff. (Prop. 3.6.5)	NOT s.eff. (Prop. 3.6.6)
If $\cap_{i\in N} P_i = \emptyset$:	—	Nash for j (Prop. 3.6.7)	—
		Nash for k depending on $R_k(s')$ and P_k (Prop. 3.6.8)	
	—	w.eff. (Prop. 3.6.9)	w.eff. (Prop. 3.6.11)
	—	s.eff. if $|N| = 2$ (Prop. 3.6.10); dep.on $(P_i)_{i\in N}$ if $|N| \geq 3$ (Ex. 3.6.1)	NOT s.eff. (Prop. 3.6.11)

Table 3.15 summarizes the results for the cases that $P_i = S\backslash\{\min \succsim_i\}$ for all $i \in N$ given by the corollaries in Subsection 3.6.1 and Subsection 3.6.2.

TABLE 3.15: Nash stability and efficiencies of min $\succsim_i$ under the condition of $P_i = S \backslash \{\min \succsim_i\}$ for all $i \in N$

	$\exists i,j \in N$, min $\succsim_i \neq$ min $\succsim_j$	$\forall i,j \in N$, min $\succsim_i =$ min $\succsim_j$
If	Nash for i depending on $R_i(s')$ (Cor. 3.6.3.1)	
$\cap_{i \in N} P_i \neq \emptyset$:	w.eff. (Cor. 3.6.6.1)	NOT w.eff. (Cor. 3.6.6.1)
	NOT s.eff. (Cor. 3.6.6.1)	NOT s.eff. (Cor. 3.6.6.1)
If	Nash for i depending on $R_i(s')$ (Cor. 3.6.8.1)	
$\cap_{i \in N} P_i = \emptyset$:	w.eff. (Cor. 3.6.11.1)	—
	s.eff. (Cor. 3.6.11.1, Prop. 3.6.12)	—

3.6.4 Verification of Propositions in Application Cases

Elmira Conflict Case III for $\cap_{i \in N} P_i \neq \emptyset$

We verify Propositions presented in refsubsection:commonly permissiblefor Case with $\cap_{i \in N} P_i \neq \emptyset$ by examining the stability analysis Elmira Conflict Case-III shown in Table 3.8. Table 3.16 summarizes the permissibility, reachability, and Nash equilibrium in the original analysis, also the status regarding Propositions for each DM for each state.

TABLE 3.16: Verification of Propositions:Elmira Conflict - Case III

		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
	M	0	0	1	0	0	0	1	0	0
Permissibility	U	1	0	1	1	1	0	1	1	1
	L	1	0	1	0	1	0	1	1	0
	M	2		4		6		8		
$PR_i(s)$	U	3,9	4,9	9	9	7,9	8,9	9	9	
	L	5	6	7	8	1	2	3	4	
PNash Equilibrium		E		E		E		E	E	E
Prop.3.6.1(Nash)				E				E		
Prop.3.6.2(Nash)		U,L			U	U,L			U,L	U
Prop.3.6.3(Nash)		M			M	M			M	M,L
Prop.3.6.4(eff.)				✓				✓		
Prop.3.6.5(eff.)					✓					✓
Prop.3.6.6(eff.)			✓				✓			

Each item in the table indicates the following; the content with asterisk is the same as in the original table 3.8.

- Permissibility*: Boolean value whether or not each state is permissible for DM i.
- $PR_i(s)$: States where DM i can unilaterally transition (UM) from each state. The number indicates the state number.
- PNash Equilibrium*: Nash holds for all $i \in N$

- Prop.1-3(Nash): E denotes equilibrium, M, U, and L indicate the DMs who obtained the stability according to the proposition.
- Prop.4-6(eff.): weak and strong efficiency hold in the state with the checkmark.

In conflicts where at least one state is permissible to all DMs, we find the following about the propositions. 1) Proposition 3.6.1 is about permissible states for all DMs; thus, Nash equilibrium is established at s_3 and s_7. 2) Propositions 3.6.2 and 3.6.3 are about other states than verified in 1) that are permissible for each DM, and these two propositions lead to Nash stability in s_1, s_5, s_8, and s_9. From 1) and 2), we can conclude that the PNash equilibria hold in s_1, s_3, s_5, s_7, s_8, and s_9. This verification result is consistent with the GMCR-PR stability analysis shown in Table 3.8.

Also, we see the weak and strong Pareto efficiency we gave a proposition for efficiency in 3.6.4 is consistent with the original results in Table 3.8.

Elmira Conflict Case P for $\cap_{i \in N} P_i = \emptyset$

Conflicts in which no state exists within the common PR for all DMs are not addressed in Sections 3.4 and 3.5. Therefore, we conduct verification of the propositions by setting up a new PR case P^9_M, P^9_U, P^9_L in the Elmira conflict.

TABLE 3.17: Elmira Conflict - Stability Analysis: P^2_M, P^2_U, P^2_L

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
M	0	0	1	0	0	0	1	0	0
U	1	0	0	1	0	0	0	0	0
L	0	0	1	0	0	0	1	0	0
PNash	✓		✓	✓	✓	✓	✓	✓	✓
PGMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
PSMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
PSEQ	✓		✓	✓	✓	✓	✓	✓	✓
Pareto	✓		✓	✓			✓		

Table 3.17 shows the stability analysis when the PR of all DMs is set to P^2. It is presented as a conflict with no single state that is commonly permissible for all DMs. The correspondence between stability and propositions in P^2_M, P^2_U, P^2_L case is presented in Table 3.18.

In conflicts where no state is permissible to all DMs, we find the following about the propositions. 1) Proposition 3.6.7 is about permissible states for DM j; thus, Nash stability holds at s_1 for **U**, s_3 for **M** and **L**, s_4 for **U**, and s_7 for **M** and **L**. 2) Propositions 3.6.8 is about other states than verified in 1) that are permissible for DM j, and this proposition leads to Nash stability for **M**, **U**, and **L**. From 1) and 2), we can conclude that the PNash equilibra hold in s_1, s_3, s_4, s_5, s_6, s_7, and s_9. This verification result is consistent with the GMCR-PR stability analysis shown in Table 3.17.

Also, we see the weak and strong Pareto efficiency we gave propositions for efficiency in 3.6.9 and 3.6.11 is consistent with the original results in Table 3.17.

In this section, the propositions presented in Section 3.6 have been examined and verified to be consistent with the results of GMCR-PR stability analyses of the Elmira conflict.

TABLE 3.18: Verification of Propositions:Elmira Conflict - P_M^2, P_U^2,P_L^2

		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
	M	0	0	1	0	0	0	1	0	0
Permissibility	U	1	0	0	1	0	0	0	0	0
	L	0	0	1	0	0	0	1	0	0
	M	2		4		6		8		
$PR_i(s)$	U	3,9	4,9	9	9	7,9	8,9	9	9	
	L	5	6	7	8	1	2	3	4	
PNash Equilibrium		E		E	E	E	E	E	E	E
Prop.3.6.7(Nash)		U		M,L	U			M,L		
Prop.3.6.8(Nash)		M,L	M,L	U	M,L	M,U,L	M,U,L	U	M,U,L	M,U,L
Prop.3.6.9(eff.)		✓		✓	✓			✓		
Prop.3.6.11(eff.)			✓			✓	✓		✓	✓

In the next section, we discuss the common ground with the consensus-building framework with simple game, based on the idea that it is possible to describe situations in which conflicts converge by setting permissible ranges.

3.7 Permissibility and Conflict Convergence

In the previous sections, we examined that introducing the permissibility into the GMCR analysis of the Elmira conflict and the Cuban Missile Crisis allows for practical first-order analysis. In the actual conflict resolution process in the Cuban Missile Crisis, it can be observed that the conflict was brought to an end by narrowing the PR between the two countries through sub rosa negotiations. This subsection discusses the GMCR-PR in relation to the committee model for conflict resolution. Suppose DMs' PRs are identified, and the DMs want to resolve the conflict. In that case, solutions to the conflict will be determined once the decision rules are provided. The mathematical model of the committee is efficiently applicable in this situation. In particular, if the conflict is between two DMs, the decision rule is inevitably "unanimity," and the conflict can be viewed as a "committee with unanimity decision rule."

The conflict resolution model based on the framework of a committee considers a negotiation process as a place where the different positions and opinions of DMs converge and describes the process of forming an efficient and rational consensus. Since "consensus" is defined as "the existence of an alternative to which all DMs agree," in the committee model, it is suitable for analyzing consensus building for conflicts among nations with the characteristics described in "rationality of a state" given in Assumption 1. Unlike pure economic competition among companies in a limited scope, rational decisions to avoid the worst-case scenario in which all DMs suffer damage due to escalation of military actions are considered to exist as preconditions for conflicts among sovereign states, even if not explicitly stated. When the object of decision-making is significant: interstate conflicts, the global environment, or world-scale economy, it is assumed that there exists an implicit agreement that the worst-case scenario, which is detrimental and irreversible to all parties involved in the conflict, should be avoided. In such a case, by incorporating the framework of the committee model into the conflict resolution model, it is possible to conduct an adequate analysis by integrating the aspects both in which the DMs pursue rational preferences and in overall efficiency is considered in the conflict. In the previous subsection, we argued that GMCR-PR is valid for the first-order analysis of conflicts. Moreover, when DMs' preferences are revealed at the end of the conflict, it can be useful as a framework for analyzing the convergence of conflicts.

We developed a framework and provided definitions reflecting the integrative perspective as GMCR-PR, based on GMCR incorporating committee model proposed by Inohara [38], by examining with applications.

First, the necessary definitions for the framework are provided, and then the framework is applied to the avoidance of the Cuban Missile Crisis and analyzed. *Simple game* [77] [82] [86] and the theory on committee [74] [93] are employed as mathematical models of consensus building, for which the basic definitions are given as follows [36].

Definition 3.7.1 (Simple Game). A *simple game* G is a pair $(N, \mathbb{W})$ in which $N = \{1, 2, \ldots, n\}$ and $\mathbb{W}$ is a collection of subsets of N that satisfies: (1) $N \in \mathbb{W}$, (2) $\emptyset \notin \mathbb{W}$ and (3) the monotonicity property: $S \in \mathbb{W}$ and $S \subseteq T \subseteq N$ implies $T \subseteq \mathbb{W}$.

Definition 3.7.2 (Unanimity Rule). In a simple game $G = (N, \mathbb{W})$, *unanimity rule* holds when $\mathbb{W} = \{N\}$.

Definition 3.7.3 (Committees). A *committee* C is four tuples: a set of DM N, a set of winning coalition $\mathbb{W}$, a set of alternatives A, and an opinion $(\succsim_i)_{i \in N}$ of DMs on the alternatives A.

$$C = (N, \mathbb{W}, A, (\succsim_i)_{i \in N}) \tag{3.12}$$

where, $(N, \mathbb{W})$ is a simple game, $2 \leq |N| \leq \infty, 2 \leq |A| \leq \infty$, which $\succsim_i$ is a linear order on A if and only if $\succsim_i$ is complete, transitive, and anti-symmetric.

Definition 3.7.4 (Permissible Range in Committees). $\boldsymbol{max} \succsim$ denotes the most preferred alternative in A in terms of $\succsim$, that is $\boldsymbol{max} \succsim = a$ if and only if $x \in A, a \succsim x$.

Definition 3.7.5 (Committees with Permissible Range). DM i's permissible range in $C = (N, \mathbb{W}, A, (\succsim_i)_{i \in N})$ is denoted by $P = (P_i)_{i \in N}$, and $C = (P)$, (but $P = (P_i)_{i \in N}$) is called a *committee with permissible range*. Moreover, assume that if there exists $\succsim \in P_i$ such that $x \succsim_i y$ for a opinion x and y, and $\boldsymbol{max} \succsim = y$, then there also exists $\succsim' \in P_i$ such that $\boldsymbol{max} \succsim' = x$. $\mathbb{P}_i$ denotes a set of all elements of DM i's permissible range.

Definition 3.7.6 (Stable Coalitions). Consider a committee $C(P)$, where $C = (N, \mathbb{W}, A, (\succsim_i)_{i \in N})$ and $\mathbb{W}_{C(P)}$. A winning coalition $\Gamma \in \mathbb{W}_{C(P)}$ is said to be stable if and only if there exists $a \in A$ such that i) $\Gamma_a = \Gamma$, and ii) for all $i \in \Gamma$ and all $b \in A \backslash \{a\}$, if $b \succsim_i a$ then $b \notin A_{C(P)}$. We represent $\overline{\mathbb{W}_{C(P)}}$ for the set of all stable coalitions in the $C(P)$, and $\overline{A_{C(P)}}$ for the set of all alternatives that are acceptable to all members for at least one stable coalition. Hence, $\overline{A_{C(P)}} = \{a \in A \mid \exists S \in \mathbb{W}_{C(P)}, \Gamma_a = \Gamma \wedge (\forall i \in \Gamma, \forall b \in A \backslash \{a\}, b \succsim_i a \to b \notin A_{C(P)}\}$, where $\Gamma_a = \{i \in \Gamma \mid a \in \mathsf{max} P_i\}, \mathbb{W}_{C(P)} = \{\Gamma \in \mathbb{W} \mid \exists a \in A, \Gamma a \in \mathbb{W}\}, A_{C(P)} = \{a \in A \mid \exists \Gamma \in \mathbb{W}, \Gamma a \in \mathbb{W}\}$.

Definition 3.7.7 (Negotiation Processes). For a committee $C = (N, \mathbb{W}, A, (\succsim_i)_{i \in N})$, a negotiation process in C is a sequence of $(P^t)_{t \in \mathcal{T}}$. P^t denotes $(P_i^t)_{i \in N}$ at time $t \in \mathcal{T}$. $P^0 = (P_i^0)_{i \in N} = (\{\succsim_i\})_{i \in N}$ represents status quo.

Definition 3.7.8 (Consensus and Consensus Building). For a committee $C = (N, \mathbb{W}, A, (\succsim_i)_{i \in N})$, a negotiation process $(P^t)_{t \in \mathcal{T}}$ in C is said to reach consensus at $t^* \in \mathcal{T}$ on $x \in A$, if and only if either i) $t^* = 0$ and $\overline{A_{C(P^0)}} = \{x\}$, or ii) for all such that $0 \leq t < t^*$, and $\overline{A_{C(P^*)}} = \{x\}$.

Remark. The sequence $(P^0, P^1, \ldots, P^{t^*})$ represents the consensus building on $\{x\}$ in C.

Definition 3.7.9 (Core of the Committee [93]). For committee C, we define the relation, $\boldsymbol{Dom}$, in the set of alternatives A. For alternatives a and b, $a\boldsymbol{Dom}b$ holds if and only if a winning coalition $S \in \mathbb{W}$ and $a \succsim_i b$ holds for any i. $a\boldsymbol{D\o m}b$ denotes that $a\boldsymbol{Dom}b$ does not hold. $\{a \mid \forall b \in A \backslash \{a\}, b\boldsymbol{D\o m}a\}$ represents the core of committee C.

Proposition 3.7.1 (Stable Alternative in Proper Simple Game [93]). Consider a committee $C(P)$, where $C = (N, \mathbb{W}, A, (\succsim_i)_{i \in N})$ and $\mathbb{W}_{C(P)}$, such that $P = (P_i)_{i \in N}$. When a simple game $G = (N, \mathbb{W})$ is proper, there exist only one $\overline{A_{C(P)}}$ at most.

Remark. From Definition 3.7.2 and Proposition 3.7.1, it is evident that there is only one stable alternative in a committee where the unanimity rule is adopted.

Proposition 3.7.2 (Stable Alternative and Core). Consider a committee $C(P)$, where $C = (N, \mathbb{W}, A, (\succsim_i)_{i \in N})$, $P = (P_i)_{i \in N}$, and $G = (N, \mathbb{W})$ is proper.
$\overline{A_{C(P^0)}} = \{x\}$ where $x \in A \Leftrightarrow x \in \boldsymbol{Core}(C)$.

The following Theorems 3.7.3, 3.7.4, and 3.7.5 hold for consensus building in the framework of committees **when unanimity rule is adopted**: a committee $C = (N, \mathbb{W}, A, (\succsim_i)_{i \in N})$ such that the simple game $G = (N, \mathbb{W})$ is unanimous, that is $\mathbb{W} = \{N\}$. [1]

Theorem 3.7.3 (Efficiency of an Alternative - Inohara 2011[36]). *There exist $(P^t)_{t \in \mathcal{T}}$, where $x \in A$ reaches a consensus at t^*, if and only if $x \in \boldsymbol{Core}(C)$.*

Theorem 3.7.4 (Stability of an Alternative - Inohara 2011[36]). *In a normal form game $G_c = (N, (T_i)_{i \in N}, (\succsim_i)'_{i \in N})$ determined from a committee C, if there exists an alternative $x \in A$ such that $\overline{A_{C(P^0)}} = \{x\}$ for DM's permissible committee $P = (P_i)_{i \in N} \in T (= \Pi_{i \in N} T_i = \Pi_{i \in N} \mathbb{P}_i)$, then P holds Nash equilibrium in G_c.*

Theorem 3.7.5 (Non-Emptiness of an Alternative - Inohara 2011[36]). *$\emptyset \neq \boldsymbol{max}(\succsim_i)_{i \in N} \subseteq \boldsymbol{Core}(C)$.*

Next, we discuss GMCR-PR using theorems, propositions, and definitions provided for the framework of the committee. GMCR-PR was characterized by a binary treatment of elements in and not in the permissible range. Therefore, the degree of preference for states (alternatives) included in the permissible range are ignored or treated as indifference. On the other hand, the framework of Committee with permissible range deals with the degree of preference for alternatives included within the permissible range. Binary information may be useful when the analysis incorporating permissivity is used as the primary analysis. Nevertheless, to find a solution to converge the conflict at the end of negotiations or the confrontations, when the DMs' preferences and permissible ranges have become more revealed, it may be necessary to deal with the states included in the permissible range.

GMCR can be interpreted in the framework of committee with permissible range shown in Definition in 3.7.3 and 3.7.5 with the identification of the corresponding elements as follows [38]:

Definition 3.7.10 (Committee with Permissible Range in GMCR). For a committee $C(P)$, where $C = (N, \mathbb{W}, A, (\succsim_i)_{i \in N})$, and a graph model : $G = (N, S, (A_i)_{i \in N}, (\succsim_i)_{i \in N})$, S is defined as $\mathbb{P} = \Pi_{i \in N} \mathbb{P}_i$, where two states $P = (P_i)_{i \in N}$ and $P' = (P'_i)_{i \in N}$ in $S = \mathbb{P}$ are identified, if $\boldsymbol{max} P_i = \boldsymbol{max} P'_i$ for all $i \in N$; for $i \in N$ and for $P = (P_i)_{i \in N}$

[1] The proofs for the three theorems are given in Appendix A.

TABLE 3.19: Cuban Missile Crisis - States After the Blockade

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
U.S. airstrike	N	Y	N	Y	Y	N	Y	Y	N	Y
U.S blockade	N	N	***Y****	***Y****	N	***Y****	***Y****	N	***Y****	***Y****
USSR withdraw	N	N	N	N	Y	Y	Y	N	N	N
USSR escalate	N	N	N	N	N	N	N	Y	Y	Y

and for $P' = (P'_i)_{i \in N}$ in $S = \mathbb{P}$, $(P, P') \in A_i \subseteq S \times S$, if and only if $P_i \neq P'_i$ and $P_j = P'_j$ for all $j \in N \backslash \{i\}$; for $i \in N$ and for $P = (P_i)_{i \in N}$ and $P' = (P'_i)_{i \in N}$ in $S = \mathbb{P}$, $P \succsim'_i P'$, if either

i) $\overline{A_{C(P)}} = \{a\}, \overline{A_{C(P')}} = \{b\}$ and $a \succsim_i b$, or

ii) $\overline{A_{C(P)}} = \overline{A_{C(P')}} = \emptyset$, or

iii) $\overline{A_{C(P)}} = \{a\}, \overline{A_{C(P')}} = \emptyset$ and $a \in \boldsymbol{max}P_i$.

Thus, Definition 3.2.4, which defines GMCR-PR, can be applied in two different methods, depending on the coarseness and fineness of the preference information available. When the information is coarse, we can use a binary permissivity as specified in Definition 3.2.3, and when the information is fine, we can use Definition 3.7.10, which considers the linear order of preference within the permissible range.

Using the framework of the mathematical model of consensus building that we have discussed, we describe the final negotiations between the United States and the Soviet Union that would have taken place to avert the worst-case scenario in the Cuban Missile Crisis. The negotiation time shall be between October 24, 1962, when the United States' naval blockade began, and October 28, 1962, when the Soviet Union offered to dismantle its missile bases. The unanimity rule shall be applied to the decision making in this negotiation. Because the negotiation occurs after the blockade, the alternatives are obtained by extracting only the ones with the option that implements the blockade from the ten feasible states; the states marked with asterisks in Table 3.19 is the target of the analysis.

The six states in which the option of the naval blockade is selected (marked with asterisk) are as follows: s_3: U.S. blockade, USSR maintains missiles, s_4: U.S. conducts both airstrikes and blockade, USSR holds missiles, s_6: U.S. blockade, USSR withdraw, s_7: U.S. blockade and airstrikes, USSR withdraw, s_9: U.S. blockade, USSR escalate: s_{10}: U.S. blockade and airstrikes, USSR escalate. Based on the framework of committee and the interpretation of the states, we can describe the negotiation as follows [50].

Example 3.7.1 (Cuban Missile Crisis-Negotiation Process-1)**.** Suppose a committee $C = (N, \mathbb{W}, A, (\succsim_i)_{i \in N})$, where $N = \{US, USSR\}$, $\mathbb{W} = \{N\}$, $A = \{3, 4, 6, 7, 9, 10\}$,$\succsim_{US} = [6, 7, 3, 4, 9, 10]$, $\succsim_{USSR} = [3, 9, 6, 4, 7, 10]$. Negotiation process at time $t = 0, 1, 2$ can be described as:

- $\boldsymbol{max}P^0_{US} = \{6\}, \boldsymbol{max}P^0_{USSR} = \{3\}$,
- $\boldsymbol{max}P^1_{US} = \{6, 7\}, \boldsymbol{max}P^1_{USSR} = \{3, 9\}$,
- $\boldsymbol{max}P^2_{US} = \{6, 7\}, \boldsymbol{max}P^2_{USSR} = \{3, 9, 6\}$.

Then, we can obtain the consensus as: $\overline{A_{C(P^2)}} = \{6\}$. From Theorem 3.7.3 and 3.7.5, this result is the ***core*** of the negotiation, in which both Pareto efficiency and Nash equilibrium can be established.

TABLE 3.20: P^3 for Games with No Dominant Strategy

Game No.	DM1				DM2				Eq	Note
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4		
58	4	2	(1)	3	4	3	(1)	2	s_1,s_2,s_4	
59	4	2	(1)	3	4	2	(1)	3	s_1,s_2,s_4	
60	4	2	(1)	3	4	(1)	2	3	s_1,s_4	
61	4	(1)	3	2	4	3	(1)	2	s_1,s_4	Stag Hunt
62	4	(1)	3	2	4	2	(1)	3	s_1,s_4	
63	4	(1)	2	3	4	2	(1)	3	s_1,s_4	
64	3	2	(1)	4	4	(1)	2	3	s_1,s_4	Luke and Matthew
65	2	3	(1)	4	4	(1)	2	3	s_1,s_4	
66	3	2	4	(1)	3	4	2	(1)	s_1,s_2,s_3	Chicken Game
67	2	3	4	(1)	3	4	2	(1)	s_1,s_2,s_3	
68	2	3	4	(1)	2	4	3	(1)	s_1,s_2,s_3	Battle of Sexes
69	2	4	3	(1)	2	3	4	(1)	s_1,s_2,s_3	
70	3	2	4	(1)	4	(1)	2	3	s_1,s_3	
71	3	2	4	(1)	3	(1)	2	4	s_1,s_3	
72	3	2	4	(1)	2	(1)	3	4	s_1,s_3	
73	2	4	3	(1)	4	(1)	2	3	s_1,s_3	
74	2	3	4	(1)	4	(1)	2	3	s_1,s_3	
75	2	4	3	(1)	3	(1)	2	4	s_1,s_3	
76	2	3	4	(1)	3	(1)	2	4	s_1,s_3	
77	2	4	3	(1)	2	(1)	3	4	s_1,s_3	
78	2	3	4	(1)	2	(1)	3	4	s_1,s_3	

Example 3.7.2 (Cuban Missile Crisis-Negotiation Process-2). We revisit the case analyzed in subsection 3.5.3, where the United States allows up to the third preference and the Soviet Union allows up to the eighth preference ranking: $P^3_{US} - P^7_{USSR}$. As Table 3.13 shows, in the GMCR analysis under the permissible range $P^3_{US} - P^7_{USSR}$, strong equilibrium and Pareto optimum are established in s_5 and s_6. Thus, it is useful as a primary analysis, but a final decision-making decision is impossible because the analysis results in precisely the same outcome for the two states of s_5 and s_6. Unlike Example 3.7.1, the status quo is assumed to be the point in time when the missile invasion by the Soviet Union is discovered.

A committee $C = (N, \mathbb{W}, A, (\succsim_i)_{i\in N})$, where $N = \{US, USSR\}$, $\mathbb{W} = \{N\}$, $A = \{1,2,3,4,5,6,7,8,9,10\}$,$\succsim_{US} = [6,5,7,1,3,2,4,9,8,10]$, $\succsim_{USSR} = [1,3,9,2,8,6,5,4,7,10]$ is given. $\boldsymbol{maxP^3_{US}} = [6,5,7]$, $\boldsymbol{maxP^7_{USSR}} = [1,3,9,2,8,6,5]$. We obtain $\overline{A_{C(P)}} = \{6\}$ from the preference relation indicated as elements in the permissible range $\boldsymbol{maxP^3_{US}}$ and $\boldsymbol{maxP^7_{USSR}}$.

We revisit the 2×2 games with no dominant strategy discussed in Section 3.3 and apply the committee framework. The analysis in Section 3.3 shows that we have states where both Nash equilibrium and Pareto efficiency hold in $P^0_1 - P^0_2$, $P^0_1 - P^3_2$, $P^3_1 - P^0_2$, $P^3_1 - P^3_2$, $P^3_1 - P^4_2$, $P^4_1 - P^3_2$, and $P^4_1 - P^4_2$. Consider describing these conflicts in a committee framework.

Within the seven pairs of permissible ranges, pairs including P^0, i.e. "permit nothing", cannot be identified in the framework that requires non-empty for the set of states for consensus, thus not applicable in the analysis of consensus building. We examine $P^3_1 - P^3_2$ and $P^3_1 - P^4_2$ in chicken game. Suppose A committee $C = (N, \mathbb{W}A, (\succsim_i)_{i\in N})$, where $N = \{US, USSR\}$, $\mathbb{W} = \{N\}$, $A = \{1,2,3,4\}$, $\succsim_1 = [3,1,2,4]$, $\succsim_2 = [2,1,3,4]$ is given.

- $P_1^3 - P_2^3$: $\boldsymbol{max}P_1^3 = \{3, 1, 2\}$, $\boldsymbol{max}P_2^3 = \{2, 1, 3\}$, $\overline{A_{C(P)}} = \{1\}$.
- $P_1^3 - P_2^4$: $\boldsymbol{max}P_1^3 = \{3, 1, 2\}$, $\boldsymbol{max}P_2^3 = \{2, 1, 3, 4\}$, $\overline{A_{C(P)}} = \{1\}$.

Table 3.20 summarizes the twenty one games without a dominant strategy, where the permissible range is P^3 for both DMs. The state with utility 1 is bracketed because it is outside the permissible range. For both DMs, Nash equilibrium and Pareto optimality are established in the permissible states.

3.8 Conclusion of the Chapter

In this chapter, the analysis capability with coarse information was discussed by introducing the concept of PR to GMCR. PR is set by placing a threshold on the original preference , whereby the DM's preference is processed as binary "permissible or impermissible" information. On the other hand, since the GMCR framework is retained, analysis using the GMCR framework is possible when more detailed information is obtained. We also examined the equilibrium established by limiting the scope of analysis. This new framework for analysis can be used for first-order analysis in conflicts and is also helpful in describing the convergence phase of conflicts because it has a common conceptual foundation with the committee framework.

By introducing the concept of PR, it becomes possible to conduct an analysis that reflects implicit assumptions that are not reflected in the basic conflict framework, such as the fact that even a DM pursuing a rational solution wants to converge conflicts and are disadvantaged by prolonged conflicts and escalation. As an extended study, it would also be possible to analyze the setting of the DM's PR by interpreting it as the DM's strategy or attitude.

In this study, we focused on the consequences of applying the new concept but have not analyzed the nature of the solutions in detail. Still, we would like to develop a generalizable theory of the relationship between rational solutions and overall efficient solutions by introducing the PR concept.

Chapter 4

Preference Order Setting for Disaster Aversion

This chapter examines a methodology to avoid the worst in severe conflicts with a high number of states where the least preferred states to avoid being known. We assume that the conflict may result in the most undesirable possibilities for all stakeholders involved in the conflict and that the damage caused by such a situation would be severe or irreversible. In such conflicts, the priority for the people involved is to avoid the worst-case scenario and then work towards a more favorable situation. We propose a method for ranking preferences in such conflicts, where **only the preference for the less favorable option is known**, while the orders are unknown. Developing preference rankings using coarse information: the "least preferred states," allows for practical first-order analysis in severe ad complex conflicts.

In game theory and other decision-making frameworks that seek solutions in conflict situations, the ranking that ordinally expresses degrees of the DM's preference for possible strategies significantly impacts the analysis results. In this chapter, we examine a new method to formulate preference rankings with the perspective of avoiding escalation of conflicts that incorporates TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) [46]. GMCR was used for the analysis, using the preference ranks obtained by both the standard method in which the analysts set the preference ranks subjectively and the new TOPSIS method, which incorporates a de-escalation perspective. As a result, it was found that the analysis method using TOPSIS ranking with de-escalation aspect was able to obtain conflict analysis results that more clearly reflected the intention of the analysis when compared with the conventional approach.

In the following sections, we examine the formulation methods of DM's preference ranking for conflict analysis with the method of option prioritization, TOPSIS as well as the new method, GMCR-DA (GMCR-Disaster Aversion). Then, we apply the methods to the Cuban Missile Crisis and Elmira Conflict, and investigated the analysis results. Finally, we discuss and evaluate the insights gained from the results of the analysis with the preference rankings obtained by these different methods.

4.1 Outline of the New Method

4.1.1 Background

The elicitation of preference order is the essential element in conflict analysis, as results showed in the sensitivity analysis conducted in Chapter 3 by applying the same preferences order for the local government as the Ministry of Environment in the Elmira conflict. In general, there are two methods to obtain the preference order in GMCR: 1) to set the rankings directly based on the information available to the analyst, and 2) to obtain the preference for all states by setting options controllable by DMs and weighting the options, premising that states are determined by combinations of these options. For conflicts with a small number of DMs and possible states, the direct method in 1) can be applicable, but it is difficult to give all the preference rankings directly for conflicts with a large number of states. Fraser and Hipel devised 2) option prioritization, which has already been described in Remark 2.1.7. As we have already examined, the option prioritization method is also not simple enough to be implemented when the preference information is not yet fully available.

4.1.2 Theoretical Novelty

We propose in this study to assume a conflict situation, where an option is known to be the most critical for the DM if it develops in a detrimental direction to the DM. In other words, the preference ranking is determined by focusing on one point: what is the worst for the DM. Generally, in conflict analysis, the goal is to find a reasonable solution to achieve higher utility. However, in real-world conflicts, the search for higher utility may lead to irreversible consequences, such as environmental destruction, loss of trust, and organizational exhaustion. In this study, we assume a situation where we do not have enough information to identify a function that will give us a precise solution on how far we can pursue rationality while preventing critical collapse. In such a situation, the method of this study, which concerns only the gravity of the elements that make up the state description, is a new proposal for a straightforward and coarse information method of conflict analysis. The analyst only needs to be able to order the options along the evaluation axis of what will cause the most severe consequences. Then only a very mechanical calculation can be used to obtain solutions.

In option prioritization, only the options are weighted, not the states, so it is less burdensome for the analyst than ranking all the states one by one. In the following subsection, we show the standard option prioritization method; subsequently, the formulation method by TOPSIS and the new method for disaster aversion are presented.

4.1.3 Information Partition and Coarseness in GMCR-DA

In GMCR-DA, the states, the reachability, and the number of criteria are left, while only the category (states) is once coarsened and is reconstructed as states by setting preferences for the options that make up the states in order of preference. We derive a preference order with the same granularity as the original number of states for the criteria. This is useful when the degree of the undesirability of options is known. The structure of information partition and coarseness is presented in Figure 4.1.

4.2 Formulating Preference Orders

4.2.1 Formulating Preference Orders by Option Prioritization

A method for eliciting preference rankings using option prioritization is presented using the case of the Elmira conflict. The summary of the conflicts and the options and states, and the graph are provided in Example 2.3.2, Table 2.3, and Figure 2.6, respectively. For the three DMs (**M, U, L**), there were five options: **M** has one option: to modify the original control order or not; **U** has three options: (1) delay the

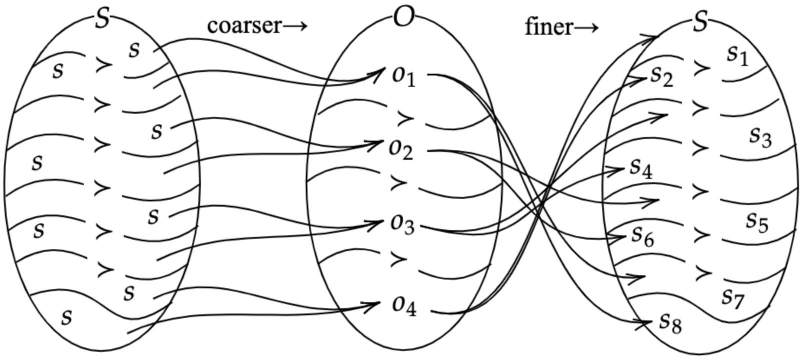

FIGURE 4.1: Coarsening and Refinement of information - GMCR DA

negotiations, (2) accept the current control order, or (3) abandon the Elmira plant. **L** has one option: whether to insist on the first control order. Tables 4.1, 4.2, and 4.3 summarize the priorities determined for each option using the option prioritization technique based on this information [98].

TABLE 4.1: Option Statement - Ministry of Environment

Option	Interpretation
-4	**M** does not want **U** to abandon its operation
3	**M** wants **U** to accept a control order
-2	**M** does not like the delay of **U**
-1	**M** does not want modify the original control order
5 IFF-1	**M** wants **L** to insist on the original order iff **M** chooses not to modify

TABLE 4.2: Option Statement - Uni Royal

Option	Interpretation
3 IFF 1	**U** will accept the control order iff **M** chooses to modify the original order
-4	**U** does not want to abandon its operation in Elmira
-5	**M** does not like the delay of **U**
2 IFF-5	**U** would like to delay iff **L** prefers not to insist on the original order

The preference ranking obtained from these option statements scored for each DM is shown in Table 4.4. For each option statement, a boolean value: $0 = false, 1 = true$, is checked for each state, and finally, the total value for each state is used to obtain the relative preference ranking. The score of each option can be obtained by equation 2.9, whereas redundant options which produce identical scores are eliminated [1].

The preference ranking obtained using option prioritization is consistent with the original preference ranking: $\succsim_M$: $s_7 \succ s_3 \succ s_4 \succ s_8 \succ s_5 \succ s_1 \succ s_2 \succ s_6 \succ s_9$. As discussed above, in the case of Elmira conflict, the option prioritization method has the advantage of making the process more visible and shareable than the direct preference ranking method for all states in conflicts with a large number of states. This method makes it suitable for cases such as the Elmira conflict, where accountability is required to DMs and other stakeholders, including local residents. On the other hand, to sort out the relationship between options and states, the analyst needs

[1]The algorithm of elimination of redundant method is provided in Appendix A

TABLE 4.3: Option Statement - Local Government

Option	Interpretation
-4	**L** does not want **U** to abandon its operation in Elmira
-1	**L** prefers not to modify the original control order
3 IF -1	**L** wants **U** to accept the original order if **M** does not modify it
5 IFF-1	**L** would insist on the original order if **M** tends to modify it
-2	**L** does not want **U** to delay the procedure
5	**L** wants to insist on the original order

TABLE 4.4: Structure of Option Statement - Ministry of Environment

States(S)	Condition Attributes (C)					Ranking
	c1=4	c2=3	c3=-2	c4=-1	c5=5 IFF-1	r_i
s_1	1	0	0	1	0	6
s_2	1	0	0	0	1	7
s_3	1	1	1	1	0	2
s_4	1	1	1	0	1	3
s_5	1	0	0	1	1	5
s_6	1	0	0	0	0	8
s_7	1	1	1	1	1	1
s_8	1	1	1	0	0	4
s_9	0	0	1	1	0	9

to have recognized their logical structure preliminarily, which can be said to be an approach in the opposite direction from GMCR, where intuitive operability was an advantage.

4.2.2 Formulating Preference Orders Applying TOPSIS

In the previous subsection, we studied the method of preference order formulation by option prioritization. In this subsection, we examine the method for conflicts with a large number of states, supported by multi-criteria decision making (MCDM). As a conceptual decision-making framework, one advantage of the GMCR is that it enables consideration using structures that are similar to the intuition of decision makers. At the same time, even when formulating preference orders that affect conflict solutions, the formulation of the preference ranking in the GMCR is left to the decision-maker or analyst's point of view. However, for this standard analysis method, an answer has yet to be provided for responding to cases in which some kind of reasoning is required as a basis for analyzing complex phenomena, or as a basis for a method of preference order formulation. When faced with multiple options in real-world social issues, and in attempting to make an optimal decision, decision-making methods that are easy for stakeholders to understand are in order. Regarding preference in the GMCR, studies have been conducted on the analysis methods where preferences are unknown [59][60] , the concept of strength is introduced to the preferences [31], and fuzzy theory is incorporated [9]. Other than TOPSIS, widely used methods in MCDM include Analytic Hierarchy Process (AHP), Elimination Choice Expressing Reality (ELECTRE), and Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE). The TOPSIS is an MCDM method devised by Hwang and Yoon [95].

In the TOPSIS, rankings are obtained through the calculation of scores and weighting of attributes. The most distinctive feature of GMCR is its ability to express the irreversibility of state transition, which is useful for decision-making where prudent judgment is required, such as when national interest is involved. TOPSIS, which considers the "most favorable solution" and the "least favorable solution" among the possible alternatives and ranks them according to the distance from the two options, can be considered to be a good fit with the preference ordering in the GMCR, which takes the degree of preference ordinal. In general, MCDM often requires complicated data processing, though, in the example considered in this present paper, the purpose is not to process empirical data but to obtain preference orders for the GMCR analysis. Considering our efforts to introduce TOPSIS into the GMCR, a study by Zhao and Xu [98] has been presented. We review the TOPSIS-GMCR method first in this subsection, then we conduct an analysis using a different method for weighting and ranking to seek a conflict resolution, primarily focusing on de-escalation in the following section. The procedure for preference determination in TOPSIS is as follows:

STEP 1 For the case to be analyzed, consider matrix D that consists of options A as m rows and decision-related attributes C as n columns, of a DMi, where $Y = 1, N = 0$.

$$D = \begin{pmatrix} x_{11} & \dots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mn} \end{pmatrix} \tag{4.1}$$

STEP 2 Normalize matrix D and obtain normalized matrix R.
$R = \left[r_{ij}\right]_{m\times n}, i = 1, \dots, m; j = 1, \dots, n,,$

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{4.2}$$

STEP 3 Determine the weights $W = (w_1, w_2, \dots, w_3)$ used to weigh each of the attributes C. With regard to weighting, a number of methods exist, such as using data processed according to the qualities of attributes C. Here, we adopt the approach described below.

$$\sum_{i=1}^{n} w_j = 1, w_j (j = 1, \dots, n) \tag{4.3}$$

STEP 4 Obtain the weighted normalized matrix $P = [p_{ij}]$.

$$p_{ij} = w_j * r_{ij}, \ \ i = 1, \dots, m; j = 1, \dots, n. \tag{4.4}$$

STEP 5 Formulate the ideal solution A^+ (Positive Ideal Solutions; PIS) and the least favorable solution A^- (Negative Ideal Solutions; NIS).

$$A^+ = (p_1^+, p_2^+, \dots, \dots, p_m^+) \tag{4.5}$$

$$A^+ = (p_1^-, p_2^-, \dots, \dots, p_m^-) \tag{4.6}$$

$$p_j^+ = \left(\substack{max \\ i} p_{ij}, \ j \in J_1; \ \substack{min \\ i} p_{ij}, j \in J_2\right) \tag{4.7}$$

$$p_j^- = \left(\substack{min \\ i} p_{ij}, \ j \in J_1; \ \substack{max \\ i} p_{ij}, j \in J_2\right) \tag{4.8}$$

J_1 is an element acting positively on each of the options$A_1, \dots, A_m$ in the attributes C, whereas J_2 is an element acting negatively.

STEP 6 For each option, determine their distance to the PIS and NIS.

$$d_i^+ = \sqrt{\sum_{j-1}^{n} (d_{ij}^+)^2} \tag{4.9}$$

$$d_i^- = \sqrt{\sum_{j-1}^{n} (d_{ij}^-)^2} \tag{4.10}$$

where

$$d_{ij}^+ = p_j^+ - pij, i = 1, \ldots, m \tag{4.11}$$

$$d_{ij}^- = p_j^- - pij, i = 1, \ldots, m \tag{4.12}$$

STEP 7 Find each distance of an option to the NIS.

$$\xi_i = \frac{d_i^-}{d_i^+ + d_i^-} \tag{4.13}$$

STEP 8 Sort the options in descending order of value ξ, obtained in STEP 7.
We apply TOPSIS to weight the options in the Elmira conflict. Assume the following weight ranking for each option for each DM.

TABLE 4.5: Weight of Options - Elmira Conflict

Weight ranking	1	2	3	4	5
Weight multiplier	$2^{(5-1)}$	$2^{(5-2)}$	$2^{(5-3)}$	$2^{(5-4)}$	$2^{(5-5)}$
w_j	.516	.258	.129	.065	.032

The resulting preference rankings and the initial preference rankings are listed in Table 4.6 . DM-TOPSIS indicates the order by TOPSIS : the ranking acquired according to the descending order of distance from Positive Ideal Solutions.

TABLE 4.6: Elmira Conflict - Preference Order by TOPSIS

	most preferred							least preferred	
M-TOPSIS	s_8	s_7	s_6	s_5	s_4	s_3	s_2	s_1	s_9
M	s_7	s_3	s_4	s_8	s_5	s_1	s_2	s_6	s_9
U-TOPSIS	s_6	s_2	s_5	s_1	s_8	s_4	s_7	s_3	s_9
U	s_1	s_4	s_8	s_5	s_9	s_3	s_7	s_2	s_6
L-TOPSIS	s_8	s_6	s_7	s_5	s_4	s_2	s_3	s_1	s_9
L	s_7	s_3	s_5	s_1	s_8	s_6	s_4	s_2	s_9

Table 4.7 shows the results of the stability analysis using the preference rankings obtained from TOPSIS. The checkmark represents that the equilibrium is established in both the original and the new analysis, D indicates that the equilibrium disappears in the new analysis, and E indicates that the equilibrium emerges in the new analysis, respectively. The Pareto efficiency of s_1, the status quo, disappears, and the equilibrium involving Nash at s_5, where L sticks to the initial order but U stalls the negotiation, disappears. On the other hand, a strong equilibrium is newly established in s_6, which is “Modify-Delay-Insist.” An equally strong equilibrium is also

TABLE 4.7: Elmira Conflict - Stability Analysis by TOPSIS Order

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
Nash					D	E		✓	✓
GMR	✓	E	E	✓	✓	E	E	✓	✓
SMR	✓	E	E	✓	✓	E	E	✓	✓
SEQ					D	E		✓	✓
CNash						E		✓	✓
CGMR	D			D		E		✓	✓
CSMR	D			D		E		✓	✓
CSEQ						E		✓	✓
Pareto	D		D	D	D	E	D	✓	

found in s_8 as in the initial analysis, indicating that the effective use of "Insist" is the key to conflict resolution. In other words, it is considered valid enough as a primary analysis with little information.

In the following subsection, the Elmira conflict is analyzed from the perspective of "avoiding the worst case scenario" using TOPSIS.

4.3 Formulating Preference Order for Aversion of Disaster - GMCR-DA

In some conflicts, it may be useful to analyze to obtain a solution to seek more benefit, but if the worst is known to be very serious, a primary analysis to avoid the worst may be necessary first. In this subsection, we present to use TOPSIS to obtain a solution that avoids the worst-case scenario by obtaining a ranking that is "far from the most undesirable state." The same procedure is used for all the calculations from Step 1 to Step 6 in the previous subsection. However, 1) In weighting each option, rank heavily the one to avoid. 2) In Step 7, find the distance from PIS instead of the distance from PIS.

Find each distance of an option to the PIS.

$$\lambda_i = \frac{d_i^+}{d_i^+ + d_i^-} \tag{4.14}$$

Elmira Conflict

The option ranking for weighting is assumed to be as in Table 4.8, and the resulting preference ranking is as in Table 4.9. The weighting for the options was not taken in the reverse order of the preferred options but instead inferred as the preferred criterion for what would be less desirable.

In Table 4.10, as in the previous table, the checkmark represents that the equilibrium is established in both the original and the new analysis, D indicates that the equilibrium disappears in the new analysis, and E indicates that the equilibrium emerges in the new analysis, respectively. Although the strong equilibrium at s_9, where U abandons the plant, is still established, we can see that the strong equilibrium involving Nash is established at s_1, s_2, s_3, and s_4, which significantly expands

TABLE 4.8: Elmira Conflict - Option Ranking for Disaster Aversion

	Modify	Delay	Accept	Abandon	Insist
M	2	3	4	1	5
U	4	3	5	2	1
L	3	2	5	1	4

TABLE 4.9: Elmira Conflict - Preference Order for Disaster Aversion

	most preferred								least preferred
M-Disaster Aversion	s_3	s_7	s_1	s_5	s_4	s_8	s_2	s_6	s_9
M	s_7	s_3	s_4	s_8	s_5	s_1	s_2	s_6	s_9
U-Disaster Aversion	s_1	s_2	s_3	s_4	s_9	s_5	s_6	s_7	s_8
U	s_1	s_4	s_8	s_5	s_9	s_3	s_7	s_2	s_6
L-Disaster Aversion	s_3	s_7	s_4	s_8	s_1	s_5	s_2	s_6	s_9
L	s_7	s_3	s_5	s_1	s_8	s_6	s_4	s_2	s_9

the solution possibilities compared to the initial stability analysis. In particular, an equilibrium is established in s_4 (Modify, Accept), where Nash was not established in the original analysis, indicating that the Ministry of Environment (**M**)'s modification policy is an essential key to the solution. Furthermore, it is important to note that the Nash equilibrium and Pareto optimum at s_5 (Insist, Delay) vanishes. The worst thing that can happen to all DMs is that **U**, which expects a modification of the original order, continues to operate while stretching out negotiations and finally abandons the plant, stops running and leaves town. All DMs must reach a consensus to find a solution and avoid a three-way game of chicken: insist on the initial order, modify the order, and extend the negotiations. From this perspective, the proposed method for disaster aversion is meaningful and valid for the primary analysis.

TABLE 4.10: Elmira Conflict - Stability Analysis
by Disaster Aversion Method

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
Nash	E	E	E	E	D			D	✓
GMR	✓	E	E	✓	D			D	✓
SMR	✓	E	E	✓	D			D	✓
SEQ	E	E	E	E	D			D	✓
CNash	E	E	E	E				D	✓
CGMR	✓	E	E	✓				D	✓
CSMR	✓	E	E	✓				D	✓
CSEQ	E	E	E	E				D	✓
Pareto	✓		✓	D	D		D	D	

Cuban Missile Crisis

We apply the preference ranking formulation for disaster aversion to the Cuban Missile Crisis for further analysis and evaluation. The options and the ten states are discussed using Table 3.10 in Subsection 3.5.1 as is. As explained in the previous section, we normalized and weighted the matrices of the two DMs. When considering the weighting of the four conditions (c1: U.S. airstrike, c2: U.S. naval blockade, c3: USSR withdraw, c4: USSR escalate) for both DMs, we must consider that, as described in the previous section, in the Cuban Missile Crisis, decision making to avoid the worst-case scenario, namely, conflict escalation caused by the Soviet Union retaliating against the U.S. airstrikes, was called for. Thus, in conducting an analysis, it is essential to account for that fact that for both countries, improving their utility through attacking was not of the highest priority. For this reason, we analyzed the preference orders based on the distance from the least favorable option (NIS), rather than the distance from the most favorable option (PIS). Regarding weighting, we used weights where each criterion's ranking (in this case, ranked in order of least favorable) was multiplied by the multiplier of the total number of criteria, and the process of normalization was carried out by setting the sum total of scores to 1. Based on the premise that both countries wanted to keep conflicts localized and avoid escalation of conflicts, the option weighting was based on the assumption that the United States wanted to prevent Soviet counterattacks the most and the Soviet Union wanted to avoid the airstrikes by the United States in addition to the naval blockade.

Using the binomial option matrix of states converted from Table 3.10 with the weighting order for each option 4.11 , we obtained the preference order presented in Table 4.12.[2]

TABLE 4.11: Cuban Missile Crisis
- Option Ranking for Disaster Aversion

	US:Airstrike	US: Blockade	USSR:Withdraw	USSR: Escalate
US	2	3	4	1
USSR	1	3	4	2

TABLE 4.12: Cuban Missile Crisis
- Scores and Preference Order for Disaster Aversion

	US			**USSR**		
state	score	DA ranking	Initial ranking	score	DA ranking	Initial ranking
s_1	1.0	1	4	1.0	1	1
s_2	.743	4	6	.435	5	4
s_3	.858	2	5	.832	2	2
s_4	.718	6	7	.414	7	8
s_5	.730	5	2	.425	6	7
s_6	.830	3	1	.800	3	6
s_7	.707	7	3	.403	8	9
s_8	.170	9	9	.200	9	5
s_9	.270	8	8	.575	4	3
s_{10}	.104	10	10	.124	10	10

[2] Calculation details are provided in Appendix B.3

The stability analysis conducted based on the new preference information obtained by the disaster aversion method is shown in Table 4.13. The checkmark represents that the equilibrium is established in both the original and the new analysis, D indicates that the equilibrium disappears in the new analysis, respectively.

TABLE 4.13: Cuban Missile Crisis - Stability Analysis by Disaster Aversion Method

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
Nash	✓				✓	✓			D	
GMR	✓				✓	✓			✓	
SMR	✓				✓	✓			✓	
SEQ	✓				✓	✓			D	
CNash	✓					✓			D	
CGMR	✓					✓			D	
CSMR	✓					✓			D	
CSEQ	✓					✓			D	
Pareto	✓					D				

In s_9 (naval blockade, escalate), where a strong equilibrium including Nash was established in the original analysis, all equilibria except GMR and SMR disappear. In contrast, the equilibria in s_1 (status quo), s_5 (airstrike, withdraw), and s_6 (naval blockade, withdraw) remain intact.

The only equilibrium other than status quo where Nash has been established is s_6, which weakens the possibility of deployment to s_9, which would trigger escalation of the conflict. In other words, if both DMs had been able to communicate their intention to avoid escalation in some way early on, we can assume that this preference ranking would have been more consistent with the situation.

4.3.1 Disaster Aversion and Permissible Range

As a supplementary analysis, this chapter compared the analysis results of the Elmira conflict and the Cuban Missile Crisis using the two methods: GMCR-DA proposed in this chapter and GMCR-PR discussed in Chapter 3.

Elmira Conflict

Assume a threshold at which only the most unfavorable state was impermissible for each DM; each DM's permissible range is P^8_M, P^8_U, and P^8_L, respectively. The worst state for each DM is: for the Ministry of Environment and the local government, Uniroyal abandons the plant (s_9); for Uniroyal, the negotiations are protracted, but the local government sticks to enforcing the original control order, and the Ministry of Environment agrees to modify it, but not to conditions acceptable to Uniroyal (s_6).

Table 4.14 integrates the stability analysis results using the disaster averse preference ranking in Table 4.10, with the results using the permissible ranges that allow for all but the least preferred states. Each symbol in Table 4.14 indicates the following:

- Checkmark: an equilibrium established in the original stability analysis (when all preference information was available).

- E: a new equilibrium established in the analysis with the disaster averse preference ranking.
- D: an equilibrium established in the original analysis that disappeared in the analysis with the disaster averse preference ranking.
- P: an equilibrium newly established in the permissible range analysis, which allows only the least preferred state to be excluded.
- DP: an equilibrium established in the original analysis that disappeared in the disaster averse preference ranking analysis and was restored in the permissible range analysis.

Also, the top three lines are a binary description of the permissible states for each DM. As can be seen from this analysis, if it is expected that **U** will use abandon as a bargaining card to intimidate, it is imperative for **M** and **L** to strongly indicate from the beginning that they will not tolerate **U**'s abandonment.

TABLE 4.14: Elmira Conflict - Stability Analysis by Disaster Aversion and Permissible Range

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
M	1	1	1	1	1	1	1	1	0
U	1	1	1	1	1	0	1	1	1
L	1	1	1	1	1	1	1	1	0
Nash	E	E	E	E	DP		P	DP	✓
GMR	✓	E	E	✓	✓		P	DP	✓
SMR	✓	E	E	✓	DP		P	DP	✓
SEQ	E	E	E	E	DP		P	DP	✓
CNash	E	E	E	E	P		P	DP	✓
CGMR	✓	E	E	✓	P		P	DP	✓
CSMR	✓	E	E	✓	P		P	DP	✓
CSEQ	E	E	E	E	P		P	DP	✓
Pareto	✓	P	✓	DP	DP		DP	DP	

Cuban Missile Crisis

As in the Elmira conflict, we compared analysis results of the Cuban Missile Crisis with the permissible range where only the least preferred states are excluded, with the analysis using the disaster averse preference ranking proposed in this chapter.

Table 4.15 summarizes the stability analysis results using the disaster averse preference ranking in Table 4.13, with the results using the permissible ranges that allow for all but the least preferred states. Each symbol in Table 4.15 indicates the following:

- Checkmark: an equilibrium established in the original stability analysis (when all preference information was available).
- E: a new equilibrium established in the analysis with the disaster averse preference ranking.

- D: an equilibrium established in the original analysis that disappeared in the analysis with the disaster averse preference ranking.
- P: an equilibrium newly established in the permissible range analysis, which allows only the least preferred state to be excluded.
- DP: an equilibrium established in the original analysis that disappeared in the disaster averse preference ranking analysis and was restored in the permissible range analysis.

Also, the top two lines are a binary description of the permissible states for each DM.

TABLE 4.15: Cuban Missile Crisis - Stability Analysis by Disaster Aversion and Permissible Range

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
U.S.	1	1	1	1	1	1	1	1	1	0
USSR	1	1	1	1	1	1	1	1	1	0
Nash	✓	P	P	P	✓	✓	P	P	DP	
GMR	✓	P	P	P	✓	✓	P	P	✓	
SMR	✓	P	P	P	✓	✓	P	P	✓	
SEQ	✓	P	P	P	✓	✓	P	P	DP	
CNash	✓	P	P	P	P	✓	P	P	DP	
CGMR	✓	P	P	P	P	✓	P	P	DP	
CSMR	✓	P	P	P	P	✓	P	P	DP	
CSEQ	✓	P	P	P	P	✓	P	P	DP	
Pareto	✓	P	P	P	P	DP	P	P	P	

In the analysis of the Cuban Missile Crisis, the analysis with the permissible range not allowing s_{10}: the United States conducts a naval blockade and airstrikes, and the Soviet Union responds with attack, will result in equilibrium in all states except s_{10}, which is difficult to provide practical information for decision making. Therefore, when the preference information available for the analysis is coarse, it is necessary to formulate a disaster aversion preference ranking or obtain additional information to narrow the permissible range.

Remark (Status Quo in Cuban Missile Crisis). In this study, the *status quo* of the Cuban Missile Crisis is defined as the point in time when the missile deployment by the Soviet Union in Cuba was discovered by the United States, as described in Chapter 3. Therefore, s_1 is NOT a state in which no conflicts have occurred, and the circumstances that led to the status quo are understood to be included in s1. In the baseline analysis presented in 3.12, we see all equilibria, including coalition stability and the Pareto optimum, are established in status quo s_1 and s_6 The conflict did not remain at s_1 but settled at s_6, since it can be as *a posteriori* presumed to be that, apart from the superficial confrontational structure as described , sub rosa negotiations were conducted in which the Soviet Union's removal of missiles in Cuba and the United States' removal of missiles in Turkey were the terms of exchange. It is certainly difficult to describe such a situation in an analysis framework that looks for a rational equilibrium.

Even in such contexts, when negotiability is present in the background of conflicts, it is useful to analyze them in conjunction with the GMCR-PR or a committee with permissible range, presented in Chapter 3. Meanwhile, if there could have been a non-explicitly shared goal behind the conflict to avoid escalation, then the GMCR-DA presented in this chapter is effective.

4.4 Conclusion of the Chapter

This chapter proposed a new method for GMCR analysis by using TOPSIS, a multivariate analysis method, to conveniently obtain the "least preferred ranking" when available information on preferences is coarse. As shown in the supplementary analysis provided in Subsection 4.3.1, GMCR-DA is generally better suited for investigation with only coarse information than GMCR-PR. When enough information to formulate a permissible range is available, it is possible to analyze the two methods concurrently to obtain deeper decision-making insights. Most notably, this method allows analysis with the coarsest information in the methodologies we propose in this thesis. We only need to know the unfavorability (favorability) of the options, which are components of a state. For the idea of considering preferences about options rather than states, we have a related discussion in Chapter 6.

Chapter 5

New Reachability by Unspecified External Factors

Thus far, we discussed methods to analyze situations in which only coarse information is available in conflict analysis through novel techniques. Two methods : GMCR with permissible range in Chapter 3, and the formulation of a preference ranking when only the avoidable worst-case scenario is known in Chapter 4, were introduced.

In this chapter, a new analysis method that is capable of describing influence of the external factors on state transitions in interstate conflicts has been developed using GMCR [48] [49] is introduced and applied. With the new method, it is possible to analyze conflict situations where **the state transitions of the primary DMs are known to be influenced by factors outside the conflict and where there is no need to specify their attributes**. Such external factors may include third parties that cannot be identified as DMs, unidentifiable third party complexes and environmental factors such as nature and global economic trends. External factors that cause changes are not treated as one of the decision-makers within the framework of the conflict, while the analytical focus is on the resulting consequences of their influence on the state transitions of the primary DMs. In other words, we assume that the primary decision-makers can be given new reachability through the influence of external factors. This path-breaking method can significantly expand the scope of analysis while building on the existing GMCR framework: it allows for the description of conflicts without identifying the origin of the influence.

First, the background and theoretical novelty of the new concept will be explained. Next, we clarify the concept by presenting conflict cases within the limited scope: 2×2 interstate conflicts based on representative preference types and the Cuban Missile Crisis described in 2×2 structure. In addition, we will show the analysis with a larger number of states: the Cuban Missile Crisis in ten states (the structure shown in Subsection 3.5.1) and Elmira conflict with nine states (the structure shown in Example 2.3.2).

5.1 Outline of the New Method

5.1.1 Background

This study examined conflicts between nations where external factors could have changed the situations. These external factors include de-escalation moves by arbitrators, interventions by some third parties who could benefit from the conflict's escalation, or environmental changes due to complex factors. In standard methods of analysis of game theory or the GMCR, DMs and state transitions are treated as a set. When a state change occurs, the DM who caused the change must be included in the conflict model as an originator of the change and its preference information. However, in an analysis, even if the state change itself may have a significant impact on the conflict, the DMs: their identifications or attributes, who initiate the change, will often not have the same importance as those of the primary DMs. For primary DMs, or in an analysis of conflicts, it is essential to know whether the significant state change will occur or not to make an initial assessment of the situation for decision making. Thus it may be more convenient to analyze who will bring it about as a separate issue.

Frameworks that incorporate uncertainty [59] [60], and fuzziness [9] have been studied as as way to deal with insufficient information about DMs' preferences. In this study, we assume a situation where the DMs' preferences are clear, while the external factors' preferences that may affect the states of conflict are not apparent. For example, For example, when there are multiple arbitrators, and the relationship between them is complex, we do not deal with these relationships but simply examine the effects. As for the motive of the impact, there may be interveners who benefit from escalation alongside bona fide arbitrators who lead de-escalation. There may be cases where the effect of possible unexpected changes in the environment can also be considered. There may be cases where the impact of possible unexpected changes in the environment can also be considered. In the simplest case, the "prisoner's dilemma " is a conflict between two prisoners, but the interrogator, who sets the rules for confession and sentence, is not treated as a DM. The framework proposed in this study is capable of describing situations in which an interrogator changes the rules of interrogation in the prisoner's dilemma.

With regard to the states of conflict, escalation and deterrence have been studied using normal form games [15] [26] [97], metagame [33] and GMCR [54]. This study focused on interstate conflicts between DMs with preferences that are considered rational. Therefore, the assumptions of the attributes of DMs as sovereign states are considered as shown in Assumption 1. After conducting GMCR stability analysis of these conflicts, we examined all cases depending on the change in reachability as a function of external factors and provided definitions related to their effects on conflict escalation. Then, we applied each of the models of de-escalation and escalation to real-world conflict cases and verified their explanation capability and suitability.

Remark (Externality in the New Concept). In this study, the term *external* refers to the general concept of elements coming or derived from a source outside the conflict consisting of states of the world described by $N, S, A_i, \succsim_i$, which are the main elements of GMCR. Thus, it is independent of the internal or external stability of the von Neumann-Morgenstern stable set, which is based on the dominance relation on imputations in a cooperative game.

5.1.2 Theoretical Novelty

Before discussing the new method, we would like to reflect on the GMCR framework. GMCR consists of four elements: $G = (N, S, (A_i)_{i\in N}, (\succsim_i)_{i\in N})$, i.e., DMs, states, DM i's transition graph, and DM i's preference over the states. In the general GMCR analysis procedure, a state is said to be generated by a combination of options that the DM can control as we reviewed in Definition 2.1.11 and 2.1.12, but the framework definition does not explicitly specify this practice. In other words, a state and state transitions can involve factors other than those caused by the actions taken by the $i \in N$. DM i's arcs $A_i \subseteq S \times S$ represent that DM i can make a unilateral move from the initial state to the terminal state. Here, there is no restrictive definition that eliminates the situation where, due to some effect produced by other than itself, DM

i can transition unilaterally from s to s'. Consider the prisoner's dilemma that we examined in Example 2.1.1 again.

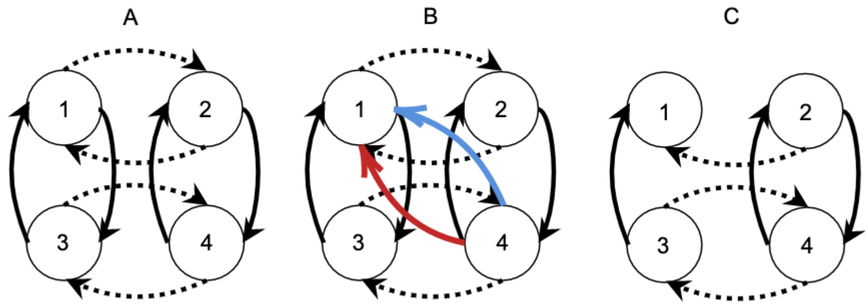

FIGURE 5.1: Graph Model of Prisoner's Dilemma - New Reachability

Now, let us assume a situation in which some circumstances outside of this conflict bring about a change in the DMs' reachability. Or, suppose a situation in which an *agent* defined in 2.1.1, who is not the *DM* defined in 2.1.2, influences the DM's unilateral move. The change will affect invalidating an arc of each of the DMs or granting them a new arc. Figure 5.1 shows the standard prisoner's dilemma (A) and the conflicts with new reachability (B, C). In each graph, the solid and dotted lines indicate the reachability of DM1 and DM2, respectively. In the graph B, red represents the reachability acquired by DM1 and blue by DM2 due to external factors; the new arcs allow them to make a transition from s_4 to s_1. Graph C shows that some external factor induces to keep the two DMs in s_1 by taking away the reachability from s_1 to s_2 or s_3. We suppose that this new reachability presented in graphs B and C occurs, for example, in a situation where the interrogator sends some signal to the two DMs to encourage them to confess or where the lawyers of both DMs cooperate to persuade them to confess. For one DM, the possible changes are four deletions and eight new acquisitions; therefore, for two DMs, a total of twenty-four changes are possible as illustrated in Figure 5.2 . The invalidation of reachability may occur for each of the four arcs that the DM originally retains. Furthermore, the acquisition of a new arc may occur for eight arcs for each DM.

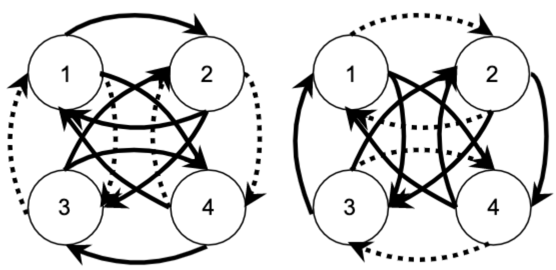

FIGURE 5.2: Graph Model of Prisoners' Dilemma - Change Possibility

In Figure 2.3, we have shown the "irreversible prisoner's dilemma", where the prisoner cannot return to the initial states once he has confessed. New reachability (addition or loss) due to external factors can be treated as a similar problem to reversibility and irreversibility of transitions. Since standard GMCR analysis methods only deal with reversibility and irreversibility for the irregular occurrence of reachability, the addition and loss of arcs proposed in this study is an entirely novel concept.

There are two theoretical points to clarify regarding this new reachability: 1) the components of the state S. 2) the controllability of state transitions by DM.

Components of a State

First, we consider the configuration of the state S. One way to describe the composition of the state S is by combining strategy options selected by the DM, as shown in Definition 2.1.12 (State in Option Form). On the other hand, it is also possible to assume a state without any restriction by strategy selection. Of these, the latter can be expressed using the description by option selection as follows.

Let E represent the set of action by external factors on the transition of DM i. The influence of the factors for $f(o_{ij})$ can be expressed as $\lambda(\epsilon)$, where if the value is 1, the influence of external factor ϵ is present in the state s; if the value is 0, it is not.

In other words, the value of 0 represents the case where the state S consists only of option selection. Assuming a situation where the state S is generated by factors that include other than DM i's option selection, the state S can be described as the product of DM i's option set O_i and the set of external factors E.

Let E be the influence of external factors in a conflict $o_{ij} \in O_i, i = 1, 2, \ldots, n$. The influence on a state is a mapping $\lambda : E \to \{0, 1\}$, such that for $i = 1, 2, \ldots, n$.

Definition 5.1.1 (External Factors)**.**

$$\lambda(o_{ij}) = \begin{cases} 1 \text{ the state is influenced by external factor,} \\ 0 \text{ otherwise.} \end{cases} \tag{5.1}$$

Definition 5.1.2 (State in Option with External Factors)**.** $f^s(O_i)$ denotes DM i's option selection corresponding to state s as in defined in Definition 2.1.12. A state can be described as a product of option selection O_i and external factors E.

Example 5.1.1 (Prisoner's Dilemma with External Factors)**.** Suppose that in the prisoner's dilemma, during the interrogation period, there was a possibility that a decision would be made to issue a pardon. It is unclear what effect the amnesty will have on the outcome of the current interrogation but that it is definitely expected to impact the severity of the prisoner's sentence. Let the option selection of the two prisoners DM1 and DM2 be NC (Not confess) or C (Confess). As an event independent of the DM's option selection, we assume the implementation of pardon E; ε_1 indicates that the pardon is implemented, and ε_2 indicates that the amnesty is not implemented. For the four states (two options for each DM), considering the influence of external factors (amnesty or not), there are a total of eight possible states as follows:

$$\begin{aligned} &N = \{DM1, DM2\}, O_i = \{NC, C\}, E = \{\varepsilon_1, \varepsilon_2\} \\ &s_1 = (NC, NC, \varepsilon_1), s_2 = (NC, NC, \varepsilon_2), s_3 = (NC, C, \varepsilon_1), s_4 = (NC, C, \varepsilon_2), \\ &s_5 = (C, NC, \varepsilon_1), s_6 = (C, NC, \varepsilon_2), s_7 = (C, C, \varepsilon_1), s_8 = (C, C, \varepsilon_2). \end{aligned}$$

In this situation, it is possible to describe the DM's tactics relating the amnesty. For example, if the DM knows that the amnesty would only cover cases that have been closed when it goes into effect, he may try to end the interrogation by confessing early for the pardon.

Controllability and Unilateral Move

If the state S consists of selections of strategy options, it is evident that its own strategy choice can only control the DM's move. In the new concept of this research, we assume that the original controllability can be removed or entirely new controllability can be added. Even if the state S is composed of external factors as well as option choices, the original controllability remains the same, and the DM can only transition unilaterally through its own option choices. The loss and gain of reachability can occur in the same way whether external factors are involved in the state S or not in the newly proposed concept.

5.1.3 Information Partition and Coarseness in GMCR with New Reachability

In the analysis based on the new reachability by external factors, **coarsening by purposefully not specifying the external factors** that affect the state transitions of DM is performed.

In the following sections, we discuss the new reachability in more detail, limiting the scope of the analysis.

5.2 New Reachability in 2 × 2 Conflicts

5.2.1 Analysis Scope

In this chapter, the analysis object is a conflict between two nations, where we assume four states as in the form of a Cooperate-Defect (CD) game, as follows: s_1: Both DMs cooperate; s_2: DM1 cooperates, DM2 defects; s_3: DM1 defects, DM2 cooperates; and s_4: both DMs defect. If we assume the most reasonable conditions of preference order for cases that are limited to interstate conflicts to be $s_3 \succ s_4$ and $s_1 \succ s_2$, i.e., that a nation seeks its own national interests but does not wish to have catastrophic conflicts and would rather make concessions than defeats, then we obtain only six applicable sets of preferences [76]:

Definition 5.2.1 (Preference Type of Nations in Interstate Conflicts)**.** a. $s_3 \succ s_1 \succ s_4 \succ s_2$, b. $s_3 \succ s_1 \succ s_2 \succ s_4$, c. $s_1 \succ s_3 \succ s_4 \succ s_2$, d. $s_1 \succ s_3 \succ s_2 \succ s_4$, e. $s_3 \succ s_4 \succ s_1 \succ s_2$, f. $s_1 \succ s_2 \succ s_3 \succ s_4$.

In the six types of preference indicated by a to f, a and b correspond to prisoner's dilemma and chicken game, respectively, in which two DMs have symmetric preferences. Considering the conflicts between nations with these six types of preferences, we have twenty-one combinations, and these twenty-one sets will be used to describe the interstate conflicts.

Definition 5.2.2 (Cooperate-Defect Game)**.** $N = \{DM1, DM2\}, S = \{s_1, s_2, s_3, s_4\}$, where s_1=(Cooperate, Cooperate), s_2=(Cooperate, Defect), s_3=(Defect, Cooperate), s_4=(Defect, Defect), DM i's preference $\succ_i$: $s_3 \succ_i s_4, s_1 \succ_i s_2$.

We first examined situations in which these conflict states changed due to the influence of external factors through an analysis of the prisoner's dilemma. In the prisoner's dilemma, the outcome rationally chosen by each DM (Nash) does not result in a desirable outcome for society as a whole (Pareto optimum).

TABLE 5.1: Stability Analysis - Prisoner's Dilemma

		s_1	s_2	s_3	s_4
Nash	DM1			✓	✓
	DM2		✓		✓
	Eq				✓
GMR	DM1	✓		✓	✓
	DM2	✓	✓		✓
	Eq	✓			✓
SMR	DM1	✓		✓	✓
	DM2	✓	✓		✓
	Eq	✓			✓
SEQ	DM1	✓		✓	✓
	DM2	✓	✓		✓
	Eq	✓			✓
Pareto		✓	✓	✓	

GMCR stability analysis of each of the eight deletions and sixteen additions of arcs, for both DM1 and DM2, showed that four among these new reachability values caused changes in Nash stability in s_4. As shown in Table 5.2 about the six changes on the prisoner's dilemma, either Nash is newly established in s_2 or s_3 in addition to s_4 by deletion, or Nash in s_4 disappears by addition, which can be interpreted as the conflicts being de-escalated by these changes.

TABLE 5.2: New Reachability and Stability of Prisoner's Dilemma

Change	Arc	Nash
Deletion		
DM1	(s_2, s_4)	s_2,s_4
DM2	(s_3, s_4)	s_3,s_4
Addition		
DM1	(s_4, s_1) ,(s_4,s_3)	$\emptyset$
DM2	(s_4, s_1) ,(s_4,s_2)	$\emptyset$

5.2.2 Interstate Conflicts

In this subsection, the new reachability discussed in the previous subsection is introduced for the twenty-one types of interstate conflicts with twenty-four arc operations. The full results of the stability analysis were classified and examined. The analysis reveals how the original conflicts' stability was affected by only one arc operation: deletion or addition. The representative features from the analysis results for these twenty-one conflicts were grouped into five categories related to the number of Nash and the state in which Nash was established in the original conflicts: 1) no Nash; 2) Nash in s_1; 3) Nash in s_4; 4) Nash in s_2 or s_3; and 5) two Nash. We used the a-f classification as the preference type, taken from the previous subsection 5.2.1.

1. No Nash : c×b
2. Nash in s_1 : (i) f×c, d×c, (ii) d×d, f×d, f×f
3. Nash in s_4 : (i) a×a, a×e, e×e (ii) a×c (iii) c×e

4. Nash in s_2 or s_3 : (i) b×a, d×a, f×a, d×b, f×b, f×e (ii) e×b, e×d
5. Two Nash : (i) b×b (ii) c×c

TABLE 5.3: Number of Nash - Original and Changed Conflicts

Original conflicts	Changed conflicts
$\emptyset$	$\emptyset, 1$
1	$\emptyset, 1, 2$
2	$1, 2$

Table 5.3 represents the number of Nash changes when the reachability is affected by external factors concerning the interstate conflicts we modeled. For example, if there is no Nash in the original conflict, no matter what changes are made to reachability, two Nash will never appear. It also means that if Nash were established in two states in the original conflict, no change in reachability would change it to a conflict without Nash.

State changes due to new reachability caused by external factors are defined as follows:

Definition 5.2.3 (State Change Types in Interstate Conflicts [48])**.**

- Escalation (+) : In addition to the Nash established in the original conflict, a Nash in s_4 is newly established.
- De-escalation (-) : The Nash established in the original conflict disappears, or a Nash is newly established in s_1.
- Divergence ($\nearrow$) : In addition to the Nash established in the original conflict, a new Nash other than s_1 or s_4 will be established.
- Conversion ($\searrow$) : In conflicts where two Nash is established, the state changes to a state where a Nash in a state other than s_1 or s_4 is established.

Table 5.4 summarizes the changes in the Nash equilibrium as a consequence of the change in transitions for each of these five categories. The impact of the change in Nash equilibrium on the escalation (+), de-escalation (-), divergence ($\nearrow$) convergence ($\searrow$) compared to the original stability is shown in the last column ($\pm$).

As shown, in Category (3), Nash was established in s_4, i.e., (D, D), of the original conflict but it disappeared when adding the new reachability to DM1 or DM2. As a result, the function of de-escalation was achieved. On the contrary, Category (1) was originally a conflict without Nash. Nonetheless, by removing the reachability (s_4, s_3) of DM2, Nash was established in s_4 and this change was interpreted as achieving escalation.

Based on the Definition 5.2.3 and these analysis results, we defined the impact of external factors on the DMs' state transitions in interstate conflicts.

Definition 5.2.4 (New Reachability in Interstate Conflicts [48])**.** A_i^A and A_i^D represent the new reachability, which is addition or deletion to the reachability in the original conflict A_i.

In conflicts, the new reachability that causes escalation is defined as follows:

Definition 5.2.5 (Escalation Reachability (**ESR**) in Interstate Conflicts [48])**.**

TABLE 5.4: Taxonomy of New Reachability - Interstate Conflicts

Category	Change	Arc	Nash	$\pm$
(1) No Nash	***Deletion***			
	DM1	(s_3, s_1)	s_3	$\nearrow$
	DM1	(s_2, s_4)	s_2	$\nearrow$
	DM2	(s_1, s_2)	s_1	$-$
	DM2	(s_4, s_3)	s_4	$+$
(2) Nash in s_1	***Deletion***			
	DM1	(s_3, s_1)	s_1, s_3	$\nearrow$
	DM1	(s_4, s_2)	s_1, s_4	$+$
	DM2	(s_2, s_1)	s_1, s_2	$\nearrow$
(3) Nash in s_4	***Addition***			
	DM1	(s_4, s_1), (s_4, s_3)	$\varnothing$	$-$
	DM1	(s_4, s_1), (s_4, s_3)	$\varnothing$	$-$
	DM2	(s_4, s_1), (s_4, s_2)	$\varnothing$	$-$
	DM2	(s_4, s_1), (s_4, s_2)	$\varnothing$	$-$
	Deletion			
	DM1	(s_1, s_3)	s_1, s_4	$-$
	DM2	(s_3, s_4)	s_3, s_4	$-$
(4) Nash in $s_2 \vee s_3$	***Addition***			
	DM1	(s_2, s_1), (s_2, s_3)	$\varnothing$	$-$
	DM2	(s_3, s_1), (s_3, s_2)	$\varnothing$	$-$
	Deletion			
	DM1	(s_1, s_3)	s_1, s_3	$-$
	DM1	(s_3, s_1)	s_2, s_3	$\nearrow$
	DM1	(s_2, s_4)	s_2, s_3	$\nearrow$
	DM1	(s_4, s_2)	s_2, s_4	$+$
	DM2	(s_1, s_2)	s_1, s_2	$-$
	DM2	(s_3, s_4)	s_2, s_3	$\nearrow$
	DM2	(s_4, s_3)	s_3, s_4	$+$
(5) Two Nash	***Addition***			
in $(s_2 \wedge s_3) \vee (s_1 \wedge s_4)$	DM1	(s_2, s_1), (s_2, s_3)	s_3	$\searrow$
	DM1	(s_4, s_3), (s_4, s_1)	s_1	$-$
	DM2	(s_3, s_1), (s_3, s_2)	s_2	$\searrow$
	DM2	(s_4, s_2), (s_4, s_1)	s_1	$-$

- A_i^{esD} and A_i^{esA} denote the escalational reachability achieved by deleting or adding arcs in A_i, respectively.
- Reachable list:

$$R_i^{esD}(s) = \left\{ s' \in S | (s, s') \in A_i^{esD} \right\} \tag{5.2}$$

$$R_i^{esA}(s) = \left\{ s' \in S | (s, s') \in A_i^{esA} \right\} \tag{5.3}$$

In conflicts, the new reachability that causes de-escalation is defined as follows:

Definition 5.2.6 (De-escalation Reachability (**DESR**) in Interstate Conflicts [48], [49]).

- A_i^{desD} and A_i^{desA} denote the de-escalational reachability achieved by deleting or adding arcs in A_i, respectively.

- Reachable list:

$$R_i^{desD}(s) = \left\{s' \in S|(s,s') \in A_i^{desD}\right\} \tag{5.4}$$

$$R_i^{desA}(s) = \left\{s' \in S|(s,s') \in A_i^{desA}\right\} \tag{5.5}$$

In conflicts, the new reachability that causes divergence is defined as follows:

Definition 5.2.7 (Divergence Reachability (**DIVR**) in Interstate Conflicts [48])**.**

- A_i^{divD} and A_i^{divA} denote the divergence reachability achieved by deleting or adding arcs in A_i, respectively.

- Reachable list:

$$R_i^{divD}(s) = \left\{s' \in S|(s,s') \in A_i^{divD}\right\} \tag{5.6}$$

$$R_i^{divA}(s) = \left\{s' \in S|(s,s') \in A_i^{divA}\right\} \tag{5.7}$$

In conflicts, the new reachability that causes convergence is defined as follows:

Definition 5.2.8 (Convergence Reachability (**CONR**) in Interstate Conflicts [48])**.**

- A_i^{conD} and A_i^{conA} denote the convergence reachability achieved by deleting or adding arcs in A_i, respectively.

- Reachable list:

$$R_i^{conD}(s) = \left\{s' \in S|(s,s') \in A_i^{conD}\right\} \tag{5.8}$$

$$R_i^{conA}(s) = \left\{s' \in S|(s,s') \in A_i^{conA}\right\} \tag{5.9}$$

We apply the framework to the prisoner's dilemma we discussed in the previous section. Suppose DMs in a the prisoner's dilemma situation want to resolve it through reconciliation. In that case, either one of them should work on external factors that will grant them the power of transition from s_4 to s_1, or there should be an environmental change that allows DM1 to transit (4,3) or DM2 to transit (4,2). Such a situation can be described as follows:

- DESR of the prisoner's dilemma
  $R_1^{desA}(s) = \{(s_4, s_1), (s_4, s_3)\}$
  $R_2^{desA}(s) = \{(s_4, s_1), (s_4, s_2)\}$

Definition 5.2.9 (Favorable New Reachability [48])**.** The new reachablity by **ESR, DESR, DIVR**, and **CONR**, which result in favorable outcomes for DM j, are denoted $R_i^{esjD}(s)$.

For example, $R_1^{es2D}(s)$ represents DM1's new ESR by deletion of arcs that is favorable for DM2.

5.3 Application to 2 × 2 Interstate Conflicts

This section applies the new framework to the Cuban Missile Crisis and discuss the interpretation. Based on the fundamental understanding discussed in subsection 3.5.1 , we analyze the conflict in a simple 2×2 structure.

We examined the explicability of external factors impact that may have occurred in the process with three possible preference scenarios and applied the DESR, ESR, and CONR defined in the previous subsection.

Four possible states for the Cuban Missile Crisis in its simplest structure can be defined as follows:

- s_1 : Both the United States and the Soviet Union make concessions and stop the conflict.
- s_2: The Soviet Union makes no concessions, and the United States stops attacking.
- s_3: The United States makes no concessions, and the Soviet Union makes concessions.
- s_4: Neither nation makes any concessions and they escalate to a full-scale conflict.

As for the United States and the Soviet Union's preferences, this conflict is often modeled and discussed as a chicken game. However, herein, based on historical records, we assumed that both DMs had deterrent preferences, hoping to avoid an irreversible worst-case scenario while simultaneously avoiding a situation in which only their side would make concessions [3]. Thus, assuming that the preferences of the two DMs were one of the following three sets: Chicken$\times$Chicken (b $\times$ b), Deterrer$\times$Chicken (c $\times$ b), or Deterrer$\times$Deterrer (c $\times$ c), we scrutinized the impact of external factors during this period. In fact, the situation could have changed among these sets during the thirteen days of negotiations. In the graphs, the solid arcs indicate the newly acquired reachability and the dotted arcs indicate the original reachability. Furthermore, where the original arc is missing, it indicates a deleted reachability.

5.3.1 Chicken $\times$ Chicken (b $\times$ b)

In the chicken game, Nash holds in s_2 and s_3. When we examined the change in reachability of this conflict due to external factors, we found that Nash only held in s_2 or s_3 for the four cases. This was interpreted as a conflict convergence to a single stable state as a result of some external factor softening the attitude of one of the DMs. Figure 5.3 is a graph model of the Cuban Missile Crisis for Chicken$\times$Chicken set , where the dotted lines indicate the arcs in the base case, and the solid lines indicate the newly acquired arcs due to external factors. Table 5.5 shows the stability analysis of the base case. In either DM1 or DM2, convergence can occur due to the addition of reachability.

In chicken game-type conflicts, CONR is established by the action of external factors on DM1 or DM2.

1. Conflict Structure: $(N, S, (A_i)_{i\in N}, (\succsim_i)_{i\in N}), N = \{1, 2\}, S = \{s_1, s_2, s_3, s_4\}$, $A_1 = \{(s_1, s_3), (s_3, s_1), (s_2, s_4), (s_4, s_2)\}$, $A_2 = \{(s_1, s_2), (s_2, s_1), (s_3, s_4), (s_4, s_3)\}$, DM_1's preference order $\succsim_1$: $s_3 \succ s_1 \succ s_2 \succ s_4$, DM_2's preference order $\succsim_2$: $s_2 \succ s_1 \succ s_3 \succ s_4$.
2. Activation of CONR: $R_1^{conA}(s) = \{(s_2, s_1), (s_2, s_3)\}$, $R_2^{conA}(s) = \{(s_3, s_1), (s_3, s_2)\}$.

Table 5.6 summarizes the stability when additional conversion reachability:$R_1^{conA}(s)$ or $R_2^{conA}(s)$ is achieved.

TABLE 5.5: Stability analysis-Cuban Missile Crisis (Chicken × Chicken-Base Case)

		s_1	s_2	s_3	s_4
Nash	US		✓	✓	
	USSR		✓	✓	
	Eq		✓	✓	
GMR	US	✓	✓	✓	
	USSR	✓	✓	✓	
	Eq	✓	✓	✓	
SMR	US	✓	✓	✓	
	USSR	✓	✓	✓	
	Eq	✓	✓	✓	
SEQ	US		✓	✓	
	USSR		✓	✓	
	Eq		✓	✓	
Pareto		✓	✓	✓	

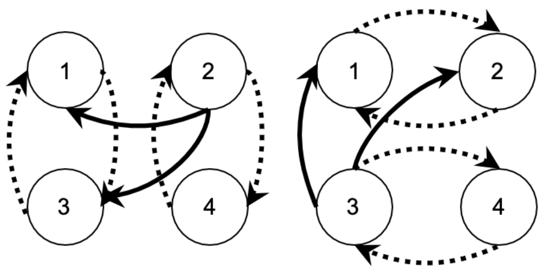

FIGURE 5.3: Graph Model of Cuban Missile Crisis - Chicken × Chicken

This conflict can be said to represent the situation just before the Cuban Missile Crisis broke out when the two nations fought for supremacy. By granting reachability (s_2, s_1) and (s_2, s_3) to DM1 and (s_3, s_1) and (s_3, s_2) to DM2, it could be interpreted that the respective allies wished to move the conflict forward by stabilizing with one Nash in s_3 or s_2, which was favorable for each of them.

5.3.2 Deterrer × Chicken (c × b)

Consider the case where a DM in either the U.S. or the Soviet Union is assumed to have a slightly more conciliatory but deterrent preference. In this combination of preferences, no Nash held. Figure 5.4 and Table 5.7 show the graph model of the Cuban Missile Crisis for the Deterrent ×Chicken set and the corresponding stability analysis. In this type of conflict, by removing the reachability of DM2, we can see that both escalation and de-escalation were possible and that the external factors' influence had considerable importance for the outcome.

In Deterrer × Chicken type conflicts, ESR and DESR are possible by the action of external factors on DM1 or DM2.

1. Conflict Structure: $(N, S, (A_i)_{i \in N}, (\succsim_i)_{i \in N}), N = \{1, 2\}, S = \{s_1, s_2, s_3, s_4\}$,
   $A_1 = \{(s_1, s_3), (s_3, s_1), (s_2, s_4), (s_4, s_2)\}$, $A_2 = \{(s_1, s_2), (s_2, s_1), (s_3, s_4), (s_4, s_3)\}$,
   DM_1's preference order $\succsim_1$: $s_1 \succ s_3 \succ s_4 \succ s_2$,
   DM_2's preference order $\succsim_2$: $s_2 \succ s_1 \succ s_3 \succ s_4$.

TABLE 5.6: Stability analysis-Cuban Missile Crisis (Chicken $\times$ Chicken-$R^{conA}(s)$)

		$R_1^{conA}(s)$								$R_2^{conA}(s)$							
		Addition (s_2, s_1)				Addition (s_2, s_3)				Addition (s_3, s_1)				Addition (s_3, s_2)			
		s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4
Nash	US			✓				✓			✓	✓			✓	✓	
	USSR		✓	✓			✓	✓			✓				✓		
	Eq			✓				✓			✓				✓		
GMR	US	✓	✓	✓		✓	✓	✓		✓	✓	✓		✓	✓	✓	
	USSR	✓	✓	✓		✓	✓	✓		✓	✓	✓		✓	✓	✓	
	Eq	✓	✓	✓		✓	✓	✓		✓	✓	✓		✓	✓	✓	
SMR	US			✓				✓		✓	✓	✓		✓	✓	✓	
	USSR	✓	✓	✓		✓	✓	✓			✓				✓		
	Eq			✓				✓			✓				✓		
SEQ	US		✓	✓				✓		✓	✓	✓		✓	✓	✓	
	USSR	✓	✓	✓		✓	✓	✓			✓	✓			✓		
	Eq		✓	✓				✓			✓	✓			✓		
Pareto		✓	✓	✓		✓	✓	✓		✓	✓	✓		✓	✓	✓	

2. Activation of ESR and DESR: $R_1^{esD}(s) = \{(s_3, s_1)\}$, $R_1^{esD}(s) = \{(s_2, s_4)\}$, $R_2^{desD}(s) = \{(s_1, s_2)\}$, $R_2^{esD}(s) = \{(s_4, s_3)\}$

In this conflict, where Nash was not established, deleting DM1's arc (s_3, s_1) resulted in a new Nash in s_3, which was favorable to DM1, and deleting (s_2, s_4) resulted in a Nash in s_2, which was favorable to DM2. Each represented an effort by the DM1 or DM2 allies. The deletion of (s_3, s_1), for example, could be interpreted as the result of moves of DM1's allies who preferred for Nash to continue in s_3 and obstructed ongoing third-party arbitration or settlement negotiation. As for the effect on DM 2's reachability, the deletion of (s_1, s_2) made s_1 Nash, which could be attributed to the arbitral factor. Further, the deletion of (s_4, s_3) made s_4 Nash, which could be attributed to the possible work of a third party who benefitted from the escalation of the conflict.

Table 5.8 summarizes the changes in stability due to the influence of the external factors.

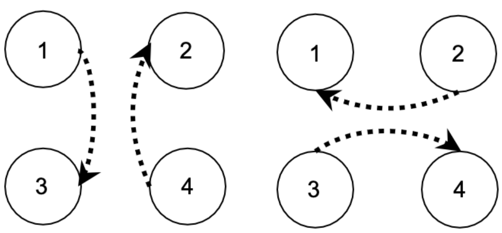

FIGURE 5.4: Graph Model of Cuban Missile Crisis - Deterrer $\times$ Chicken

5.3.3 Deterrer $\times$ Deterrer (c $\times$ c)

1. Conflict Structure: $(N, S, (A_i)_{i \in N}, (\succsim_i)_{i \in N}), N = \{1, 2\}, S = \{s_1, s_2, s_3, s_4\}$, $A_1 = \{(s_1, s_3), (s_3, s_1), (s_2, s_4), (s_4, s_2)\}$, $A_2 = \{(s_1, s_2), (s_2, s_1), (s_3, s_4), (s_4, s_3)\}$,

TABLE 5.7: Stability analysis-Cuban Missile Crisis (Deterrer × Chicken-Base Case)

		s_1	s_2	s_3	s_4
Nash	US	✓			✓
	USSR		✓	✓	
	Eq				
GMR	US	✓		✓	✓
	USSR	✓	✓	✓	
	Eq	✓		✓	
SMR	US	✓		✓	✓
	USSR	✓	✓	✓	
	Eq	✓		✓	
SEQ	US	✓		✓	✓
	USSR	✓	✓	✓	
	Eq	✓		✓	
Pareto		✓	✓		

DM_1's preference order $\succsim_1$: $s_1 \succ s_3 \succ s_4 \succ s_2$,
DM_2's preference order $\succsim_2$: $s_1 \succ s_2 \succ s_4 \succ s_3$.

2. Activation of DESR: In Deterrer × Deterrer type conflicts, DESR is possible by the action of external factors on DM1 or DM2. $R_1^{desA}(s) = \{(s_4, s_1), (s_4, s_3)\}$ $R_2^{desA}(s) = \{(s_4, s_1), (s_4, s_2)\}$

Table 5.9 shows the result of the stability analysis of the deterrent game.

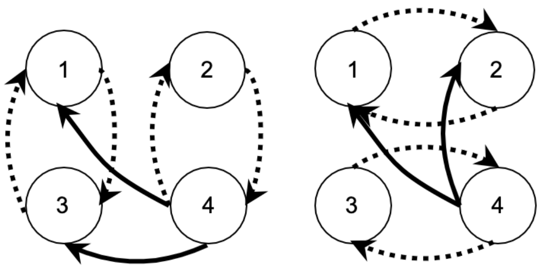

FIGURE 5.5: Graph Model of Cuban Crisis - Deterrer × Deterrer

As the results show, even though both sides had a preference for s_1 as the highest priority, Nash held not only in s_1 but also in s_4, leaving them with the risk of the worst-case scenario. The following changes are possible by considering the impact of external factors.

In the deterrent game, Nash in s_4 disappeared if one of the DMs acted on an external factor that gave it the power of transition (s_4, s_1), or if an environmental change occurred that gave DM1 the power to move (s_4, s_3) or DM2 the power to move (s_4, s_2). Figure 5.5 shows the graph in the case that both sides have the deterrent preference.

This type of conflict, in which both sides have deterrent preferences, is probably the most appropriate representation of the state of conflict between the United States and the Soviet Union in the Cuban Missile Crisis. We can consider that the impact of external factors on DM1 (s_4, s_1), (s_4, s_3), and DM2 (s_4, s_1), (s_4, s_2), respectively, showed the movement of allies and arbitrators, and finally, the conflict has settled at (s_4, s_1). Table 5.10 shows the results of stability analysis considering the influence of external factors on Deterrer × Deterrer structure.

TABLE 5.8: Stability analysis-Cuban Missile Crisis (Deterrer × Chicken-$R^{desD}(s)$,$R^{esD}(s)$)

		$R_1^{esD}(s)$								$R_2^{desD}(s)$				$R_2^{esD}(s)$			
		Deletion (s_3, s_1)				Deletion (s_2, s_4)				Deletion (s_1, s_2)				Deletion (s_4, s_3)			
		s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4
Nash	US	✓		✓	✓	✓	✓		✓	✓			✓	✓			✓
	USSR		✓	✓			✓	✓		✓	✓	✓			✓	✓	
	Eq			✓			✓			✓							
GMR	US	✓		✓	✓	✓	✓	✓	✓	✓			✓	✓		✓	✓
	USSR	✓	✓	✓			✓	✓		✓	✓	✓		✓	✓	✓	
	Eq	✓		✓			✓	✓		✓				✓		✓	
SMR	US	✓		✓	✓	✓	✓	✓	✓	✓			✓	✓		✓	✓
	USSR	✓	✓	✓			✓	✓		✓	✓	✓		✓	✓	✓	
	Eq	✓		✓			✓	✓		✓				✓		✓	
SEQ	US	✓		✓	✓	✓	✓	✓	✓	✓			✓	✓		✓	✓
	USSR	✓	✓	✓			✓	✓		✓	✓	✓		✓	✓	✓	
	Eq	✓		✓			✓	✓		✓				✓		✓	
Pareto		✓	✓			✓	✓			✓	✓			✓	✓		

It is known that during the period between the Soviet Union's deployment of missiles and the resolution of the conflict, there were a number of moves, including intervention by the United Nations and under-the-table contacts between the two countries [51] [67] [85]. It is believed that the parties' final decision was made under the influence of the consensus of allied nations outside the conflict and international public opinion.

The framework with new reachability due to external factors has been applied to conflicts with a simple 2×2 structure, but the application results are also presented for the Cuban Missile Crisis (ten states) and the Elmira conflict (nite states) in the following subsections.

5.4 Application to Larger Conflicts

5.4.1 Cuban Missile Crisis - Ten States

In the Cuban Missile Crisis, s_8, s_9, and s_{10}, i.e., when there is a severe threat that the Soviet Union will launch further attacks and escalate the conflict, consider operations to restore the situation under the influence of external arbitral factors. In Figure 5.6, the solid line shows the original arcs for the United States and the dotted line for the Soviet Union. Regarding the impact of external arbitral factors, the red lines represent the new arcs acquired by the United States, and the blue lines represent the new arcs acquired by the Soviet Union. Each arc depicts a situation in which the DM was given the new reachability to return to s_1. The impact of the external arbitration factors on the reachability in the conflict can be described to add the three arcs: $A_{US}^{des} = \{(s_8, s_1), (s_9, s_1), (s_{10}, s_1\})$, and $A_{USSR}^{des} = \{(s_8, s_1), (s_9, s_1), (s_{10}, s_1\})$, respectively.

The stability analysis conducted based on the new reachability obtained due to the external factors is shown in Table 5.11. The checkmark represents that the equilibrium is established in both the original and the new analysis, D indicates that the equilibrium disappears in the new analysis under the influence on the United

TABLE 5.9: Stability analysis-Cuban crisis
(Deterrer × Deterrer-Base Case)

		s_1	s_2	s_3	s_4
Nash	US	✓			✓
	USSR	✓			✓
	Eq	✓			✓
GMR	US	✓		✓	✓
	USSR	✓	✓		✓
	Eq	✓			✓
SMR	US	✓		✓	✓
	USSR	✓	✓		✓
	Eq	✓			✓
SEQ	US	✓			✓
	USSR	✓			✓
	Eq	✓			✓
Pareto		✓			

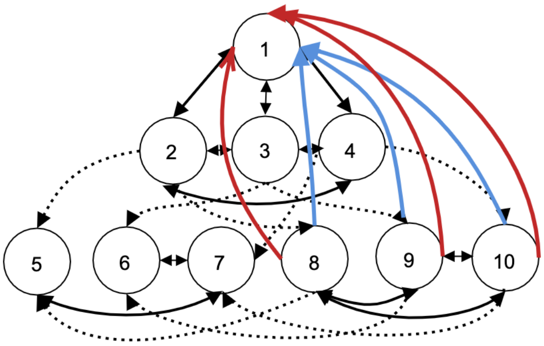

FIGURE 5.6: Graph Model of Cuban Missile Crisis
-Influenced by External Factors

States, and DE indicates an equilibrium re-appears under the influence only on the Soviet Union, respectively. The strong equilibrium involving Nash at s_9 is no longer established as intended in these operations, and we can expect a de-escalating intervention effect.

5.4.2 Elmira Conflict - Nine States

Next, we examined the situation under which external factors affect DMs' state transitions for the Elmira conflict. The Elmira conflict has a structure in which Uniroyal, unwilling to accept a control order that is unfavorable to itself, continues to negotiate insinuatingly, threatening the abandonment of the plant. However, if Uniroyal abandoned the factory, its corporate attitude would be questioned, and its reputation would be damaged locally and widely. If it were a global company, the bad publicity would be reported all over the world instantaneously, and the stock price would probably plummet. Here, we assume a situation in which stakeholders who are not directly involved in the negotiations, such as Uniroyal's headquarters and shareholders, want the plant not to be abandoned. Suppose such external factors can change Uniroyal's transition between s_1 and s_9 reversible. Figure 5.7 shows the graph of Elmira conflict with external factors; the green arc represents the new reachability which **U** was given reversibility of this transition.

TABLE 5.10: Stability analysis-Cuban Missile Crisis
(Deterrer × Deterrer-R^{desA})

		$R_1^{desA}(s)$								$R_2^{desA}(s)$							
		Addition (s_4, s_1)				Addition (s_4, s_3)				Addition (s_4, s_1)				Addition (s_4, s_2)			
		s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4
Nash	US	✓				✓				✓			✓	✓			✓
	USSR	✓			✓	✓			✓	✓				✓			
	Eq	✓				✓				✓				✓			
GMR	US	✓		✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	USSR	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓		✓
	Eq	✓		✓	✓	✓		✓	✓	✓	✓		✓	✓	✓		✓
SMR	US	✓				✓				✓	✓	✓	✓	✓		✓	✓
	USSR	✓	✓		✓	✓	✓		✓	✓				✓			
	Eq	✓				✓				✓				✓			
SEQ	US	✓				✓			✓	✓			✓	✓	✓		✓
	USSR	✓			✓	✓		✓	✓	✓				✓			✓
	Eq	✓				✓			✓	✓				✓			✓
Pareto		✓				✓				✓				✓			

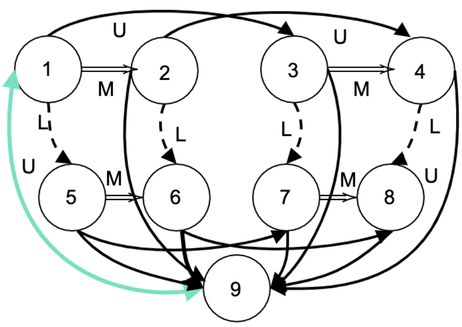

FIGURE 5.7: Elmira Conflict
- Influenced by External Factors

The results of the stability analysis based on the new reachability are shown in Table 5.12. The checkmark represents that the equilibrium is established in both the original and the new analysis, D indicates that the equilibrium disappears in the new analysis.

5.4.3 Application to State Transition Time Analysis

The novel approach can be applied in GMCR state transition time analysis; external factors affect not only reachability but also transition time when the elements can influence DMs to speed up or slow down its transition time. This section examines the circumstances under which external factors affect state transition times and applies the new concept to the Elmira conflict for analysis.

We define the following extended concepts 5.4.1, 5.4.2 based on the overview of *State transition time analysis* [39] and provided Definitions 2.3.14. The lists are prepared, assuming that external factors grant the power to DM i to transition "faster" or "slower." Subscript f and l denote the identifying information that external factors affect DM i's transition speed faster or slower.

TABLE 5.11: Cuban Missile Crisis with New Reachability - Stability Analysis

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
Nash	✓				✓	✓			D	
GMR	✓				✓	✓			DE	
SMR	✓				✓	✓			D	
SEQ	✓				✓	✓			D	
CNash	✓					✓			D	
CGMR	✓					✓			D	
CSMR	✓					✓			D	
CSEQ	✓					✓			D	
Pareto	✓					✓				

TABLE 5.12: Elmira Conflict with External Factors - Stability Analysis

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
Nash					✓			✓	D
GMR	✓			✓	✓			✓	✓
SMR	✓			✓	✓			✓	D
SEQ					✓			✓	D
CNash								D	D
CGMR	✓			✓				✓	D
CSMR	✓			D				D	D
CSEQ								D	D
Pareto	✓		✓	✓	✓		✓	✓	

DM i's time reachable list from $s \in S$ with faster movability under the influence of external factors are as follows:

Definition 5.4.1 (Time Reachable List with Faster Movability)**.**

i. DM i's time reachable list with faster movanility under the influence of external factors: For $i \in N, s \in S$, and $T \subset N \backslash \{i\}$, from s to s' against the move T , where DM i is given faster movability by external factors:

$$tR^f_{i,T}(s) = \{s' \in R_i(s) \mid \forall j \in T, \forall s'' \in R_j(s), tt_j(s,s'') \geq tt_i(s,s')\}. \tag{5.10}$$

ii. DM i's reachable list under the influence of external factors from s to s' aginst the credible move by T, where DM i is given faster movability by external factors:

$$tcR^f_{i,T}(s) = \{s' \in R_i(s) \mid \forall j \in T, \forall s'' \in R^+_j(s), tt_j(s,s'') \geq tt_i(s,s')\}. \tag{5.11}$$

iii. DM i's time improvement list under the influence of external factors from s to s' against the moves by T, where DM i is given faster movability by external factors:

$$tR^{f+}_{i,T}(s) = tR^{f}_{i,T}(s) \cap R^{+}_{i}(s). \tag{5.12}$$

iv. DM i's time improvement list under the influence of external factors from s to s' against the credible moves by T, where DM i is given slower movability by external factors :

$$tcR^{f+}_{i,T}(s) = tcR^{f}_{i,T}(s) \cap R^{+}_{i}(s). \tag{5.13}$$

DM i's time reachable list with slower movability under the influence of external factors from $s \in S$ are as follows:

Definition 5.4.2 (Time Reachable List with Slower Movability).

i. DM i's time reachable list with slower movability under the influence of external factors: For $i \in N, s \in S$, and $T \subset N \backslash \{i\}$, from s to s' against the move T , where DM i is given slower movability by external factors:

$$tR^{l}_{i,T}(s) = \{s' \in R_i(s) \mid \forall j \in T, \forall s'' \in R_j(s), tt_j(s,s'') \leq tt_i(s,s')\}. \tag{5.14}$$

ii. DM i's reachable list under the influence of external factors from s to s' against the credible move by T, where DM i is given slower movability by external factors:

$$tcR^{l}_{i,T}(s) = \{s' \in R_i(s) \mid \forall j \in T, \forall s'' \in R^{+}_j(s), tt_j(s,s'') \leq tt_i(s,s')\}. \tag{5.15}$$

iii. DM i's time improvement list under the influence of external factors from s to s' against the moves by T, where DM i is given slower movability by external factors:

$$tR^{l+}_{i,T}(s) = tR^{l}_{i,T}(s) \cap R^{+}_{i}(s). \tag{5.16}$$

iv. DM i's time improvement list under the influence of external factors from s to s' against the credible moves by T, where DM i is given slower movability by external factors :

$$tcR^{l+}_{i,T}(s) = tcR^{l}_{i,T}(s) \cap R^{+}_{i}(s). \tag{5.17}$$

We examine Elmira conflict again. The contamination victims are the residents, and the entire local economy may suffer significant damage if Uniroyal abandons and retires the plant. Therefore, we assume the situation where the local government could accelerate its actions through the strong encouragement of residents, the local business community, and other stakeholders and was able to take quicker action than Uniroyal (**U**) and the Ministry of the Environment (**M**). More specifically, we envision a situation where stakeholders can lobby their legislators, who represent their interests, against the decisions of the local government (**L**) so that **L** can make and enforce decisions on this environmental issue more quickly. When portraying such a situation, it would be very complicated to analyze all the stakeholders who influence the policies of the state government, such as residents, the local business community, and the power relations in the legislature, by considering them as DMs. It would be unnecessary to position them as DMs if we only want to portray their impacts. On the other hand, it is also true that **L**, unlike **U**, a private company, has such insider stakeholders in its internal structure of decision making. When depicting such a situation, this method using the concept of "external factors" is suitable because it focuses only on the influence of external factors on primary DMs' transitions.

Figure 5.8 depicts the graph of Elmira conflict where **L** moves faster than **M** and **U**. A Stability analysis using the newly defined concepts is presented in Table 5.13 [1].

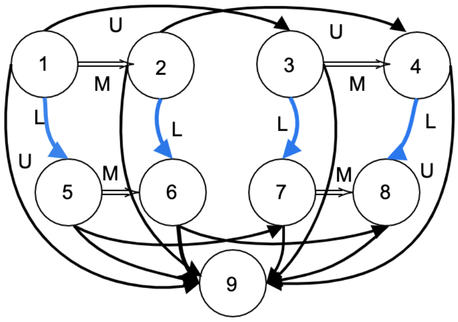

FIGURE 5.8: Elmira Conflict
-Transition Time Influenced by External Factors

As for the stability analysis results when there is a change in **L**'s transition time, we should pay attention to the changes that occurred in the Pareto states where any equilibrium was not established in the original GMCR analysis: s_3 and s_7. s_3 is the state when **U** accepts the initial control order without any modification by **M** or insistence by **L**. s_7 is the state when **U** accepts the initial control order without any modification by **M** but with insistence by **L**. In both cases, the initial control order is accepted by **U** without modification; it can be interpreted that **L**'s faster move could successfully block **U**'s lobbying or negotiation with **M** to modify the order.

5.5 Nash Stability for New Reachablity by External Causes

5.5.1 Proposition

With respect to deletion and addition of state transitions by unspecified external causes, the following proposition holds about the consequent stability change:

Proposition 5.5.1. If state s is Nash stable for DM i before deletion, then state s is Nash stable for DM i after deletion. If state s is Nash stable for DM i after addition, then state s is Nash stable for DM i before addition.

Proof. Let $R_i(s)$ and $\underline{R}_i(s)$ be the reachable lists from s for DM i before and after deletion, respectively. When $R_i^+(s) = \{s' \in R_i(s) \mid s' \succ_i s\} = \emptyset$, then $\underline{R}_i^+(s) = \{s' \in \underline{R}_i(s) \mid s' \succ_i s\} = \emptyset$, because $R_i(s) \subseteq \underline{R}_i(s)$.

Let $R_i(s)$ and $\overline{R}_i(s)$ be the reachable lists from s for DM i before and after addition, respectively. When $\overline{R}_i^+(s) = \{s' \in \overline{R}_i(s) \mid s' \succ_i s\} = \emptyset$, then $R_i^+(s) = \{s' \in R_i(s) \mid s' \succ_i s\} = \emptyset$, because $\overline{R}_i(s) \subseteq R_i(s)$. □

5.5.2 Verification of Proposition in Application Cases

The change in Nash equilibrium due to deletion and addition of reachability can be easily verified in Table 5.4: Taxonomy of New Reachability-Interstate Conflict.

[1]The calculation results were obtained by using Inooka's python program [43]. Supplementary information is given in B.2

TABLE 5.13: Elmira Conflict with External Factors
t-GMCR Stability Analysis

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
Nash					✓			✓	✓
tNash					✓	✓	✓	✓	✓
tNash-c					✓			✓	✓
GMR	✓			✓	✓			✓	✓
tGMR	✓			✓	✓	✓	✓	✓	✓
tcGMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
tGMR-c					✓			✓	✓
tcGMR-c	✓	✓	✓	✓	✓			✓	✓
SMR	✓			✓	✓			✓	✓
tSMR					✓	✓	✓	✓	✓
tcSMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
tSMR-c					✓			✓	✓
tcSMR-c	✓	✓	✓	✓	✓			✓	✓
SEQ					✓			✓	✓
tSEQ					✓	✓	✓	✓	✓
tcSEQ		✓	✓		✓	✓	✓	✓	✓
tSEQ-c					✓			✓	✓
tcSEQ-c		✓	✓		✓			✓	✓
Pareto	✓		✓	✓	✓		✓	✓	

5.6 Conclusion of the Chapter

In this chapter, we proposed a new framework for describing the impact of external factors outside the scope of conflicts on DMs' reachability and conflict stability that are not handled by current standard analytical methods. This new framework will make it possible to analyze conflicts involving local governments, which have essential authority but for which it is difficult to obtain information on their detailed preferences or to describe conflicts in which factors are not a person or group, such as climate change or oil prices, but the effects of these factors affect the state transitions of the conflicts' primary DMs.

In the future, we would like to extend the definitions developed here to general conflicts without limitation of preference setting, to make it a more versatile framework, and further study the stability when some DMs are affected by external influences while others are not.

Chapter 6

Conclusion

In this study, we have proposed three new methods for conflict analysis that can be applied when only coarse information is available. In Chapter 3, we proposed a new GMCR framework that incorporates the concept of the permissible range; in Chapter 4, we proposed a method for the case where preferences are known only for options that can lead to the worst possible outcome. In Chapter 5, we proposed a method for the case where the DM may be influenced by factors other than the DM concerning state transitions, and the elements other than the DM cannot or need not be specified. All three new methods employ the GMCR framework for analysis. Therefore, the same structure can be used for fine and coarse information, both within and among the methods.

In Chapters 3, 4, and 5, we have presented the stability analysis results by the three methods, applying them to the Cuban Missile Crisis and the Elmira environmental dispute, respectively. In terms of the relationship between the three frameworks, GMCR-PR and GMCR-DA, as discussed in Subsection 4.3.1, GMCR-DA uses options and is therefore generally more suitable for analyses with only coarse information than GMCR-PR, which requires preference information. For situations where states are formulated by factors other than DM's options, using the *New reachability* framework introduced in Chapter 5 for the conflict description is convenient.

This chapter provides a general discussion of this research by outlining the relationship between the axis of "fineness/coarseness of information" and the three new methods, and the overall analytical picture consisting of combinations of fineness of information and methods. The three new methods were not developed from the outset to be interrelated but rather to extend the theory in a derivative of the analysis using the GMCR as the basic framework. Hence, it is essential to show the relationship between the methods again as a conclusion in this paper.

6.1 The New Concepts in the Overall Structure

In this section, we review the relationship between the four elements of GMCR (DM, states, DM i's graph, DM i's preferences) and the three new concepts.

Figure 6.1 shows the new concept extended in this study within the basic structure of GMCR: each of states S, state transitions A_i, and DM i's preferences $\succsim_i$ is subdivided from (i) to (vi) for the different classifications that had been used for analysis.

The relationship between the general method of analysis and the new concepts proposed in this paper can be captured as follows:

- *N* In this study, elements other than DM are interpreted in new frameworks, but the definitions and methods of analysis for DM remain the same as in general analysis.

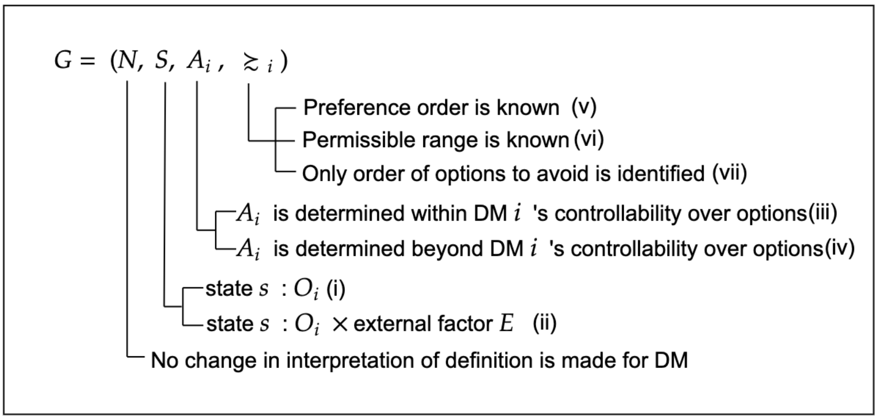

FIGURE 6.1: Theoretical Structure

S A state can be considered in two ways: O_i, as a set of option selection by DM i (i), as in Definition 2.1.12, or as a state generated by taking into account other factors E in addition to O_i, the set of option selection by DM i (ii). It is the same as the basic method of analysis that only the feasible case is adopted in the analysis for each of them.

Remark. The composition of the state S has already been discussed in Subsubsection 5.1.2.

A_i For DM i's graph A_i, there are two possible cases according to the DMi's controllability over the state transition : (iii) DM i's transitions are possible based on the concept that state S is a set of option selection,i.e.(i), and (iv) DM i's transitions are possible as in the definition $A_i \subseteq S \times S$, without any constraints.

Remark. The controllability has already been discussed in Subsubsection 5.1.2.

$\succsim_i$ For preferences, we assumed three cases: (v) the general analysis method, where the preference order for all states is known, (vi) the permissible range for preferences is known, and (vii) the ranking for the options to be avoided is known.

The GMCR-PR proposed in Chapter 3 was a framework for (vi), and the GMCR-DA in Chapter 4 was a framework for (vii). The new reachability proposed in Chapter 5 was a framework for the case (iv). For (vii), it is necessary to assume (i) since the state S must consist of option selection. Other than that, there are no restrictions, and any combination among these conceptual methods can be considered.

6.2 Fine and Coarse Information

6.2.1 GMCR Incorporating Permissible Range

In GMCR-PR, fine and coarse information can be assumed and analyzed for preference information. The relationship between the coarse preference information described by the information partitioning with PR setting and the fine preference information in linear order is clarified in Subsections 3.2.1 and Example 3.2.1.

In Figure 6.2, the upper figure A-B-C is the same as in Figure 3.1, and the lower figure C′-B′-A′ shows the coarsening and refinement of information in the opposite direction to the upper figure. In the lower figure, the initial preference information in C′, $s_1 \succ s_2 \succ s_3 \sim s_4$ is changed by coarsening to B′, P_a^2, $\mathcal{P}_a^2 = \{\{s_1, s_2\}, \{s_3, s_4\}\}$, $\overline{P}_a^2 = \{s_1, s_2\}$. Then A′ shows that the finest information partition occurred by the new detailed preference information. In C′, s_3 and s_4 are classified as impermissible, which is *equivalent* in the framework of GMCR-PR. In A′, where more detailed information was available for the analysis, the preference order for s_3 and s_4 can be reversed as a consequence of the equivalence in C′ and B′.

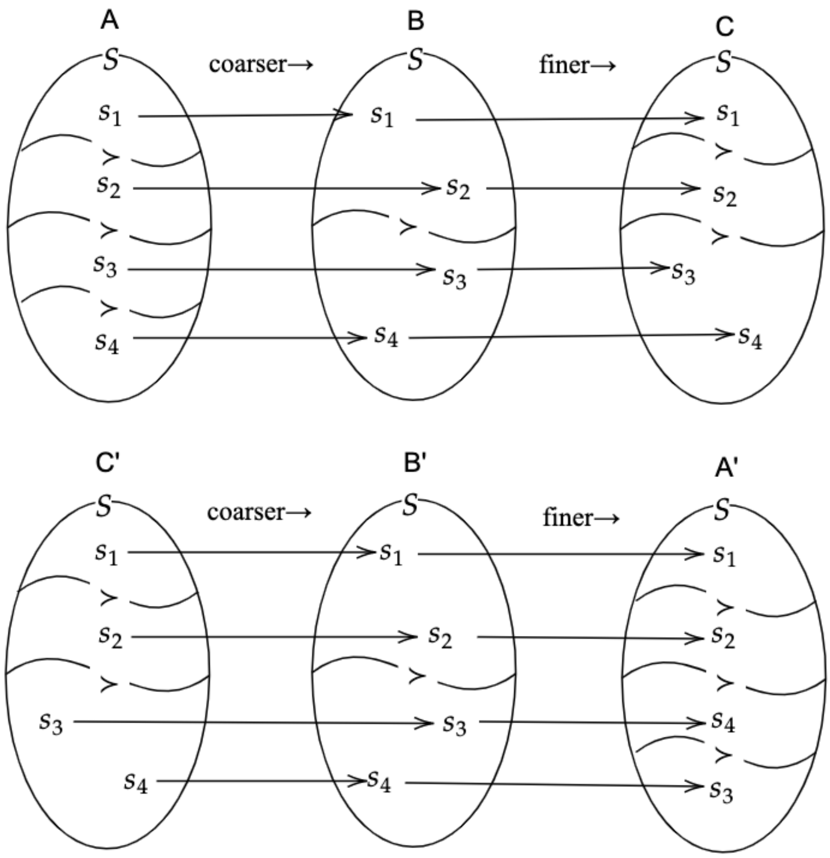

FIGURE 6.2: Coarsening and Refinement of Preference Information-2

Remark (Partition and Cells). In a set S, refinement of a non-repeating, non-empty subset s_k denotes a partition of the set S, and the subset s_k is a cell.

$$S = S_1 \cup S_2 \cup S_3 \cdots \cup S_n, \;\; S_k \cap S_l = \emptyset (k \neq l). \tag{6.1}$$

Remark (Equivalence Relation). If a relation R in GMCR-PR on a set S follows Definition 3.2.3, then this relation R can be said to be an *equivalence relation* because R is reflexive, symmetric, and transitive.

Remark (Equivalent Class and Quotient Set). Let the relation R of permissibility in GMCR-PR be an equivalence relation on the set S. (x, y) satisfies R can be denoted by $x \sim_R y$. The image $q(S)$ of the map $q : S \to P(S)$ that maps $x \in S$ to the *equivalence class* $\{y \in S \mid y \sim_R x\} \in P(S)$ of x is the *quotient set* S/R by R of S.

The equivalence relation R_q defined by the quotient map $q : S \to S/R$ is equivalent to the original equivalence relation R. If the equivalence relation R_q is equivalent to R, the graph of Rq, i.e., $C_q = \{(x, y) \in S \times S \mid q(x) = q(y)\}$ is equal to the graph of R.

If the relation R is an equivalence relation, it is reflexive, symmetric, and transitive; thus, the following lemmas for the general binary relation also hold for PR information.

Lemma 6.2.1. $\forall q \in P, q \in P_q$.

Lemma 6.2.2. $\forall q, x, y \in S, x, y \in P_q \Rightarrow xRy$.

Lemma 6.2.3. $\forall q, r \in S, qRr \Rightarrow Pq = Pr$.

Lemma 6.2.4. *S/R is a partition of S.*

6.2.2 Preference Order Setting for Disaster Aversion

In the GMCR-DA, the TOPSIS method was used to establish the most unfavorable state by ranking the options with the most negative impact. Since available options for DM are components of states but not subsets states; there is no binary relationship between states and options. Hence, unlike the relationship between states, the information partition in Definition 2.1.4 is not possible, and the coarsening and refinement of information shown in Definition 2.1.2 is not applicable.

Nevertheless, from the practical standpoint of the possibility of primary analysis with less information, the new method is sufficiently useful. As shown in Definition 2.1.12, when h options are available for a DM, a state is an h-dimensional column vector with the value of either selecting the option (1) or not selecting the option (0). Thus, the number of all states is $2^O = 2^h$. Since $2^h > h$, it is easier to formulate a ranking for options than it is to formulate a preference ranking for all states. In other words, if only insufficient information on the preferences of all states, which are more numerous is available, to formulate a preference ranking, it is sufficient to have information on the unfavorable order of the options, which are fewer in number. Hence analysis with "rougher" information is possible in GMCR-DA.

Remark (Permissible Range for Options). As a more straightforward way to analyze using options that are generally fewer than the states, it is possible to apply the concept of permissible range by treating the option order in the same way as the preference order in GMCR. For example, suppose there is insufficient information for the analysis. In that case, it is possible to transform the “Option ranking for disaster aversion” in Table 4.8 into the direct target of setting PR , provided in Table 6.1.

Let OP_i^n denote that DM i permits up to its n^{th} option, and for **M**, **U** and **L**, OP_M^n, OP_U^n and OP_U^n, respectively. The rankings are presented in an unfavorable order in Table 6.1; $OP_i^{n\backslash iv}$ represents that DM i allows up to $|O| - n^{th}$ less preferred option, where $|O|$ indicates the total number of options available to the DMs.

TABLE 6.1: Elmira Conflict - Option Ranking for Disaster Aversion-2

	o_1	o_2	o_3	o_4	o_5
	Modify	Delay	Accept	Abandon	Insist
M	2	3	4	1	5
U	4	3	5	2	1
L	3	2	5	1	4

Assume that the option PR for the three DMs are $OP_M^{2\backslash iv}$, $OP_U^{2\backslash iv}$, and $OP_L^{2\backslash iv}$. Figure 6.3 illustrates the option PR situation with the option PR information.

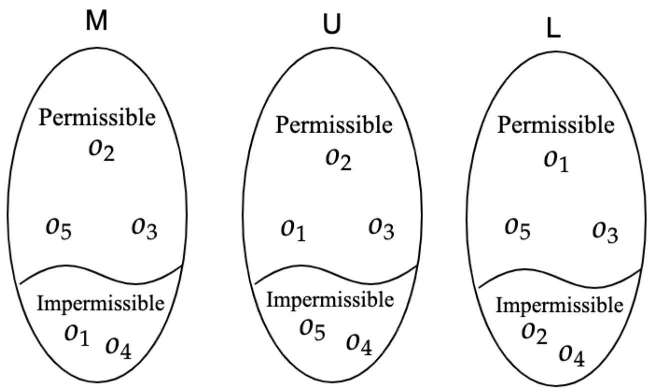

FIGURE 6.3: Elmira-Option PR: $OP_M^{2\backslash iv}$, $OP_U^{2\backslash iv}$, $OP_L^{2\backslash iv}$

If there is little information about the status of the dispute, and all we know is that **U** might withdraw without resolving the contamination, the coarsest criterion for this dispute would be: permit U's "abandon" or not. Likewise, in the Cuban Missile Crisis, the coarsest criteria for decision-making would be to permit or not / intend or not the escalation of the conflict by additional Soviet attacks.

6.2.3 New Reachability by Unspecified External Factors

By definition, DM i's set of oriented arcs contains the movements in one step controlled by DM i, whereas the controllability by DM i has been clarified in Definition 2.2.3 for the unilateral move in option form. The new reachability proposed in this research is a new concept that extends Definition 2.2.3 in two points: the constitutive elements of the state s and the controllability. Figure 6.4 is a comprehensive illustration of the coarseness and fineness of the information and the controllability of the movement by DM i, based on the prisoner's dilemma with external factors presented in Example 5.1.1.

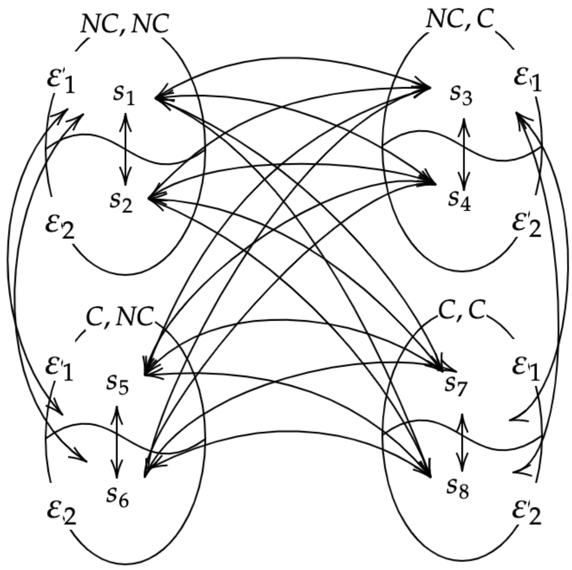

FIGURE 6.4: Prisoner's Dilemma - Extension

$$N = \{DM1, DM2\}, O_i = \{NC, C\}, E = \{\varepsilon_1, \varepsilon_2\}$$
$$s_1 = (NC, NC, \varepsilon_1), s_2 = (NC, NC, \varepsilon_2), s_3 = (NC, C, \varepsilon_1), s_4 = (NC, C, \varepsilon_2),$$
$$s_5 = (C, NC, \varepsilon_1), s_6 = (C, NC, \varepsilon_2), s_7 = (C, C, \varepsilon_1), s_8 = (C, C, \varepsilon_2).$$

In Figure 6.4, the original four states (NC, NC), (NC, C), (C, NC), and (C, C) generated by choice of strategy is used as a base. States 1 to 8 are obtained by considering the effects of external factor E such that effective: ε_1, and ineffective: ε_2, over the four original states. In other words, if the states are not affected by external factors and consist only of options that DM i can control, there are four states, s_2, s_4, s_6, and s_8, and if external factors are involved, there are eight states, s_1 to s_8.

Suppose that if one of the prisoners confesses, the interrogation would proceed quickly, and the case may be eligible for a full pardon. However, the prisoners shall not be informed about how many fewer days of interrogation they must complete being eligible for a pardon. Figure 6.5 summarizes the three situations: the original prisoner's dilemma, when there is a possibility of pardon, and when the timing of the completion of interrogation is an issue for the application of pardon.

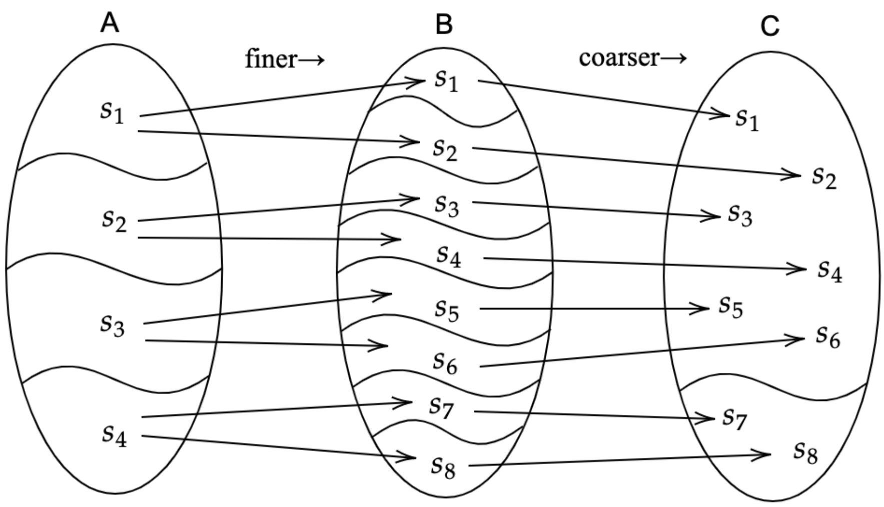

FIGURE 6.5: Prisoner's Dilemma - Information Partition

A shows the original four states, and B shows the eight states assuming the possibility of amnesty. C is the partition into states s_1 to s_6, where either prisoner could become eligible for a pardon by confessing earlier, and states s_7 and s_8, where neither prisoner confesses and would not be eligible even if a pardon were implemented. As long as the definitions of states and reachability are clear, any of these three situations can be analyzed for stability using the GMCR framework.

6.3 Novelty and Contribution

6.3.1 Overview

This study aimed to propose decision-making and conflict analysis methods that allow analysis with less information and as intuitive treatment of preferences and possible states as possible.

In GMCR, which was used as the basic framework in this study, the components of conflicts are defined simply, and the validity of the definition is not often examined, while numerous studies extend and elaborate the concept of preferences.

There is more research on the application than concepts and improvement of the framework; the improvement and extension of frameworks also tend to be more complex. Whereas, substantially no studies reexamine the nature and validity of the definitions: DM, states, and state transitions themselves.

In contrast to optimizing within a single system, it is essential to be able to make broad decisions with coarse information when making decisions in an ever-changing environment of interdependent relationships among individuals or nations. In *GMCR Incorporating Permissible Range* in Chapter 3, we introduced the concept of "permissible range" to show how to analyze with coarse information. We consider that when making a decision, the first thing that comes to mind in the absence of sufficient information might be, "What is the worst that could happen based on the information available at the time, and what is the acceptable range ?, and "What kind of states would be absolutely unacceptable?" Suppose, at least at that point, it is clear what is permissible (or impermissible). In that case, the information should be more reliable for analysis than linear order of preferences based on ambiguous survey results. In this study, we showed that the intuitively available evaluation of "permissible or impermissible" can be processed as binary information and operationalized within the GMCR stability analysis framework.

In *Preference Order Setting for Disaster Aversion* in Chapter 4, we proposed a technique to obtain a preference ranking farthest from the worst-case scenario by weighting the options that make up the scenario when the worst-case scenario is identified. By using this technique, decision analysis in a way that makes the most of intuition is possible.

In *New Reachability by Unspecified External Factors* in Chapter 5, we proposed a method for analyzing states in which factors not controllable by the DM are involved, focusing only on the effects received, even without detailed information about the external factors. In proposing this new method, the concept of state and the concept of state transition in GMCR were examined and restructured so that the novel concept, which has not been used ever before, can be applied in the framework of GMCR. For example, the influence of external factors can be applied to GMCR that considers the state transition time. It is now possible to describe a situation in which the speed becomes faster or slower due to the influence of external factors rather than the user's own intention.

6.4 Further Research

We will continue to deepen our research on the concepts and frameworks in two directions in the future: 1) Exploring and extending the newly proposed methodologies in GMCR framework, and 2) Scrutinizing and clarifying the interrelationship between the concepts of game theory and GMCR. 3)Verification of conflict resolution models from the perspective of modal logic and investigation of computational algorithms.

6.4.1 Exploring and Extending the New Methodologies in GMCR

Application and Validation of the New Concepts to Other Extended Framework in GMCR

In this study, only one case of application to other extended concepts in GMCR was taken up, the case of exercising the influence of external factors on state transition time [39]. Many other possible applications include external factors influencing

DMs' behavior through coalition [40] or preference changes [45].

Exploring the Possibility of Extending the Concept of DM

This study reexamined the four main elements of conflict: DM, states, state transitions of DM, and preferences, then established new concepts for states, state transitions, and preferences. Among the new concepts regarding the changes in state transitions due to the influence of external factors, we focused only on the changes that occurred as a consequence. In contrast, the agent, the source of the impact, was wholly excluded from the analysis. In the future, we would like to consider constructing a system that positions the agent within a comprehensive framework.

New Analytical Framework by Options as "Coarse Information"

In this study, we focused on the interpretation of the essential components of GMCR. We clarified that the state could be viewed in two ways: one is composed of options alone, and the other includes other elements. However, since the options were regarded as an element that constitutes the state, applying the new frameworks in this study to the options only suggested the application of PR in this chapter, as shown in Figure 6.3. We would like to study further the new conceptual framework of analysis that is possible by using "coarse information" at the level of decomposition into options and external factors.

Resolution of Conflict Analysis by Fineness and Coarseness of Information

We proposed a new perspective of coarseness and fineness of information in conflict analysis. From this perspective, we would like to examine the development of a new framework using the concept of "resolution:" high and low resolution with fine and coarse information. How does the aspect of stability analysis change when the "resolution" of information about a conflict gradually increases from coarse to fine, and which information and how much of it is clear enough to obtain valid decision-making results? The results and applicability will be the next theme.

6.4.2 Scrutinizing and Clarifying the Interrelationship between the Concepts of Game Theory and GMCR.

Although GMCR is a framework derived from non-cooperative game, it has not had much interaction with researches in game theory and is currently developing on its own. A more comprehensive, flexible, and logically robust framework may be explored by studying the relationship between the concepts proposed in GMCR and game theory, which has a broad spectrum of achievements in previous studies.

Relevance Assessment of Cooperative Game Concepts

This research is based on GMCR, which treats non-cooperative situations in an ordinal and discrete manner. For this reason, we do not adopt the concept of placing the aggregate utility in cardinal terms and considering its imputation. As mentioned in 3.7, when the preferences of DMs have become apparent at the end of the conflict, the transition from a non-cooperative situation to a cooperative one can be described by reconsidering the conflict or negotiation as a committee.

This approach is conditional on establishing a specific direction of agreement as a rule among DMs when the conflict is coming to convergence.

We would like to enrich the methodologies of conflict resolution in the future by referring to Greenberg [29] and Chwe [18] on forming coalitions in the shifting phase from non-cooperative to cooperative situations.

Decision-Makers' Behavioral Principle

Concerning the principle of DM's behavior, GMCR assumes credibility in the basic framework (2.2.2), and sanctionability of blocking by time-credible move in t-GMCR (2.3.3). As an extended concept, there is research on frameworks that incorporate DM's responses (devoting, aggressive, indifferent) that take into account *attitudes* toward self and others (positive, negative, neutral) [41] [42].

Furthermore, forming a *self-enforcing agreement* [87] that may elicit *joint deviations* [12] can also be considered one of the causes of the "convergences of DM's PR " in Chapter 3 in the convergent situation of conflicts. GMCR's analytical framework includes concepts of credibility and sanctionability for DM behavior, but these norms of behavior are not necessarily associated with self-enforcement. Research that places the concept of self-enforcement within the frameworks proposed in this study may be an issue in the future.

6.4.3 Conflict Resolution Models from the Perspective of Modal Logic

Generally, there is not much theoretical discussion about the definitions of "state" or "state transition" in conflict resolution studies; the mainstream research is on the extension of the framework. Eliminating redundant option statements in option prioritization, which we covered in 4.2.1 and Appendix A, is based on *rough set theory* [73]; it is close to the idea of coarse decision theory, our basic concept in this study.

Meanwhile, we believe that the conceptual aspects of essential elements of game theory and GMCR still have room to be revisited. In some cases, it is more natural to include elements other than DM's controllable option selection to describe a particular "state."

We would like to review the issue from the viewpoint of *Modal logic* as the next research project. State recognition and information partitioning discussed in this research may correspond to *frame problems* [24] in information processing, and coarse information analysis can be considered in contrast to the *zooming* [68] problem in *granularized possible worlds* [68]. It is physically difficult to collect, recognize and use all the information in the real world to make optimal decisions. Thus it is desirable to have a mechanism to make appropriate decisions based on partial information. It is worth noting that McCarthy and Hayes [64] raised this limit to the perception of infinite information in the real world as a *frame problem* in the early days of AI research; later, Dennett [19] redefined it as a *new frame problem* from epistemology.

For the future, we aim to implement a flexible, logically correct, and computable decision-making system that is highly applicable to the real world while also incorporating the aspects of modal logic. We have already started to study the mechanism of state generation by option selection based on *four-valued logic*, which is *paraconsistent*, instead of two-valued logic. Starting the analysis with the *paraconsistent* view towards the status quo can be a framework that encompasses the contradictions that may be caused by additional information flowing into the framework. We believe it is a more conservative valid method of describing the real world.

Appendix A

Supplementary Figures

1. The algorithm of option statement reduction method mentioned in subsection 4.2.1 can be illustrated as follows [99].

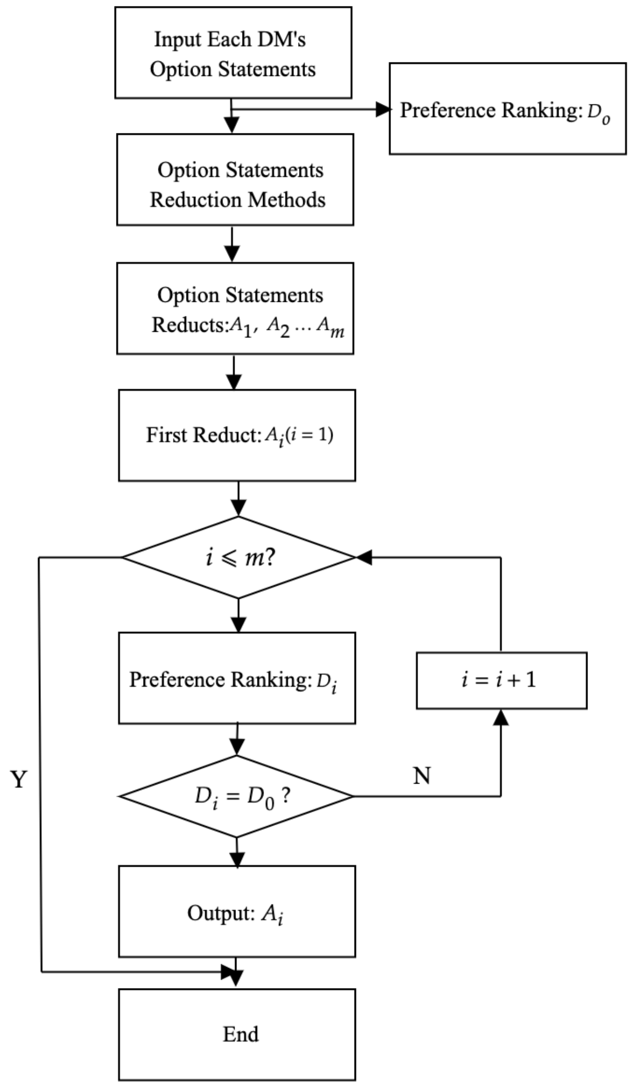

FIGURE A.1: Algorithm of Option Statement Reduction

Appendix B

Calculation Process and Results

B.1 Games with No Dominant Strategy-Stability Analysis with GMCR-PR

The check mark indicates equilibrium holds in the state, in combinations of PRs of each DM:O and O and P^0 to P^4.

TABLE B.1: Games with No Dominant Strategy
Stability Analysis with GMCR-PR:No.58

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O	✓	✓	✓	✓	✓		✓									✓	✓	✓	✓	
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^1	p^1	✓	✓	✓	✓	✓		✓	✓				✓	✓			✓	✓	✓	✓	
p^1	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^1	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓	
p^1	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^2	p^0	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^2	p^1	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^2	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^2	p^3	✓	✓	✓	✓	✓		✓									✓	✓	✓	✓	✓
p^2	p^4	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^3	p^1	✓	✓	✓	✓	✓		✓	✓								✓	✓	✓	✓	
p^3	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	
p^3	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.2: Games with No Dominant Strategy
Stability Analysis with GMCR-PR:No.59

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O	✓	✓	✓	✓	✓		✓									✓	✓	✓	✓	
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^1	p^1	✓	✓	✓	✓	✓		✓	✓				✓	✓			✓	✓	✓	✓	
p^1	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^1	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓	
p^1	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^2	p^0	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^2	p^1	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^2	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^2	p^3	✓	✓	✓	✓	✓		✓									✓	✓	✓	✓	✓
p^2	p^4	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^3	p^1	✓	✓	✓	✓	✓		✓	✓								✓	✓	✓	✓	
p^3	p^2	✓	✓	✓	✓	✓		✓									✓	✓	✓	✓	✓
p^3	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.3: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.60

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O	✓	✓	✓	✓	✓											✓	✓	✓	✓	
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^1	p^1	✓	✓	✓	✓	✓		✓	✓				✓	✓			✓	✓	✓	✓	
p^1	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^1	p^3	✓	✓	✓	✓	✓							✓	✓			✓	✓	✓	✓	
p^1	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^2	p^0	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^2	p^1	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^2	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^2	p^3	✓	✓	✓	✓	✓							✓				✓	✓	✓	✓	✓
p^2	p^4	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^3	p^1	✓	✓	✓	✓	✓		✓	✓								✓	✓	✓	✓	
p^3	p^2	✓	✓	✓	✓	✓		✓									✓	✓	✓	✓	✓
p^3	p^3	✓	✓	✓	✓	✓											✓	✓	✓	✓	✓
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.4: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.61

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O	✓	✓	✓	✓	✓											✓	✓	✓	✓	
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^1	p^1	✓	✓	✓	✓	✓		✓	✓				✓	✓			✓	✓	✓	✓	
p^1	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^1	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓	
p^1	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^2	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^2	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	
p^2	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓	
p^2	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^3	p^0	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^3	p^1	✓	✓	✓	✓	✓						✓	✓	✓	✓		✓	✓	✓	✓	
p^3	p^2	✓	✓	✓	✓	✓						✓	✓	✓	✓		✓	✓	✓	✓	
p^3	p^3	✓	✓	✓	✓	✓											✓	✓	✓	✓	✓
p^3	p^4	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.5: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.62

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O	✓	✓	✓	✓	✓											✓	✓	✓	✓	
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^1	p^1	✓	✓	✓	✓	✓		✓	✓				✓	✓			✓	✓	✓	✓	
p^1	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^1	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓	
p^1	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^2	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^2	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	
p^2	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓	
p^2	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^3	p^1	✓	✓	✓	✓	✓		✓	✓								✓	✓	✓	✓	
p^3	p^2	✓	✓	✓	✓	✓		✓									✓	✓	✓	✓	✓
p^3	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓	✓
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓	✓
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.6: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.63

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O	✓	✓	✓	✓	✓											✓	✓	✓	✓	
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^1	p^1	✓	✓	✓	✓	✓		✓	✓				✓	✓			✓	✓	✓	✓	
p^1	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^1	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓	
p^1	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	
p^2	p^0	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^2	p^1	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^2	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^2	p^3	✓	✓	✓	✓	✓		✓									✓	✓	✓	✓	✓
p^2	p^4	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^3	p^0	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^3	p^1	✓	✓	✓	✓	✓						✓	✓	✓	✓		✓	✓	✓	✓	
p^3	p^2	✓	✓	✓	✓	✓							✓				✓	✓	✓	✓	✓
p^3	p^3	✓	✓	✓	✓	✓											✓	✓	✓	✓	✓
p^3	p^4	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.7: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.64

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O	✓	✓	✓	✓	✓											✓	✓	✓	✓	✓
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓
p^1	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓
p^1	p^2	✓	✓	✓	✓			✓					✓				✓	✓	✓	✓	✓
p^1	p^3	✓	✓	✓	✓							✓	✓	✓	✓		✓	✓	✓	✓	✓
p^1	p^4	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓
p^2	p^0	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^2	p^1	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^2	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^2	p^3	✓	✓	✓	✓	✓							✓				✓	✓	✓	✓	✓
p^2	p^4	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^3	p^1	✓	✓	✓	✓	✓		✓	✓								✓	✓	✓	✓	
p^3	p^2	✓	✓	✓	✓	✓		✓									✓	✓	✓	✓	✓
p^3	p^3	✓	✓	✓	✓	✓											✓	✓	✓	✓	✓
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.8: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.65

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O	✓	✓	✓	✓	✓											✓	✓	✓	✓	✓
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓
p^1	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓
p^1	p^2	✓	✓	✓	✓			✓					✓				✓	✓	✓	✓	✓
p^1	p^3	✓	✓	✓	✓							✓	✓	✓	✓		✓	✓	✓	✓	✓
p^1	p^4	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓
p^2	p^0	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓
p^2	p^1	✓	✓	✓	✓	✓		✓	✓		✓	✓	✓	✓	✓		✓	✓	✓	✓	✓
p^2	p^2	✓	✓	✓	✓			✓					✓				✓	✓	✓	✓	✓
p^2	p^3	✓	✓	✓	✓							✓	✓	✓	✓		✓	✓	✓	✓	✓
p^2	p^4	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^3	p^1	✓	✓	✓	✓	✓		✓	✓								✓	✓	✓	✓	
p^3	p^2	✓	✓	✓	✓	✓		✓									✓	✓	✓	✓	✓
p^3	p^3	✓	✓	✓	✓	✓											✓	✓	✓	✓	✓
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.9: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.66

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^2		✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^3		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓					
p^1	p^4		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^1		✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	
p^2	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓					
p^2	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓						
p^3	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						
p^3	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.10: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.67

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^2		✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^3		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓					
p^1	p^4		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^0		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^1		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓			✓			
p^2	p^2		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓			✓			
p^2	p^3		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^2	p^4		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓						
p^3	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓						
p^3	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.11: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.68

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^0	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^2		✓				✓	✓	✓	✓		✓	✓	✓	✓	✓		✓			
p^1	p^3		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓					
p^1	p^4		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^0		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^1		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓			✓			
p^2	p^2		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^3		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^2	p^4		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓						
p^3	p^2		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^4	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.12: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.69

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓		✓	✓		
p^0	p^2		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^0	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
p^1	p^1	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓		
p^1	p^2		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓			✓			
p^1	p^3	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓						
p^1	p^4	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
p^2	p^0		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^1		✓				✓	✓	✓	✓		✓	✓	✓	✓	✓		✓			
p^2	p^2		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^3		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^2	p^4		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓					
p^3	p^2		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓		✓	✓		
p^4	p^2		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^4	p^3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.13: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.70

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓	✓	✓	✓							✓		✓	✓					
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	
p^1	p^1		✓	✓				✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^2		✓	✓	✓			✓		✓			✓				✓	✓	✓	✓	✓
p^1	p^3		✓	✓								✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^4		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	
p^2	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^2	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^2	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓						
p^3	p^2	✓	✓	✓	✓	✓		✓					✓		✓						
p^3	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.14: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.71

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓	✓	✓	✓							✓		✓	✓					✓
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^1		✓	✓	✓		✓	✓	✓	✓			✓	✓		✓	✓	✓	✓	✓	✓
p^1	p^2		✓	✓	✓	✓		✓		✓			✓			✓	✓	✓	✓	✓	✓
p^1	p^3		✓	✓								✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^4		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^1	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^2	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	
p^2	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1	✓	✓	✓	✓	✓		✓	✓		✓	✓	✓	✓		✓					✓
p^3	p^2	✓	✓	✓	✓	✓		✓					✓		✓						
p^3	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.15: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.72

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓	✓	✓								✓	✓	✓	✓					✓
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^0	p^2	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^1		✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^1	p^2		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^3		✓	✓								✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^4		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^1	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		✓	✓	✓	✓	✓	✓
p^2	p^2	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓					✓
p^3	p^2	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓					
p^3	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^4	p^2	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.16: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.73

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓		✓	✓					✓		✓		✓	✓					
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
p^1	p^1	✓	✓	✓	✓	✓		✓	✓		✓	✓	✓	✓	✓			✓	✓	✓	
p^1	p^2	✓	✓	✓	✓	✓		✓					✓		✓			✓	✓	✓	✓
p^1	p^3	✓	✓	✓	✓	✓					✓	✓	✓	✓	✓	✓		✓	✓	✓	✓
p^1	p^4	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
p^2	p^0		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^1		✓			✓		✓	✓	✓	✓	✓	✓	✓	✓	✓		✓		✓	
p^2	p^2		✓		✓	✓		✓		✓	✓		✓		✓	✓		✓		✓	✓
p^2	p^3		✓									✓	✓	✓	✓	✓		✓		✓	
p^2	p^4		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓						
p^3	p^2	✓	✓	✓	✓	✓		✓					✓		✓						
p^3	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.17: Games with No Dominant Strategy Stability Analysis with GMCR-PR:No.74

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓		✓	✓							✓		✓	✓					
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^1		✓	✓		✓		✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^2		✓	✓	✓	✓		✓		✓			✓		✓	✓	✓	✓	✓	✓	✓
p^1	p^3		✓	✓								✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^4		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^0		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^1		✓			✓		✓	✓	✓	✓	✓	✓	✓	✓	✓		✓		✓	
p^2	p^2		✓		✓	✓		✓		✓	✓		✓		✓	✓		✓		✓	✓
p^2	p^3		✓									✓	✓	✓	✓	✓					
p^2	p^4		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓						
p^3	p^2	✓	✓	✓	✓	✓		✓					✓		✓						
p^3	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.18: Games with No Dominant Strategy
Stability Analysis with GMCR-PR:No.75

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓		✓	✓					✓		✓		✓	✓					✓
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
p^1	p^1	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		✓
p^1	p^2	✓	✓	✓	✓	✓		✓			✓		✓		✓			✓	✓	✓	✓
p^1	p^3	✓	✓	✓	✓	✓					✓	✓	✓	✓	✓	✓		✓	✓	✓	✓
p^1	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
p^2	p^0		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^1		✓		✓		✓	✓	✓	✓	✓		✓	✓	✓	✓		✓			✓
p^2	p^2		✓		✓	✓		✓		✓	✓		✓		✓	✓		✓		✓	✓
p^2	p^3		✓									✓	✓	✓	✓	✓		✓		✓	
p^2	p^4		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓					✓
p^3	p^2	✓	✓	✓	✓	✓		✓					✓		✓						
p^3	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.19: Games with No Dominant Strategy
Stability Analysis with GMCR-PR:No.76

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓		✓	✓							✓		✓	✓					✓
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^0	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^1		✓	✓	✓			✓	✓	✓			✓	✓		✓	✓	✓	✓	✓	✓
p^1	p^2		✓	✓	✓	✓		✓		✓			✓			✓	✓	✓	✓	✓	✓
p^1	p^3		✓	✓								✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^4		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^2	p^0		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^1		✓		✓		✓	✓	✓	✓	✓		✓	✓	✓			✓			
p^2	p^2		✓		✓	✓	✓	✓		✓	✓		✓		✓	✓		✓		✓	✓
p^2	p^3		✓									✓	✓	✓	✓	✓		✓		✓	
p^2	p^4		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓					✓
p^3	p^2	✓	✓	✓	✓	✓		✓					✓		✓						
p^3	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^4	p^2	✓	✓	✓	✓	✓		✓					✓				✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.20: Games with No Dominant Strategy
Stability Analysis with GMCR-PR:No.77

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓		✓						✓		✓	✓	✓	✓					✓
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^0	p^2	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
p^1	p^1	✓	✓	✓	✓		✓	✓	✓	✓	✓		✓	✓	✓			✓	✓		✓
p^1	p^2	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓		✓
p^1	p^3	✓	✓	✓	✓	✓					✓	✓	✓	✓	✓	✓		✓	✓	✓	✓
p^1	p^4	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
p^2	p^0		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^1		✓		✓		✓	✓	✓	✓	✓		✓	✓	✓	✓		✓			✓
p^2	p^2		✓				✓	✓	✓	✓		✓	✓	✓	✓	✓		✓			
p^2	p^3		✓									✓	✓	✓	✓	✓		✓			
p^2	p^4		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓					✓
p^3	p^2	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓					
p^3	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^4	p^2	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

TABLE B.21: Games with No Dominant Strategy
Stability Analysis with GMCR-PR:No.78

		s_1					s_2					s_3					s_4				
		Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto	Nash	GMR	SMR	SEQ	Pareto
O	O		✓		✓								✓	✓	✓	✓					✓
p^0	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^0	p^2	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^0	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^1	p^0		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^1		✓	✓	✓		✓	✓	✓	✓			✓	✓		✓	✓	✓	✓	✓	✓
p^1	p^2		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^3		✓	✓								✓	✓	✓	✓	✓	✓	✓	✓	✓	
p^1	p^4		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓			✓	✓	✓	
p^2	p^0		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^2	p^1		✓		✓		✓	✓	✓	✓	✓		✓	✓	✓	✓		✓			✓
p^2	p^2		✓				✓	✓	✓	✓		✓	✓	✓	✓	✓		✓			
p^2	p^3		✓									✓	✓	✓	✓	✓		✓			
p^2	p^4		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓			
p^3	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^3	p^1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓					✓
p^3	p^2	✓	✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓					
p^3	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓					
p^3	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
p^4	p^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^1	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓
p^4	p^2	✓	✓	✓	✓							✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^3	✓	✓	✓	✓	✓						✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p^4	p^4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

B.2 Elmira Conflict - State Transition Time Analysis Under Influence of External Factors

A python program developed by Inooka [43] was used to calculate the GMCR stability analysis considering the state transition time. In the Spyder environment, the calculation results were obtained by running Inooka's python programs:

`input_time.py, setop.py, matrixop.py, timeop.py, time_stabilities.py`

Variable inputs are as follows:

```
DM=[1, 2, 3]
States=[1, 2, 3, 4, 5, 6, 7, 8, 9]
Graphs=[array([[0, 1, 0, 0, 0, 0, 0, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0],
        [0, 0, 0, 1, 0, 0, 0, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0],
        [0, 0, 0, 0, 0, 1, 0, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 1, 0],
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        [0, 0, 0, 0, 0, 0, 0, 0, 0]]),
       array([[0, 0, 1, 0, 0, 0, 0, 0, 1],
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        [0, 0, 1, 0, 0, 0, 0, 0, 0],
        [0, 0, 0, 1, 0, 0, 0, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0]])]
Times=[array([[x, 2, x, x, x, x, x, x, x],
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        [x, x, x, x, x, x, x, x, x],
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        [x, x, x, x, x, x, x, 2, x],
        [x, x, x, x, x, x, x, x, x],
        [x, x, x, x, x, x, x, x, x]], dtype=object),
       array([[x, x, 2, x, x, x, x, x, 2],
        [x, x, x, 2, x, x, x, x, 2],
        [x, x, x, x, x, x, x, x, 2],
```

```
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[x, x, x, x, x, 1, x, x, x],
[x, x, x, x, x, x, 1, x, x],
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[x, x, x, 1, x, x, x, x, x],
[x, x, x, x, x, x, x, x, x]], dtype=object)]
```

```
m,u,l=2,2,1
```

B.3 Preference Order Setting for Disaster Aversion

Binarization

US	Airtrike	Blockade	Withdraw	Attack
1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	1	1	0	0
5	1	0	1	0
6	0	1	1	0
7	1	1	1	0
8	1	0	0	1
9	0	1	0	1
10	1	1	0	1
計	6	6	3	3

Square

US	Airstrike	Blockade	Withdraw	Attack
1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	1	1	0	0
5	1	0	1	0
6	0	1	1	0
7	1	1	1	0
8	1	0	0	1
9	0	1	0	1
10	1	1	0	1
Total	6	6	3	3
√	2.45	2.45	1.73	1.73
Weight	0.267	0.133	0.067	0.533

Normalization

States	C1	C2	C3	C4
1	0.000	0.000	0.000	0.000
2	0.109	0.000	0.000	0.000
3	0.000	0.054	0.000	0.000
4	0.109	0.054	0.000	0.000
5	0.109	0.000	0.038	0.000
6	0.000	0.054	0.038	0.000
7	0.109	0.054	0.038	0.000
8	0.109	0.000	0.000	0.308
9	0.000	0.054	0.000	0.308
10	0.109	0.054	0.000	0.308

max	0.109	0.05443311	0.03849002	0.30792014
min	0.000	0	0	0

FIGURE B.1: Cuban Missile Crisis - US Scores-1

Distance from Max and Min values

1	0.012	0.003	0.001	0.095	0.11111111	0.33333333
2	0.000	0.003	0.001	0.095	0.09925926	0.31505438
3	0.012	0.000	0.001	0.095	0.10814815	0.32885886
4	0.000	0.000	0.001	0.095	0.0962963	0.31031645
5	0.000	0.003	0.000	0.095	0.09777778	0.31269438
6	0.012	0.000	0.000	0.095	0.10666667	0.32659863
7	0.000	0.000	0.000	0.095	0.09481481	0.30792014
8	0.000	0.003	0.001	0.000	0.00444444	0.06666667
9	0.012	0.000	0.001	0.000	0.01333333	0.11547005
10	0.000	0.000	0.001	0.000	0.00148148	0.03849002

1	0.000	0.000	0.000	0.000	0	0
2	0.012	0.000	0.000	0.000	0.01185185	0.10886621
3	0.000	0.003	0.000	0.000	0.00296296	0.05443311
4	0.012	0.003	0.000	0.000	0.01481481	0.12171612
5	0.012	0.000	0.001	0.000	0.01333333	0.11547005
6	0.000	0.003	0.001	0.000	0.00444444	0.06666667
7	0.012	0.003	0.001	0.000	0.0162963	0.12765695
8	0.012	0.000	0.000	0.095	0.10666667	0.32659863
9	0.000	0.003	0.000	0.095	0.09777778	0.31269438
10	0.012	0.003	0.000	0.095	0.10962963	0.33110365

St	Score
1	1.000
2	0.743
3	0.858
4	0.718
5	0.730
6	0.830
7	0.707
8	0.170
9	0.270
10	0.104

FIGURE B.2: Cuban Missile Crisis - US Scores-2

Binarization

USSR	Airstrike	Blockade	Withdraw	Attack
1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	1	1	0	0
5	1	0	1	0
6	0	1	1	0
7	1	1	1	0
8	1	0	0	1
9	0	1	0	1
10	1	1	0	1

Square

USSR	Airstrike	Blockade	Withdraw	Attack
1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	1	1	0	0
5	1	0	1	0
6	0	1	1	0
7	1	1	1	0
8	1	0	0	1
9	0	1	0	1
10	1	1	0	1
Total	6	6	3	3
√	2.45	2.45	1.73	1.73
Weight	0.533	0.133	0.067	0.267

Normalization

States	C1	C2	C3	C4
1	0.000	0.000	0.000	0.000
2	0.218	0.000	0.000	0.000
3	0.000	0.054	0.000	0.000
4	0.218	0.054	0.000	0.000
5	0.218	0.000	0.038	0.000
6	0.000	0.054	0.038	0.000
7	0.218	0.054	0.038	0.000
8	0.218	0.000	0.000	0.154
9	0.000	0.054	0.000	0.154
10	0.218	0.054	0.000	0.154

max	0.218	0.05443311	0.03849002	0.15396007
min	0.000	0	0	0

FIGURE B.3: Cuban Missile Crisis - USSR Scores-1

Distance from Max and Min values

1	0.047	0.003	0.001	0.024	0.076	0.275
2	0.000	0.003	0.001	0.024	0.028	0.168
3	0.047	0.000	0.001	0.024	0.073	0.269
4	0.000	0.000	0.001	0.024	0.025	0.159
5	0.000	0.003	0.000	0.024	0.027	0.163
6	0.047	0.000	0.000	0.024	0.071	0.267
7	0.000	0.000	0.000	0.024	0.024	0.154
8	0.000	0.003	0.001	0.000	0.004	0.067
9	0.047	0.000	0.001	0.000	0.049	0.221
10	0.000	0.000	0.001	0.000	0.001	0.038

1	0.000	0.000	0.000	0.000	0	0
2	0.047	0.000	0.000	0.000	0.0474	0.217732422
3	0.000	0.003	0.000	0.000	0.003	0.054433105
4	0.047	0.003	0.000	0.000	0.0504	0.224433443
5	0.047	0.000	0.001	0.000	0.0489	0.221108319
6	0.000	0.003	0.001	0.000	0.0044	0.066666667
7	0.047	0.003	0.001	0.000	0.0519	0.227710017
8	0.047	0.000	0.000	0.024	0.0711	0.266666667
9	0.000	0.003	0.000	0.024	0.0267	0.163299316
10	0.047	0.003	0.000	0.024	0.0741	0.272165527

St	Score
1	1.000
2	0.435
3	0.832
4	0.414
5	0.425
6	0.800
7	0.403
8	0.200
9	0.575
10	0.124

FIGURE B.4: Cuban Missile Crisis - USSR Scores-2

Bibliography

[1] Mubarak S Al-Mutairi, Keith W Hipel, and Mohamed S Kamel. "Fuzzy preferences in conflicts". In: *Journal of Systems Science and Systems Engineering* 17.3 (2008), pp. 257–276. ISSN: 1861-9576. DOI: 10.1007/s11518-008-5088-4. URL: https://doi.org/10.1007/s11518-008-5088-4.

[2] Nabil I. Al-Najjar and Mallesh M. Pai. "Coarse decision making and overfitting". In: *Journal of Economic Theory* 150.C (2014), pp. 467–486. URL: https://EconPapers.repec.org/RePEc:eee:jetheo:v:150:y:2014:i:c:p:467-486.

[3] Graham T. Allison and P. Zelikow. *Essence of decision; explaining the Cuban missile crisis*. English. Little, Brown Boston, 1971, p. 27. ISBN: 0316034363.

[4] Kenneth J. Arrow and Gerard Debreu. "Existence of an Equilibrium for a Competitive Economy". In: *Econometrica* 22.3 (1954), pp. 265–290. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/1907353.

[5] Robert J. Aumann. "Existence of Competitive Equilibria in Markets with a Continuum of Traders". In: *Econometrica* 34.1 (1966), pp. 1–17. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/1909854.

[6] Robert J. Aumann. "Agreeing to Disagree". In: *The Annals of Statistics* 4.6 (1976), pp. 1236–1239. ISSN: 00905364. URL: http://www.jstor.org/stable/2958591.

[7] Yurii Averboukh and Artem and Baklanov. "Stackelberg Solutions of Differential Games in the Class of Nonanticipative Strategies". In: *Dynamic Games and Applications* 4.1 (2014), pp. 1–9. ISSN: 2153-0785. DOI: 10.1007/s13235-013-0077-8. URL: http://10.0.3.239/s13235-013-0077-8https://dx.doi.org/10.1007/s13235-013-0077-8.

[8] M. Abul Bashar, D. Marc Kilgour, and Keith W. Hipel. "Fuzzy Preferences in the Graph Model for Conflict Resolution". In: *IEEE Transactions on Fuzzy Systems* 20.4 (2012), pp. 760–770. DOI: 10.1109/TFUZZ.2012.2183603.

[9] M Abul Bashar et al. "Interval fuzzy preferences in the graph model for conflict resolution". In: *Fuzzy Optimization and Decision Making* 17.3 (2018), pp. 287–315. ISSN: 1568-4539. DOI: 10.1007/s10700-017-9279-7. URL: http://10.0.3.239/s10700-017-9279-7https://dx.doi.org/10.1007/s10700-017-9279-7.

[10] Sean G. Bernath Walker, Keith William Hipel, and Takehiro Inohara. "Attitudes and preferences: Approaches to representing decision maker desires". In: *Applied Mathematics and Computation* 218.12 (2012), pp. 6637–6647. ISSN: 0096-3003. DOI: https://doi.org/10.1016/j.amc.2011.11.102. URL: https://www.sciencedirect.com/science/article/pii/S0096300311014482.

[11] Sean G. Bernath Walker, Keith William Hipel, and Takehiro Inohara. "Attitudes and preferences: Approaches to representing decision maker desires". In: *Applied Mathematics and Computation* 218.12 (2012), pp. 6637–6647. ISSN: 0096-3003. DOI: 10.1016/J.AMC.2011.11.102.

[12] B.Douglas Bernheim, Bezalel Peleg, and Michael D Whinston. "Coalition-Proof Nash Equilibria I. Concepts". In: *Journal of Economic Theory* 42.1 (1987), pp. 1–12. ISSN: 0022-0531. DOI: https://doi.org/10.1016/0022-0531(87)90099-8. URL: https://www.sciencedirect.com/science/article/pii/0022053187900998.

[13] E. Borel. "The Theory of Play and Integral Equations with Skew Symmetric Kernels". In: *Econometrica* 21.1 (1953), pp. 97–100. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/1906946.

[14] Steven Brams and Peter C Fishburn. *Approval voting*. Springer Science & Business Media, 2007.

[15] Steven J Brams. "Theory of moves". In: *American Scientist* 81.6 (1993), pp. 562–570.

[16] Steven J Brams and Donald Wittman. "Nonmyopic equilibria in 2×2 games". In: *Conflict Management and Peace Science* 6.1 (1981), pp. 39–62.

[17] Hans Carlsson and Eric van Damme. "Global Games and Equilibrium Selection". In: *Econometrica* 61.5 (1993), pp. 989–1018. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/2951491.

[18] Michael Suk-Young Chwe. "Farsighted Coalitional Stability". In: *Journal of Economic Theory* 63.2 (1994), pp. 299–325. ISSN: 0022-0531. DOI: https://doi.org/10.1006/jeth.1994.1044. URL: https://www.sciencedirect.com/science/article/pii/S0022053184710441.

[19] Daniel Dennett. "Cognitive Wheels: The Frame Problem of AI". In: *Minds, Machines and Evolution*. Ed. by Christopher Hookway. Cambridge University Press, 1984.

[20] F. Y. Edgeworth. "Mathematical Psychics, an Essay on the Application of Mathematics to the Moral Sciences". In: *Revue Philosophique de la France Et de l'Etranger* 12 (1881), pp. 536–539.

[21] Ronald Fagin et al. "Common knowledge revisited". In: *Annals of Pure and Applied Logic* 96.1 (1999), pp. 89–105. ISSN: 0168-0072. DOI: https://doi.org/10.1016/S0168-0072(98)00033-5. URL: https://www.sciencedirect.com/science/article/pii/S0168007298000335.

[22] Liping Fang, Keith W. Hipel, and D. Marc Kilgour. "Conflict models in graph form: Solution concepts and their interrelationships". In: *European Journal of Operational Research* (1989). ISSN: 03772217. DOI: 10.1016/0377-2217(89)90041-6.

[23] Liping Fang, Keith W. Hipel, and D. Marc Kilgour. *Interactive decision making : the graph model for conflict resolution*. Wiley series in systems engineering. J. Wiley, 1993, pp. 7 –9. URL: https://ci.nii.ac.jp/ncid/BA21188518.

[24] Charles J Fillmore et al. "Frame semantics". In: *Cognitive linguistics: Basic readings* 34 (2006), pp. 373–400.

[25] N. Fraser and K. Hipel. "Solving Complex Conflicts". In: *IEEE Transactions on Systems, Man, and Cybernetics* 9 (1979), pp. 805–816.

[26] Niall M Fraser and Keith W Hipel. "Dynamic modelling of the cuban missile crisis". In: *Conflict Management and Peace Science* 6.2 (1982), pp. 1–18.

[27] Niall M. Fraser and Keith W. Hipel. *Conflict analysis : models and resolutions*. eng. Literaturverz. S. 361 - 371. 1984.

[28] Allan Gibbard. “Manipulation of Voting Schemes: A General Result”. In: *Econometrica* 41.4 (1973), pp. 587–601. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/1914083.

[29] Joseph Greenberg. “An application of the theory of social situations to repeated games”. In: *Journal of Economic Theory* 49.2 (1989), pp. 278–293. ISSN: 0022-0531. DOI: https://doi.org/10.1016/0022-0531(89)90082-3. URL: https://www.sciencedirect.com/science/article/pii/0022053189900823.

[30] Paul Halmos. *Naive Set Theory*. Reprinted by Springer-Verlag, Undergraduate Texts in Mathematics, 1974. Van Nostrand, 1960. ISBN: 0387900926.

[31] Luai Hamouda, D. Marc Kilgour, and Keith W. Hipel. “Strength of preference in the graph model for conflict resolution”. In: *Group Decision and Negotiation* (2004). ISSN: 09262644. DOI: 10.1023/B:GRUP.0000045751.21207.35.

[32] Keith W. Hipel and Liping Fang. “The Graph Model for Conflict Resolution and Decision Support”. In: *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 51.1 (2021), pp. 131–141. DOI: 10.1109/TSMC.2020.3041462.

[33] N. Howard. “Paradoxes of Rationality: Theory of Metagames and Political Behavior. Cambridge”. In: *American Behavioral Scientist* 6 (), pp. 948–948. DOI: 10.1177/000276427201500663.

[34] Takehiro Inohara. “Generalizations of the Concept of Core of Simple Games and Their Characterization in Terms of Permission of Voters”. In: *Appl. Math. Comput.* 132.1 (Oct. 2002), 47–62. ISSN: 0096-3003. DOI: 10.1016/S0096-3003(01)00175-8. URL: https://doi.org/10.1016/S0096-3003(01)00175-8.

[35] Takehiro Inohara. “Consensus building and the Graph Model for Conflict Resolution”. In: Nov. 2010, pp. 2841 –2846. DOI: 10.1109/ICSMC.2010.5641917.

[36] Takehiro Inohara. *Goi-Keisei Gaku (Consensus Building)*. keisoshobo, Mar. 2011, pp. 103–122.

[37] Takehiro Inohara. “Majority decision making and the Graph Model for Conflict Resolution”. In: *2011 IEEE International Conference on Systems, Man, and Cybernetics* (2011), pp. 2702–2707.

[38] Takehiro Inohara. “Stability of consensus as a decision technology for service management”. In: *ICSSSM11*. 2011, pp. 1–5. DOI: 10.1109/ICSSSM.2011.5959468.

[39] Takehiro Inohara. “State transition time analysis in the Graph Model for Conflict Resolution”. In: *Applied Mathematics and Computation* (2016). ISSN: 00963003. DOI: 10.1016/j.amc.2015.11.020.

[40] Takehiro Inohara and Keith W. Hipel. “Coalition analysis in the graph model for conflict resolution”. In: *Systems Engineering* 11.4 (2008), pp. 343–359. ISSN: 10981241. DOI: 10.1002/sys.20104.

[41] Takehiro Inohara, Keith W. Hipel, and Sean Walker. “Conflict analysis approaches for investigating attitudes and misperceptions in the War of 1812”. In: *Journal of Systems Science and Systems Engineering* (2007). ISSN: 10043756. DOI: 10.1007/s11518-007-5042-x.

[42] Takehiro Inohara, Saied Yousefi, and Keith W. Hipel. “Propositions on interrelationships among attitude-based stability concepts”. In: *2008 IEEE International Conference on Systems, Man and Cybernetics*. 2008, pp. 2502–2507. DOI: 10.1109/ICSMC.2008.4811671.

[43] Takumi Inooka. *Stability Concept and Matrix Representation of Graph Models for Conflict Resolution Considering State Transition Time and Coalition*. 2016.

[44] Ludovic A Julien. "A note on Stackelberg competition". In: *Journal of Economics* 103.2 (2011), pp. 171–187.

[45] Saori Katakura and Takehiro Inohara. "A Graph Model for Conflict Resolution Considering Preference Changes". In: vol. 2009f. 0. The Japan Society for Management Information, 2009, pp. 78–78. DOI: 10.11497/jasmin.2009f.0.78.0. URL: https://ci.nii.ac.jp/naid/130004606271/.

[46] Yukiko Kato. "Formulating Preference Orders for Conflicts De-escalation". In: *2020 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*. 2020, pp. 472–479. DOI: 10.1109/SMC42975.2020.9282892.

[47] Yukiko Kato. "Binary Processing of Permissible Range in Graph Model of Conflict Resolution". In: *2021 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*. 2021, pp. 685–690. DOI: 10.1109/SMC52423.2021.9658701.

[48] Yukiko Kato. "New Reachability via the Influence of External Factors for Conflict Escalation and De-escalation". In: *2021 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*. 2021, pp. 813–819. DOI: 10.1109/SMC52423.2021.9659282.

[49] Yukiko Kato. "State transition for de-escalationin the graph model for conflict resolution framework". In: *JSIAM Letters* 13.0 (2021), pp. 60–63. ISSN: 1883-0609. DOI: 10.14495/jsiaml.13.60. URL: https://www.jstage.jst.go.jp/article/jsiaml/13/0/13_60/_article.

[50] Yukiko Kato. "Mathematical Models for Conflict Resolution and Consensus Building in International Relations". Japanese, in: *Mathematics of Elections, Voting, and the Public Choice*, ed. by Tatsuo Oyama. book, Kyoritsu Shuppan, Mar. 2, 2022. Chap. 15, p. 352. ISBN: 978-4320019560.

[51] Robert F Kennedy 1925-1968. *Thirteen days : a memoir of the Cuban missile crisis*. First edition. New York : W.W. Norton, 1969., 1969. URL: https://search.library.wisc.edu/catalog/999466729602121.

[52] D Marc Kilgour. "Equilibria for far-sighted players". In: *Theory and Decision* 16.2 (1984), pp. 135–157.

[53] D Marc Kilgour. "Anticipation and stability in two-person noncooperative games". In: *Dynamic models of international conflict* (1985), pp. 26–51.

[54] D Marc Kilgour, Keith W Hipel, and Liping Fang. "The graph model for conflicts". In: *Automatica* 23.1 (1987), pp. 41–55.

[55] Hanbin Kuang et al. "Grey-Based Preference in a Graph Model for Conflict Resolution With Multiple Decision Makers". In: *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 45 (2015), pp. 1254–1267.

[56] Hanbin Kuang et al. "Strategic analysis of a brownfield revitalization conflict using the grey-based graph model for conflict resolution". In: *EURO Journal on Decision Processes* 3 (May 2015). DOI: 10.1007/s40070-015-0042-4.

[57] Thierry Lafay. "A linear generalization of Stackelberg's model". In: *Theory and decision* 69.2 (2010), pp. 317–326.

[58] David K. Lewis. "Convention: A Philosophical Study". In: *Synthese* 26.1 (1973), pp. 153–157.

[59] Kevin Li, Takehiro Inohara, and Haiyan Xu. "Coalition analysis with preference uncertainty in group decision support". In: *Applied Mathematics and Computation* 231 (Mar. 2014), 307–319. DOI: 10.1016/j.amc.2013.12.143.

[60] Kevin Li et al. "Preference Uncertainty in the Graph Model for Conflict Resolution". In: *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactionson* 34 (July 2004), pp. 507 –520. DOI: 10.1109/TSMCA.2004.826282.

[61] Kevin W. Li, Takehiro Inohara, and Haiyan Xu. "Coalition analysis with preference uncertainty in group decision support". In: *Applied Mathematics and Computation* 231 (2014), pp. 307–319. ISSN: 00963003. DOI: 10.1016/j.amc.2013.12.143.

[62] R. D. Luce and H. Raiffa. *Games and Decisions*. New York: Wiley, 1957.

[63] Michael Mandler. "Coarse, Efficient Decision-Making". In: *Journal of the European Economic Association* 18.6 (2020), pp. 3006–3044. ISSN: 1542-4766. DOI: 10.1093/jeea/jvaa002. URL: https://doi.org/10.1093/jeea/jvaa002.

[64] John McCarthy and Patrick J. Hayes. "Some Philosophical Problems from the Standpoint of Artificial Intelligence". In: *Machine Intelligence 4*. Ed. by B. Meltzer and D. Michie. reprinted in McC90. Edinburgh University Press, 1969, pp. 463–502.

[65] Erik Mohlin. "Optimal categorization". In: *Journal of Economic Theory* 152 (2014), pp. 356–381. ISSN: 0022-0531. DOI: https://doi.org/10.1016/j.jet.2014.03.007. URL: https://www.sciencedirect.com/science/article/pii/S0022053114000477.

[66] Stephen Morris and Hyun Song Shin. "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks". In: *The American Economic Review* 88.3 (1998), pp. 587–597. ISSN: 00028282. URL: http://www.jstor.org/stable/116850.

[67] D. Munton et al. *The Cuban Missile Crisis: A Concise History*. Oxford University Press, 2007. ISBN: 9780195178593. URL: https://books.google.co.jp/books?id=TaYMAAAAYAAJ.

[68] Tetsuya Murai et al. "Granular Reasoning Using Zooming In & Out". In: *International Workshop on Rough Sets, Fuzzy Sets, Data Mining, and Granular-Soft Computing*. Springer. 2003, pp. 421–424.

[69] J F Nash. "Equilibrium Points in N-Person Games." In: *Proceedings of the National Academy of Sciences of the United States of America* 36.1 (1950), pp. 48–9. ISSN: 0027-8424. DOI: 10.1073/pnas.36.1.48. URL: http://www.ncbi.nlm.nih.gov/pubmed/16588946http://www.pubmedcentral.nih.gov/articlerender.fcgi?artid=PMC1063129.

[70] John Nash. "Non-Cooperative Games". In: *Annals of Mathematics* 54.2 (1951), pp. 286–295. ISSN: 0003486X. URL: http://www.jstor.org/stable/1969529.

[71] John von Neumann. "Zur Theorie der Gesellschaftsspiele". In: *Mathematische Annalen* 100 (1928), pp. 295–320.

[72] John von Neumann, Oskar Morgenstern, and Ariel Rubinstein. *Theory of Games and Economic Behavior (60th Anniversary Commemorative Edition)*. Princeton University Press, 1944. ISBN: 9780691130613. URL: http://www.jstor.org/stable/j.ctt1r2gkx.

[73] Zdzisław Pawlak. "Rough Sets". In: *Rough Sets and Data Mining* 11.5 (1997), pp. 3–7. DOI: 10.1007/978-1-4613-1461-5_1.

[74] Bezalel Peleg. *Game Theoretic Analysis of Voting in Committees*. Jan. 2000. DOI: 10.2307/1960622.

[75] Xiaoyong Peng et al. "Representing ordinal preferences in the decision support system GMCR II". In: *1997 IEEE International Conference on Systems, Man, and Cybernetics. Computational Cybernetics and Simulation*. Vol. 1. 1997, 809–814 vol.1. DOI: 10.1109/ICSMC.1997.626196.

[76] Sam Perlo-Freeman et al. "The topology of conflict and co-operation". In: *U of the West of England, Dept of Economics, Discussion Paper* 609 (2006).

[77] K.G. Ramamurthy. *Coherent Structures and Simple Games*. Linthicum, Md, 1994.

[78] A. Rapoport and M. Guyer. *A Taxonomy of 2 X 2 Games, By Anatol Rapoport and Melvin Guyer*. Bobbs-Merrill Reprint Series in the Social Sciences, S617. 1966. URL: https://books.google.co.jp/books?id=7Ea8XwAACAAJ.

[79] Leandro Chaves Rêgo and Andrea Maria dos Santos. "Probabilistic Preferences in the Graph Model for Conflict Resolution". In: *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 45 (2015), pp. 595–608.

[80] THOMAS C. Schelling. *Arms and Influence*. Yale University Press, 1966. ISBN: 9780300002218. URL: http://www.jstor.org/stable/j.ctt5vm52s.

[81] Ulrich Schwalbe and Paul Walker. "Zermelo and the Early History of Game Theory". In: *Games and Economic Behavior* 34.1 (2001), pp. 123–137. ISSN: 0899-8256. DOI: https://doi.org/10.1006/game.2000.0794. URL: https://www.sciencedirect.com/science/article/pii/S0899825600907942.

[82] L. S. Shapley. "Simple games: An outline of the descriptive theory". In: *Behavioral Science* 7.1 (1962), pp. 59–66. DOI: https://doi.org/10.1002/bs.3830070104. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/bs.3830070104. URL: https://onlinelibrary.wiley.com/doi/abs/10.1002/bs.3830070104.

[83] Herbert A Simon. "A Behavioral Model of Rational Choice". In: *The Quarterly Journal of Economics* 69.1 (1955), pp. 99–118. ISSN: 0033-5533. DOI: 10.2307/1884852. URL: https://doi.org/10.2307/1884852.

[84] Christopher A Sims. "Implications of rational inattention". In: *Journal of Monetary Economics* 50.3 (2003), pp. 665–690. ISSN: 0304-3932. DOI: https://doi.org/10.1016/S0304-3932(03)00029-1. URL: https://www.sciencedirect.com/science/article/pii/S0304393203000291.

[85] Theodore C. Sorensen. *Kennedy*. English. [1st ed.] Harper Row New York, 1965, viii, 783 p.

[86] ALAN D. Taylor and WILLIAM S. Zwicker. *Simple Games: Desirability Relations, Trading, Pseudoweightings*. Princeton University Press, 1999. ISBN: 9780691001203. URL: http://www.jstor.org/stable/j.ctv18zhdsm.

[87] L. G. Telser. "A Theory of Self-Enforcing Agreements". In: *The Journal of Business* 53.1 (1980), pp. 27–44. ISSN: 00219398, 15375374. URL: http://www.jstor.org/stable/2352355.

[88] Marc Trachtenberg. "The Influence of Nuclear Weapons in the Cuban Missile Crisis". In: *International Security* 10.1 (1985), p. 162. ISSN: 01622889, 15314804. URL: http://www.jstor.org/stable/2538793.

[89] Amos Tversky and Daniel Kahneman. "Judgment under Uncertainty: Heuristics and Biases". In: *Science* 185.4157 (1974), pp. 1124–1131. DOI: 10.1126/science.185.4157.1124. URL: https://www.science.org/doi/abs/10.1126/science.185.4157.1124.

[90] Haiyan Xu et al. *Conflict Resolution Using the Graph Model: Strategic Interactions in Competition and Cooperation*. Jan. 2018. ISBN: 978-3-319-77669-9. DOI: 10.1007/978-3-319-77670-5.

[91] Haiyan Xu et al. "Stackelberg stability in the graph model for conflict resolution: Definition and implementation". In: *Innovation for Systems Information and Decision Meeting*. Springer. 2020, pp. 77–92.

[92] Akira Yamazaki, Takehiro Inohara, and Bunpei Nakano. "Comparability of coalitions in committees with permission of voters by using desirability relation and hopefulness relation". In: *Applied Mathematics and Computation - AMC* 113 (2000), pp. 219–234. DOI: 10.1016/S0096-3003(99)00089-2.

[93] Akira Yamazaki, Takehiro Inohara, and Bunpei Nakano. "New interpretation of the core of simple games in terms of voters' permission". In: *Applied Mathematics and Computation* 108.2-3 (2000), pp. 115–127. ISSN: 0096-3003. DOI: 10.1016/s0096-3003(99)00008-9.

[94] Akira Yamazaki, Takehiro Inohara, and Bunpei Nakano. "Symmetry of simple games and permission of voters". In: *Applied Mathematics and Computation - AMC* 114 (2000), pp. 315–327. DOI: 10.1016/S0096-3003(99)00126-5.

[95] K Paul Yoon and Ching-Lai Hwang. *Multiple attribute decision making: an introduction*. Sage publications, 1995.

[96] L A Zadeh. "Fuzzy sets". In: *Information and Control* 8.3 (1965), pp. 338–353. ISSN: 0019-9958. DOI: https://doi.org/10.1016/S0019-9958(65)90241-X. URL: https://www.sciencedirect.com/science/article/pii/S001999586590241X.

[97] Frank Zagare. "A Game-Theoretic History of the Cuban Missile Crisis". In: *Economies* 2.1 (2014), pp. 20–44. ISSN: 2227-7099. DOI: 10.3390/economies2010020. URL: http://www.mdpi.com/2227-7099/2/1/20.

[98] Shinan Zhao and Haiyan Xu. "A Novel Preference Elicitation Technique Based on a Graph Model and Its Application to a Brownfield Redevelopment Conflict in China". In: *International Journal of Environmental Research and Public Health* 16 (Oct. 2019), p. 4088. DOI: 10.3390/ijerph16214088.

[99] Shinan Zhao and Haiyan Xu. "A Novel Method for Eliminating Redundant Option Statements in the Graph Model for Conflict Resolution". In: *Lecture Notes in Business Information Processing*. Springer International Publishing, 2020, pp. 45–55. ISBN: 1865-1348. DOI: 10.1007/978-3-030-48641-9_4. URL: http://10.0.3.239/978-3-030-48641-9_4https://dx.doi.org/10.1007/978-3-030-48641-9_4.